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TEACHING THE FOUNDATIONS OF QUANTUM MECHANICS IN SECONDARY SCHOOL: A PROPOSED CONCEPTUAL STRUCTURE

(Enseñando los fundamentos de la Mecánica Cuántica en la Escuela Media: una Estructura Conceptual Propuesta)

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Abstract

This paper is part of a doctoral thesis that investigates Basic Quantum Mechanics (QM) teaching in high school. A Conceptual Structure of Reference (CSR) based on the Path Integral Method of Feynman (1965) was rebuilt and a Proposed Conceptual Structure for Teaching (PCST) (Otero, 2006, 2007) the basics of Quantum Mechanics at secondary school was designed, analysed and carried out. This PCST does not follow the historical route and it is complementary to the canonical formalism. The concepts: probability distribution, quantum system, $x(t)$ alternative, amplitude of probability, sum of probability amplitude, action, Planck's constant, and classic-quantum transition were rebuilt with the students. Mathematical formalism was avoided by using simulation software assistance. The Proposed Conceptual Structure for Teaching (PCST) is described and some results from the test carried out by the class group are discussed. This information allows the analysis of the Conceptual Structure Effectively Reconstructed (CSER) to be initiated with the students.

Keywords: Quantum Mechanics, High School, Feynman's Paths Integral Method.

Resumen

Este trabajo es parte de una tesis doctoral que investiga la enseñanza de los fundamentos de la Mecánica Cuántica en la Escuela media. Se reconstruyó una estructura Conceptual de Referencia (ECR) basada en el método de la Integral de Camino de Feynman (1965) y se diseñó, analizó e implementó una Estructura Conceptual Propuesta para Enseñar (ECPE) (Otero, 2006, 2007) fundamentos de Mecánica Cuántica en la escuela media. Esta ECPE no sigue el camino histórico y es complementaria al formalismo canónico. Los conceptos: distribución de probabilidad, sistema cuántico, trayectoria alternativa $x(t)$, amplitud de probabilidad, suma de amplitud de probabilidad acción, constante de Planck y transición clásica-cuántica se reconstruyeron con los estudiantes. Se describe la Estructura Conceptual Propuesta para Enseñar (ECPE) y se discuten algunos resultados de la evaluación realizada por el grupo de clase. Esta información permite el análisis de la Estructura Conceptual Efectivamente Reconstruida (ECER) con los estudiantes.

Palabras-clave: Mecánica Cuántica, Enseñanza Secundária, Método de la Integral de Camino de Feyman.

Theoretical Framework

In physics, there are a lot of conceptual fields (Vergnaud, 1990) in which at least one Conceptual Structure of Reference (CSR) can be distinguished and recognized (Otero, 2006, 2007). When a physics teacher invites his students to study a specific conceptual field, he/she adopts, in a somewhat explicit way, a particular Conceptual Structure of Reference (CSR). A CSR is a set of concepts, the relationships among them, the principles, the knowledge claims, and the explanations relative to a conceptual field, accepted by the scientific community of reference. This investigation rebuilt a Conceptual Structure of Reference (CSR) based on Feynman's Paths Integral Method (1965). A detailed analysis of this CSR can be seen in Arlego (2008). The full proposal of adapting a conceptual organization for high school students can be found in Fanaro, Otero (2008) and Fanaro, Arlego, Otero (2007). The CSR adopted will be partially or fully reconstructed by a class group, or by someone who tries to study it in high school, or at the basic or advanced courses at university.

Any attempt at reconstruction originates a different conceptual structure, as much for the components as for the relationship between them. In a more or less explicit way, each teacher of a certain group will reconstruct or select – based on an existing structure – one conceptual structure to be taught, and, in the best of the cases, he/she will invite the class to study it. We named this other structure: Proposed Conceptual Structure for Teaching (PCST) (Otero, 2006, 2007). It is a set of concepts, the relationships between them, knowledge claims, principles, situations, and explanations related to a certain conceptual field, that the teacher must reconstruct based on a Conceptual Structure of Reference (CSR). The teacher aims at transforming the scientific knowledge and at reconstructing it in a certain context of a given Institution (Otero, 2006, 2007).

The SCPT design requires multiple actions: analysing and selecting the key concepts of the conceptual field that would be reconstructed in the class group (CG) and creating appropriate situations to use the software that simulates the Double Slit Experiment (DSE). Also, suitable parameters have been chosen to avoid actions that could blurr the study. For instance, in certain approach configurations (zoom lens), the software shows the effects of diffraction, making it difficult to distinguish between small balls and electrons. The energy of electrons, the width and the separation of the slits were properly chosen. Thus, when covering each one of them sequentially, the curves are similar – except for the scale – to the ones obtained with small balls. Two simulations using Modellus[1] were specially developed for the didactic sequence. The possible results for each simulation procedure were analysed beforehand and the students' actions were anticipated.

SCR and SCPT are partly related to the idea of cognitive structure as it has been proposed by Ausubel and Novak together with Vergnaud's ideas about conceptual fields and concepts. The structures are systems (components + organization) that include key concepts, such as the relationships, fundamental principles, explanations and explanatory mechanisms that tie them together. When we adopted Vergnaud's ideas about concepts and conceptualization, we included language, referents and operational invariants that are involved in conservation of the forms to organize the action. This idea of concepts related to the action in all their variations, allows building a bridge to the underlying emotions and feelings, also included in the conceptual structure. The conceptual structures are inseparable from the set of problems and situations that give sense to them.

The Proposed Conceptual Structure for Teaching has the following components (PCST):

Situations: Vergnaud's idea of situation.
Key Concepts: These are the main concepts that must be built. These concepts are produced by the proposed situation and without them the problem established in the situation cannot be resolved.
Key Principles: These are knowledge propositions accepted as true and are not deduced from others.
Key Questions: The situations given by the teacher are problem-situations. These situations have a set of questions that must be discussed with good interaction between the members of the class.

