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Deep inelastic scattering from polarized spin-1/2 hadrons at low x from string theory

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Abstract: We study polarized deep inelastic scattering of charged leptons from spin-1/2 hadrons at low values of the Bjorken parameter and large 't Hooft coupling in terms of the gauge/string theory duality. We calculate the structure functions from type IIB superstring theory scattering amplitudes. We discuss the role of the non-Abelian Chern-Simons term and the Pauli term from the five-dimensional SU(4) gauged supergravity. Furthermore, the exponentially small-x regime where Regge physics becomes important is analyzed in detail for the antisymmetric structure functions. In this case the holographic dual picture of the Pomeron exchange is realized by a Reggeized gauge field. We compare our results with experimental data of the proton antisymmetric structure function g_1, obtaining a very good level of agreement.

Keywords: AdS-CFT Correspondence, Gauge-gravity correspondence

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1 Introduction

The idea of this work is to study polarized deep inelastic scattering (DIS) of charged leptons off spin-1/2 hadrons, in order to investigate properties of the hadronic tensor at small values of the Bjorken parameter. We consider large values of the 't Hooft coupling λ and the planar limit of the gauge theory, within the framework of the gauge/string theory duality. We carry out first principles calculations starting from type IIB superstring theory scattering amplitudes. Alternatively, we show how to approach the problem by deriving heuristic Lagrangians for the symmetric and the antisymmetric contributions. We first introduce the heuristic approach which is more intuitive, and then we describe the formal string theoretical derivation. The parametric region we focus on is $x \ll 1/\sqrt{\lambda}$, where type IIB supergravity does not give an accurate description of the holographic dual DIS process, hence it is necessary to consider string theory. Furthermore, we investigate the region where the Bjorken parameter becomes exponentially small, which allows us to compare our results for the antisymmetric structure function g_1 with recent experimental data of electron-proton DIS.

The DIS differential cross-section of a charged lepton off a hadron is proportional to the contraction of the leptonic tensor, which is obtained from perturbative QED, and the hadronic tensor, where non-perturbative QCD effects are essential. The hadronic tensor

of a spin-1/2 hadron is usually written in terms of symmetric (S) and antisymmetric (A) tensors under Lorentz indices exchange [1, 2][1]

$$
\begin{aligned}
W_{\mu\nu} &= W^{(\mathrm{S})}_{\mu\nu}(q,P) + i\, W^{(\mathrm{A})}_{\mu\nu}(q,P,S)\,, \\
W^{(\mathrm{S})}_{\mu\nu} &= \left(\eta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\left[F_1(x,q^2) + \frac{1}{2}\frac{S\cdot q}{P\cdot q} g_5(x,q^2)\right], \\
&\quad - \frac{1}{P\cdot q}\left(P_\mu - \frac{P\cdot q}{q^2}q_\mu\right)\left(P_\nu - \frac{P\cdot q}{q^2}q_\nu\right)\left[F_2(x,q^2) + \frac{S\cdot q}{P\cdot q} g_4(x,q^2)\right] \\
&\quad - \frac{1}{2P\cdot q}\left[\left(P_\mu - \frac{P\cdot q}{q^2}q_\mu\right)\left(S_\nu - \frac{S\cdot q}{P\cdot q}P_\nu\right) + \left(P_\nu - \frac{P\cdot q}{q^2}q_\nu\right)\left(S_\mu - \frac{S\cdot q}{P\cdot q}P_\mu\right)\right] \\
&\quad g_3(x,q^2)\,, \\
W^{(\mathrm{A})}_{\mu\nu} &= -\frac{\varepsilon_{\mu\nu\rho\sigma}q^\rho}{P\cdot q}\left\{S^\sigma g_1(x,q^2) + \left[S^\sigma - \frac{S\cdot q}{P\cdot q}P^\sigma\right] g_2(x,q^2)\right\} - \frac{\varepsilon_{\mu\nu\rho\sigma}q^\rho P^\sigma}{2P\cdot q} F_3(x,q^2)\,,
\end{aligned}
\tag{1.1}
$$

where $\eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1)$, P_μ and S_μ are the four-momentum and the spin vector of the incident hadron, respectively. The four-momentum of the virtual photon is denoted by q_μ. The symmetric structure functions are F_1, F_2, g_3, g_4 and g_5, while F_3, g_1 and g_2 are the antisymmetric ones. For electromagnetic DIS in QCD the non-preserving parity structure functions g_3, g_4, g_5 and F_3 vanish. In fact, we consider electromagnetic DIS not precisely for QCD but for an IR deformation of $\mathcal{N}=4$ SYM theory. The last is a chiral theory, therefore it may lead to a non-vanishing F_3. In this sense this result is in perfect agreement with respect to the glueball case presented in reference [3]. The condition for F_3 to be non-vanishing is that the IR deformation of $\mathcal{N}=4$ SYM theory must be such that there are massless Nambu-Goldstone modes associated to the spontaneously broken R-symmetry [4]. We will assume this property in the present approach.

The Bjorken parameter is defined as

$$
x = -\frac{q^2}{2P\cdot q}\,, \tag{1.2}
$$

being the physical range $0 \leq x \leq 1$, in the DIS limit $q^2 \gg P^2$ while x is kept fixed. From the Cutkosky rules for scattering amplitudes, based on S-matrix theory, one can derive the optical theorem leading to the following relations

$$
W^{\mu\nu}_{(\mathrm{S})} = 2\pi \mathrm{Im}\left[T^{\mu\nu}_{(\mathrm{S})}\right], \qquad W^{\mu\nu}_{(\mathrm{A})} = 2\pi \mathrm{Im}\left[T^{\mu\nu}_{(\mathrm{A})}\right], \tag{1.3}
$$

where the tensor $T^{\mu\nu}$ is defined by the time-ordered expectation value of two electromagnetic currents inside the hadron

$$
T^{\mu\nu} \equiv i\int d^4x e^{iq\cdot x}\langle P|\hat{\mathrm{T}}\{J^\mu(x)J^\nu(0)\}|P\rangle\,. \tag{1.4}
$$

This relates DIS to forward Compton scattering (FCS), which is what one calculates. DIS and FCS are schematically shown in figure 1.

[1] We use the notation for the hadronic tensor as in reference [8], having some sign differences with respect to [1, 2].

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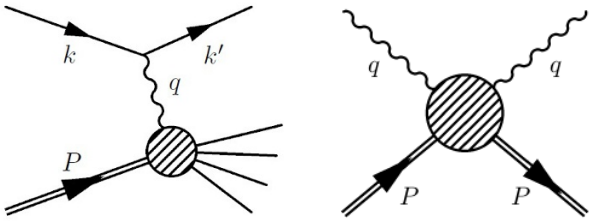

(a) (b)

Figure 1. Schematic representation of DIS (a) and FCS (b) processes. k and k' denote the four-momenta of the incoming and outgoing leptons in DIS.

In the pioneering work by Polchinski and Strassler [5], the symmetric structure functions F_1 and F_2 of DIS of a charged lepton off a spin-1/2 hadron have been calculated in the supergravity regime, i.e. for $1/\sqrt{\lambda} \ll x < 1$. The spin-1/2 hadron can be holographically represented by a dilatino wave-function in the bulk of $\mathrm{AdS}_5 \times S^5$ in type IIB supergravity, with the inclusion of an IR cut-off $z_0 = 1/\Lambda$. This scale breaks conformal invariance in the IR of the holographic dual gauge field theory, inducing color confinement. The holographic dual gauge theory corresponds to the planar limit of $\mathcal{N} = 4$ SU(N) supersymmetric Yang-Mills (SYM) theory in four dimensions, with an IR cut-off scale Λ. In the UV this gauge theory is conformal. The holographic dual process is schematically represented in figure 2. The result from [5] for the symmetric structure functions is

$$2F_1 = F_2 = \pi\, A'\, \mathcal{Q}^2 \left(\frac{\Lambda^2}{q^2}\right)^{\tau-1} x^{\tau+1} (1-x)^{\tau-2}\,, \tag{1.5}$$

where A' is a dimensionless constant, $\mathcal{Q}$ is a charge eigenvalue under the U(1) $\subset$ SU(4) symmetry group, and τ is the twist of the incident hadron, $\tau = \Delta - s$, being Δ the conformal dimension and s the spin (in the present case $s = 1/2$).

Furthermore, also in the supergravity regime ($1/\sqrt{\lambda} \ll x < 1$), in reference [6] polarized DIS structure functions considering a spin-1/2 hadron have been studied using the AdS/CFT duality. The results are [6]

$$2F_1 = F_2 = F_3 = 2g_1 = g_i\,, \quad g_2 = \left(\frac{1}{2x}\frac{\tau+1}{\tau-1} - \frac{\tau}{\tau-1}\right) g_1\,, \tag{1.6}$$

where $i = 3, 4, 5$. The explicit form of F_2 is given in equation (1.5). The functions F_3, g_3, g_4 and g_5 are similar to F_2 since the dilatino is a right-handed fermion in the massless limit. However, as we shall show in section 2, g_3, g_4 and g_5 vanish at leading order in $1/N$ for $x \ll 1/\sqrt{\lambda}$. Further calculations in this regime for non-forward Compton scattering have been done in [7]. A study of neutral spin-1/2 hadrons (similarly to case of charged spin-1/2 hadrons considered in [6]) is presented in reference [8] for this regime of the Bjorken parameter.

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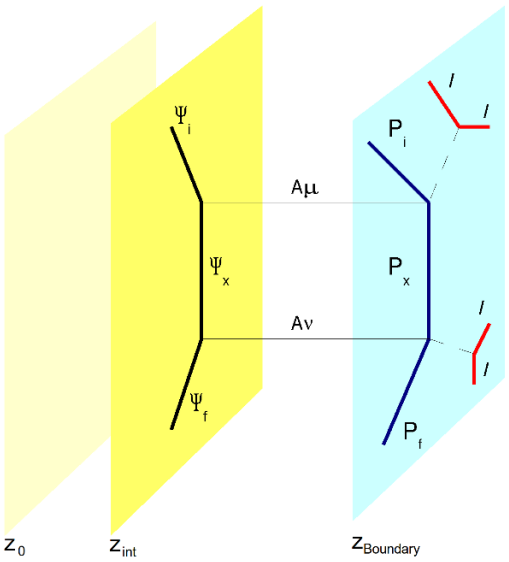

Figure 2. Schematic picture of the s-channel diagram corresponding to the holographic dual description of forward Compton scattering in the $1/\sqrt{\lambda} \ll x < 1$ regime. The incoming and outgoing spin-1/2 hadrons with four-momenta P_i and P_f are represented by blue lines in the boundary theory. Their corresponding dual dilatino fields in the bulk are denoted by Ψ_i and Ψ_f, respectively. Gauge fields A_μ and A_ν couple to the $J^\mu(x)$ and $J^\nu(0)$ electromagnetic currents in the boundary gauge field theory. z_0 is the IR cut-off and $z_{\rm int}$ is where the graviphoton-dilatino interaction takes place. Red lines denote leptons (l), while dashed lines indicate virtual photons.

On the other hand, in a completely different physical regime as it is the exponentially small-x region, the proton F_2 structure function has been investigated by Brower and collaborators in [9], by using the BPST-Pomeron techniques developed by Brower, Polchinski, Strassler and Tan within the gauge/string theory duality framework [10]. The authors of reference [9] have found that the BPST kernel fits remarkably well the region where the four-momentum transfer q^2 of the virtual photon is large, and also it works surprisingly well for small values of q^2, as low as $q^2 = 0.1$ (GeV/c)^2. They fit their result for F_2 to the combined H1-ZEUS small-x data of the inclusive DIS cross sections measured by H1 and ZEUS Collaborations in neutral and charged current unpolarized $e^\pm p$ scattering at HERA [11–13], in the range 0.1 (GeV/c)$^2 \leq q^2 \leq 400$ (GeV/c)^2, and for $10^{-6} \leq x \leq 10^{-2}$. For large q^2 conformal symmetry dominates, while near to the IR the hard-wall cut-off becomes important. This behavior is reflected on the results presented in [10]. In addition, in the case of $\mathcal{N} = 4$ SU(N) SYM theory considering polarized DIS also from a spin-1/2 hadron, a heuristic calculation based on the AdS/CFT duality has been developed in [4]. The result is that the Reggeized virtual photon leads to the polarized structure functions F_3 and g_1. For exponentially small x it has been obtained that $g_1 \approx (1/x)^{1-1/(2\sqrt{\lambda})}$.

