

# Incoherent Pion Production in Antineutrino-Nucleus Scattering within a Fermi Smearing Approach

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In this paper, we compute the charged current (CC) cross section of the background processes  $\bar{\nu}_\mu A \rightarrow \mu^+(A-1)N'\pi$  which are involved in the measurement of the oscillation probability  $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ , for the CP-mirror processes of  $\nu_\mu \rightarrow \nu_e$ . We develop a model that takes into account: binding effects, nucleon smearing, and final state interactions (FSI) between nucleons-pions and the residual nucleus. It was also suitable in describing other channels as  $\nu_\mu A \rightarrow \mu^-(A-1)N'\pi$ , keeping covariance, gauge invariance and partially unitarity.

Our calculations are compared with other dynamical models that have introduced the  $\Delta(1232)$  resonance but inconsistently, and contrasted with experimental data obtained at CERN PS in Gargamell chamber.

**KEYWORDS:** antineutrino,  $\Delta$  resonance, cross section

## 1. Introduction

In previous works were performed computations for processes  $\nu_\mu N \rightarrow \mu^-(\nu)N'\pi$  in the case of free and nucleons, using a dynamic effective Lagrangian model with nucleons ( $N$ ),  $\pi$ ,  $\sigma$ ,  $\rho$ ,  $\omega$  mesons and the isobar  $\Delta(1232)$  resonance. Those computation have been compared with scattering experiments on Hydrogen and Deuterium [1, 2]. Also this reaction was calculated in nuclei [3].

Now we intended to compute, within the same model, the CC cross section of the processes  $\bar{\nu}_\mu A \rightarrow \mu^+(A-1)N'\pi$ , main background of  $\bar{\nu}_\mu A \rightarrow \mu^+(A-1)N'$ , important at the moment of estimating the oscillation probability  $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$  measurement for the CP-mirror process for  $\nu_\mu \rightarrow \nu_e$ . So, having computations of  $\nu$  and  $\bar{\nu}$  CC  $1\pi$  cross sections within the same model, one should expect that could be used in a future determination of the value of CP violation phases.

## 2. A Description of the $\Delta(1232)$ resonance

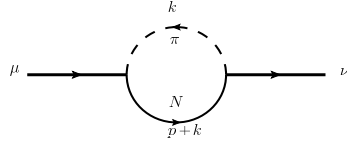
Being the  $\Delta(1232)$  a spin 3/2 resonance, in its propagation between interaction vertices we can not avoid virtual 1/2 states. There is a family of free possible Lagrangians which depends on a parameter ( $A$ ) related with contact transformations that change the 1/2 components, so the unperturbed propagator will be:

$$G_{\alpha\beta}^0(\mathbf{p}) = \frac{\not{p} + m_\Delta^0}{\mathbf{p}^2 - (m_\Delta^0)^2} \left\{ -g_{\alpha\beta} + \frac{1}{3}\gamma_\alpha\gamma_\beta + \frac{2}{3(m_\Delta^0)^2}p_\alpha p_\beta - \frac{1}{3m_\Delta^0}(p_\alpha\gamma_\beta - \gamma_\alpha p_\beta) \right. \\ \left. - \frac{b(\not{p} - m_\Delta^0)}{3(m_\Delta^0)^2} \left[ \gamma_\alpha p_\beta - (b-1)\gamma_\beta p_\alpha + \left( \frac{b}{2}\not{p} + (b-1)m_\Delta^0 \right) \gamma_\alpha\gamma_\beta \right] \right\}, \quad (1)$$

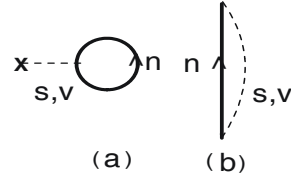
with  $b = \frac{A+1}{2A+1}$ . The dressed one satisfies:

$$(G^{-1})^{\mu\nu}(p) = (G_0^{-1})^{\mu\nu}(p) + i\Sigma^{\mu\nu}(p),$$

where the one-loop self energy is shown in Fig.1 .



