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Inconsistency of the 'spin-3/2 gauge invariant' interaction of Rarita–Schwinger fields

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Abstract

We perform the Dirac quantization of Rarita–Schwinger fields interacting with a spinor and the first derivative of a pseudoscalar field. We achieve the calculations for two forms of this interaction: first we review the conventional coupling of lowest derivative order, reproducing the well known inconsistencies in its anticommutator algebra. Then, we perform the analysis on the next order term popularly known as 'spin-$3/2$ gauge invariant interaction', which is claimed to be free of these inconsistencies. Nevertheless we find that the direct application of the Dirac formalism leads to inconsistencies in complete analogy to the previous case. This is of high relevance in the particle phenomenology field, where these interactions are used to interpret experimental data involving $\Delta(1232)$ resonances.

Keywords: constraints, Dirac brackets, positive nondefinite metric

1. Introduction

The problem of setting consistent interactions for higher spin fields has been a much debated subject for several decades, both in the quantum field theory and particle phenomenology communities. In phenomenology, interactions of the Rarita–Schwinger (RS) fields are crucial for interpreting experimental data involving spin $3/2$ resonances, most notably the $\Delta(1232)$ (see [1] for a review of most phenomenological difficulties). In particular, for the interaction between $\Delta(1232)$, nucleons and pions, the interactions mostly used are the 'conventional' interaction introduced in [2] (equation (1) below) and the so called 'spin-$3/2$ gauge invariant' interaction proposed in [3] (equation (2) below). The latter suffers for a number of shortcomings as we expose below, but nevertheless it became quite popular due to the belief (to be analyzed here) that it avoids the inconsistencies that plagued interactions of RS fields since its inception. These inconsistencies occur when there are background fields and lead to indefinite metrics in the Fock space [4, 5]. Surprisingly, severe difficulties arise even at the classical level. For instance, when critically large magnetic fields are present the RS field coupled minimally to the electromagnetic (EM) field propagates acausally [6]. Something analogous happen when the RS field is coupled to Dirac and pseudoscalar fields via the 'conventional' coupling in the presence of a critical gradient of the pseudoscalar [7]. At the classical level, in the case of EM coupling, the inconsistency can be avoided if certain non-minimal couplings are used instead [8].

These difficulties are tightly related to the occurrence of constraints. Since vector-spinor fields contains both a spin 3/2 sector and two spin 1/2 ones, the correct description of spin 3/2 degrees of freedom requires projection onto the first sector. Nevertheless, the complete space is needed to invert the propagator, so virtual spin 1/2 states do also propagate. When quantizing this theory second class constraints arise, which amount to projecting out the Hilbert space sectors corresponding to the lower spin. But interactions, in general, change the constraints quite drastically making field anticommutators dependent on the dynamics [4, 5]. That is why one talks of 'quantizing' the interaction.

The problem was first described in [4], for the RS field minimally coupled to the EM field. Then, in [2] it was claimed that a linear coupling to a spinor and the derivative of a scalar ($g_{00} = 1,\ g_{ij} = -\delta_{ij}$ and isospin omitted)

$$\mathcal{L}_{\rm NEK} = g\bar{\Psi}^{\mu}\left(g_{\mu\nu} + \left[\frac{1}{2}(1 + 4z)a + z\right]\gamma_{\mu}\gamma_{\nu}\right)\psi\partial_{\nu}\phi + \text{h.c.}, \tag{1}$$

(where the value $z = \frac{1}{2}$ was chosen from field consistency theoretical arguments) would be free of such problems, but it was shown by Hagen [5] that this is not the case. Later, it was shown [7] that in the presence of scalar gradients non-causal propagation arises. The source of the problem was made clear in [9]: they have shown that any coupling leading to linear constraints on the fermion degrees of freedom leads to indefinite anticommutators due to the existence of negative parameters (which they called 'negative masses') in the kinetic terms of the Lagrangian, which are always present for the RS fields. This may be construed as a consequence of the reintroduction of the spin 1/2 sector, to which correspond such parameters, which are projected out in the free theory.

More recently, [3] proposed a new interaction which is derivative in the RS field. The most general such term preserving chiral symmetry can be written as ($\epsilon^{0123} = 1,\ \gamma_5 = \mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3$)

$$\mathcal{L}_P = g\bar{\Psi}^{\mu}\left(g_{\mu\sigma} + \left[\frac{1}{2}(1 + 4z)a + z\right]\gamma_{\mu}\gamma_{\sigma}\right)\epsilon^{\sigma\nu\lambda\rho}\gamma_5\gamma_{\lambda}(\partial_{\rho}\psi)(\partial_{\nu}\phi) + \text{h.c.} \tag{2}$$

If the off-shell parameter z is set to $z = -\frac{1}{2}$ this is the interaction proposed in [3]. We show in appendix A that this value is indeed needed for consistency, so we will not consider other values. Recall that in the RS formalism the parameter a is unobservable and thus arbitrary. The interaction (2) for $z = -1/2$ is just the one proposed in [3] as rewritten in [10] more generally to restore its A dependence. This interaction with z chosen as above has the property of projecting out spin 1/2 virtual state of the propagator in elastic amplitudes at tree level, but the off-shell sector of spin 1/2 is potentially present, and manifests itself in radiative amplitudes, as stated below. It has been argued in [3] that such interactions are free of the above mentioned problems and so it became quite popular for the description of Δ resonances. Nonetheless, we showed recently that in the presence of the EM coupling (unavoidable in this context, since the Δ is charged) the consistency problem remains even for this new proposed interaction: when the EM interaction is introduced, renormalization considerations force to reintroduce an interaction of the form $\mathcal{L}_{\rm NEK}$ [11]. The interaction obtained from $\mathcal{L}_P$ does not eliminate spin $1/2$ virtual states in all circumstances, radiative processes for instance exhibit a spin $1/2$ 'background'. Also, the new interaction is not superior even phenomenologically, since a background compatible with exchange of virtual spin $1/2$ is indeed observed, and the use of $\mathcal{L}_{\rm NEK}$ vertexes is found to fit better the data than the $\mathcal{L}_P$ ones [12]. Finally, this new interaction presents also problems with the coexistence with the EM gauge invariance [11].