In the present work we derive explicitly all the structure functions for a spin-1/2 hadron in the low-x regime. For small but not exponentially small x, in addition to a heuristic derivation, we carry out a detailed top-down string theory calculation from closed strings scattering amplitudes which constitutes the first complete derivation of this kind for spin-1/2 hadron in the low-x regime. This approach leads to effective Lagrangians from

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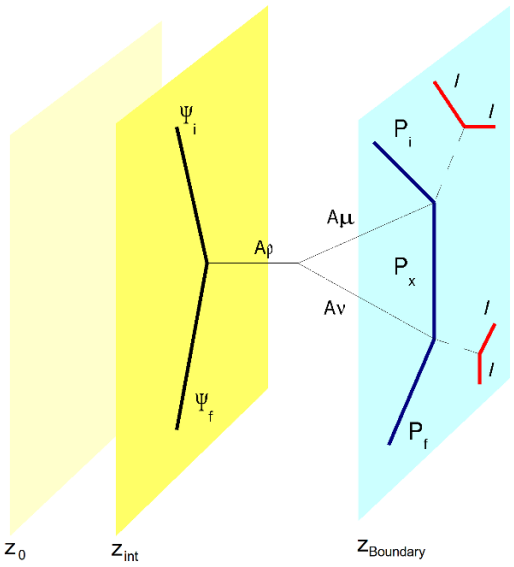

Figure 3. Schematic picture of the t-channel diagram corresponding to the holographic dual description of forward Compton scattering in the $x \ll 1/\sqrt{\lambda}$ regime. The incoming and outgoing spin-1/2 hadrons with four-momenta P_i and P_f are indicated with blue lines in the boundary theory. Their corresponding dual fields in the bulk are denoted by Ψ_i and Ψ_f, respectively. Gauge fields A_μ and A_ν couple to the $J^\mu(x)$ and $J^\nu(0)$ electromagnetic currents in the boundary gauge field theory.

which one can construct the leading-diagram contributions which are t-channel Feynman diagrams in the bulk theory. Their corresponding schematic representation is shown in figure 3. This is related to the Feynman diagrams presented in figure 4, corresponding to the calculations of the symmetric and antisymmetric structure functions in the range $\exp(-\sqrt{\lambda}) \ll x \ll 1/\sqrt{\lambda}$ that we introduce in sections 2 and 3, respectively.

Furthermore, for the exponentially low-x regime, generalizing the BPST-Pomeron approach, we consider a Reggeized gauge field and derive the antisymmetric structure functions. Then, we compare with experimental data. We fit our results[2] of g_1 to the data of the corresponding structure function of the proton at 190 GeV measured by the SMC Collaboration [14], and by the COMPASS Collaboration with beam energies of 160 GeV and 200 GeV reported in [15] and [16], respectively. In these cases we consider data within the $x < 0.01$ region. Following [16], in our figures 6 and 7 we also include data from the SMC [14], EMC [17, 18], HERMES [19], SLAC E143 [20], E155 [21] and CLAS [22] Collaborations, at $q^2 > 1$ (GeV/c)^2. Also, we consider the very recent data (2017) from the COMPASS Collaboration [23], where the photon virtuality is $q^2 < 1$ (GeV/c)^2, while $4 \times 10^{-5} < x < 4 \times 10^{-2}$. The chi-square value per degree of freedom that we obtain for our best fit corresponding to the conformal model is $\chi^2_{\text{d.o.f.}} = 1.140$, while for the hard-wall model our fit gives $\chi^2_{\text{d.o.f.}} = 1.074$. In both cases we fit the structure function g_1 against data from the COMPASS Collaboration [23]. Thus, our predictions lead to a very good fit as we shall discuss in detail in section 5. Also we have calculated the structure function F_3.

[2]As we shall explain in sections 4 and 5 we consider two different models, namely: a conformal model with no IR cut-off and the hard-wall model that we have already described.

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The holographic dual model corresponding to the planar limit of $\mathcal{N}=4$ SYM theory is represented by a solution of type IIB supergravity on $AdS_5 \times S^5$. The metric can be written as

$$ds^2 = \frac{R^2}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2\right) + R^2 d\Omega_5^2\,, \tag{1.7}$$

with radius $R = (4\pi\,\lambda\,\alpha'^2)^{1/4}$. The ten-dimensional indices are denoted by $M, N, \cdots = 0, \ldots, 9$, the AdS_5 ones are $m, n, \cdots = 0, \ldots, 4$, the flat four-dimensional indices are $\mu, \nu, \cdots = 0, \ldots, 3$, while the S^5 indices are $a, b, \cdots = 1, \ldots, 5$. The region $z \to 0$ corresponds to the UV. In the IR we assume the cut-off $z_0 = 1/\Lambda$.

In [3] we have calculated holographically the structure function $F_3(x, q^2)$ for glueballs of $\mathcal{N}=4$ SYM theory. This has also been done at strong coupling and at low x. Other very interesting developments from first principles calculations for scalar and polarized vector mesons have been done in [24–27], as well as $1/N$ corrections for glueballs [28], scalar mesons [29] and vector mesons [30]. The development of a unified description of the Regge physics and the BFKL Pomeron using the AdS/CFT duality has been done in [10]. Further developments including the eikonal approach, have been presented in [31–40]. Other aspects of applications of the AdS/CFT correspondence to DIS processes can be found in [41–46].

The work is organized as follows. In section 2 we focus on the calculation of the symmetric structure functions for spin-1/2 hadrons. In sections 2 and 3 we calculate the structure functions both from the heuristic point of view and from the type IIB superstring theory scattering amplitudes. All this corresponds to low but not exponentially low x. In section 4 we consider the calculation of g_1 in the exponentially small region of the Bjorken parameter, extending the BPST Pomeron techniques to the Reggeized gauge field. In section 5 we analyze our results and make comparison to the existing experimental data for g_1.

2 DIS from spin-1/2 hadrons at low x: the graviton exchange contribution

In this section we focus on the calculation of the symmetric structure functions for DIS of charged leptons from spin-1/2 hadrons at low x. The dual holographic calculation involves a graviton exchange in the t-channel as shown in figure 4(a) (also see figure 3).

For the Bjorken parameter x within the parametric region $\lambda^{-1/2} \ll x < 1$, at strong coupling and for large N, double-trace operators dominate the operator product expansion (OPE) of two electromagnetic currents inside the hadron. The scattering is produced from the entire hadron. In this regime the holographic dual description can be done in terms of the calculation of the s-channel in type IIB supergravity schematically shown in figure 2. Beyond that regime, at low x (more precisely when $x \ll \lambda^{-1/2}$) the holographic dual description of DIS requires considering the dynamics of type IIB superstring theory on the $AdS_5 \times S^5$ background. In particular, for values of the Bjorken parameter in the $\exp\left(-\lambda^{1/2}\right) \ll x \ll \lambda^{-1/2}$ range it is possible to carry out the holographic dual description in terms of scattering amplitudes of closed strings propagating in ten-dimensional space-time [5]. In fact, as argued in [5, 10], the dominant t-channel contribution to the DIS

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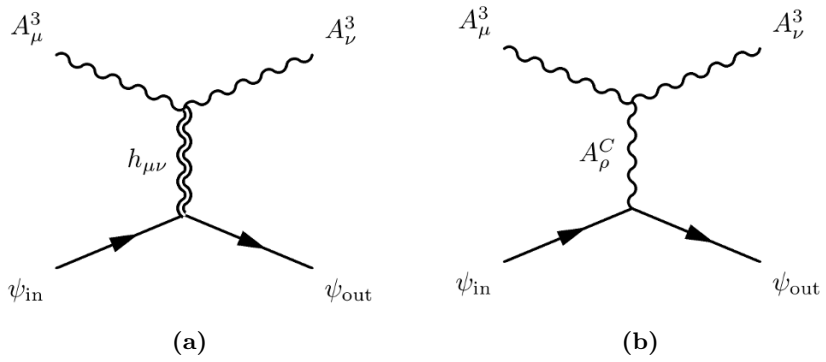

Figure 4. t-channel holographic dual representation of forward Compton scattering at tree-level. Figure (a) shows the exchange of a graviton in the AdS bulk, leading to the calculation of symmetric structure functions. Figure (b) indicates the Chern-Simons interaction in top vertex and the propagation of a bulk-to-bulk gauge field, leading to the anti-symmetric structure functions.

process is well described by local flat-space scattering amplitudes. Therefore, an effective Lagrangian can be built out from the local string theory scattering amplitude. Then, in order to obtain the dual FCS amplitude from which one can derive the structure functions, we have to take the imaginary part and integrate over the full $\text{AdS}_5 \times S^5$. Since we focus on a spin-1/2 hadron, the dual closed string modes are associated with the ten-dimensional dilatino $\Psi(x^M)$.

On the other hand, the relevant effective Lagrangian can also be constructed in a heuristic way [3][3] (also see [7]), which basically involves two steps. Firstly, we have to consider the five-dimensional supergravity interactions together with the graviton propagator. Secondly, we need to combine them by taking a local limit and interpreting the resulting expression of the propagator as coming from the α'-dependent pre-factor of the string theory scattering amplitude.[4] This leads to the so-called ultra-local approximation of the scattering amplitude.

In both frameworks, i.e. the heuristic and the first-principle gauge/string theory dual approaches, it is possible to calculate the symmetric structure functions of the spin-1/2 hadron. In the string theory scattering amplitude approach, the DIS process is related to the choice of the external modes: while the ten-dimensional dilatino field is given by a Neveu-Schwarz-Ramond (NS-R) field, we consider the photon to be a particular polarization state of the graviton NS-NS mode as in [5]. In the heuristic approach, the external states are described by Kaluza-Klein (KK) modes corresponding to ten-dimensional modes. At low x, the four-dimensional center-of-mass (CM) energy s is very high since

$$s \equiv -(P+q)^2 \approx -q^2 - 2P\cdot q = -q^2\left(1-\frac{1}{x}\right) \approx \frac{q^2}{x}, \tag{2.1}$$

[3]Strictly speaking this method only gives the AdS_5 contribution, thus we have to multiply by an *ad hoc* contribution from the integration on S^5 which only gives an overall factor. The dependence on the S^5 radius is accounted for by using dimensional analysis.

[4]Details are given in reference [3].

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where we have used the fact that in this regime $-P^2 \ll q^2 \ll -P\cdot q$. This implies that the ten-dimensional Mandelstam variable $\tilde{s}$ becomes very large. Thus, the leading contribution to the scattering process comes from the t-channel exchange. When the exchanged field carries spin j this contribution is proportional to $\tilde{s}^j$. Consequently, the dominant process in this context is the t-channel Reggeized graviton exchange where $j \approx 2$.

In the next subsection we derive an effective Lagrangian in a heuristic approach. In subsection 2.2 we carry out the formal derivation of the Lagrangian starting from the four-point type IIB superstring theory scattering amplitude, with one NS-R, one R-NS and two NS-NS fields. In subsection 2.3 we explicitly obtain the symmetric structure functions.

2.1 Heuristic derivation of the effective Lagrangian

In order to construct the heuristic effective Lagrangian leading to the symmetric part of the hadronic tensor $W^{\mu\nu}$ at low x we need to consider a t-channel five-dimensional SU(4) gauged supergravity tree-level diagram, as shown in figure 4(a). This maximally supersymmetric supergravity is obtained from dimensional reduction of type IIB supergravity on S^5 [47–51]. This spontaneous compactification of type IIB supergravity leads to a five-dimensional Chern-Simons term [48–50] that will be very important in the calculation of antisymmetric structure functions described in section 3. We will follow closely the steps described in our previous paper [3], however there is a crucial difference now, namely: instead of using a dilaton wave-function, in the present heuristic case we must consider the wave-function of a dilatino field $\psi(x,z)$, representing the spin-1/2 hadron.