**Fig.1**  $\pi N$  loop contribution to the  $\Delta$  resonance Self energy.



**Fig.2** (a)  $\Sigma_{RHA}$ . (a)+(b)  $\Sigma_{HartreeFock}$ .

The  $G_{\alpha\beta}^0(p)$  satisfies the Ward identity but the dressed one does not. To render again gauge invariance we must add vertex corrections representing the coupling of the photon in “all ways” to the self energy contribution. It is possible to simplify this situation around the resonance region to get a gauge invariant amplitude in the presence of finite width effects. That means: for  $W_{\pi N} \equiv \sqrt{(p_N + p_\pi)^2} \approx m_\Delta = 1232 MeV$ , the dressed propagator could be replaced by the bare one, (1) with a complex mass  $m_\Delta + i\Gamma_\Delta$  with a constant  $\Gamma_\Delta$  and an effective  $g \pi N \Delta$  coupling constant [4, 5] this is called the complex mass scheme (CMS ) approximation. As a result, the transition amplitude has a pole located at the complex mass position being no longer singular and  $G_{CMS}^{\mu\nu}$  satisfies Ward identity.

In this paper we will go to higher energies and we make  $G \simeq G(\tilde{m} \rightarrow m_\Delta + i\Gamma_\Delta)$  [6] where  $\tilde{m}$  is the effective mass, which describes data below and above the resonance energy region. Amplitudes can be built with A-independent Feynman rules [4] or with the same value of A everywhere. An usual mistake is to fix  $A = -1$  to obtain the simplest propagator (1) and at the same time fix  $A = -1/3$  for simplest interaction vertexes, which represents an important inconsistency in the model. In our computations we are going to use the  $A = -1/3$  value everywhere. Also we present results for  $A = -1$  for which  $b = 0$  in the propagator (1), but  $A = -1/3$  in the interaction vertexes to show the effect of using an inconsistent model. Those results will be referred as “full” (F) and “trimmed” (T) propagator respectively.

### 3. Nuclear Effects

Now we have  $\pi$  production as a result of the process  $\bar{\nu}A \rightarrow l\pi(A - 1)N$ , being  $A(A - 1)$  the initial (final) nucleus (incoherent scattering). Using the nuclear matter plus impulse (IA) approximations the differential cross section reads:

$$\frac{d\sigma_{\bar{\nu},A}}{d^3k} = 2 \left( 1 - \frac{|\mathbf{k}| \cos \theta_{\bar{\nu},\mathbf{k}}}{E(k_{\bar{\nu}})} \right) n_A(\mathbf{k}) \sum_m d\sigma(\bar{\nu}, N)^{CM},$$

where  $N$  is the initial bounded nucleon and there is a sum over residual nucleon spin, therefore we have a relation between  $\bar{\nu}A$  and  $\bar{\nu}N$  cross sections. We introduce binding effects in nuclei within the mean field theory of QHD I with scalar (s) and vector(v) mesons in the relativistic Hartree approximation (RHA) [7](see Fig.2). The nucleon field then reads

$$\psi(x) = \int d^3p \sum_{m_s m_t} \sqrt{\frac{m_N^*}{(2\pi)^3 E^*(\mathbf{p})}} \left[ u(\mathbf{p} m_s m_t) a_{\mathbf{p} m_s m_t} e^{ip \cdot x} + b_{\mathbf{p} m_s m_t}^\dagger v(\mathbf{p} m_s m_t) e^{-ip \cdot x} \right],$$

with  $p_0 \equiv \Sigma_{RHA}^V + E^*(\mathbf{p})$ ,  $E^*(\mathbf{p}) = \sqrt{\mathbf{p}^2 + m_N^{*2}}$ , and  $m_N^* \equiv m_N + \Sigma_{RHA}^S$  ( $m_N^* < m_N$ ) computed self consistently [7], that is the RHA is obtained adding self energies to all order through the self consistent determination of  $M^*$ . To take into account the effects of the medium in the  $\Delta$ , we assume the same scalar vector and self energies to correct the mass and the particle energy [8,9](universality approach). To compute the momentum distribution in a nuclear matter, we assume  $2p2h + 4p4h$  ground state correlations (GSC) in the fundamental state introduced within a perturbation theory

$$n(\mathbf{k}) = \langle \tilde{0} | a_{\mathbf{k}m}^\dagger a_{\mathbf{k}m} | \tilde{0} \rangle, \quad \int d^3k n_A(\mathbf{k}) = \frac{A}{4},$$