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[1] Modellus version 2.5 Created by Victor Duarte Teodoro, João Paulo Duque Viera; Filipe Costa Clérigo, Faculty of Sciences and Technology, New University, Lisbon, Portugal

Emotions: These are dynamic body dispositions determining our actions domain (Maturana, 1995). When we participate in conversations this participation affects our emotions, and our emotions are affected by our conversations. The Proposed Conceptual Structure for Teaching (PCST) is an invitation to the students. They are invited to enter a knowledge domain where the denial of others is avoided, and an emotional dynamic adequate to knowledge construction is built.

Actions: Are to be understood in three dimensions: biological, mental and acting. In the Proposed Conceptual Structure for Teaching (PCST) we emphasize the acting dimension. We are interested in the class group's members actions related to knowledge. We need to anticipate which actions are appropriate in the knowledge domain that has been built. The different meanings of these concepts flow from the system of actions related to them in every domain and situation.

Explanations: these are knowledge claims obtained by using valid mechanisms of inference. They can be generalizations or deductions. The explanations built into a domain will change our actions and our emotions.

Explanation Mechanism: is the procedure or set of actions accepted in an explanation domain as a good method to generate valid assertions of knowledge.

Language: refers to the several semiotic modes to establish and to describe the concepts and the objects of every knowledge domain.

The teacher and his class group will reconstruct the PCST in a given and specific institution, generating the Conceptual Structure Effectively Reconstructed (CSER). A CSER is the set of concepts, relationships among them; principles, knowledge claims and explanations relative to a certain conceptual field that are reconstructed by the class group. The teacher and the students interact in conversations characterized by an adapted emotional dynamic. Every member of the class group will relate to a personal conceptual structure and a unique network of meanings, personal and private. Simultaneously, the class group conversations lead to the construction of a meaning network, which is shared and public. This meaning network is a consensual product; it has been called “process of meaning negotiation". This negotiation process can be more or less explicit and more or less conscientious, depending on the professionalism of the teacher, and the distance among CSR, PCST and CSER.

The Conceptual Structure of Reference: Path Integral Formulation of quantum mechanics

Basics Concepts of the Path Integral formulation

The Path Integral Formulation (PIF) of quantum mechanics was developed by Richard Feynman in 1948 and it is presented in detail in his book, written in collaboration with A. R, Hibbs in 1965. The PIF is equivalent to the canonical treatment of quantum mechanics (operator formalism), previously developed by E. Schrödinger, W. Heisenberg y P. M. Dirac in 1925-1926. Both formulations have shown to be complementary in the sense that some type of problems, such as central potentials, are simpler to treat in the canonical formalism, whereas the PIF makes it possible to treat advanced topics in a simpler way as the case of the standard model of elementary particles (Ryder, 1996).

The concept of action plays a central role in the PIF. Therefore we are going to review the principle of least action of classical mechanics. For the sake of simplicity, we are going to consider the motion of a macroscopic particle in one dimension (x), under the influence of a Potential V(x). As it is known, given initial and final states, $I = x_i(0)$ and $F = x_f(T)$, respectively, the solution of Newton's second law provides a definite trajectory connecting both states, which is in agreement with the observed motion. This problem can be formulated as well by means of the principle of least action in the following way. The action $S[x(t)]$:

$S[x(t)] = \int_0^T L[x(t)]dt$, where $L[x(t)] = \frac{1}{2}m\left(\frac{dx(t)}{dt}\right)^2 - V(x(t))$ is the Lagrangian of the system

For each $x(t)$, which describes a possible alternative from the initial to the final state, the action takes a value. The principle of least action states that from all possible trajectories, the classical trajectory, $x_{cl(t),}$ is the one that minimizes the action, i.e. S[xcl(t)] is minimum. The equivalence between the principle of least action and Newton's second law can be found in textbooks on classical mechanics (for instance Goldstein, 1966).

As it is known, the laws of classical mechanics are not valid for arbitrarily small masses. In this case, the laws of quantum mechanics provide the adequate description (Shankar, 1980). These laws describe accurately the behaviour of systems at atomic scale and include classical mechanics as a particular case when masses become macroscopic. The main paradigm change of quantum mechanics, regarding classical mechanics, is that given some initial (I) and final (F) conditions, the concept of a definite function $x(t)$ which connects both states is no longer valid. Instead, quantum mechanics makes predictions about the probability to arrive at F starting from I. In the PIF, the calculation of this probability involves the evaluation of all alternative $x(t)$, connecting the initial and the final states. This is particularly relevant in the atomic domain. The PIF of quantum mechanics for the one-dimensional problem that we are considering here can be formulated this way:

1. The action S[x(t)], corresponding to each possible trajectory x_α(t) that connects an initial state I = x_i (0) with a final one F = x_f (T).
2. This action is associated with a modulus one complex number Z_α = exp(i S[x(t)] / ħ), where ħ = h / 2 π, being h = 6.625 x10^{-34} J.s, Planck's constant.
3. The probability, P[I → F], to arrive at F starting from I is the square of the absolute value of the probability amplitude, A[I → F], i.e.: P[I → F] = $|A[I \rightarrow F]|^2$, where A[I → F] is obtained by summing all the Z_α:

$$A[I \rightarrow F] \propto \sum_{\text{all x (t)s}} \exp(i\,S[x(t)]\;/\hbar), \quad (1)$$

The previous formula has the following interpretation: each alternative path x_α(t) that connects I to F contributes to the amplitude of probability A[I → F] with a modulus one complex number, being its phase (argument), given by S[x_α(t)] / ħ. The resulting probability amplitude is the resultant complex number obtained by summing each one of the Z_α, associated with each possible alternative, multiplied by an adequate proportionality constant. That is to say, for alternative events that the total amplitude is obtained by summing the amplitudes for each alternative. Finally, the probability to arrive at F starting from I is obtained by squaring |A[I → F]|.

It is possible to generalize the PIF for particle systems and fields, even in a relativistic domain (Zee, 2003). On the other hand, the equivalence between the PIF and the canonical operator formalisms is shown in textbooks of quantum mechanics (e.g. Shankar, 1980). In practice, the sum over all paths can be evaluated only in simple cases, as the free particle or the harmonic oscillator. In these cases, an analytical expression is obtained after having been discretized and taken limits in a procedure considered a generalization of the method to obtain ordinary integrals. In fact, this is the origin of the term path integrals. In more complex cases, and usually the most interesting ones, approximate methods, such as series expansion (about cases with exact solution) or statistical evaluation of the sums (Monte Carlo methods), among others, are the only way to partially solve the problem (Ryder, 1996). One of the advantages of PIF is the possibility to obtain some non-perturbative formal results, like quantum field theories renormalizabiliy proofs. On the contrary, these results are difficult to obtain within the canonical formalism.