The relevant part of the maximally supersymmetric supergravity action on AdS_5, with indices $m,n = 0,\ldots,4$, is given by the expression [50]

$$S_{5d} = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g_{AdS_5}}\left(\mathcal{R} - \bar{\psi}\,\gamma^m D_m\psi - \frac{1}{4}\left(F^A_{mn}\right)^2 + \cdots\right), \tag{2.2}$$

where $2\kappa_5^2 = 16\pi^2/N^2$ is the Newton constant in five dimensions (we set $R=1$), F^A_{mn} is the non-Abelian gauge field strength associated with the gauge field A^A_m, and $\mathcal{R}$ is the Ricci scalar in five dimensions. Also we use the definition $\gamma^m = e^m_{\hat{m}}\gamma^{\hat{m}}$, where $\gamma^{\hat{m}}$ are the flat-space Dirac matrices ($\hat{m} = 0,\ldots,4$) and $e^m_{\hat{m}}$ is the vielbein. Dots include kinetic and interaction terms which are not relevant for our present analysis.

At high energy the leading diagram is given by the t-channel exchange of a graviton. Since the graviton couples to the energy-momentum tensors T^{ψ}_{mn} and T^A_{mn} given by[5]

$$T^{\psi}_{mn} = \bar{\psi}\,\gamma_{(m}\partial_{n)}\psi\,, \qquad T^A_{kl} = g^{pq}F_{kp}F_{lq} - \frac{1}{4}g_{kl}F_{pq}F^{pq}\,, \tag{2.3}$$

the corresponding amplitude has the form

$$\mathcal{A} = \kappa_5^2\int d^5x\,d^5x'\,T^{\psi}_{mn}(x)\,G^{mnkl}(x,x')\,T^A_{kl}(x')\,, \tag{2.4}$$

[5]Note that the fluctuations of the fields are normalized with an extra factor $\sqrt{2}\kappa_5$. We only write the quadratic terms of the energy-momentum tensors.

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where $G^{mnkl}(x,x')$ denotes the AdS_5 graviton propagator whose relevant terms can be expressed as

$$G^{mnlk}(x,x') = \left(g^{mk}g^{nl} + g^{ml}g^{nk} - \frac{2}{3}g^{mn}g^{kl}\right) G_{\rm grav}(x,x') + \dots\,, \tag{2.5}$$

being $G_{\rm grav}(x,x')$ some function whose explicit form we dot not need.

Gathering all the information we obtain the following integrand

$$T^{\psi}_{mn}(x)\, G^{mnlk}(x,x')\, T^{A}_{kl}(x) = 2\, G_{\rm grav}(x,x')\, F_{mp}(x')\, F^{p}_{n}(x')\, \bar{\psi}(x)\gamma^{n}\partial^{m}\psi(x)\,, \tag{2.6}$$

plus $\mathcal{O}(t)$ terms. We only consider the leading terms in $\tilde{s} = -\tilde{u}$, since in the $x \ll \lambda^{-1/2}$ regime we have $\tilde{s} \gg \tilde{t}$.

In order to obtain the effective action one would have to integrate the effective Lagrangian obtained from equation (2.6) over the full $AdS_5 \times S^5$. The sphere reduction gives a numerical constant C. Then, we need to multiply it by the superstring theory pre-factor

$$\tilde{s}^2\, \mathcal{G}(\alpha',\tilde{s},\tilde{t},\tilde{u}) = -\frac{\alpha'^3\tilde{s}^2}{64} \prod_{\chi=\tilde{s},\tilde{t},\tilde{u}} \frac{\Gamma\left(-\alpha'\chi/4\right)}{\Gamma\left(1+\alpha'\chi/4\right)}\,. \tag{2.7}$$

The effective action is

$$S^{(\rm S)}_{\rm eff} = 2\,\kappa_5^2\, {\rm Im}\left[\tilde{s}^2\,\mathcal{G}(\alpha',\tilde{s},\tilde{t},\tilde{u})\right] C \int d^5x \sqrt{g_{AdS_5}}\; F_{mp}F_{n}{}^{p}\, \bar{\psi}\gamma^{(m}\partial^{n)}\psi\,. \tag{2.8}$$

By plugging the solutions for $\psi(x^\mu,z)$ and $A_m(x^\mu,z)$ in equation (2.8) we can evaluate the on-shell action and then take its imaginary part. This leads to the dilatino (symmetric) structure functions that will be calculated in subsection 2.3. In the next subsection we show how to derive the effective action from first principles, starting from the scattering amplitude of four closed strings in type IIB superstring theory.

2.2 Derivation from the string theory scattering amplitude

In the $e^{-\sqrt{\lambda}} \ll x \ll \lambda^{-1/2}$ regime we can obtain the spin-1/2 hadronic tensor by calculating a certain tree-level four-point string theory scattering amplitude in ten-dimensional flat-space. This was motivated in [5]. Once the local flat-space amplitude is obtained one can derive an effective Lagrangian, which is then integrated over the $AdS_5 \times S^5$ space after the inclusion of the curved-space wave-functions of the dilatinos and the gravi-photons. The five-dimensional spin-1/2 and gauge fields which we have used in the previous section are specific KK modes of these ten-dimensional excitations reduced on S^5. The external states are given by two dilatinos and two gravitons. The details of the decomposition are given below. In other words, we are interested in a closed string amplitude with two modes from the NS-R sector and the other two from the NS-NS sector.

Following the KLT relations [52, 53] the closed-string theory scattering amplitude factorizes in terms of open-string amplitudes as

$$\mathcal{A}(1,2,\tilde{3},\tilde{4}) = 4\; i\; \kappa_{10}^2\, \mathcal{G}(\alpha',\tilde{s},\tilde{t},\tilde{u})\, K^{\rm bos}_{op}(1,2,3,4) \otimes K^{\rm fer}_{op}(\tilde{3},1,2,\tilde{4})\,, \tag{2.9}$$

where K_{op} are open string kinematic factors. Particle numbers with a tilde indicate fermionic modes. For these particular combinations of modes these factors can be found in [53, 54]. The relevant terms take the form

$$K_{op}^{\rm bos}(1,2,3,4) = \xi_1^M \xi_2^N \xi_3^P \xi_4^Q \left[-1/4\, \tilde{s}\, \tilde{u}\, \eta_{MN}\eta_{PQ} + \cdots\right], \tag{2.10}$$

and

$$K_{op}^{\rm fer}(\tilde{3},1,2,\tilde{4}) = \xi_1^{M'} \xi_2^{N'} \bar{u}_3^\alpha u_4^\beta \left[\tilde{s}\left(k^2_{M'}(\Gamma_{N'})_{\alpha\beta} - k^1_{N'}(\Gamma_{M'})_{\alpha\beta} - \eta_{M'N'}(\Gamma^P)_{\alpha\beta}k^2_P\right) + \cdots\right], \tag{2.11}$$

where dots indicate sub-leading terms in the dual DIS process. ξ_i and u_i are the boson and fermion polarizations, respectively, while Γ^N indicates the ten-dimensional gamma matrices. The spinor indices are denoted by α, β and the ten-dimensional bosonic indices are denoted by M, N. In the notation of equations (2.10) and (2.11) the ten-dimensional Mandelstam variables are defined as

$$\tilde{s} = -(k_1+k_4)^2\,,\ \tilde{t} = -(k_1+k_2)^2\,,\ \tilde{u} = -(k_1+k_3)^2\,, \tag{2.12}$$

where k_1 and k_2 are the momenta associated to the bosonic modes, while k_3 and k_4 are the ones associated to the fermionic modes. Also, the closed-string graviton and dilatino polarizations are given by [55]

$$h_i^{MN} \equiv \xi_i^M \otimes \xi_i^N\,, \quad (\Gamma^M)^\alpha_\beta \Psi_i^\beta \equiv u_i^\alpha \otimes \xi_i^M\,. \tag{2.13}$$

Thus, to leading order in $\tilde{s}$ the corresponding amplitude becomes

$$\begin{aligned}\mathcal{A}(1,2,\tilde{3},\tilde{4}) = 4\, i\, \kappa_{10}^2\, \mathcal{G}\, \tilde{s}^2\, \overline{\Psi}_3 \left[\tilde{u}(h_1\cdot h_2)\not{k}_1 + \tilde{s}(h_1\cdot h_2)\not{k}_2\right. \\ \left. +\, 2\ \tilde{u}(k_2\cdot h_1\cdot h_2\cdot \Gamma) + 2\ \tilde{s}(k_1\cdot h_2\cdot h_1\cdot \Gamma)\right]\Psi_4\,.\end{aligned} \tag{2.14}$$

In this expression we can set $k^1_M h_2^{MN} = k^2_M h_1^{MN} = 0$ since both graviton states correspond to the ingoing dual photon and its complex conjugate associated with the outgoing one, respectively. Thus, from now on we will neglect the first two terms in equation (2.14). Then, the effective Lagrangian associated with this scattering amplitude can be written by replacing momenta with derivatives, giving the following structure

$$-\, i\, \kappa^2\, (\partial_P h_{MN})\ (\partial_Q h^{MN})\ \overline{\Psi}\, \Gamma^{(P}\, \partial^{Q)}\Psi\,. \tag{2.15}$$

Next, we need to obtain a curved-space version of (2.15) and rewrite it in terms of the five-dimensional fields. The decomposition of the fields is given by [47]

$$\Psi(x^m,\Omega) = \sum_\Delta \psi_\Delta(x^m)\otimes\eta_\Delta(\Omega)\,, \quad h^{ma} = \sum_k A_k^m(x^n)\, Y_k^a(\Omega)\,, \tag{2.16}$$

where Δ and k are integers, $\eta_\Delta(\Omega)$ are eigenfunctions of the Dirac operator on S^5 and $Y_k^a(\Omega)$ are the corresponding vector spherical harmonics. In the holographic dual DIS process calculation, we focus on a particular value of Δ (note that henceforth we write

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$\psi_\Delta \equiv \psi$). Also, the massless gauge field has the lowest vector spherical harmonics, which are given by the Killing vectors K_a^A on S^5. Moreover, when considering gauged supergravity both vector fields carry a gauge group index[6] A. Thus, we can write

$$\Psi(x^M) \to \psi(x^m) \otimes \eta(\Omega)\,, \quad h_{MN} \to h_{ma} = A^A_{(m}(x^n)\, K^A_{a)}\,. \tag{2.17}$$

Plugging these expressions in the effective Lagrangian and integrating over $\mathrm{AdS}_5 \times S^5$ we obtain the effective on-shell action (2.8), where C is defined by considering the normalization condition

$$\int d^5\Omega\, \sqrt{g_{S^5}}\, \bar{\eta}(\Omega)\, \eta(\Omega)\, K^a\, K_a = C\,. \tag{2.18}$$

2.3 Symmetric structure functions

In this section we calculate the symmetric structure functions of a spin-1/2 hadron (which is assumed to be dual to a dilatino bound state as in [5]) in the $e^{-\sqrt{\lambda}} \ll x \ll \lambda^{-1/2}$ regime. We follow the conventions of reference [5]. We consider the AdS_5 metric given in (1.7). In the hard-wall model a radial cut-off is included at $z_0 = \Lambda^{-1}$, in order to account for the IR confinement scale Λ in the dual field theory. For energy larger than Λ the theory becomes approximately conformal.

In order to compute the hadronic tensor we need to obtain the effective action (2.8) evaluated on-shell. Since the AdS process is dual to the FCS, the imaginary part gives the DIS hadronic tensor as follows

$$S^{(\mathrm{S})}_{\mathrm{eff}} \equiv n_\mu n^*_\nu\, \mathrm{Im}\, \left[T^{\mu\nu}_{(S)}\right] = \frac{1}{2\pi} n_\mu n^*_\nu\, W^{\mu\nu}_{(S)}\,. \tag{2.19}$$

The incoming and outgoing gauge fields are given by the non-normalizable solutions of the Einstein-Maxwell equations in AdS. By imposing the appropriate boundary conditions

$$A^3_\mu(z \to 0) = n_\mu e^{iq\cdot x}\,, \quad A^3_z(z \to 0) = 0\,, \tag{2.20}$$

the solutions are given by

$$A^3_\mu = n_\mu\, e^{iq\cdot x}\, qz\, K_1\,(qz)\,, \quad A^3_z = i(n\cdot q)\, e^{iq\cdot x}\, z\, K_0(qz)\,, \tag{2.21}$$

where K_i denotes the Bessel functions of the second kind. Note that without loss of generality we can choose a transversal polarization for the virtual photon. Thus, from now on we take $n \cdot q = 0$ and in particular we set $A^3_z = 0$.