$$|\tilde{0}\rangle = \mathcal{N} \left[ |0\rangle + \frac{1}{(2!)^2} \sum_{p's,h's} c_{p_1 p_2 h_1 h_2} |p_1 p_2 h_1 h_2\rangle + \frac{1}{(4!)^2} \sum_{p's,h's} c_{p_1 p_2 p_3 p_4 h_1 h_2 h_3 h_4} |p_1 p_2 p_3 p_4 h_1 h_2 h_3 h_4\rangle \right],$$

$$c_{p_1 p_2 h_1 h_2} = -\frac{\langle p_1 p_2 h_1 h_2 | \hat{V} | 0 \rangle}{E_{p_1 p_2 h_1 h_2}}, \quad c_{p_1 p_2 p_3 p_4 h_1 h_2 h_3 h_4} = \frac{\langle 0 | \hat{V} | p_1 p_2 h_1 h_2 \rangle \langle p_1 p_2 h_1 h_2 | \hat{V} | p_1 p_2 p_3 p_4 h_1 h_2 h_3 h_4 \rangle}{E_{p_1 p_2 h_1 h_2} E_{p_1 p_2 p_3 p_4 h_1 h_2 h_3 h_4}},$$

which induces corrections to the uncorrelated distribution as

$$n^{m_i}(\mathbf{p}) = \frac{3N^{m_i}}{4\pi p_F^3} \theta(1 - p) + \delta n^{(2)}(\mathbf{p}) + \delta n^{(4C)}(\mathbf{p}).$$

$N^{m_i}$ ,  $m_t = \frac{1}{2}$ ,  $m_n = \frac{-1}{2}$  are the number of protons and neutrons, while  $\delta n^{2,4C}$  the corrections on the impulse distribution due to the presence of  $2p2h$  and  $4p4h$ .

FSI on nucleons (Toy Model!) is introduced by using effective fields also for the final  $N'$  and using the simplest version of the Eikonal approximation for pions. Assuming an average travel distance performed by  $\pi$  in the nuclei, constant nucleonic density and the  $\Delta$ -h model for the pion selfenergy, then for the  $\pi$ -optical potential we have

$$\phi_\pi^*(\mathbf{r}) \sim e^{-i\mathbf{p}_\pi \cdot \mathbf{r}} e^{-i\lambda(s)|\mathbf{p}_\pi| \langle d \rangle}, \quad \lambda(s) = \frac{2}{9} \left( \frac{f_{\pi N \Delta}}{m_\pi} \right)^2 \frac{m_N^2 \rho_0 T}{s(\sqrt{s} - m_\Delta^* + 1/2\Gamma_\Delta^*)}, \quad (2)$$

where  $\langle d \rangle = \sqrt{R^2 - 2/3} \langle r \rangle^2$ ,  $R = r_0 A^{1/3}$ ,  $\langle r \rangle = cA^{1/3}$ , and  $T$  the isospin.

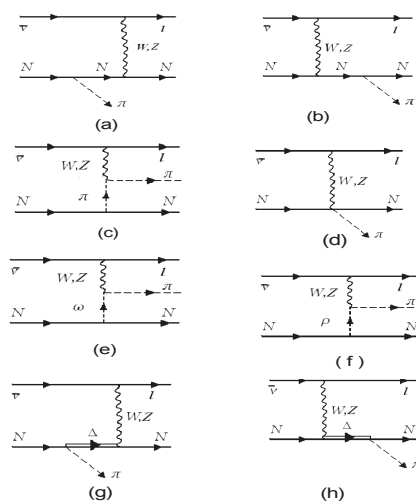


Fig.3 Contribution to de amplitude for  $\bar{\nu}N \rightarrow lN'\pi$

## 4. Results

We compute the CC cross section of the  $\bar{\nu}N \rightarrow \mu^+\pi^-N'$  process and analyse the reactions

$$(1) \quad \bar{\nu}A \rightarrow \mu^+(A-1)n\pi^-,$$

$$(2) \quad \bar{\nu}A \rightarrow \mu^+(A-1)p\pi^-.$$

The different contributions to the amplitude are shown in Fig.3. Where the elementary amplitude is splitted as  $\mathcal{M} = \mathcal{M}_B + \mathcal{M}_R$  ( $B \equiv a,b,c,d,e,f,g$  ;  $R \equiv h$ ), each with V-A contributions. Via CVC we set the vector constants [10], while axial are built from chiral Lagrangians based on spin-parity arguments [11, 12].  $\sigma_{\bar{\nu}A}/A$  results for (1) and (1)+(2) are shown in Fig.4 and 5 respectively.