Figure 1 shows a schematic representation of the sum process that illustrates our previous reasoning. As it can be observed, paths nearby to $x_{cl(t)}$ coherently contribute to the sum, i.e. with the same phase, whereas the others cancel each other and do not contribute to the probability amplitude.

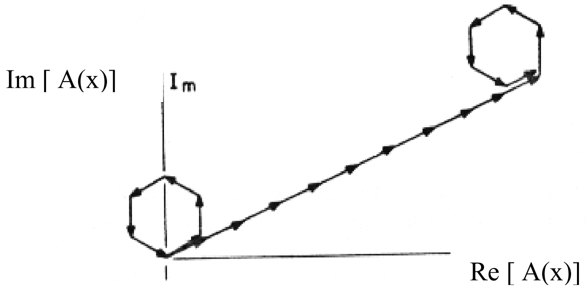


Figure 1: Schematic representation of the sum of probability amplitudes associated with alternative paths. The central straight line represents the coherent contributions of $x_{cl(t)}$ and its surroundings. The other paths cancel each other and do not contribute to the sum.

In the general case of an arbitrary potential, in the atomic domain, all paths contribute to the sum. However, for the free particle (V=0), even at an atomic scale, the classical trajectory, $x_{cl(t)}$, and its surroundings, still keep a special role. Except that now this region around $x_{cl(t)}$ can be large, as compared to the classical case. In fact, the probability amplitude (Eq.(1)) for arbitrary mass (even at an atomic level) can be expressed in an exactly factorized form, including only the contribution of classical action: $S[x_{cl}]$ in the exponential, multiplied by a factor C in which the coherent contributions of neighbouring paths around classical trajectory are considered, i.e.

$$A(I\rightarrow F) = C \exp (i\, S_{cl} / \hbar) \quad (V = 0 \text{ and arbitrary mass}) \quad (2)$$

According to the correspondence principle, the laws of quantum mechanics reproduce classical mechanics results in the limit of large values of the action S, in relation to ħ, i.e. in practice we can consider $\hbar \rightarrow 0$ for macroscopic objects. One of the most significant advantages of the PIF is the possibility to explain the transition from macroscopic to microscopic behavior in simple terms, as follows:

Let us consider a macroscopic object with mass m from the PIF of quantum mechanics point of view. Let us also suppose several arbitrary trajectories $x_1(t)$, $x_2(t)$, . . ,$x_n(t)$, that contribute to the sum in Eq(1). They contribute to the amplitude: $\exp (i\, S[x_1(t)] / \hbar) + \exp (i\, S[x_2(t)] / \hbar). +. . + \exp (i\, S[x_n(t)] / \hbar)$. Since m is macroscopic, each phase in the exponential is very large and, in average, they will cancel each other (notice that this is an statistical argument). However, this cancellation will not happen with all trajectories. What happens with $x_{cl(t)}$, the classical function? As it is known, the action takes its minimum value in this case. Therefore, in a region that is extremely close to the classical function, macroscopically indistinguishable, all paths (functions x(t)) contribute with approximately the same phase (coherently). It follows that, in the case of a macroscopic mass, the probability amplitude is dominated by the classical trajectory and we reproduce classical results via the PIF.

How can it introduce the principles of quantum mechanics by means of PIF concepts?

Start with the problem of a free particle, with an arbitrarily small mass (e.g. the electron mass). According to previous discussions about the double slit experiment it seems clear that at a quantum level the adequate question is: Which the probability there is to arrive at the final state F = (T, xf), starting from an initial state I = (0, xi)? Following the Sum All Alternatives (SAA) technique:

1. For each possible *x*(*t*) connecting the initial *I* and the final *F* states, calculate the corresponding action *S,* as we have proceeded previously.
2. Associate to this action *S*, a unitary vector in the plane with an angle (measured from the positive *x* axis) given by $S / \hbar$. Notice the didactic decision of replacing complex numbers by vectors in the plane, which is equivalent for the purposes of this work and more accessible to secondary school students.
3. By summing up all vectors associated with all possible paths connecting both states, and squaring its module, the probability to arrive at *F*, starting from *I* is obtained.

Figure 2 shows a position-time plot, depicting some possible paths connecting initial and final states, along with the classical path for the free particle.

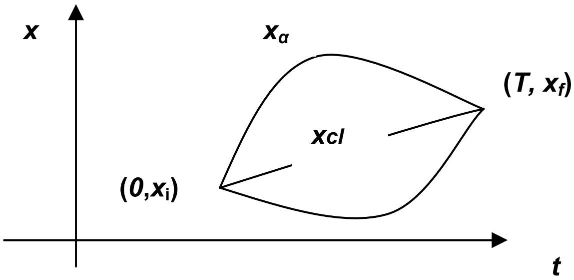


Figure 2: Some of the paths that contribute to the sum in the PIF of quantum mechanics. Each path has a definite value for the action. It is associated with a unitary vector in the plane, representing the probability amplitude

It is important to emphasize to the students the fundamental principle: each one of the alternative paths *x*(*t*) connecting the initial state *I* with the final state F contributes to the probability amplitude with a unitary vector but with a different angle, given by $S / \hbar$. The probability amplitude is the resulting vector obtained by summing each vector associated with each different alternative. Finally, the probability to go from *I* to *F* is obtained by squaring the module of the probability amplitude.