Now, let us consider the dilatino. We briefly describe the corresponding type IIB supergravity solution following [5]. In the conformal region we can write the dilatino wavefunction as in equation (2.17). The $\psi(x, z)$ solution satisfies the Dirac equation in five dimensions. Factorizing out the spinor harmonic $\eta(\Omega)$ on the sphere, the five-dimensional solution with four-momentum P_μ is

$$\psi = e^{iP\cdot x}\, C'\, z^{5/2}\, [J_{\tau-2}(Pz)P_+ + J_{\tau-1}(Pz)P_-]\, u(P)\,, \tag{2.22}$$

[6] If the full isometry group of the sphere $\mathrm{SO}(6) \sim \mathrm{SU}(4)$ is gauged the index A runs from 1 to 15.

where C' is a normalization constant, $\tau = \Delta - 1/2 = mR + 3/2$ is the twist of the corresponding QFT operator, and the four-dimensional chirality projectors are defined as $P_\pm \equiv \frac{1}{2}\left(1 \pm \gamma^5\right)$ with $\gamma^5 \equiv \gamma^{\hat{z}}$. Also, u and $\overline{u}$ are Dirac spinors in four dimensions.

The leading terms in the near-boundary expansion are given by[7]

$$\begin{aligned}\psi_i &\approx e^{iP\cdot x}\frac{c_i}{\Lambda^{3/2}}(z/z_0)^{\tau+1/2}\left[P_+ + \frac{Pz}{2(\tau-1)}P_-\right]u_i(P)\,,\\ \overline{\psi}_i &\approx e^{-iP\cdot x}\frac{c_i^*}{\Lambda^{3/2}}(z/z_0)^{\tau+1/2}\,\overline{u}_i(P)\left[P_- + \frac{Pz}{2(\tau-1)}P_+\right],\end{aligned} \tag{2.23}$$

where c_i is some dimensionless constant.

Since we consider the $\tilde{t} \to 0$ and $\tilde{s} \to \infty$ limit, we can expand the string theory scattering amplitude pre-factor as in [5]. Thus, by taking the imaginary part we can rewrite it as a sum over the excited states in the form

$$\mathrm{Im}_{\mathrm{exc}}\left[G(\alpha',\tilde{s},\tilde{t},\tilde{u})\ \tilde{s}^2\right]|_{\tilde{t}\to 0} = \frac{\pi\alpha'}{4}\sum_{m=1}^{\infty}\delta\left(m - \frac{\alpha'\tilde{s}}{4}\right)(m)^{\alpha'\tilde{t}/2}\,, \tag{2.24}$$

where the last factor can be ignored in the region of interest since $\alpha'\tilde{t} \sim \mathcal{O}(\lambda^{-1/2})$. This sum can be approximated by an integral for $x \ll \lambda^{-1/2}$. Recall that the relation between the ten-dimensional Mandelstam variables and the four-dimensional ones is

$$\alpha'\tilde{s} \approx \alpha' s\, z^2/R^2\,, \tag{2.25}$$

plus corrections from the radial and S^5 coordinates which can be neglected.

Note that once the fermion solutions are inserted, the objects with spinor indices in the leading term give a factor

$$\overline{u}_i P_- \gamma_{\hat{\mu}} P_+ u_i = \overline{u}_i \gamma_{\hat{\mu}} P_+ u_i = -i(P_\mu + S_\mu)\,, \tag{2.26}$$

where S_μ is the spin polarization vector. However, the second term is actually misleading and should be omitted. The graviton exchange of the dual calculation corresponds to the energy-momentum tensor term in the current-current OPE (on the QFT side). Thus, terms proportional to S_μ should not be present in the expectation value. From the holographic dual approach it is necessary to go back to the full expression for the spin-1/2 solution and "undo" the local approximation for the t-channel diagram. Then, the z-integral (with the correct integration limits $0 < z < z_0$) can be computed in the relevant low-momentum limit of the graviton mode, giving a vanishing result (as opposed to the contributions proportional to $P_\mu P_\nu$). The details of the computation are very similar to those of reference [6] where the authors study the elastic form factors for the conserved current. Note that this observation implies that in the string theory regime the structure functions $g_{3,4,5}(x, q^2)$ will vanish at leading order in the $1/N$ expansion.

[7] The second terms in both equations (2.23) give P^2/q^2 sub-leading contributions to the symmetric structure functions, thus we will not consider them in the following calculations.

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Now, plugging all these elements in the effective action and carrying out the integrals over $AdS_5 \times S^5$ we find

$$n_\mu n_\nu^* T^{\mu\nu}_{(S)} = n_\mu^* n_\nu \frac{\pi\,|c_i|^2 C}{2\sqrt{4\pi\lambda}} \left(\frac{\Lambda^2}{q^2}\right)^{\tau-1} q^{-2} \times \left[\eta^{\mu\nu}\frac{(P\cdot q)^2}{q^2} I_{1,2\tau+3} + P^\mu P^\nu (I_{0,2\tau+3} + I_{1,2\tau+3})\right] \tag{2.27}$$

where

$$I_{j,n} = \int_0^\infty dw\; w^n K_j^2(w) = 2^{n-2}\frac{\Gamma(\nu+j)\Gamma(\nu-j)\Gamma(\nu)^2}{\Gamma(2\nu)}\,,\;\; \nu = \frac{1}{2}(n+1)\,,\;\; I_{1,n} = \frac{n+1}{n-1} I_{0,n}\,. \tag{2.28}$$

Writing the above expressions in terms of the Bjorken parameter $x = -\frac{q^2}{2(P\cdot q)}$ and comparing with the structure of hadronic tensor (1.1), we obtain the following symmetric structure functions for the spin-1/2 hadron

$$F_1\left(x, q^2\right) = \frac{1}{x^2}\left(\frac{\Lambda^2}{q^2}\right)^{\tau-1} \frac{\pi^2 |c_i'|^2 C}{4(4\pi\lambda)^{1/2}} I_{1,2\tau+3}\;\;,\;\; F_2\left(x, q^2\right) = 2x\frac{2\tau+3}{\tau+2} F_1(x, q^2)\,, \tag{2.29}$$

together with $g_3 = g_4 = g_5 = 0$. Note that the x and q^2 dependence of F_1 and F_2 structure functions agree with the ones obtained by Polchinski and Strassler for the (scalar) glueball in the same parametric regime, by interchanging Δ and τ. Also, the above equation (2.29) gives a generalization of the Callan-Gross relation of a spin-1/2 hadron. In addition, there are no contributions to the antisymmetric structure functions coming from the t-channel graviton exchange. In the next section we shall see how they appear in a different way.

3 DIS from spin-1/2 hadrons at low x: the gauge field exchange contribution

We now describe the calculation of the antisymmetric contributions to the hadronic tensor and derive the corresponding structure functions for polarized DIS of charged leptons from spin-1/2 hadrons at low x and at strong 't Hooft coupling in the large N limit of the $\mathcal{N} = 4$ SU(N) SYM theory with an IR cut-off. The corresponding dual holographic calculation is dominated by the exchange of a gauge field in t-channel within the AdS space, as shown in figure 4(b). A heuristic analysis has been done in [4] for the F_3 function, while in our previous paper [3] we have done a first principles calculation from type IIB superstring theory four-point scattering amplitude for glueballs.

From a heuristic viewpoint, one can understand that the antisymmetric contribution arises due to the Chern-Simons term in the five-dimensional SU(4) gauged supergravity action. In our conventions, it can be written as

$$S_{\rm CS} = \frac{i\;\kappa}{96\pi^2} d_{ABC} \int d^5x\, \varepsilon^{mnopq}\, A_m^A\, \partial_n A_o^B\, \partial_p A_q^C\,, \tag{3.1}$$

where A, B, C stand for the SU(4) gauge group indices, ε^{mnopq} is the Levi-Civita symbol, k an integer and d_{ABC} is the completely symmetric symbol. By coupling the matter fields to the A_m^3 gauge field (the gravi-photon), throughout the exchange of a spin-one field,

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the antisymmetric F_3 and g_1 structure functions are obtained. The exchanged gauge field cannot be A^3_m because the interaction term includes the d_{ABC} symmetric symbol [3, 57].

In fact, there are two tree-level t-channel Feynman diagrams contributing to the coupling of the dilatino to the Chern-Simons term. One involves the exchange of a gauge field A^C_m associated to an S^5 isometry. This coupling also appears in the dilaton DIS [3]. The second diagram comes from the so-called Pauli term, which was discussed in the holographic dual description of DIS in reference [8], but only for the $\lambda^{-1/2} \ll x < 1$ regime. In the non-Abelian case, it takes the form

$$S_P = \beta^A \int d^5x \sqrt{-g_{\rm AdS}}\, F^A_{mn}\, \bar{\psi}\, [\gamma^m, \gamma^n]\, \psi\,, \tag{3.2}$$

for some constants β^A. The interaction also occurs through the exchange of a gauge field. However, note that it is present even when the dilatino is not charged under the usual isometries.

In terms of the superstring scattering amplitudes, the antisymmetric contribution leading to F_3 and g_1 comes from the R-R sector of the closed string. This occurs in a way similar to the dilaton case. Recall that the massless five-dimensional gauge field A_m that emerges after the S^5 spontaneous compactification is a linear combination of the graviton h_{MN} and a particular mode of the R-R self-dual five-form field strength [47]. Therefore, it is important to consider that the incoming dual non-Abelian gauge states contain modes from both the NS-NS and the R-R sectors.

3.1 Heuristic derivation of the effective Lagrangian

We are now interested in the calculation of the t-channel supergravity process at tree-level, but in this case the exchanged field has spin one. By looking at the figure 4(b) the top-vertex interaction is given by non-Abelian Chern-Simons term (3.1). The two diagrams that we will analyze are schematically represented by the Feynman diagram shown in figure 4(b).

The Chern-Simons term involves the full set of non-Abelian gauge fields A^C_m associated to the SU(4) symmetry group. We focus on processes where the non-normalizable mode dual to the virtual photon is A^3_m, since this mode couples to the electromagnetic current of the AdS boundary gauge theory. The completely symmetric symbol d_{ABC} in equation (3.1) is then restricted to the form d_{33C}. Thus, it is easy to see that the t-channel propagating gauge boson A^C_m can only have color numbers $C = 8$ or $C = 15$ (see for example [3, 4]). These indices are associated with two diagonal matrices in the Lie algebra of SU(4). The idea is to write a heuristic effective Lagrangian, for which we have to consider the corresponding t-channel gauge field propagator and couple the A^C_m coming from the Chern-Simons current to the dilatino current. The amplitude can be written as

$$\mathcal{A} = \kappa_5^2 \int d^5x\, d^5x'\ \mathcal{J}^m_C(x)\, G^{CD}_{mn}(x, x')\, J^n_D(x')\,, \tag{3.3}$$

where $\mathcal{J}^m_C$ denotes the Chern-Simons current and J^n_D is given by

$$\mathcal{J}^m_C(x) = \frac{i}{6}\, d_{ABC}\, \varepsilon^{mnopq}\, \partial_n A^A_o\, \partial_p A^B_q\,, \quad J^n_D(x') = -\mathcal{Q}_D\, \bar{\psi}\gamma^n\psi\,, \tag{3.4}$$

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respectively, while $G^{CD}_{mn}(x,x')$ is the gauge field propagator in AdS_5, whose relevant part at high energy can be expressed as

$$G^{CD}_{mn}(x,x') = g_{mn}\,\delta^{CD}\,G_{\rm gauge}(x,x') + \cdots\,, \tag{3.5}$$

with $G_{\rm gauge}(x,x')$ being some function which is not relevant for the present calculation. The charge Q_D in the dilatino current is related to the eigenvalue equation for the ten-dimensional wave-function $\Psi(x^m,\Omega)$

$$K^a_D\partial_a\Psi(x^m,\Omega) = -\mathcal{Q}_D\Psi(x^m,\Omega)\,, \tag{3.6}$$

being $D = 8, 15$ the Lie algebra indices corresponding to the matrices T_D.