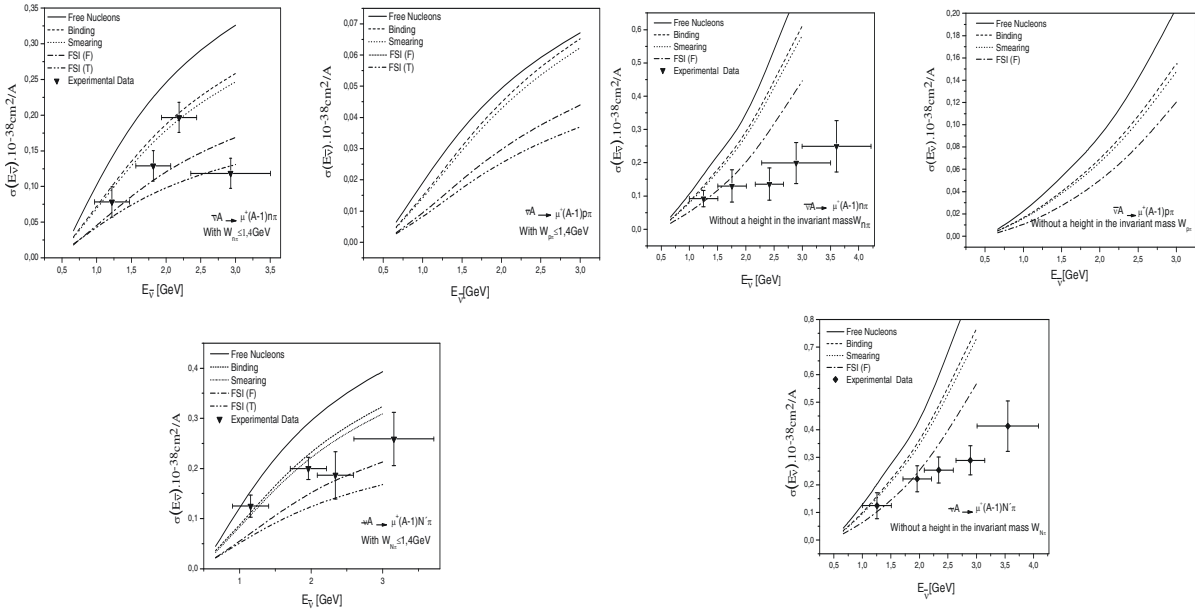


Fig.4 Data on propane gas ( $\langle E_{\bar{\nu}} \rangle \gg 1.5 GeV$ ) [13]

Fig.5 Idem Fig.4 without cuts.

## 5. Conclusion

In this paper we have evaluated the  $\bar{\nu}A \rightarrow \mu^+(A-1)\pi^-N'$  cross section, main background of the  $\bar{\nu}A \rightarrow \mu^+(A-1)N'$  reaction used to detect the arrived  $\bar{\nu}$  in neutrino oscillation experiments. The developed model is based on chiral hadronic effective Lagrangians which includes  $N$  nucleons,  $\pi$ ,  $\rho$ ,  $\omega$  mesons and the  $\Delta(1232)$  resonance. Obtained results (shown in Figs. 4 and 5) were compared with available experimental data. Those comparisons show that it is not possible to describe the data without introducing nuclear effects, although reported experimental papers mention that have been simulated. Also, the effect of using different values of  $A$  to obtain the simplest propagator and interaction vertexes at the same time, it is shown. Third graph in Figs.4 and 5 below 2 GeV, show that proton channel (2) is also acceptably reproduced. We have also seen that our description is satisfactory for final invariant  $\pi N$  mass near  $\Delta(1232)$  resonance energy region, but above, when we remove the restriction  $W_{\pi N} \leq 1.4 GeV$ , it is expected that we should include in the model more energetic resonances at amplitude level which could interfere with the  $\Delta$  to give the right tendency of the data cross section. Also, although in the case of the  $\Delta$  resonant contribution (R) rescattering effects are included by the one loop self-energy correction, we should also include the rescattering for the non-resonant contributions (B) to get full unitarity.

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