Interpretation of Double Slit Experiment for electrons

Consider a source that emits electrons at an average velocity v. Also suppose that for electrons, the detection screen, as well as the intermediate screen is impenetrable, i.e. V=∞, (except for the slits). In the rest of the space electrons are free, and therefore valid Eq.(2). The treatment will be simplified, since that it ignores the diffraction of electrons on each slit, and it supposes that each slit is a new radial source of electrons at the average velocity v. The question is: Which is the probability for an electron to arrive at a point on the detection screen, localized at a distance *x* from its centre, having started from the source? For this, suppose the amplitudes that to arrive at *x* passing through one, of the slits O_1 or the other one O_2, respectively are:

$$A(O_1 \rightarrow x) = C \exp (i\, S_{cl}\, [O_1 \rightarrow x] / \hbar\ ) \text{ and } A(O_2 \rightarrow x) = C \exp (i\ _{Scl}\, [O_2 \rightarrow x] / \hbar\ )$$

According to PIF, the resulting amplitude in this case, denoted by A(x), is the sum of the amplitudes that arrive at *x* from one and the other slit, i.e.:
$A(x) = A(O_1 \rightarrow x) + A(O_2 \rightarrow x) \Rightarrow A(x) = C(\exp (i\, Scl\, [O_1 \rightarrow x]/\hbar) + \exp (i\, Scl\, [O_2 \rightarrow x]/\hbar)$.

The classical action is $S_{cl}\, [O_{1,2} \rightarrow x] = ½.m.v^2_{1,2}\, .\, T_{1,2} = ½\, .m.\, v.\, R_{1,2}$, being $T_{1,2}$ the one that takes an electron, with an average velocity v, to arrive at *x* from each slit, and R_{12} the distances from the slits O_{12} to x (see Figure 3). The probability to arrive at *x* will be given by:

$$P(x) = |A(x)|^2 = |C(\exp i\,\alpha\,R_1 + \exp i\,\alpha\,R_2)|^2 \;;\quad \text{where } \alpha = m\,.\,v/(2.\hbar) \qquad (4)$$

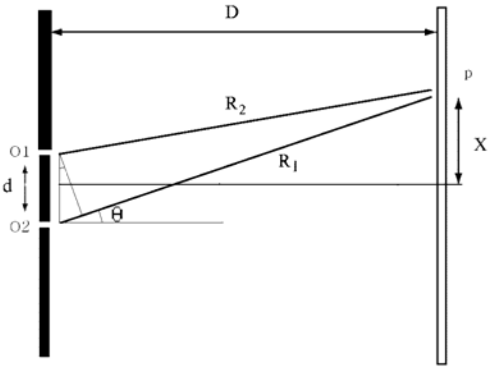


Figure 3: Schematic representation of the Double Slit Experiment.

By expanding the square in Eq.(4) we can easily obtain:

$$P(x) = 2\,C_2\,(1 + \mathrm{Cos}[\alpha\,(R_1 - R_2)]) = 4\,C\,\mathrm{Cos}^2[\alpha\,(R_1 - R_2)/2] \qquad (5)$$

Considering that the detection screen is far from the slits, we can check in Figure 3 that the following approximation is valid: $R_1 - R_2 \sim d$ sen $\theta \sim d$ tg $\theta = d\,x\,/\,R$, being d the separation between slits. Replacing in Eq.(5):

$$P\,(x) \;= c'\,\cos 2\,[(p\,/\,\hbar)\;\;(d\,x\,/\,(4\,R))], \qquad (6)$$

where $p = mv$ is the momentum of the electron and c'= 4 C2 is, for our purposes, an irrelevant factor. The usefulness of this expression is that students will now be able to understand the origin of square cosine-type distribution of relative probabilities, observed in the interference pattern of real experiments. From Eq.(6) we can note that the distribution of maxima and minima on the screen depends on two sets of parameters. One of them is related with experimental setup parameters: $d\,x\,/\,(4\,R)$. The other one depends on the intrinsic quantum nature of the electron: $p\,/\,\hbar$. Students should be motivated to reconstruct the before mentioned interference pattern by means of the evaluation of Eq.(6) and its subsequent analysis with software assistance.

It is very instructive for students, to recognize Eq.(6) as the type of equation they would obtain for the interference pattern in a double slit experiment with classical electromagnetic waves or mechanical waves of wavelength λ. By comparing these equations they should obtain a complete agreement by associating λ and p by means of: $\lambda = h\,/\,p$. De Broglie's formula, expresses the intrinsic ondulatory character of the electron and the matter in general.

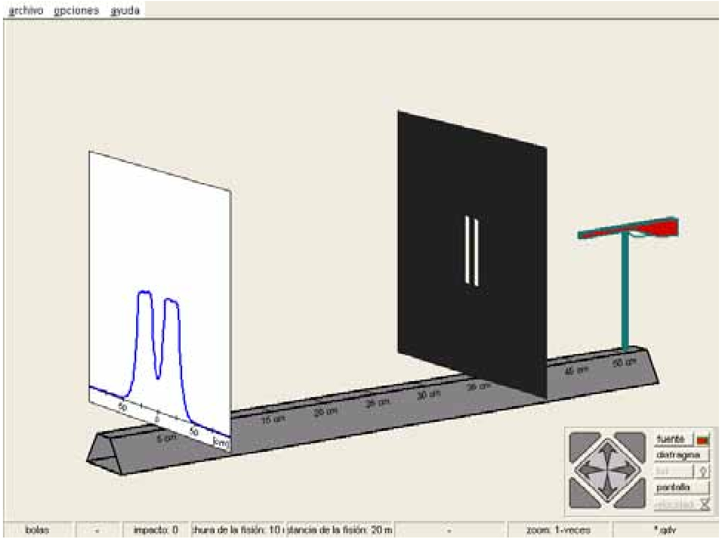
The Proposed Conceptual Structure to Teaching (PCST)

The PCST was designed for a physics course af the final year of high school with a science orientation. The group had thirty (30) 17-18 years old students It was a well performing group. The curriculum establishes two one-hour periods of Physics a week. The students had the required physics and mathematical knowledge: Classical mechanics, vectors and trigonometrical functions. The habitual work style of these students -who had been working in groups, was maintained.