Though it remains the interesting theoretical possibility that in absence of EM interactions $\mathcal{L}_P$ be indeed consistent, there are reasons to strongly suspect that it is not the case. Indeed, in spite of being inspired in a 'gauge invariance' of the kinetic Δ term, $\mathcal{L}_P$ can be obtained by simply invoking the next order interaction (in derivatives) to $\mathcal{L}_{\rm NEK}$, which was not considered in [2]. We would thus expect a somewhat more involved but otherwise analogous constraint structure. In fact, the theory exhibit the same linear constraints in fermionic degrees of freedom described in [9], so the same positivity issue should have to arise. In [3] there is some argumentation in favor of the consistency of $\mathcal{L}_P$, but while the constraint arguments showing $\mathcal{L}_{\rm NEK}$ inconsistency are developed with certain detail, the same analysis was not performed for $\mathcal{L}_P$. Instead, a 'Stückelberg parameter' is introduced in order to render the massive theory 'spin-$3/2$ gauge invariant'. Nevertheless, that's not the right procedure since a Stückelberg variable is not a parameter but a dynamical field [13] (see appendix C). So, a complete constraints analysis treating both $\mathcal{L}_{\rm NEK}$ and $\mathcal{L}_P$ on the same footing is desirable. This is done in this paper.

The paper is organized as follows: first we will review the Dirac quantization for the RS field. Then, we will apply it to $\mathcal{L}_{\rm NEK}$, reproducing the classical result by Hagen [5], which has been obtained with the action principle. Finally, we will apply the same scheme to $\mathcal{L}_P$ and will show that the same positivity issues arise. We then briefly draw our conclusions.

2. Dirac quantization for the RS field

We will perform quantization via the Dirac bracket formalism generalized to include fermions (see appendix B). To do so, we will introduce intermediate level brackets for the trivial constraints following [15] (which used them for the free theory) and [16] (which used them for the RS coupled to EM fields). This is algebraically much easier than using the Dirac formalism for the whole set of constraints and eases comparison with [3], where the same procedure is followed. We will call 'first level brackets' to the brackets after the imposition of the constraints on the spatial components of the RS field, and 'second level brackets' to those obtained after the elimination of the constraint on the Dirac field. First, we will reproduce the quantization of $\mathcal{L}_{\rm NEK}$ performed in [5] with the action principle, and then we will quantize $\mathcal{L}_P$.

The general Lagrangian for the interacting RS, scalar and spinor fields (the interaction between the scalar and spinor fields are not of interest in this context) reads:

$$\mathcal{L} = \mathcal{L}_{\rm RS} + \mathcal{L}_\psi + \mathcal{L}_\phi + \mathcal{L}_P \quad \text{or} \quad \mathcal{L}_{\rm NEK}, \tag{3}$$

where

$$\mathcal{L}_{\rm RS} = \bar{\Psi}^{\mu}\Lambda_{\mu\nu}\Psi^{\nu}, \qquad \mathcal{L}_{\psi} = \bar{\psi}({\rm i}\partial\!\!\!/ - m_{\psi})\psi, \qquad \mathcal{L}_{\phi} = 1/2(\partial_{\mu}\phi\partial^{\mu}\phi - m_{\phi}^{2}\phi), \tag{4}$$

with

$$\Lambda_{\mu\nu} = -({\rm i}\partial\!\!\!/ - m)g_{\mu\nu} - {\rm i}a(\partial_{\mu}\gamma_{\nu} + \partial_{\nu}\gamma_{\mu}) - {\rm i}B(a)\gamma_{\mu}\partial\!\!\!/\gamma_{\nu} - mC(a)\gamma_{\mu}\gamma_{\nu}, \tag{5}$$

where $a \neq -\frac{1}{2}$, $B(a) = \frac{3}{2}a^{2} + a + \frac{1}{2}$, $C(a) = 3a^{2} + 3a + 1$. The structure of constraints is greatly simplified when $A = -1$: in that case $\Lambda_{00} = 0$, so $\mathcal{L}_{\rm RS}$ becomes independent of $\dot{\Psi}^{0}$. The condition that interactions do not reintroduce a dynamics for Ψ^{0} constraints the possible values for z: $\frac{1}{2}$ in $\mathcal{L}_{\rm NEK}$ and $-\frac{1}{2}$ for $\mathcal{L}_{P}$ (see appendix A). Then we have ($\epsilon^{\mu\nu\lambda\rho}\gamma_{5}\gamma_{\lambda} = {\rm i}/2\{-{\rm i}\sigma^{\mu\rho}, \gamma^{\nu}\}$)

$$\begin{aligned} \Lambda^{\mu\nu} &= -\epsilon^{\mu\nu\lambda\rho}\gamma_{5}\gamma_{\lambda}\partial_{\rho} + {\rm i}m\sigma^{\mu\nu}, \\ \mathcal{L}_{P} &= -g\bar{\Psi}_{\mu}[\Lambda^{\mu\nu}(m=0)\psi]\partial_{\nu}\phi + \text{h.c.}, \\ \mathcal{L}_{\rm NEK} &= g\bar{\Psi}_{\mu}{\rm i}\sigma^{\mu\nu}\psi\partial_{\nu}\phi + \text{h.c.} \end{aligned} \tag{6}$$

Next we define the momenta $\Pi_{f,f^{\dagger}} = \frac{\partial\mathcal{L}}{\partial\dot{f},\dot{f}^{\dagger}}$ (see appendix B) and using that $\epsilon^{0ijk}\gamma_{5}\gamma_{k} = \epsilon_{ijk}\gamma_{5}\gamma_{k} = \sigma_{ij}\gamma_{0}$ we get

$$\Pi_{\Psi_{0}} = 0, \qquad \Pi_{\Psi_{0}^{\dagger}} = 0, \tag{7}$$

$$\Pi_{\Psi_{i}} = -\Psi_{k}^{\dagger}\sigma_{ki}, \qquad \Pi_{\Psi_{i}^{\dagger}} = 0, \tag{8}$$

$$\Pi_{\psi} = {\rm i}\psi^{\dagger} + \begin{pmatrix} 0 \\ g\Pi_{\Psi_{j}}(\partial_{j}\phi) \end{pmatrix}, \qquad \Pi_{\psi^{\dagger}} = \begin{pmatrix} 0 \\ g\Pi_{\Psi_{j}^{\dagger}}(\partial_{j}\phi) \end{pmatrix}, \tag{9}$$

$$\Pi_{\phi} = -g\Psi_{i}^{\dagger}\begin{pmatrix} \gamma_{i}\psi \\ -\sigma_{ij}(\partial_{j}\psi) \end{pmatrix} + \dot{\phi}, \tag{10}$$

where the upper value in brackets corresponds to the $\mathcal{L}_{\rm NEK}$ interaction while the lower one to the $\mathcal{L}_{P}$ case.