The rest of the computation is analogous to the symmetric case. After including the string pre-factor times a constant $\tilde{C}$ coming from S^5 integration, and performing the curved-space integrals, we obtain

$$S^{(\rm A)}_{\rm eff} = -i\,\frac{1}{6}\mathcal{Q}^C d_{ABC}\;{\rm Im}\left[\mathcal{G}\,\tilde{s}^2\right]\tilde{C}\int d^5x\,\varepsilon^{mnopq}\,\partial_m A^A_n\,\partial_o A^{*B}_p\;\bar{\psi}\gamma_q\psi\,. \tag{3.7}$$

A similar method can be used for the Pauli term contribution.

3.2 Derivation from the string theory scattering amplitude

Now, we formally derive the effective Lagrangian which permits to obtain the antisymmetric structure functions from type IIB superstring theory. For that we first obtain the string theory scattering amplitude that we need in order to construct the associated effective Lagrangian relevant for the antisymmetric contribution. The only difference is that in the present case the four-point scattering amplitude must contain external states coming from the R-R sector. The reason for the presence of the R-R sector is that the massless gauge fields A^C_m of the five-dimensional SU(4) gauge supergravity are constructed as linear combinations of two low-lying KK modes on S^5, coming from both NS-NS and R-R string states. The former is a graviton perturbation h_{MN}, while the second one corresponds to a R-R four-form field perturbation $\mathcal{C}_{M_1\cdots M_4}$. This is described in detail in [47] and reviewed in our previous work where we have investigated the dilaton case related to the DIS from glueballs [3].

The relevant four-point amplitudes can be written as one of the two forms

$$\mathcal{A}\,(\text{R-R},\text{R-R},\text{NS-R},\text{NS-R})\quad\text{or}\quad\mathcal{A}\,(\text{NS-NS},\text{R-R},\text{NS-R},\text{R-NS})\,,$$

where the first two external states correspond to the gauge fields in both cases. The two amplitudes above are important. We explicitly calculate the first one and show that it leads exactly the effective action (3.7) associated with the coupling between the Chern-Simons term and the minimal coupling of the dilatinos with the gauge field. Then, we argue why the second amplitude should lead to the case where this minimal coupling is replaced by the Pauli term.

Next, we want to obtain the scattering amplitude for two NS-R and two R-R states following the same steps as in section 2.2. Due to the KLT relations between open and

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closed superstring amplitudes, one can see that the amplitude we are interested in is given by [52, 53]

$$\mathcal{A}(\tilde{1},\tilde{2},\tilde{3},\tilde{4}) = -i\ \kappa^2 \mathcal{G}(\alpha',\tilde{s},\tilde{t},\tilde{u})\, K^{\rm fer}_{op}(\tilde{1},\tilde{2},\tilde{3},\tilde{4}) \otimes K^{\rm fer}_{op}(\tilde{3},1,2,\tilde{4})\,, \tag{3.8}$$

where the italic numbers stand for the R-R fields. The first kinematic factor is

$$K^{\rm fer}_{op}(\tilde{1},\tilde{2},\tilde{3},\tilde{4}) = \frac{\tilde{s}}{2}\,\bar{u}_1\Gamma^M u_2\,\bar{u}_3\Gamma_M u_4\,, \tag{3.9}$$

and second one is given in equation (2.11). The dilatino polarizations are given in equation (2.13), and the polarizations of the closed-string four-form field are given in terms of the open-string ones by

$$u_i^\alpha \otimes \bar{u}_i^\beta = (\mathcal{C}_Q\Gamma_{i(5)})^{\alpha\beta}\,, \quad \text{with} \quad \Gamma_{i(5)} = (\mathcal{F}_i)_{M_1\cdots M_5}\Gamma^{M_1\cdots M_5}\,, \tag{3.10}$$

in the conventions of [53], being $\mathcal{C}_Q$ the charge conjugation matrix. After some algebra, we obtain the leading amplitude in this regime

$$\mathcal{A}(\tilde{1},\tilde{2},\tilde{3},\tilde{4}) = -i\ \kappa^2 \mathcal{G}(\alpha',\tilde{s},\tilde{t},\tilde{u})\,\tilde{s}^2\frac{16}{15}(\mathcal{F}_3)_{MM2\cdots M_5}(\mathcal{F}_4)_N^{\ M_1\cdots M_5}\bar{\Psi}_1\gamma^{(N}k_2^{M)}\Psi_2\,, \tag{3.11}$$

from which the effective Lagrangian can be constructed. For that purpose we consider the curved metric together with the S^5-reduction of the different fields, which are rewritten as an expansion in modes over the S^5. The dilatino expansion is given in equation (2.16), while the 5-form field strength perturbation is [3, 47, 57]

$$\mathcal{F}_{mnabc} \sim 5(1+*)\,\partial_{[n}A^B_{m]}\,Z^B_{abc}\,, \quad \text{with} \quad Z^A_{abc} \equiv \epsilon_{abcde}\,\nabla^d K^{eA}\,, \tag{3.12}$$

where ϵ_{abcde} is the Levi-Civita tensor on the sphere.

The charges $\mathcal{Q}^C$ come from the harmonic spinors transformation under the corresponding S^5 isometries, while the d_{ABC} symbol emerges from a Killing vector identity [3]. Then, multiplying by the string pre-factor we obtain the effective action with the following structure

$$-\,i\,d_{ABC}\,\mathcal{Q}^C\,{\rm Im}\left[\mathcal{G}\,\tilde{s}^2\right]\int d^5\Omega\,\sqrt{g_{S^5}}\,\bar{\eta}(\Omega)\,\eta(\Omega)\ \int d^5x\,\epsilon^{mnopq}\,\partial_m A^A_n\,\partial_o A^{B*}_p\,\bar{\psi}\gamma_q\psi\,. \tag{3.13}$$

The dependence on the fields and the Mandelstam variables of this result fully agrees with equation (3.7).

The fact that this particular amplitude leads to the effective action (3.7) could have been anticipated by looking at the three-point string theory scattering amplitudes. As it has been carefully analyzed in our previous paper [3], one can see that the $\mathcal{A}$(R-R, R-R, NS-NS) $\sim \mathcal{A}(\mathcal{F},\mathcal{F},h)$ leads to an interaction term of the form of the Chern-Simons term in the supergravity action. One obtains this precisely when the external states have the particular polarizations indicated above. Thus, the Feynman diagram associated with the minimal coupling comes from an amplitude where the graviton state (with one index on AdS_5 and the other on S^5) propagates in the t-channel. In this heuristic approach, the two incoming dual gauge fields are modes of the self-dual five-form field strength.

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Then, the dilatino and dilaton IR vertices come from the string theory scattering amplitudes $\mathcal{A}$(NS-NS,NS-NS,NS-NS) $\sim \mathcal{A}(h,\phi,\phi)$ and $\mathcal{A}$(NS-NS,NS-R,NS-R) $\sim \mathcal{A}(h,\Psi,\Psi)$, respectively.

For the Pauli term, on the other hand, one can use a similar reasoning. It is not difficult to see that from the three-point scattering amplitude $\mathcal{A}$(R-R,R-NS,NS-R) $\sim \mathcal{A}(\mathcal{F},\overline{\psi},\psi)$, supplemented with the corresponding polarizations, one can derive a five-dimensional effective Lagrangian of the form of the Pauli interaction term (3.2), at least in the AdS_5 space. This is so because in terms of the ten-dimensional fields the effective Lagrangian is of the form

$$L_{\mathcal{F}\overline{\Psi}\Psi} \propto \mathcal{F}_{MNOPQ}\,\overline{\Psi}\Gamma^{MNOPQ}\Psi\,, \tag{3.14}$$

and then we only have to take two indices on AdS_5 and the other three on S^5. Therefore, we can consider that in this case the exchanged gauge field is a five-form field strength mode. Then, since in this case the top vertex is derived from the Chern-Simons term, we conclude that it should be possible to explicitly obtain the effective action coming from the Pauli interaction term by studying the $\mathcal{A}$(NS-NS, R-R, NS-R, R-NS) $\sim \mathcal{A}(\mathcal{F},h,\overline{\Psi},\Psi)$ four-point amplitude.

3.3 Antisymmetric structure functions

Contribution of the Chern-Simons term. In this subsection we explicitly derive the antisymmetric structure functions of the spin-1/2 hadron. We need to evaluate the effective action on-shell, and then use the holographic relation

$$-iS^{(\mathrm{A})}_{\mathrm{eff}} \equiv n_\mu n^*_\nu \,\mathrm{Im}\left[T^{\mu\nu}_{(\mathrm{A})}\right] = \frac{1}{2\pi} n_\mu n^*_\nu W^{\mu\nu}_{(\mathrm{A})}\,. \tag{3.15}$$

Both the heuristic and the string-amplitude approaches give the same effective action. Let us consider equation (3.7). The AdS_5 solutions are given in equation (2.21) for the incoming gauge field A^3_m and in equation (2.23) for the dilatino. Using equations (2.24) and (2.25) to evaluate the imaginary part of the string pre-factor, we obtain

$$n_\mu n^*_\nu \,\mathrm{Im}\left[T^{\mu\nu}_{(\mathrm{A})}\right] = \varepsilon^{\mu\nu\rho\sigma} n_\mu n^*_\nu q_\rho P_\sigma q^{-2} \mathcal{Q}\,\frac{\pi\,|c_i|^2}{12\sqrt{4\pi\lambda}}\left(\frac{\Lambda^2}{q^2}\right)^{\tau-1}\mathcal{I}_\tau\,, \tag{3.16}$$

where the charge is defined as $\mathcal{Q} \equiv d_{33C}\mathcal{Q}^C$. We also define

$$\mathcal{I}_\tau \equiv \int d\omega\,\omega^{2\tau+2} K_0(\omega)K_1(\omega) = \frac{\sqrt{\pi}}{4}\frac{\Gamma^2(\tau+1)\,\Gamma(\tau+2)}{\Gamma\left(\tau+\frac{3}{2}\right)}\,. \tag{3.17}$$

Finally, comparing with equation (1.1) and using the relation (1.3) we obtain the Chern-Simons (CS) term contribution to the antisymmetric structure functions

$$F_3^{\mathrm{CS}}\left(x,q^2\right) = \frac{1}{x}\left(\frac{\Lambda^2}{q^2}\right)^{\tau-1}\mathcal{Q}\frac{\pi^2|c_i|^2}{6\sqrt{4\pi\lambda}}\mathcal{I}_\tau\,, \tag{3.18}$$

and $g_1^{\mathrm{CS}}\left(x,q^2\right) = g_2^{\mathrm{CS}}\left(x,q^2\right) = 0$.

Note that there is no contribution proportional to S_μ due to the R-current conservation. This is similar to the case of the symmetric part described in section 2. However, in the

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antisymmetric part there is an important difference. There are examples of holographic dual models similar to $\mathcal{N}=4$ SYM in the UV but where the R-symmetry is spontaneously broken in the IR.[8] As noted in [4], in those models our computation actually leads to a non-zero result for g_1, more precisely we have

$$g_1^{\rm CS}(x,q^2) = \frac{1}{2}\, F_3^{\rm CS}(x,q^2) \propto \frac{1}{x}\,. \tag{3.19}$$

With respect to $g_1^{\rm CS}$, from now on we assume that we work with a model of this kind. This will be important in order to analyze the phenomenological implications of our results for the antisymmetric contributions in section 5.

Let us briefly comment on the results we have obtained so far. We have obtained new relations of the Callan-Gross type for the antisymmetric structure functions in the range $\lambda^{-1/2} \gg x \gg e^{-\sqrt{\lambda}}$. They can be compared for example with the corresponding ones in the $1 > x \gg \lambda^{-1/2}$ region [6]. We find that the relation $F_3 = 2g_1$ holds for low x. In addition, the structure function g_2 vanishes for $\lambda^{-1/2} \gg x \gg e^{-\sqrt{\lambda}}$, which could seem to be surprising since in the $x \sim 1$ regime g_2 is of the same order as F_3, however, this is in agreement with the discussion presented in reference [4]. We should emphasize that in the string theory regime the structure functions F_3 and g_1 are of the same order as F_2. A similar behavior was found in the scalar case for F_3 [3]. Also, it is important to note that, in contrast to the symmetric case, the antisymmetric structure functions we have obtained are proportional to the dilatino charge $\mathcal{Q}_C$.