The didactic sequence had thirteen lessons. The material was handed out at each period balancing appropriately the new features and problem introduction. The classes were recorded in audio together with the conversations in each work group. The main situation in the didactic sequence allows the "unexpected" distribution of electrons in the Double Slit Experiment to be explained. Using the Sum All the Alternatives (SAA) method, the expression for the probability curve of electrons to arrive at a certain point of the screen was obtained. This allowed an approach to the basic Quantum Mechanics Principles. Before applying the SAA Method to explain the results of the double slit experiment, the behaviour of the electrons and the free particle were studied. This helped the students to understand the special characteristics of the electrons' microscopic world and their differences in relation to particles of greater mass. When the students recognized the interference phenomenon for electrons, and they analysed the relationship between the interference pattern detection and the mass, an associated wavelength was assigned to the electron and to all the matter. The sequence had the following stages:

1- Double Slit Experiment (DSE) with small balls and electrons

The students imagined and predicted the results of this experience when small balls were used. Afterwards, the DSE with small balls was simulated using the "Doppelspalt"[2]. This Software permits the impacts on the screen to be observed, generates the histogram of frequencies and visualizes the theoretical curve of frequencies distribution, named P(x) or probability curve.



bolas | - | impacto: 0 | chura de la fisión: 10 | stancia de la fisión: 20 m | - | zoom: 1-veces | *.gdv

Figure 4: Screen using small balls with the separation slits to have two maximums.

The students compared their predictions for the results of the experiment with the simulated results. They solved a set of tasks analysing the effect on the form of the curve when the distance between the slits and the slits width were changed. The group was guided to agree and establish the principle:

When both slits are open, the resulting curve is the sum of the individual curves

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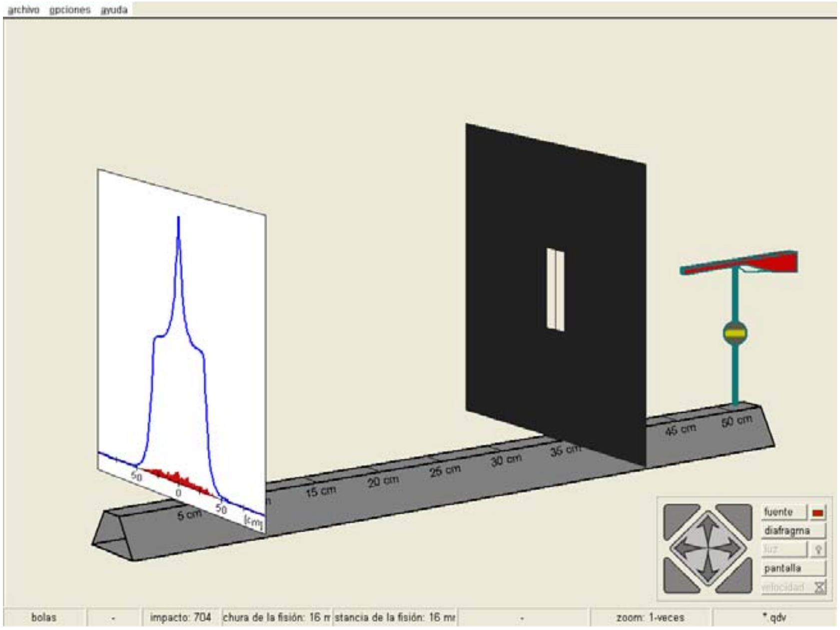
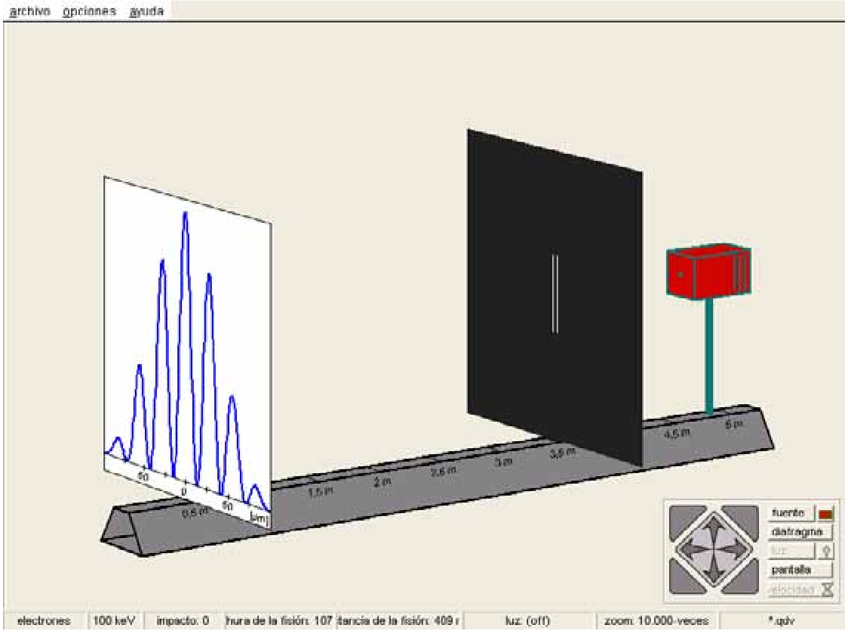
[2] "Double Slit Experiment"(2003). By Muthsam, K (Version 3.3, translated to Spanish by Wolfamann y Brickmann) Physics Education Research Group of the University of Munich. Obtenido en Internet de http://www.physik.uni-muenchen.de/didaktik/Downloads/doppelspalt/dslit.html



bolas | - | impacto: 704 | chura de la fisión: 16 m | stancia de la fisión: 16 mr | - | zoom: 1-veces | *.qdv

Figure 5: Screen using small balls with larger slits having a single central maximum

Soon the students solved the situation about the Double Slit Experiment simulation (DSE) in which they chose electrons instead of small balls. The simulation allowed students to consider that the interference pattern cannot be explained by the classical theory neither by the naive idea that the electrons are like small balls.



electrones | 100 keV | impacto: 0 | hura de la fisión: 107 | tancia de la fisión: 409 r | luz: (off) | zoom: 10.000-veces | *.qdv

Figure 6: Function $P(x)$ electrons are used like-projectiles with both slits open

A perturbation takes place; generating the necessity to look for an explanation for the unexpected behaviour of electrons. The group accepted and established another key principle in the sequence:

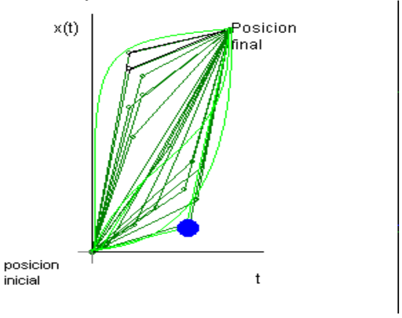
When both slits are open and even when the electrons arrive in discreet units, the resulting curve is similar to an interference pattern.