Whenever a degree of freedom f is such that $\dot{f}$ cannot be solved in terms of f and Π_{f}, constraints arise. So

$$\chi_{0}(x) \equiv \Pi_{\Psi_{0}}(x) = 0, \qquad \chi_{0^{\dagger}}(x) \equiv \Pi_{\Psi_{0}^{\dagger}}(x) = 0, \tag{11}$$

$$\chi_{i}(x) \equiv \Pi_{\Psi_{i}} + \Psi_{k}^{\dagger}\sigma_{ki} = 0, \qquad \chi_{i^{\dagger}}(x) \equiv \Pi_{\Psi_{i}^{\dagger}} = 0, \tag{12}$$

$$\chi_{\psi} \equiv \Pi_{\psi} - {\rm i}\psi^{\dagger} - \begin{pmatrix} 0 \\ g\Pi_{\Psi_{j}}(\partial_{j}\phi) \end{pmatrix} = 0, \tag{13}$$

$$\chi_{\psi^{\dagger}} \equiv \Pi_{\psi^{\dagger}} - \begin{pmatrix} 0 \\ g\Pi_{\Psi_{j}^{\dagger}}(\partial_{j}\phi) \end{pmatrix} = 0 \tag{14}$$

are primary constraints. For the RS field Ψ in (12) as usually done with the Dirac field, one can eliminate $\Psi^{\dagger}$ in terms of Π directly and using the identity $\sigma_{ij}\left(\frac{i}{2}\gamma_{k}\gamma_{j}\right) = \delta_{ik}$ we get

$$\Psi_i^\dagger = -\frac{\mathrm{i}}{2}\Pi_{\Psi_k}\gamma_i\gamma_k. \tag{15}$$

Then by using the fundamental Poisson brackets given in appendix B we get the nonzero first level brackets

$$\begin{aligned}\{\Psi_i(x),\, \Psi_j^\dagger(y)\}_I &= \frac{\mathrm{i}}{2}\gamma_j\gamma_i\delta^{(3)}(x-y),\\ \{\Psi_0(x),\, \Pi_{\Psi_0}(y)\}_I &= \delta^{(3)}(x-y),\\ \{\phi(x),\, \Pi_\phi(y)\}_I &= \delta^{(3)}(x-y),\end{aligned} \tag{16}$$

where Ψ_0, Π_{Ψ_0} are not affected by the eliminated constraint and for the pseudoscalar field it coincides with the fundamental one since we have not a constraint. Note that this algebra could at first be achieved via the Dirac procedure (see appendix B), as done in [3, 16]. As can be seen form equation (13) for the $\mathcal{L}_{\mathrm{NEK}}$ interaction, we could get $\psi = -\mathrm{i}\Pi_\psi$ in analogy with (15) and using the fundamental brackets we get

$$\{\psi(x),\, \psi^\dagger(y)\}_I = -\mathrm{i}\delta^{(3)}(x-y), \tag{17}$$

but this procedure is no more valid for $\mathcal{L}_P$, since it connects ψ, Π_ψ,and $\Pi(\text{or } \Psi^\dagger)$. Let us then introduce the second level brackets eliminating the conjugate momenta of the spinor field by using the Dirac formalism. The obtained results will be also valid for $\mathcal{L}_{\mathrm{NEK}}$ making $g = 0$ since with this value we get the right constraints for this case in equations (13) and (14). In order to find the second level brackets we need

$$\{\chi_\psi(x),\, \chi_\psi^\dagger(y)\}_I = -\mathrm{i}\delta^{(3)}(x-y), \qquad \{\chi_\psi(x),\, \chi_\psi^\dagger(y)\}_I^{-1} = \mathrm{i}\delta^{(3)}(x-y) \tag{18}$$

and the Dirac brackets for the scalar, spinor and RS fields look like

$$\begin{aligned}\{\Psi_i(x),\, \Psi_j^\dagger(y)\}_{\mathrm{II}} &= \left(\frac{\mathrm{i}}{2}\gamma_j\gamma_i - \mathrm{i}g^2(\partial_i\phi(x))(\partial_j\phi(y)\right)\delta^{(3)}(x-y),\\ \{\Psi_i(x),\, \psi^\dagger(y)\}_{\mathrm{II}} &= -\mathrm{i}g(\partial_i\phi(y))\delta^{(3)}(x-y),\\ \{\psi(x),\, \Psi_i^\dagger(y)\}_{\mathrm{II}} &= \mathrm{i}g(\partial_i\phi(x))\delta^{(3)}(x-y),\\ \{\psi(x),\, \psi^\dagger(y)\}_{\mathrm{II}} &= -\mathrm{i}\delta^{(3)}(x-y),\\ \{\Psi_0(x),\, \Pi_{\Psi_0}(y)\}_{\mathrm{II}} &= \delta^{(3)}(x-y),\\ \{\phi(x),\, \Pi_\phi(y)\}_{\mathrm{II}} &= \delta^{(3)}(x-y),\end{aligned} \tag{19}$$

for the $\mathcal{L}_P$ interaction, while setting $g = 0$ we get those for the $\mathcal{L}_{\mathrm{NEK}}$ case. Now the other primary constraints χ_0, $\chi_{0^\dagger}$, cannot be eliminated as above. Then, we impose the condition to be preserved in time, that is $\theta_0(x) \equiv \{\Pi_0(x), H\}_{\mathrm{II}} = 0$ and get new nontrivial secondary constraints. The Hamiltonian density reads

$$\begin{aligned}\mathcal{H}(x) &= \Pi_\mu(x)\dot{\Psi}^\mu(x) + \Pi_\phi(x)\dot{\phi}(x) + \Pi_\psi(x)\dot{\psi}(x) + \text{h.c.} - \mathcal{L}(x)\\ &= -\Psi_i^\dagger\epsilon_{ijk}\gamma_5\partial_k\Psi_j + \Psi_i^\dagger\begin{pmatrix}\mathrm{i}g\sigma_{ij}\gamma_0\psi\partial_j\phi\\ -g\epsilon_{ijk}\gamma_5(\partial_j\psi)(\partial_k\phi)\end{pmatrix} + \text{h.c.} - \mathrm{i}m\bar{\Psi}_i\sigma_{ij}\Psi_j\\ &\quad + \mathrm{i}\bar{\psi}\boldsymbol{\nabla}\cdot\boldsymbol{\gamma}\psi + \bar{\psi}m_\psi + 1/2\boldsymbol{\nabla}\phi\cdot\boldsymbol{\nabla}\phi + 1/2m_\phi^2\phi^2\\ &\quad + \Psi_0^\dagger\left[(\sigma_{ji}\partial_j - m\gamma_i)\Psi_i + \begin{pmatrix}-g\gamma_i\psi\partial_i\phi\\ -g\sigma_{ji}(\partial_j\psi)(\partial_i\phi)\end{pmatrix}\right] + \text{h.c.}\end{aligned} \tag{20}$$

and thus we get

$$\theta_0 = \{\Pi_{\Psi_0}, H\}_{\text{II}} = (\partial_j \Psi_i^\dagger \sigma_{ji} + m\Psi_i^\dagger \gamma_i) + \begin{pmatrix} g\psi^\dagger \gamma_i \partial_i \phi \\ -g(\partial_j \psi^\dagger)(\partial_i \phi)\sigma_{ji} \end{pmatrix},$$
$$\theta_{0^\dagger} = \{\Pi_{\Psi_0^\dagger}, H\}_{\text{II}} = (\sigma_{ji}\partial_j - m\gamma_i)\Psi_i + \begin{pmatrix} -g\gamma_i \psi \partial_i \phi \\ -g\sigma_{ji}(\partial_j \psi)(\partial_i \phi) \end{pmatrix}. \tag{21}$$