Contribution of the Pauli term. Up to this point we have not considered the Pauli interaction arising from the Lagrangian (3.2). The computation is analogous to the one made in the previous section. One arrives to an $F_3^P(x,q^2)$ structure function that behaves in a similar way as the one we found in equation (3.18) with two differences: firstly instead of $d_{33C}\mathcal{Q}^C$ the new structure function is proportional to $d_{33C}\beta^C$, and secondly there is an extra twist-dependent factor $(\tau-1)$. The relation between the Chern-Simons (CS) and the Pauli (P) contributions to the g_1 structure function can be summarized as

$$\frac{g_1^P}{d_{33C}\beta^C} \propto \frac{g_1^{\rm CS}}{d_{33C}\mathcal{Q}^C}(\tau-1)\,, \tag{3.20}$$

and similarly for F_3, which means that the Pauli contribution becomes more important for hadrons with larger twist.

It is interesting to remark that from a holographic dual string theory perspective the mechanism which leads to the antisymmetric structure functions at low x is very different with respect the one in the $1 > x \gg \lambda^{-1/2}$ range studied in [6] and [8]. In the latter regime, F_3, g_1 and g_2 (together with g_3, g_4 and g_5) come from the right handed nature of the (massless) dilatino solution in AdS_5 near the boundary. On the other hand, in the low-x regime, they come from the non-Abelian Chern-Simons and Pauli terms. As it has been shown in [3] this also happens in the dilaton case.

The total contribution to the antisymmetric structure functions is given by the sum to $F_3 = F_3^{\rm CS} + F_3^P$ and $g_1 = g_1^{\rm CS} + g_1^P$. Note that both the Chern-Simons and Pauli

[8] For a specific example see [4] and references therein.

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contributions lead to the same dependence on the Bjorken parameter as well as on the virtual photon momentum for each antisymmetric function. Thus, for each function the only difference is a multiplying constant which can be fixed through an overall fitting. We shall do this in section 5.

4 DIS from spin-1/2 hadrons at exponentially small x: the Regge region

In this section we consider the parametric region where the Bjorken parameter becomes exponentially small. In this regime the so-called ultra-local approximation does not hold. In terms of the four-dimensional center of mass energy and the 't Hooft coupling, this occurs when both s and λ are large but satisfy the relation

$$\frac{\log(s)}{\lambda^{1/2}} = \text{constant}\,. \tag{4.1}$$

We can understand this by looking at the factor $m^{\alpha'\tilde{t}/2}$ in the imaginary part of the string theory scattering amplitude pre-factor in equation (2.24), which within this parametric region cannot be replaced by a constant. The point is that even if the four-dimensional Maldestam variable t vanishes, the ten-dimensional one $\tilde{t}$ could be non-vanishing [5]. Moreover, in terms of our field solutions the radial component[9] corresponds to a Laplacian acting on the mode that propagates in the t-channel.

Let us briefly review the general ideas used in this context. First, it is useful to define a spin-j second order differential operator according to

$$\Delta_j = z^2\partial_z^2 + (2j-3)z\partial_z + j(j-4)\,. \tag{4.2}$$

This operator can only differ from the actual Laplacian only by a (j-dependent) constant, i.e. $\nabla_j^2 = \Delta_j + f(j)$. These constants can be fixed by looking at the supergravity equations of motion. The case $j = 2$ corresponds to the propagation of a graviton in the t-channel and it gives $f(2) = 0$. On the other hand, $f(1) = 3$ corresponds to the propagation of a gauge field. In terms of the scattering amplitude, we see that the introduction of the differential operator effectively breaks the local approximation. It leads to an effect of diffusion in the z-direction, which modifies the amplitude dependence on q^2. As we will see, it also gives an $\mathcal{O}\left(\lambda^{-1/2}\right)$ correction to the exponent of $1/x$ in the structure functions. There are different ways of dealing with this operator, which are described in detail in references [10, 31] for $j \approx 2$, which is associated with a Reggeized graviton exchange. This is the relevant case for the symmetric structure functions. On the other hand, the spin-$j \approx 1$ exchange has been studied in references [3, 4], and it is important for the antisymmetric structure functions. Let us briefly explain how it works. We can assume that the operator Δ_j acts on a generic

[9] Here we neglect the contributions from the angular directions on the S^5. These differential operators are present for $\tilde{s}$ as well, but they can be neglected in comparison to the four-dimensional contribution proportional to s.

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field $\Phi(z)$. Then,

$$
\begin{aligned}
(\alpha' \tilde{s})^{\alpha' \tilde{t}/2} \Phi(z) &= (\alpha' \tilde{s})^{\rho \nabla_j^2/4} \Phi(z) \\
&= \int dz' \, (\alpha' \tilde{s})^{\rho[\Delta_j + f(j)]/4} \delta(z - z') \Phi(z') \\
&= \int \frac{dz'}{z'} \left(\frac{z'}{z}\right)^{j-2} \frac{\left[e^{\zeta\rho}\right]^{\frac{1}{4}(f(j)-4)}}{\sqrt{\pi\zeta\rho}} \, e^{-\frac{1}{\zeta\rho}\log^2(z/z')} \Phi(z') \,,
\end{aligned}
\tag{4.3}
$$

where we have defined

$$
\rho \equiv 2/\sqrt{\lambda} \,, \;\; \zeta \equiv \log \alpha' \tilde{s} = \log\left(\alpha' z z' s\right) \,,
\tag{4.4}
$$

and inserted a Dirac delta function written in the form

$$
z' \, \delta(z - z') = \int \frac{d\nu}{2\pi} \left(\frac{z'}{z}\right)^{j-2+i\nu} = \int \frac{d\nu}{2\pi} e^{(j-2+i\nu)(u-u')} \,,
\tag{4.5}
$$

where $z = e^{-u}$. This is equivalent to considering the identity written in terms of eigenfunctions of the operator Δ_j. However, for non-conformal backgrounds one has to impose suitable boundary conditions. In the hard-wall model where there is a cut-off at $z = z_0$ we can impose Neumann-type conditions. Thus, only a different linear combination of the previous eigenfunctions survives. This is taken into account by the following replacement

$$
e^{i\nu u} \to e^{i\nu u} + \left(\frac{\nu - 2i}{\nu + 2i}\right) e^{-i\nu u} \,.
\tag{4.6}
$$

The only change in the final expression analogous to (4.3) is

$$
e^{-\frac{1}{\zeta\rho}\log^2(z/z')} \to e^{-\frac{1}{\zeta\rho}\log^2(z/z')} + \mathcal{F}(z, z', \zeta) \, e^{-\frac{1}{\zeta\rho}\log^2(zz'/z_0^2)} \,.
\tag{4.7}
$$

The function $-1 < \mathcal{F}(z, z', \zeta) < 1$ is defined as in [9] (equation (5.8) of that reference).

The expressions we have obtained are related to the so-called Pomeron exchange. By inserting the particular case $j \approx 2$ and $j \approx 1$ into the respective on-shell effective actions we obtain the explicit integral form for the leading contributions to the amplitude of the dual FCS process. Schematically, the general amplitude takes the form

$$
\mathcal{A} = 2s \int d^2b \int dz dz' \, P_A(z) P_\psi(z') \, \chi(z, z', s, b)|_{t=0} \,,
\tag{4.8}
$$

in this kinematic regime where $P_A(z)$ contains the information on the gauge field wavefunction and $P_\psi(z)$ contains that of the dilatino,[10] while χ is the corresponding modified propagator. By taking the imaginary part, this propagator corresponds to the conformal and the hard-wall Pomeron kernels. We present the explicit forms of all these quantities for the cases of interest in the following section. Although in the DIS context we have been dealing only with the imaginary part of $\mathcal{A}$, the expression in equation (4.8) is valid for the real part as well. It has also been extended to non-zero t, which corresponds to non-zero

[10] In particular, in the present context it is proportional to the generic $\Psi(z')$ defined above.

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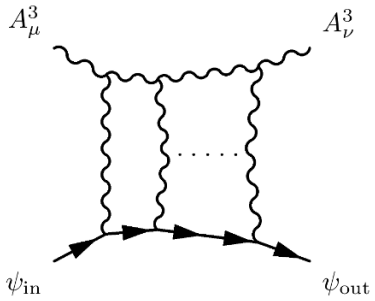

Figure 5. Ladder diagrams re-summed by the eikonal exponentiation, corresponding to a multi-Pomeron exchange.

transverse momentum transfer in the flat directions, and it has also been written in the impact-parameter space. The form of these amplitudes shows a formal similarity to the weak-coupling BFKL results [10, 31].

Under certain conditions, the analysis described in the previous paragraphs has been extended in order to include an infinite number of $1/N^2$ corrections. This has been achieved in the context of the eikonal approximation, which gives the sum of all the t-channel ladder diagrams shown in figure 5. The flat-space eikonal approximation is well-known, and its extension to the curved AdS_5 background for the graviton and a ($j \approx 2$) Pomeron exchanges was investigated in several important papers [9, 32–34]. In the eikonal regime, by including the higher orders in the ladder expansion, it leads to an exponentiation of the amplitude of equation (4.8) in the form

$$\mathcal{A} = 2is \int d^2b \int dz dz' \, P_A(z) \, P_\psi(z') \, \left[1 - \exp\left(i \, \chi(z, z', s, b)|_{t=0}\right)\right], \tag{4.9}$$

where b is the impact parameter. The previous case corresponds to the first non-trivial term in the power series expansion of the exponential. We should stress that in the present work we only consider a single Pomeron exchange. The following contributions are then associated with multi-Pomeron exchanges, each order being suppressed by a $1/N^2$ factor. The implications of these results in the context of the unitarity restrictions and the saturation regime were studied in [35].

The kind of factors in equation (4.3) present in the amplitude are crucial when analyzing both the x and q^2 dependence of the DIS amplitude and the structure functions. The Bjorken parameter is contained in the ζ-variable since for low x one has

$$\zeta = \log\left(\alpha' z z' s\right) \approx \log\left(z z' \frac{\rho}{2} \frac{q^2}{x}\right). \tag{4.10}$$

Thus, we straightforwardly see that the behavior $1/x^2$ and $1/x$ of the different structure functions at small but not exponentially small values of x has to be modified by an extra factor

$$\left[e^{\zeta \rho}\right]^{\frac{1}{4}(f(j)-4)} \sim \left(\frac{1}{x}\right)^{-\frac{1}{2\sqrt{\lambda}}(4-f(j))}. \tag{4.11}$$

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For $f(j) < 4$, this implies that the increase of the amplitudes and the structure functions when $x \to 0$ becomes softened. The actual correction depends on the spin of the propagating field. For the Reggeized graviton, $j \approx 2$ hence $f(j) \approx 0$ (up to $\mathcal{O}(\lambda^{-1/2})$ terms), leading to a correction in the exponent of $-\frac{2}{\sqrt{\lambda}}$. This correction affects all the functions contained in the symmetric part of the hadronic tensor. On the other hand, when the exchanged field is a Reggeized gauge field $j \approx 1$, which implies that $f(j) \approx 3$. Thus, the correction is somewhat less important, namely: $-\frac{1}{2\sqrt{\lambda}}$. Therefore we conclude that, for exponentially small x as $x \to 0$, the antisymmetric structure functions obtained in this work grow faster than F_2 in the same parametric regime.