The probability curve cannot be obtained by adding the individual curves produced when the slits are opened one at a time. Then it would be inadequate to consider electrons as particles. This novel way of understanding electrons leads us to introduce the "quantum system" idea. Also, it showed that a probabilistic formulation was necessary to explain the pattern interference obtained in the double slit experiment with electrons (according to the experimental fact that these arrive at the screen as measurable and discreet units).

2- Analysing and using the SAA technique with free electrons

The sequence emphasizes the probabilistic character of predictions as the central aspect of the quantum theory. To help the students to use the "Method of multiple ways of Feynman for Quantum mechanics" the complex numbers representation is replaced by a vectorial one. The method can be applied to any physical system, like the free particle. A key didactic decision has been to start off in the free particle case, getting up the properties of quantum systems. This example joins the most general properties of these systems. The method to calculate the probability was called "Sum all the Alternatives" (SAA) and it has been presented to the students in the following steps:

1- There is not a unique form, but multiple forms to connect initial state I with the final state F - using a lot of *x (t)* - all equally possible (In order to simplify some functions were only drawn to connect the initial state with the end - straight sections-. These are the only functions in which the software used by the students allows modelling).



Then, each possible *x(t)* has an associated numerical value called action, represented by "S". The action is related to the kinetic average energy (of movement) and potential average energy (of the position with respect to other bodies with which it interacts).

$$S= (E_k - E_p)\ T$$

When the particle is "free", it is not in the presence of forces and it has zero potential energy. Then, in this case the action is directly:

$$S = E_k\ T$$

$$S = ½mv^2\ T$$

2- Using the action S, a vector on the plane is constructed, it has module one and angle of measurement S/ (respect to positive x-axis). This vector is called "Probability amplitude". The denominator of this quotient is $\hbar$ =h/2p, where h = 6.625x10-34 Js is Plank's constant and it is one of basic constant in Physics.
That is to say:

Every x(t) has a value of S
using this S, a vector is constructed:

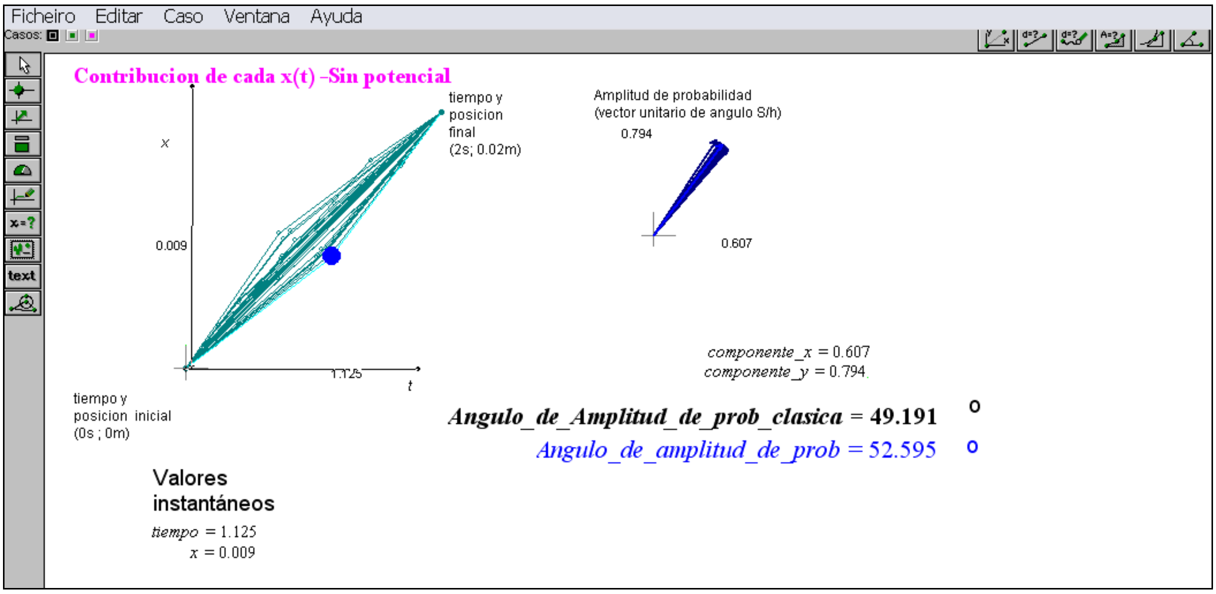
Amplitude Vector associated to each x(t) $\Downarrow$

$$(\cos\frac{S}{\hbar};sen\frac{S}{\hbar})$$

3-All the amplitude associated vectors at the different functions that connect both states initial and final are added. The Sum vector (head to tail method) is called:
"Total Probability Amplitude"

Total Probability Amplitude = Sum of all associated vectors
4- The MODULE of total probability amplitude is calculated (that is the resultant vector of the sum) and it is elevated to the square. This calculation represents the probability of arriving at the final state F, having starting from the initial state I.

In the double slit experiment electrons could be considered free from the instant they leave the source until they arrive at the screen. We could suppose they are sent at time intervals as long as there is no interaction. The analysis of the free electron allows: a) validating the technique, b) generating later on an explanation of the position of maximum and minimum obtained in the first simulation. The students were helped to apply the technique SAA to the free electron, using a Modellus[3] simulation that was specifically developed for this situation.



componente_x = 0.607
componente_y = 0.794

Angulo_de_Amplitud_de_prob_clasica = 49.191 °
Angulo_de_amplitud_de_prob = 52.595 °

Valores instantáneos
tiempo = 1.125
x = 0.009

Figure 7: Screen view of the first simulation. Selecting different functions *x(t)* that connect the initial and the final states, the simulation shows the angles on the Cartesian plane and the angle value of this vector in sexagesimal degrees. The probability amplitude vectors are drawn simultaneously for each function *x(t)* selected.