Observe that θ_0 and $\theta_{0^\dagger}$ are the coefficients in (20) of Ψ_0 and $\Psi_{0^\dagger}$ respectively, so the latter are indeed Lagrange multipliers. Observe that $\{\Pi_0, \theta_0\}_{\text{II}} = \{\Pi_{0^\dagger}, \theta_0\}_{\text{II}} = \{\Pi_0, \theta_{0^\dagger}\}_{\text{II}} = \{\Pi_{0^\dagger}, \theta_{0^\dagger}\}_{\text{II}} = 0$ but for $m \neq 0$ (as will be shown) $\{\theta_0, \theta_{0^\dagger}\}_{\text{II}} \neq 0$. By imposing $\{\theta_0, H\}_{\text{II}} = 0$ and $\{\theta_{0^\dagger}, H\}_{\text{II}} = 0$ in order to preserve θ_0, $\theta_{0^\dagger}$ in time, we get tertiary constraints proportional to Ψ_0 and $\Psi_0^\dagger$ giving nonzero brackets with $\Pi_0 = 0$ and $\Pi_0^\dagger = 0$. Nevertheless the only effect of those tertiary constraints is to determine Ψ_0 and $\Psi_0^\dagger$, which are no dynamical as was seen above, so the relevant Dirac algebra reduces to θ_0 and $\theta_{0^\dagger}$. Consequently, the only nontrivial bracket to consider in the Dirac procedure is $\{\theta(x)_0, \theta_{0^\dagger}(y)\}_{\text{II}}$, which can be obtained from equation (21) and is different for $\mathcal{L}_{\text{NEK}}$ and $\mathcal{L}_P$.

2.1. Conventional coupling

For the $\mathcal{L}_{\text{NEK}}$ conventional coupling the second level brackets are obtained from (19) setting $g = 0$ and read

$$\{\Psi_i(x), \Psi_j^\dagger(y)\}_{\text{II}} = \frac{\mathrm{i}}{2}\gamma_j\gamma_i\delta^{(3)}(x-y),$$
$$\{\Psi_0(x), \Pi_{\Psi_0}(y)\}_{\text{II}} = \delta^{(3)}(x-y),$$
$$\{\psi(x), \psi^\dagger(y)\}_{\text{II}} = -\mathrm{i}\delta^{(3)}(x-y),$$
$$\{\phi(x), \Pi_\phi(y)\}_{\text{II}} = \delta^{(3)}(x-y), \tag{22}$$

from which we can get using (21)

$$\{\theta_0(x), \theta_{0^\dagger}(y)\}_{\text{II}} = \frac{3\mathrm{i}}{2}\delta^3(x-y)\left(m^2 - \frac{2g^2}{3}(\nabla\phi)^2\right). \tag{23}$$

Observe that the quantity between parenthesis in the rhs, which is reported in [3, 5], can become zero at the classical level for certain values of the gradient. In equation (30) of [3] it appears in the path integral where the theory is supposed to be quantized. Here only it is stated that a kind of noncovariance in the measure (where the expressions have several misprints, see appendix C) usually happens and may be canceled, and raises the question (without answering it) if such cancellation verifies or not in this case. The only condition asked to quantize the theory in [3] is that $R(x) \equiv 3/2\mathrm{i}\left(m^2 - \frac{2g^2}{3}g^2(\nabla\phi(x))^2\right) \neq 0$, to avoid a violation of DOF counting. But the problem is more serious: as shown in [4, 5], the actual problem is that R flips sign, making the Hilbert space non positive-definite. So, the theory was not actually quantized in [3], the signature problem means that it is not possible [4, 5]. To show the existence of this signature problem we will construct the simplest nontrivial anticommutator, the one between of spinor fields, and evaluate it between states on a classical scalar background. The Dirac brackets between ψ fields that take into account the secondary constraints are, by using equations (21) and (22),

$$\{\psi(x),\,\psi^{\dagger}(y)\}_{\rm D} = \{\psi(x),\,\psi^{\dagger}\}_{\rm II} - \int {\rm d}^3z\ {\rm d}^3z'\{\psi(x),\,\theta(z)\}_{\rm II}(\{\theta_0(z),\,\theta_{0^\dagger}(z')\}_{\rm II})^{-1}\{\theta^{\dagger}(z'),\,\psi^{\dagger}(y)\}_{\rm II} \tag{24}$$

$$= -{\rm i}\frac{\delta^3(x-y)}{1-\frac{2g^2}{3m^2}(\nabla\phi)^2}, \tag{25}$$

while the corresponding quantum anticommutator will be

$$[\psi(x),\,\psi^{\dagger}(y)]_+ = \hbar\frac{\delta^3(x-y)}{1-\frac{2g^2}{3m^2}(\nabla\phi)^2}. \tag{26}$$

We will see that the study of the sign definiteness of the space of states, required for consistency, reduces to the possibility of $R(x)$ vanish when evaluated for quantum states.

Now, let $|f\rangle$ be a coherent state for the ϕ field, such that $(\nabla\phi(x))|f\rangle = (\nabla f(x))|f\rangle$, being $f(x)$ a c-number function. The calculation of norms of one-particle spinor states on such background will lead to consider the quantity

$$\langle f|[\psi(x),\,\psi^{\dagger}(y)]_+|f\rangle = \hbar\frac{\delta^3(x-y)}{1-\frac{2g^2}{3m^2}(\nabla f(x))^2}\langle f||f\rangle \tag{27}$$

which is not positive definite. Observe that if the gradient is nonzero there is always a reference frame where the norm will flip sign, this conclusion also was arrived at in [5] from a different procedure. This is the real concern at the quantum level, beyond any consideration of loss of degrees of freedom of the classical theory in a zero measure region of the configuration space.

2.2. Spin-3/2 gauge invariant coupling

We will show, with an argument absolutely parallel to the followed in the previous subsection (which leads to the same results of [5]) that the theory is not quantizable for the $\mathcal{L}_P$ interaction, so it is pointless to develop Feynman rules as in [3]. Of course, in absence of backgrounds the quantization is the same as the corresponding to the free RS theory, and perturbative expansions of both $\mathcal{L}_{\rm NEK}$ and $\mathcal{L}_P$ are unproblematic.