Having described the general considerations, let us write down the explicit expressions for the structure functions at tree-level and in the hard-wall model. Due to the energy-momentum tensor conservation we have seen that for small values of x the symmetric part of the hadronic tensor is dominated by the universal contributions associated with F_1 and F_2. Inserting the corresponding (imaginary part of) the $j \approx 2$ kernel one obtains the following expression for the F_2 [9]

$$F_2(x,q^2) \sim \int \frac{dz}{z}\frac{dz'}{z'} P_A(z,q^2) P_\psi(z') (zz'q^2) \frac{e^{\zeta(1-\rho)}}{\sqrt{\zeta}} \left(e^{-\frac{\log^2(z/z')}{\rho\zeta}} + \mathcal{F}(z,z',\zeta) e^{-\frac{\log^2\left(zz'/z_0^2\right)}{\rho\zeta}} \right). \tag{4.12}$$

The wave-function dependent terms are defined as

$$P_A(z,q^2) = (qz)^2 \left(K_1^2(qz) + K_0^2(qz)\right), \; P_\psi(z') = z^{-3}|f_+(z')|^2 \sim (z'\Lambda)^{2\tau-2}, \tag{4.13}$$

being $f_+(z')$ given in terms of the initial state $\psi(x,z) = e^{iP\cdot x}\left[f_+(z)P_+ + f_-(z)P_-\right]u(P)$ as equation (2.22). In the last formula we have written its near-boundary expansion. The analogous expression for $2xF_1(x,q^2)$ can be obtained by omitting the K_0^2 term in P_A.

For the case of the antisymmetric contributions one has to use the $j \approx 1$ kernel. In this context it is interesting to consider situations in which the R-symmetry is spontaneously broken in the IR in such a way that it allows for a contribution to the $g_1(x,q^2)$ structure function. In such case, g_1 does not vanish, moreover it is related to F_3 through $2g_1 = F_3$. Assuming that the kernels can be approximately described in the same way, this leads to the holographic dual description of the structure function g_1 at low x:

$$\begin{aligned} g_1(x,q^2) = \frac{\mathcal{Q}\pi^2}{24} \int dy\, dy'\, \mathcal{P}_A(y,q) P_\psi(y') \\ \times \frac{e^{\zeta(1-\rho/4)}}{\sqrt{\pi\zeta\rho}} \left(e^{-\frac{\sqrt{\lambda}}{8\zeta}(y-y')^2} + \mathcal{F}(y,y',\zeta) e^{-\frac{\sqrt{\lambda}}{8\zeta}(y+y')^2} \right), \end{aligned} \tag{4.14}$$

with $y = -2\log(z)$. Note that the P_A factor has to be replaced by

$$\mathcal{P}_A(z,q^2) = (qz)^3 K_1(qz) K_0(qz), \tag{4.15}$$

reflecting the use of the Chern-Simons term. For more details of the computation we refer the interested reader to references [3, 4]. Equation (4.14) is very similar to the one obtained for F_3 in the scalar case in [3].

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Equations (4.12) and (4.14) are difficult to evaluate analytically. In the next section we describe a possible way to extract the relevant information, from which we perform a very interesting phenomenological analysis.

5 Analysis of the results and conclusions

In reference [5] Polchinski and Strassler have shown that in the region $1 > x \gg \lambda^{-1/2}$ the results obtained from the computation of the s-channel amplitude in the supergravity approximation are different from the QCD expectations in the parton model for weak coupling. This is partly because in the planar limit, where the supergravity approximation holds, particle creation in the bulk becomes suppressed. It means that the virtual photon interacts with the entire hadron since the latter does not effectively contain partons in that limit. The structure functions show a behavior of the form $(\Lambda^2/q^2)^{\tau-1}$. This is related to the fact that the hadron wave-function is localized near $z_0 = \Lambda^{-1}$. Thus, in order for inelastic scattering to occur, the string (which holographically represents the hadron) must tunnel to the region near the boundary ($z < q^{-1}$).

Let us very briefly consider the x-dependence of the structure functions in the region $1 > x \gg \lambda^{-1/2}$. They are somehow similar to bell-shaped curves, with maxima around $x_\star \sim 0.6$ which are larger than the experimental observations. In this regime, these results have been extended for charged and neutral polarized spin-1/2 hadrons [6, 8], and also for scalar and polarized vectors mesons for different Dp-brane models, both in the Abelian and non-Abelian cases [24, 25]. However, by considering supergravity one-loop corrections in this x-regime very interesting results have been found [28, 29]. Particularly, the first moments of the structure function F_2 for the pion [24, 25] can be compared with the lattice QCD ones [56], having found discrepancies under 20%, while the supergravity one-loop level calculation improves quite significantly that prediction reaching an accuracy of 1.27 % or better. Similarly, it occurs for the first three moments of F_1 of the rho meson, which gives tree-level results with accuracy of 20% [24, 25], while for the one-loop level calculations the agreement with respect to lattice QCD results [56] reaches an accuracy of 3% [30].[11]

On the other hand, at low x the holographic dual calculation must include the exchange of excited string states in the t-channel. This approach leads to important new insights on Regge physics since the AdS/CFT duality provides a unified description for both the soft Pomeron and the BFKL Pomeron [10]. In QCD when q^2 becomes small, it is not possible to think of the constituents of a hadron as approximately free partons. Confinement as well as saturation effects become important. These phenomena are related to modifications of the Pomeron kernel and the inclusion of multi-Pomeron exchange respectively. In reference [9] it has been done a remarkable comparison with $F_2(x, q^2)$ data for the proton obtained by the HERA Collaboration [11–13].

[11] It is interesting to also compare the level of accuracy for other physical observables calculated in terms of the AdS/CFT duality. For instance in the bottom-up AdS/QCD model observables obtained by the calculation of two-point functions lead to an overall fitting of 5% or better [58, 59]. For observables depending on four-point functions, for instance for the $\Delta I = 1/2$ rule for the kaon decay the level of agreement is about 25% or better [60, 61].

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Also the gauge/string theory duality has been applied to other scattering processes such as deeply virtual Compton scattering (DVCS), double diffractive Higgs production, generalized parton distributions (GPD) and form factors.[12]

5.1 Structure functions results at low x

In the first part of this work we have computed both the symmetric and antisymmetric structure functions of a polarized spin-1/2 hadron in the low x regime. The target is represented by a dilatino KK mode in the dual type IIB superstring theory on $AdS_5 \times S^5$, with an IR deformation. This has been done in two separate but equivalent ways: from superstring theory scattering amplitudes and in a heuristic way developed from the supergravity interactions.

For the symmetric structure functions we have shown that F_1 and F_2 behave as x^{-2} and x^{-1}, respectively, and found a new generalized Callan-Gross relation given by equation (2.29). It is analogous to the one found for the scalar glueball [5] and also in the meson case [26]. Moreover, the $g_{3,4,5}$ vanish in this regime, in contrast to the results obtained in the $1 > x \gg \lambda^{-1/2}$ regime [6]. Although it is not obvious at first sight from the gravity computation, from the CFT point of view we know that the dominant contribution to the JJ OPE is proportional to the energy-momentum tensor. Since the $g_{3,4,5}$-terms in the hadronic tensor are proportional to the spin vector S^μ, they cannot appear at this order.

By considering the t-channel $j \approx 1$ exchange, we have also described the leading contributions to the antisymmetric part of the hadronic tensor. This leads to $g_1 \sim x^{-1}$ (and the same for F_3). Strictly speaking, an argument similar to that of the previous paragraph implies that in the hard-wall model g_1 should vanish. However, there is an important difference: here the relevant term comes from the $JJ \sim J$ term in the OPE. If one considers a QFT where the R-symmetry is spontaneously broken in the IR, it is possible to obtain a non-vanishing $g_1 = \frac{1}{2}F_3$. On the other hand we find that the structure function g_2 vanishes in this regime. Recall that in the parton model g_2 also vanishes, moreover there is no simple interpretation for this function in the parton model [62]. In the following we shall concentrate on the comparison of our results for the phenomenologically relevant structure function g_1 with respect to experimental data.

5.2 New predictions for g_1 and comparison with COMPASS data

In the hard-wall model, symmetric structure functions at low-x and exponentially small-x depend on a finite set of parameters as shown in equation (4.12). Since the z and z' integrals are difficult solved analytically, it has been proposed to approximate the P_i factors by Dirac delta functions supported at appropriate reference scales [9]

$$P_\psi(z') \approx \frac{1}{q'}\delta(z' - 1/q')\,, \quad P_A(z) \approx \frac{1}{q}\delta(z - 1/q)\,, \tag{5.1}$$

where q' is some scale of order of the hadron mass. Using this approximation the integrals in equation (4.12) can be performed, obtaining an expression that has four free parameters,

[12] See for example [36–40] and references therein.

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namely: an overall constant, z_0, ρ and q'. These parameters can be fixed by fitting F_2 to data. In fact, there is considerable amount of experimental results from HERA obtained from DIS off protons at low x which have been used in order to fit the structure function F_2. This has been done by considering the H1-ZEUS data [11–13] for $x < 10^{-2}$. In [9] the authors fitted F_2 for the conformal and the hard-wall models. In the conformal model their fit leads to

$$\rho = 0.7740 \pm 0.0103\,, \;\; q' = 0.5575 \pm 0.0432\,\text{GeV}\,, \tag{5.2}$$

with a reduced chi-square $\chi^2_{\text{d.o.f.}} = 0.75$. On the other hand, their best fit for F_2 using the hard-wall model gives [9]

$$\rho = 0.7792 \pm 0.0034\,, \;\; q' = 0.4333 \pm 0.0243\,\text{GeV}\,, \;\; z_0 = 4.96 \pm 0.14\,\text{GeV}^{-1}\,, \tag{5.3}$$

with $\chi^2_{\text{d.o.f.}} = 1.07$. Interestingly, their values of the parameters are reasonable since ρ lies between the strong/weak coupling transition, q' is of order of the proton mass, while $z_0 \sim \mathcal{O}(\Lambda^{-1}_{\text{QCD}})$ as expected in hard-wall model.

Now, we can obtain new predictions for the g_1 structure function by using the formal expressions derived in our present work. Considering the approximation (5.1) in equation (4.14) we obtain the following expression for the antisymmetric structure function g_1 which holds for both the conformal ($\mathcal{F} = 0$) and the hard-wall kernels

$$\begin{aligned} g_1(x,q^2) &= \mathcal{C}\,\rho^{-1/2} e^{\zeta(1-\rho/4)} \left\{ \frac{\exp\left[-\frac{\log^2(q/q')}{\rho\zeta}\right]}{\sqrt{\zeta}} + \mathcal{F}(q,q',\zeta)\frac{\exp\left[-\frac{\log^2(qq'z_0^2)}{\rho\zeta}\right]}{\sqrt{\zeta}} \right\} \\ &\approx \mathcal{C}\,\frac{\rho^{1/2}}{2x}\frac{q}{q'} e^{\zeta(-\rho/4)} \left\{ \frac{\exp\left[-\frac{\log^2(q/q')}{\rho\zeta}\right]}{\sqrt{\zeta}} + \mathcal{F}(q,q',\zeta)\frac{\exp\left[-\frac{\log^2(qq'z_0^2)}{\rho\zeta}\right]}{\sqrt{\zeta}} \right\}, \end{aligned} \tag{5.4}$$

where we have used that $e^{\zeta} \approx \frac{1}{\sqrt{\lambda}}\frac{1}{x}\frac{q}{q'}$. Next, we carry out the comparison with experimental data. For that, firstly note that three of the four free parameters in equation (5.4) are already determined by the previous fitting of the structure function F_2 done in [9] and shown in equation (5.3) (or (5.2) for the conformal model). Therefore, the only unknown free parameter is the overall constant $\mathcal{C}$.