The proposed situations using the software and representing on the Cartesian plane several vectors associated to functions *x(t)* near to and distant from the classical one, allowed the CG agreed the following conclusions:

- The action S is minimum for the classical functional relation *x(t)* -a straight line- if it is compared with other arbitrary functional relations *x(t).*

- The angles of the amplitude vectors associated with those paths *x(t)* near the classical path $x_{clas}(t)$ are similar. However, the angles of the vectors associated to the *x(t)* placed far from the classical path are different from each other. This means that only a set of paths “around” the classical path contributes to the sum. Paths situated too far from the classical one, have associated vectors in different directions that will be annulled in the sum.

- When the particle mass increases, there are fewer vectors to add in the sum, because up to the near paths, they are annulled. For a macroscopic particle, in the borderline case, only the classical path $x_{clas}(t)$ is contributing to the sum.

---

[3] MODELLUS TM versión 2.5 Developed by Victor Duarte Teodoro, Joao Paulo Duque Viera; Filipe Costa Clérigo Faculty of Sciences and Technology Nova University, Lisbon, Portugal. Available in the web: http://phoenix.sce.fct.unl.pt/modellus

3- Applying the SAA method to reconstruct the interference pattern with electrons

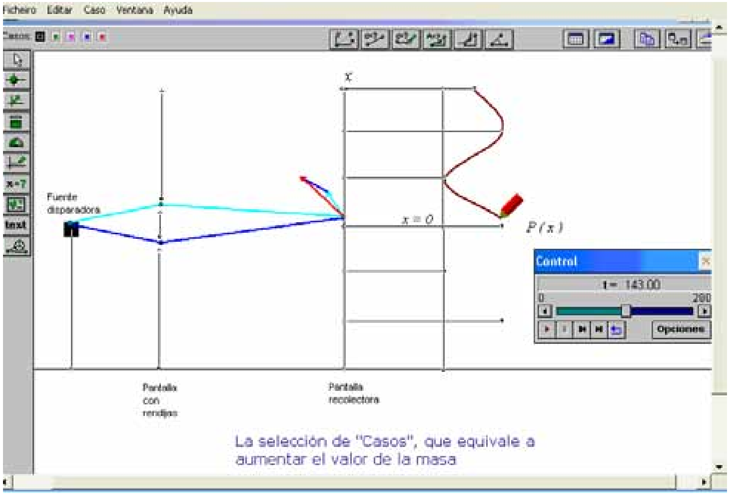
The previous stage allowed to establish, in the CG, the SAA method, and to justify an exact expression calculating the amplitude in the free particle case. Here, the conventional action plays a central role; this expression can be calculated using Feynman's method as the canonical formalism (Arlego, 2008). Soon we can ask the key question again: How probable is it that an electron starting from the source arrives at a distance *x* to the center of the screen? The answer is got by applying the method SAA to the DSE with electrons for certain experimental dispositions: the separation of the slits, the distance between the source and the screen and the electron's speed. Then, adopting a geometric-vectorial frame, some trigonometrical properties and sum of vectors the next expression of P(x) is obtained:

$$P(x) \sim \cos^2\left(\frac{md}{4\hbar T}x\right)$$

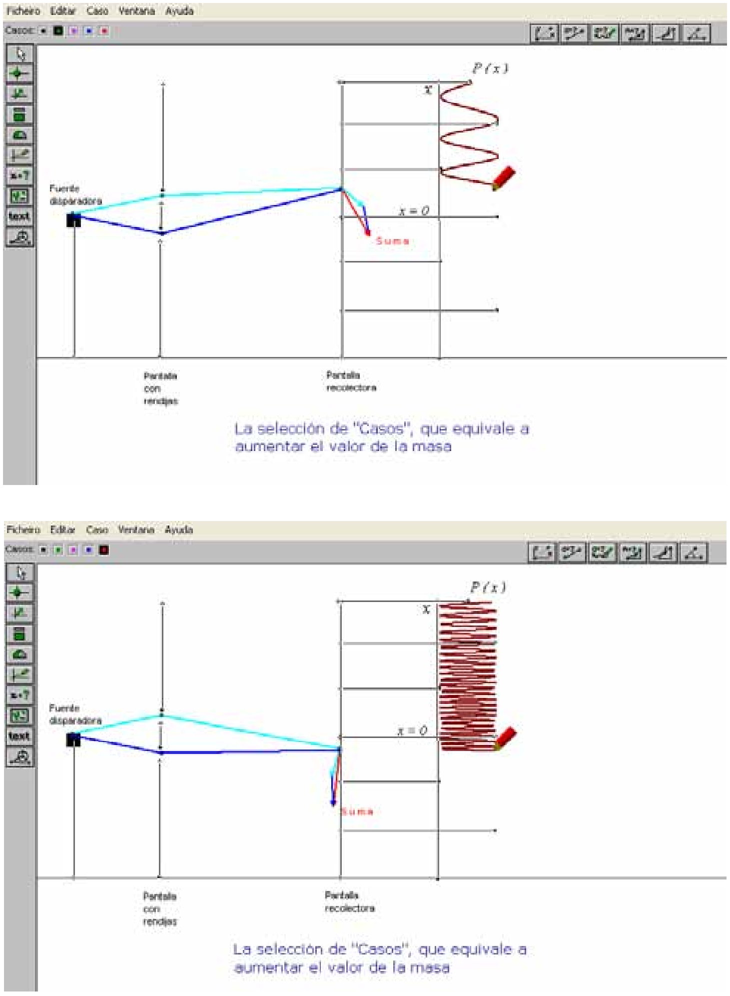
The students discussed and analysed in their group the applied procedures and the functional form of the expression P(x). Using this mathematical expression and certain experimental characteristics (separation distance, time, etc.) given by the teacher, they made an approximated graphical representation of P(x). They used the values of the independent variable suggested by the teacher observing the maximum and minimum. Without this orientation to help them the graph construction would have turned the students aside from the basic aim: to recognise that the graph has almost the same form of the graph of P(x) obtained by the first simulation. This result returns to the generating question of the sequence: How to explain the maximum and minimum of interference?