Using the second level brackets (19) in (21) we get

$$\{\theta_0(x),\,\theta_{0^\dagger}(y)\}_{\rm II} = \frac{3{\rm i}m^2}{2}\left(1-\frac{2g^2}{3}(\boldsymbol{\nabla}\phi)^2\right)\delta^3(x-y) \tag{28}$$

which, very similarly to the previous case, can vanish for certain values of the gradient at the classical level if $m \neq 0$. It is interesting to analyze the zero mass limit since it corresponds to a gauge invariant theory under the transformation $\Psi_\mu \to \Psi_\mu + \partial_\mu\epsilon$, where ϵ is an arbitrary spinor for $\mathcal{L}_{\rm RS}$. Naively, one should expect (28) to vanish as a consequence of this gauge invariance thus rendering θ_0 and $\theta_{0^\dagger}$ first class constraints, in spite there is no general proof that every gauge invariance leads to first class constraints [17]. As a matter of fact, there are some counterexamples when the gauge symmetry acts trivially [18] and in this case the gauge symmetry acts trivially on the scalar and spinor fields. To check it rigorously, observe that (see (6) and $S^0 \equiv g\Lambda^{0\nu}(m=0)\psi\partial_\nu\phi$)

$$Q = \int d^3x S^0(x) = -\int d^3x g\gamma_0\sigma_{ji}(\partial_j\psi)(\partial_i\phi)(x) \tag{29}$$

and $Q^\dagger$ should be the generators of the gauge symmetry due to the interaction, but its bracket with all fields vanishes identically (using (19))

$$\{Q, \Psi^\dagger_\mu(y)\}_{\rm II} = 0, \tag{30}$$

$$\begin{aligned}&\{Q, \phi^\dagger(y)\}_{\rm II} = 0,\\ &\{Q, \psi^\dagger(y)\}_{\rm II} = 0.\end{aligned} \tag{31}$$

So, although the first terms of θ_0 in equation (21) $\Lambda^{0\nu}(m=0) = -\epsilon^{0\nu\lambda\mu}\gamma_5\gamma_\lambda\partial_\mu = -\gamma_0\sigma_{ji}\partial_j$ in the massless limit act as the generator of $\Psi_\mu \to \Psi_\mu + \partial_\mu\epsilon$, the last term in spite of being nonzero, does not generate any gauge transformation. By the same token, the scalar and spinor fields are neutral under this gauge transformation, which suggests that in a consistent gauge theory should decouple from the RS field. Observe that $\{Q, Q^\dagger\} = 0$ in spite of $\{S^0, S^{0\dagger}\} \neq 0$.

To check the signature of the Hilbert space let us proceed analogously as in our previous subsection for $\mathcal{L}_{\rm NEK}$. We intend firstly calculate $\{\psi(x), \psi^\dagger(y)\}_D$ as in equation (25) but using now the equation (28) and the commutators of equations (19). We get

$$\{\psi(x), \psi^\dagger(y)\}_D = -i\delta^3(x-y), \tag{32}$$

that is unaffected by the secondary constraint and then, we must pursue looking for possible problems for

$$\begin{aligned}\{\Psi_i(x), \Psi_j^\dagger(y)\}_D = &\{\Psi_i(x), \Psi_j^\dagger(y)\}_{\rm II}\\ &- \int d^3z d^3z' \{\Psi_i(x), \theta_0(z)\}_{\rm II}\{\theta_0(z), \theta_{0^\dagger}(z')\}^{-1}_{\rm II}\{\theta_{0^\dagger}(z'), \Psi_j^\dagger(y)\}_{\rm II}.\end{aligned} \tag{33}$$

Now by using the corresponding entry for the $\mathcal{L}_P$ case in equation (21), the corresponding brackets (19) and equation (28) we get

$$\begin{aligned}\{\Psi_i(x), \Psi_j^\dagger(y)\}_D = &\left[\frac{i}{2}\gamma_j\gamma_i - ig^2(\partial_i\phi(x))(\partial_j\phi(y))\right]\delta^{(3)}(x-y)\\ &+ \int d^3z \left\{\partial_i^x - img^2(\partial_i\phi(x))(\partial_k\phi(z))\gamma_k + \frac{i}{2}m\gamma_i\right\}\delta^{(3)}(x-z)\\ &\times \frac{\frac{-2i}{3m^2}}{\left(1 - \frac{2g^2}{3}(\nabla\phi(z))^2\right)}\\ &\times \left\{\partial_j^y - img^2(\partial_l\phi(z))\gamma_l(\partial_j\phi(y)) + \frac{i}{2}m\gamma_j\right\}\delta^{(3)}(z-y),\end{aligned} \tag{34}$$

where the integral over z' was absorbed by the $\delta(z-z')$ in equation (28) and where the property $\partial_i^z\delta^3(z-x) = -\partial_i^x\delta^3(z-x)$ was used. We can arrange the Dirac bracket as

$$\{\Psi_i(x),\,\Psi_j^\dagger(y)\}_D = \left[\frac{\mathrm{i}}{2}\gamma_j\gamma_i - \mathrm{i}g^2(\partial_i\phi(x))(\partial_j\phi(y))\right]\delta^{(3)}(x-y) + \left\{\partial_i^x - \mathrm{i}mg^2(\partial_i\phi(x))(\partial_k\phi(x))\gamma_k + \frac{\mathrm{i}}{2}m\gamma_i\right\} \times \frac{\frac{-2\mathrm{i}}{3m^2}}{\left(1-\frac{2g^2}{3}(\nabla\phi(x))^2\right)} \times \left\{-\partial_j^x - \mathrm{i}mg^2(\partial_l\phi(x))\gamma_l(\partial_j\phi(x)) + \frac{\mathrm{i}}{2}m\gamma_j\right\}\delta^{(3)}(x-y). \tag{35}$$

Note that in the case of free RS fields ($g = 0$) our result coincides with that in [16] ($e = 0$).