The g_1 structure function of the proton has been measured by the SMC Collaboration [14], also more recently by the COMPASS Collaboration with the beam energies of 160 GeV and 200 GeV [15, 16]. The corresponding sets of data can be found in the mentioned experimental references. Since the calculations performed in the previous sections are valid for the low-x regime, we consider data within the $x < 0.01$ region. Therefore, we have nineteen experimental values that can be used to fit equation (5.4). Proceeding in this way we obtain the constant value: $\mathcal{C} = 0.0195 \pm 0.0024$ for the conformal model, and $\mathcal{C} = 0.0191 \pm 0.0023$ for the hard-wall model. Both fits give $\chi^2_{\text{d.o.f.}} = 0.27$, which indicates that our model is over-fitting the data set, thus this is not a good fit. In figure 6 the experimental data presented in [16] and our first fit for the conformal model are shown. For completeness we included experimental points up to $x = 0.035$ obtained by the SMC [14], EMC [17, 18], HERMES [19], SLAC E143 [20], E155 [21] and CLAS [22] collaborations, at

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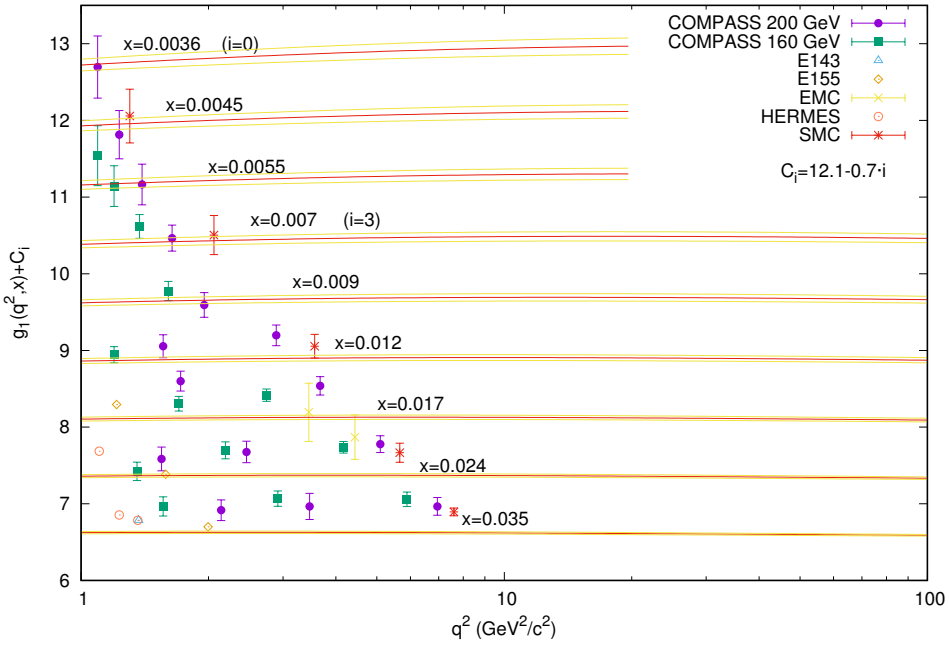

Figure 6. Red curves display our fit for the g_1 structure function as a function of q^2 for different values of the Bjorken variable, compared to the experimental data presented in [16] and references therein, while the yellow lines are the values of g_1 considering the error on $\mathcal{C}$. The best fit corresponds to $x < 0.01$ (larger values of the Bjorken parameter are shown for completeness) obtaining a constant $\mathcal{C} = 0.0195 \pm 0.0024$ with a $\chi^2_{\text{d.o.f.}} = 0.27$. Note that following reference [16] for each value of x we are adding a constant $C_i = 12.1 - 0.7i$ to the g_1 data and the corresponding curve.

$q^2 > 1$ (GeV/c)^2. The fit is better in the region $x < 0.01$ and then it drifts apart from the experimental data as x increases, i.e. where the Pomeron approach does not hold.

It is very interesting to compare with the most recent data from the COMPASS Collaboration. They have published new and more precise data for the g_1 structure function of the proton [23] for photon virtuality $q^2 < 1$ (GeV/c)^2 and for the Bjorken parameter $4 \times 10^{-5} < x < 4 \times 10^{-2}$. This region seems more suitable for our analysis given that the Pomeron formalism describes DIS processes at very low values of x. However, one should keep in mind that we have considered q to be much larger than the Λ scale. Thus, it seems reasonable to consider only the data set where $q^2 > q'^2$, being the latter approximately 0.2-0.3 (GeV/c)^2 according to (5.2) and (5.3). In this way, we consider fifteen points, which represent half of the data presented in [23], and obtain the following results for the fits:

$$\text{conformal model}: \ \mathcal{C} = 0.011 \pm 0.002\,, \ \chi^2_{\text{d.o.f.}} = 1.140 \tag{5.5}$$

$$\text{hard} - \text{wall model}: \ \mathcal{C} = 0.012 \pm 0.002\ , \ \chi^2_{\text{d.o.f.}} = 1.074\,. \tag{5.6}$$

As we can see, for this new data set we obtain a very good fit. The value of the parameter $\mathcal{C}$ does not change significantly, being always $\mathcal{C} \simeq 0.01$. This is a very interesting prediction for the proton structure function g_1, and together with expression (5.4) it represents the main result of this work. As expected, the confining model gives a more accurate description

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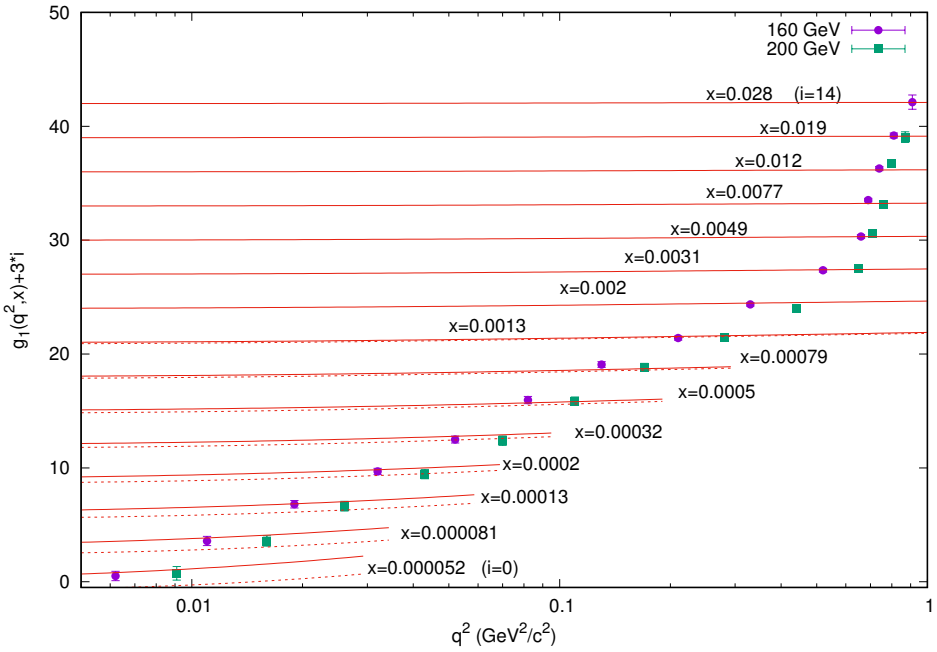

Figure 7. Our best fit of the structure function g_1 carried out with the newest data presented in [23]. Solid curves correspond to the best fit for the conformal model, with $\mathcal{C} = 0.0112 \pm 0.0020$ and a $\chi^2_{\text{d.o.f.}} = 1.140$, while dotted lines correspond to the best fit for the hard-wall model with $\mathcal{C} = 0.0120 \pm 0.0020$ and a $\chi^2_{\text{d.o.f.}} = 1.074$. For values above $x = 0.0013$ we only show the conformal model because at the scale used in the plot there are no visible differences. In this case we do not show the error on $\mathcal{C}$ because it is negligible in the displayed scale. Note that for each row corresponding to different x value, we are adding the constant $3i$.

in the region where q and q' become comparable [9]. As an aside, we should say that, rather surprisingly, including all data points from [23] it still renders an acceptable fit (with $\chi^2_{\text{d.o.f.}} = 0.911$ and $\mathcal{C} = 0.0114 \pm 0.0011$), but only for the conformal model. This is not so in the confining case. Figure 7 displays the experimental data presented in [23] together with our best fits. It can be seen that the hard-wall model gives the best fit for the points with larger values of q^2, while the conformal model gives an acceptable fit for the full data set. Finally, by using our results (5.5) and (5.6) we can predict the behavior of $g_1(x, q^2)$ for different values q^2 in the small-x regime. This is shown in figure 8.

In order to make the comparison between our fitted g_1 function and the experimental data simpler, figures 6 and 7 are displayed in a similar way as the corresponding figures of references [16] and [23], respectively. We briefly comment on the range of validity of the fits we have done. The first set of data described in figure 6 corresponds to larger values of x and a relatively broad range of q^2. On the other hand, the new set of data shown in figure 7 corresponds to much lower values of the Bjorken parameter, although the q^2-range is much smaller (notice the logarithmic scale for q^2 in both figures). The ideal situation where our results should fit better data would be for low x and large photon virtuality. It would be very interesting to have experimental data in that parametric range. We should also empha-

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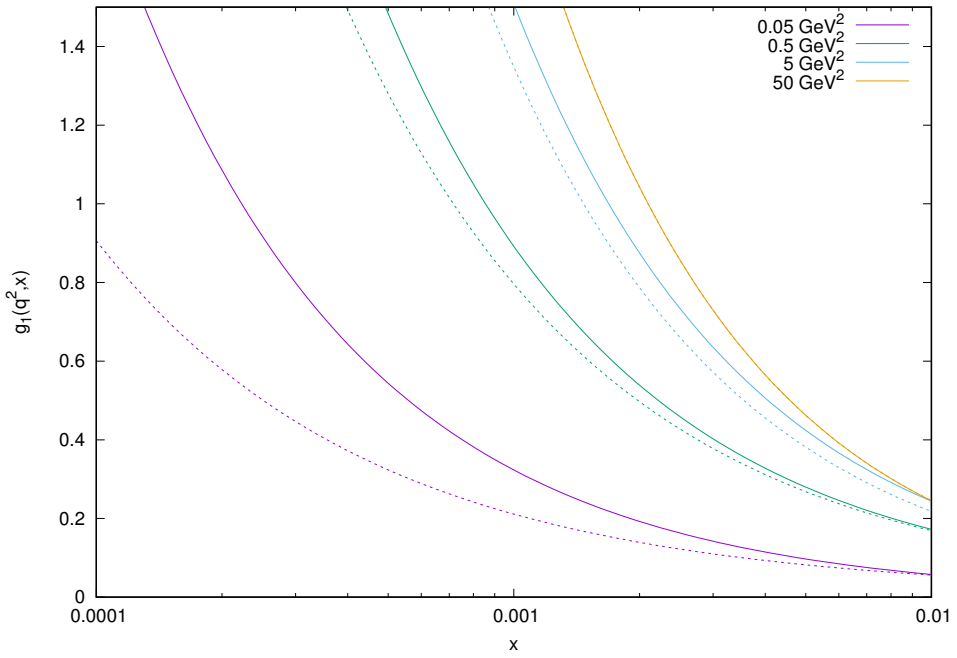

Figure 8. Our results for the structure function g_1. Solid lines correspond to the conformal model. Dotted lines correspond to the hard-wall model. The values of the photon virtuality q^2 are indicated.

size that the amount of experimental data for g_1 at present is much less than the available data for F_2. Thus, our predictions for g_1 in terms of the comparison with experimental data could possibly be improved, depending on the availability of new data in future.

There is an important and very interesting aspect to emphasize. We know that in QCD the electromagnetic DIS leads to a vanishing F_3 structure function. This changes drastically when considering DIS for weak interactions mediated by $W^{\pm}$ or Z^0 gauge bosons. Thus we should stress that although QCD and this IR-deformed $\mathcal{N} = 4$ SYM theory we consider can have a number of analogous properties in the planar limit, as QFTs they are different. One important difference is that $\mathcal{N} = 4$ SYM theory is chiral. The R-symmetry current associated with the global $\mathrm{U}(1)_R \subset \mathrm{SU}(4)_R$ is promoted to become a gauge symmetry in order to describe the electric current. Therefore, our prediction for a non-vanishing F_3 is entirely related to an IR deformation of $\mathcal{N} = 4$ SYM theory.

On the other hand, we conclude that the present results for g_1 fit very well the experimental data as shown in figure 7. Let us emphasize that our knowledge this is the first fully string-theoretical derivation and comparison with experimental data from a calculation of g_1 obtained by using the gauge/string theory duality framework, where non-perturbative physics plays a major role. Also, it is important to remark that the fits for g_1 presented above are totally compatible with those of F_2 obtained by Brower et al. [9]. Therefore, this work also contributes to the understanding of a unified picture of the Pomeron physics by adding relevant results from the holographic dual description of the antisymmetric structure functions.

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