The difficulty in analyzing the signature for the RS states is that the constraint θ_0 for the $\mathcal{L}_P$ case should be enforced (recall that even the free theory includes negative norm states, which are eliminated only when the constraints are imposed). Let us create a single RS particle at rest ($\vec{p} = 0$) in presence of a scalar background of constant gradient: for definiteness and simplicity let us impose $f(x) = Ax_1$, being $|(A, 0, 0)\rangle$ a coherent state such that $\nabla\phi|(A, 0, 0)\rangle = (A, 0, 0)|(A, 0, 0)\rangle$, and absence of any Dirac field quanta. We then built the state $\alpha_i^\dagger\Phi_i|(A, 0, 0)\rangle$ where α_i is a vector-spinor coefficient as those appearing in the second quantization expansion of $\Psi_i(x)$, being $\Phi_i = \int \mathrm{d}^3x\mathrm{e}^{\mathbf{x}\cdot 0}\Psi_i(x)$ are creation operators of RS quanta at rest, where to achieve normalization, a regulator volume V should be used. When $\vec{p} = 0$ and in absence of nucleon quanta the constraint θ_0 implies then $\gamma_i\alpha_i = 0$. One such state is $\vec{\alpha} = (\gamma_2\chi, \gamma_1\chi, 0)$ for some constant nonzero spinor χ, where on time $\vec{\alpha}^\dagger = (-\chi^\dagger\gamma_2, -\chi^\dagger\gamma_1, 0)$. Let us calculate the norm

$$\langle(A, 0, 0)|(\Phi_i^\dagger\alpha_i)(\alpha_j^\dagger\Phi_j)|(A, 0, 0)\rangle = \alpha_j^\dagger\langle(A, 0, 0)|\mathrm{i}\{\Phi_j, \Phi_i^\dagger\}_D|(A, 0, 0)\rangle\alpha_i, \tag{36}$$

where $\{\Phi_i, \Phi_j^\dagger\}_D = \int \mathrm{d}^3x\mathrm{d}^3y\{\Psi_i(x), \Psi_j^\dagger(y)\}_D$. Since once all field operators act on the states no x or y dependence results in the rhs of (35) except for the dirac Delta from the bracket, the integration over x and y of the delta results in a factor V (the regulated volume) which we absorb in the normalization of χ. We thus get

$$\langle(A, 0, 0)|(\Phi_i^\dagger\alpha_i)(\alpha_j^\dagger\Phi_j)|(A, 0, 0)\rangle = 2\left(1 + \frac{\frac{1}{2}g^2A^2}{1-\frac{2}{3}g^2A^2}\right)\chi^\dagger\chi\langle(A, 0, 0)||(A, 0, 0)\rangle \tag{37}$$

which clearly becomes negative for A large enough.

Before ending, let us remark that the quantization procedure in [3] is flawed, since the invoked decoupling of the introduced auxiliary fields does not verify (see appendix C). So, the path integration in [3] reaches a dead end analogous to that for $\mathcal{L}_{\mathrm{NEK}}$ in the same reference.

3. Concluding remarks

We have shown that the so called spin-3/2-gauge-invariant coupling $\mathcal{L}_P$ to the RS field presents inconsistencies analogous to the ones found by Johnson and Sudarshan [4] and by Hagen [5] with the usual π-derivative $\mathcal{L}_{\mathrm{NEK}}$ interaction. This proves that consistency conjecture of $\mathcal{L}_P$ stated in [3] is incorrect. Observe that the main argument in [3] to claim consistency of $\mathcal{L}_P + \mathcal{L}_{\mathrm{RS}}$ is gauge invariance under $\Psi_\mu \to \Psi_\mu + \partial_\mu\epsilon$ in the massless limit.

Nevertheless we have shown that the gauge invariance of this interaction is trivial, in the sense that the gauge transformation does not transform the scalar and spinor field (observe that the 'current' S_P^μ is conserved identically, without imposing the equations of motion on the scalar and spinor fields). The mass term breaks this invariance anyway. Our treatment cannot be taken to the massless limit: the Dirac bracket contains the mass as divissor and our proof of signature problems uses the rest frame which makes sense only in the massive case, but possibly inconsistencies will arise even at the massless limit since the gauge invariant interaction is not trivial in spite of the scalar and Dirac fields being neutral under the gauge invariance.

Our result has a great relevance in the hadron phenomenology community, since often the consistency issue is invoked in evaluations of work done with $\mathcal{L}_{\rm NEK}$. Recall that $\mathcal{L}_{\rm NEK}$ and $\mathcal{L}_P$ are used to interpret accelerator data, estimate parameters for resonances and other critical tasks in phenomenology. The present work shows that there is no basis to dismiss work done with interaction $\mathcal{L}_{\rm NEK}$ or prefer the use of $\mathcal{L}_P$ in hadron phenomenology on the basis of their (in)consistency.

On the other hand, the decades old problem of finding consistent interactions for spin $3/2$ fields still remains.

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Appendix A. Restrictions for *z*

Note that in equation (3) the interaction can be expressed as $\mathcal{L}_{{\rm NEK},P} = \bar{\Psi}_\mu S^\mu + {\rm h.c.}$ and let us discuss about the structure of the S^μ. Observe that in the free RS lagrangian in (4), if $a = -1$ (see equation (5)), there is no term containing $\dot{\Psi}^0$. So, the equation of motion for Ψ^0 is a true constraint, and Ψ^0 has no dynamics. It is necessary that interactions do not change that, or there will be no projection of degrees of freedom, and so no hope to get rid from the unwanted negative-norm sector. The contribution from interactions to such equations of motion will come from S^0. The condition that no term containing $\dot{\Psi}^0$ will arise is that S^0 contains no time derivative of any of the other fields of the theory. Indeed, suppose that $S^0(\dot{\chi})$ and consider its equation of motion ($\mathcal{L} = \mathcal{L}_{\rm RS} + \mathcal{L}_N + \mathcal{L}_\pi + \bar{\Psi}^\mu S_\mu + \bar{S}_\mu \Psi^\mu$):

$$\frac{\partial\mathcal{L}}{\partial\chi} - \frac{\rm d}{{\rm d}t}\frac{\partial\mathcal{L}}{\partial\dot{\chi}} = \frac{\partial\mathcal{L}}{\partial\chi} - \frac{\rm d}{{\rm d}t}\left(\frac{\partial\mathcal{L}_{\rm RS} + \mathcal{L}_N + \mathcal{L}_\pi}{\partial\dot{\chi}}\right) - \frac{\rm d}{{\rm d}t}\left(\frac{\partial S^0}{\partial\dot{\chi}}\bar{\Psi}^0 - \frac{\partial S^i}{\partial\dot{\chi}}\bar{\Psi}^i\right)$$
$$= \frac{\partial\mathcal{L}}{\partial\chi} - \frac{\rm d}{{\rm d}t}\left(\frac{\partial\mathcal{L}_N + \mathcal{L}_\pi}{\partial\dot{\chi}}\right) - \frac{\rm d}{{\rm d}t}\left(\frac{\partial S^0}{\partial\dot{\chi}}\bar{\Psi}^0 - \frac{\partial S^i}{\partial\dot{\chi}}\bar{\Psi}^i\right), \quad (38)$$

for some field $\chi = \psi, \phi$, since $\frac{\partial\mathcal{L}_{\rm RS}}{\partial\dot{\chi}} = 0$ for all fields. It is apparent that if $\frac{\partial S^0}{\partial\dot{\chi}} \neq 0$, a contribution proportional to $\dot{\Psi}^0$ will arise. This condition fixes the off-shell parameter for $\mathcal{L}_{\rm NEK}$ (in coincidence with determinations by other means in [2, 3]) as well as $\mathcal{L}_P$ (only after this value is fixed $\mathcal{L}_P$ coincides to the interaction proposed in [3]). Indeed, for $\mathcal{L}_{\rm NEK}$ ($a = -1$):

$$\mathcal{L}_{\rm NEK} = \bar{\Psi}_\nu g(g^{\nu\mu} - (z + 1/2)\gamma^\nu\gamma^\mu)\psi(\partial_\mu\phi) + g\bar{\psi}(\partial_\mu\phi)(g^{\mu\nu} - (z + 1/2)\gamma^\mu\gamma^\nu)\Psi_\nu$$

and so:

$$S^0_{\rm NEK} = g\left(\frac{1}{2} - z\right)\dot{\phi}\psi - g\left(\frac{1}{2} + z\right)\gamma^0\gamma^i\partial_i\phi\psi \tag{39}$$

so we get $z = \frac{1}{2}$, and thus

$$S^0_{\rm NEK} = -g\gamma^i\gamma^0\psi(\partial_i\phi), \tag{40}$$

$$S^{\dagger 0}_{\rm NEK} = g(\partial_i\phi^\dagger)\bar{\psi}\gamma^i. \tag{41}$$

$\mathcal{L}_P$ can be written for $a = -1$ as

$$\begin{aligned}\mathcal{L}_P = g\epsilon^{\rho\alpha\beta\nu}&[\bar{\Psi}^\mu(g_{\mu\rho} - (1/2 + Z)\gamma_\mu\gamma_\rho)\gamma_5\gamma_\alpha(\partial_\nu\phi)(\partial_\beta\psi)]\\ &+ (\partial_\beta\bar{\psi})(\partial_\nu\phi^\dagger)\gamma_5\gamma_\alpha(g_{\mu\rho} - (1/2 + Z)\gamma_\mu\gamma_\rho)\Psi^\mu],\end{aligned} \tag{42}$$

now should be $z = -\frac{1}{2}$ to avoid time derivative of fields coupled to Ψ_0 then

$$\mathcal{L}_P = g\epsilon^{\mu\nu\alpha\beta}(\bar{\Psi}_\nu\gamma_5\gamma_\alpha(\partial_\mu\psi)(\partial_\beta\phi) + (\partial_\mu\bar{\psi})(\partial_\beta\phi^\dagger)\gamma_5\gamma_\alpha\Psi_\nu) \tag{43}$$

so we get

$$S^0_P = g(\partial_i\bar{\psi})(\partial_k\phi)\epsilon^{i0jk}\gamma_j\gamma_5, \tag{44}$$

$$S^{\dagger 0}_P = g(\partial_n\phi)\epsilon^{l0mn}\gamma^0\gamma_5\gamma_m(\partial_l\psi). \tag{45}$$

We thus see that indeed, within the interaction $\mathcal{L}_P$ there is a coupling to the spin 1/2 sector. This coupling is not noticeable at tree level of pure hadron coupling, but shows up in radiative amplitudes and at loop level, as was shown recently in [11].

Appendix B. Dirac brackets for fermions

Fermion fields have not a classical limit. However, in order to cope with constrained fermion systems and path integrals in supergravity and string theory, in the 70s emerged a description of fermion degrees of freedom in the pseudoclassical limit $\hbar \to 0$. It has been shown that in this limit fermions are described by Grassmann (anticommuting) variables [20]. A generalized Poisson pseudoclassical bracket is defined for describing pseudoclassical field systems including both c-number and Grassmann fields. These are the fundamental brackets adequate to apply the Dirac quantization scheme in boson-fermion systems. Let f and g be fields, λ a parameter, and $\epsilon(h) = 0$ if h is bosonic, $\epsilon(h) = 1$ if it is fermionic. Then the Poisson bracket {,} obeys

$$\{f, g\} = (-1)^{\epsilon(f)\epsilon(g)+1}\{g, f\}, \tag{46}$$

$$\{f + h, g\} = \{f, g\} + \{h, g\}, \tag{47}$$

$$\{f, \{g, h\}\} = f\{g, h\} + (-1)^{\epsilon(f)\epsilon(g)}\{f, h\}g, \tag{48}$$

$$\{f, \lambda\} = 0. \tag{49}$$

Then, for each field $f, f^\dagger$ in the theory, we define its canonical momentum as

$$\Pi_f = \frac{\partial\mathcal{L}}{\partial\dot{f}}, \;\; \Pi_{f^\dagger} = \frac{\partial\mathcal{L}}{\partial\dot{f}^\dagger} \tag{50}$$

with the caution that, if f is fermionic, the derivative with respect to it is also anticommuting, so it is important to distinguish between left or right derivation. We will adopt right derivation, that is, the derivative of $\mathcal{C}q$ with respect to the Grassmann variable q will be always $\mathcal{C}$, but the derivative of $q\mathcal{C}$ will be -1 if $\mathcal{C}$ is a Grassmann number, and 1 if it is a c-number. Observe that for bilinear Lagrangians like $\mathcal{L}_{\mathrm{RS}}$, if all derivatives act on the fields on the right (our case, [15, 19]) canonical momenta are the same one should obtain from treating Ψ^μ as c-numbers, but in the case of symmetrized lagrangians like in [3, 16] one must be careful in considering the anticommutation between the Grassmann derivative and Ψ^μ in the terms in which derivatives act on $\bar{\Psi}^\mu$. Observe also that if one defines canonical momenta as left derivatives instead of right, the RS lagrangian would be minus our $\mathcal{L}_{\mathrm{RS}}$ in order to yield a positive definite spectrum.

We define the 'Poisson brackets' fulfilling the properties (46)–(49) as

$$\{f(x),\, g(y)\} = \int \mathrm{d}^3 z \left[\frac{\partial f(x)}{\partial \Psi(z)} \frac{\partial g(y)}{\partial \Pi_\Psi(z)} + (-1)^{\epsilon(f)\epsilon(g)+1} f \leftrightarrow g \right. \\ \left. + \frac{\partial f(x)}{\partial \Psi^\dagger(z)} \frac{\partial g(y)}{\partial \Pi_{\Psi^\dagger(z)}} + (-1)^{\epsilon(f)\epsilon(g)+1} f \leftrightarrow g \right], \tag{51}$$

from which the so called 'fundamental (equal time) Poisson breackets' are obtained:

$$\{f(x),\, \Pi_f(y)\} = \delta^3(x-y). \tag{52}$$

These are not always compatible with (50) since f and Π_f are not in general independent variables appearing constraints $\Omega_k = 0$ relating them.

To construct a consistent Poisson algebra, the Dirac procedure is used: the fundamental Poisson brackets between constraints are calculated using (52)

$$C_{kl}(x,\, y) = \{\Omega_k(x),\, \Omega_l(y)\} \tag{53}$$

and the linearity property (47) is used. So we get the new 'Dirac' brackets as follows:

$$\{f(x),\, g(y)\}_D = \{f,\, g\} \\ - (-1)^{\epsilon(f)\epsilon(g)+1} \int \mathrm{d}^3 z\ \mathrm{d}^3 z' \{f(x),\, \Omega_k(z)\}\ C^{-1}(z,\, z')_{kl}\ \{\Omega_l(z'),\, g(y)\}. \tag{54}$$

These Dirac brackets are also Poisson brackets in the sense that they obey (46)–(49), but they are now consistent with the constraints.

For the RS case the Poisson Bracket between a field f, $g = \Psi_{\mu,m}$, $\Pi_{\mu,m} = \partial\mathcal{L}/\partial(\dot{\Psi}^\mu_m)$, or $\Psi^\dagger_{\mu,m}$, $\Pi_{\Psi^\dagger \mu,m} = \partial\mathcal{L}/\partial(\dot{\Psi}^\dagger)^\mu_m$ is

$$\{f(x)_{\mu,m},\, g(y)^\nu_n\} \\ = \int \mathrm{d}^3 z \left[\frac{\partial f(x)_{\mu,m}}{\partial \Psi(z)_{\alpha,a}} \frac{\partial g(y)^\nu_n}{\partial \Pi_\Psi(z)^\alpha_a} + f \leftrightarrow g + \frac{\partial f(x)_{\mu,m}}{\partial \Psi^\dagger(z)_{\alpha,a}} \frac{\partial g(y)^\nu_n}{\partial \Pi_{\Psi^\dagger}(z)^\alpha_a} + f \leftrightarrow g \right], \tag{55}$$

where a, m, n = 1, 2, 3, 4 are spinor matrix indexes while α, μ, $\nu = 0, 1, 2, 3$ are Lorentz ones. The fundamental equal time Poisson brackets for the RS field are

$$\{\Psi(x)_{\mu,m},\, \Pi(y)^\nu_n\} = g^\nu_\mu \delta_{m,n} \delta^3(x-y), \\ \{\Psi^\dagger(x)_{\mu,m},\, \Pi^\dagger(y)^\nu_n\} = g^\nu_\mu \delta_{m,n} \delta^3(x-y), \tag{56}$$

with other combinations of Ψ, Π, $\Psi^\dagger$, $\Pi^\dagger$ vanishing, and where we have used that $\frac{\partial f(x)}{\partial f(y)} = \delta(x-y)$, while for the Dirac spinor ψ and the scalar ϕ will be

$$\{\psi(x)_m,\, \Pi_\psi(y)_n\} = \delta_{m,n}\delta^3(x-y),$$
$$\{\phi(x)_{\mu,m},\, \Pi_\phi(y)\} = \delta^3(x-y). \tag{57}$$

Appendix C

We will devote this appendix to the path integral treatment of RS interactions in [3]. Let us start with $\mathcal{L}_{\mathrm{NEK}}$. Observe that in expression (29) of [3] the square root in the lhs is not carried to the rhs, nor to expression (30). Observe also that in the simplification of the determinant (29) invoked in (30) there should be a factor R^2 which is field dependent and thus cannot be ommitted. The precise form of the determinant is however unimportant for the development of [3], since it is not used to derive any result.

On the other hand, the treatment of $\mathcal{L}_P$, whithout introducing Stückelberg parameters (our treatment), leads to a determinant very simmilar to that in equation (29) of [3], whith $R = \frac{3im^2}{2}\left(1 - \frac{2g^2}{3}g^2(\nabla\phi)^2\right)$ and $-g\sigma_{ji}(\partial_j)(\partial_i\phi)$ instead of $g\gamma_i\partial_i\phi$ in the matrix positions (3, 5) and (5, 3), where ∂_j acts on the Dirac delta. The reduction is even more involved than in the case of $\mathcal{L}_{\mathrm{NEK}}$. But [3] did not produce such an expression, since they tried another way introducing an auxiliary field ξ. They did so through a reasoning in line with Stückelberg formalism, but they could have produced alternatively the expression (45) by exponentiating part of the measure, in the spirit of the Faddeev and Popov method. Whatever the method employed, the result is that the problematic constraint $\Pi_0 = 0$ together with $\theta_4 = 0$ are traded into first class constraints (thus dissapearing from the determinant in the Dirac algebra), at the price of adding the Stückelberg field ξ. But in passing from expression (45) and (46) it is stated without proof that, due to the gauge invariance, ξ decouples. Observe that the mentioned gauge invariance is no longer the corresponding to the massless case, but the subtler invariance shown in expression (A2) of [3], which is a more restricted kind of gauge transformation since the gauge parameter is no longer arbitrary but obeys a nontrivial ecuation of motion [13]. Thus, there is no reason to think that the functional integration over ξ leads to its decoupling nor to the δ functionals which lead to the reported Feynman rules. As a mater of fact, the statement that gauge invariance (A2) implies the decoupling of ξ makes no sense: it is the coupling to ξ what compensates the symmetry breaking of the mass term making the theory gauge invariant [13]. If ξ is integrated out properly, obviously they must have had arrived to the complicated measure mentioned at the begining of this paragraph, since the original theory should be recovered.

To see how absurd this result is, observe that we could take the limit $\mathcal{L}_P \to 0$, so (46) would imply that the free RS is equivalent to a theory without the constraints $\Pi_0 = 0$ and $\theta_4 = 0$. This is in contradiction with the developement of section 2 of [3] itself. This mistake carries over to the Feynman rules, as stated above. The right procedure would be, if one is to work with first class constraints instead of second class ones, to retain the field ξ, whith their own Feynman rules, and then make the gauge choice at the level of the Hilbert space. In fact, other authors have done generalizations of the Stückelberg formalism to RS fields in the past [14] and found it to be quite tricky, requiring for instance the introduction of two spin $1/2$ Stückelberg fields instead of one.

So, once this flaw in the quantization procedure is pointed out, we get to a situation simmilar to that after expression (30) of [3]: a functional integration with a complicated measure with wich it is very difficult to proceed with path integrals. Since afterwards we have shown that the theory is not quantizable (since the Hilbert space is not positive definite nor semidefinite) there is no point in trying to get any Feynman rules or quantization, by any means.

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