4- The Classic-quantum transition in the Double Slit Experiment

A simulation with Modellus was generated to show that the ratio between the mass and Planck’s constant generates, or not, the interference pattern. Fixing the rest of the parameters, it was observed how every larger value of the mass affected the P(x) curve. The software also draws the associated vector to each alternative – started through one slit or the other –, the extreme vector and the curve. The following figures show the interference pattern disappearing when the mass increases, making evident the transition between the quantum mechanics and the classical mechanics.



La selección de "Casos", que equivale a aumentar el valor de la masa



La selección de "Casos", que equivale a aumentar el valor de la masa

La selección de "Casos", que equivale a aumentar el valor de la masa

Figure 8 Screens showing a Double-Slit Experiment simulation

Coming back to the DSE phenomenon and according to the electrons arriving at the screen one at a time; the students were invited to analyse results of the DSE obtained by Tonomura in 1974. They looked at a series of successive photographs of a collector screen. Against this background, the concept of wavelength associated to electrons: $\lambda \propto h/(m.\ v)$ was defined, discussed and justified. After, the concept was generalized for all particles: matter behavior is not that of classical particles, and is not that of classical waves as well. Matter behavior is actually well described by the Quantum Theory.

Immediately the question was put to the students why quantum interference is not detected if the experiment is carried out with small balls. The students were invited to analyse the relationship between the associated wavelength and the interference pattern. Why it does not happen with the small balls while it is possible to detect it with electrons. In this last case, the quotient between Planck's constant and the mass is extremely small, due to the value of h; therefore, the associated wavelength is too small, and the maximums and minimums on the curve P(x) are indistinguishable, obtaining an average curve similar to the classical curve. The sequence finished analysing the role of Planck's constant as a fundamental constant in nature, to establish if the quantum behavior was evident or not.

Quantum Mechanics teaching in secondary school

Several studies in physics teaching and the curricular programs in many countries are suggesting introducing the basics of quantum theory in secondary school. In Argentina, the school physics program establishes that students must know basic concepts of quantum theory. However, if these topics are occasionally studied, it is done within chemistry courses and not in physics (Fanaro, Moreira, Otero, 2006; Fanaro, Otero, Moreira, 2007; Fanaro, Otero, Arlego, 2007). What are the reasons for this school resistance while in successive educational reforms quantum mechanics concepts are included? One reason is related to the teacher's difficulties and to the scientific community's difficulties in communicating their own knowledge. Essentially, the problem is in the teacher's preparation process. One study (Gonzalez, Fernandez y Solbes, 2000) related to teacher's knowledge, showed that teachers had studied "topics concerned with the contents of introductory books of modern physics". The researchers emphasized that teachers have limited viewpoints about quantum mechanics. The teachers relate between quantum mechanic concepts and classical ideas in an undifferentiated way. This is a big obstacle in the teaching process in school.

Another reason for not teaching quantum concepts at secondary school is the mathematical complexity involved. Some people think that the quantum theory can only be understood through manipulation of the mathematical contents. In the few attempts to introduce quantum physics at this school level, the mathematical formalism hides the fundamental principles in a small set of mathematical equations and discourages the teaching of quantum physics.

Our proposal is an introduction to quantum mechanics that does not follow the historical route and is an alternative to and complementary to the canonical formalism (Arlego, 2008). We built a Conceptual Structure of Reference based on the Feynman's Integral Path Method (1965), and we designed a Proposed Conceptual Structure for Teaching (PCST) (Otero, 2006, 2007) that has been implemented in secondary school. The objective is to rebuild with the students the main ideas without using the mathematical formalism and with simulation software assistance. Next, the students' answers to the exam are analysed and the results obtained and concepts, principles and explanations students should have learned are discussed.

Analysing the exam results

The test (see Appendix) was given to detect and describe the knowledge transformations that students were able to carry out. The students were faced with situations that required that:

- They could justify – using the principles and information related to DSE and the SAA – the electrons quantum behaviour.
- They could draw rough graphs showing the results of the DSE when different projectiles are used, comparing them with a given curve and justifying the obtained curve.
- They could calculate the associated wavelength predicting if an interference pattern would be detected or not.
- They could describe the performed tasks to apply the SAA to the DSE.
- They could anticipate the simulations' results for certain initial conditions.

The first question was if the students believed that the given propositions would be true or false. The second question was about the students' ability to draw the curve P(x) that would be obtained in the DSE when electrons and small balls are used, having as reference the protons curve showed in the test. Also, it was about similarities and differences between curves generated using different projectiles. The third question, was about calculating and understanding the meaning of the wavelength associated to the projectile used in the DSE. The fourth one was about applying the SAA to explain the DSE results, and about how the maximum and minimum interference pattern was obtained applying the SAA technique. The last question was about calculating the value of the mass of a "free" particle or quantum system in certain initial conditions, using Planck's constant to decide if it was a quantum case or not, and explaining the simulation results using the vectors associated to the nearby classical path. The reliability of this instrument was tested with the Cronbach *alpha parameter, in this case (*Moreira and Lang da Silveira, 1993).

The following table summarizes the concepts; the fundamental principles and relationships students had to learn. Various items in the test are related to the same concept, the relations between items and concepts are displayed in the last column of the table.

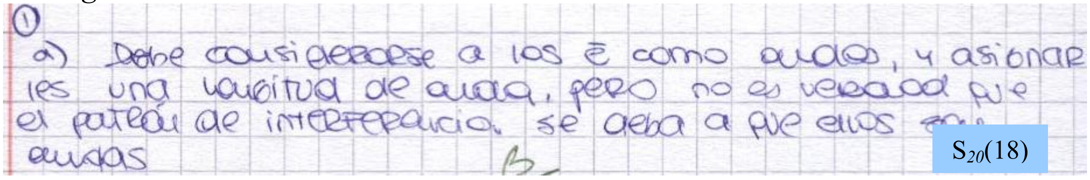
Key concepts, principles and explanations	Students Actions	Test Item
1- Electron = "quantum system"	To accept the special behaviour of the electron.	1.a)
2- SAA to free electron	**2.1- To describe the electron movement.** **2.2 –To apply the SAA technique to obtain the value of the electron mass in the established initial conditions (initial and final state, and angle of amplitude vector for classic *x(t)*) and to decide if it is a quantum system, or not.**	1.d) 5.a) 5.b)
3- The SAA applied to DSE	**To describe the SAA procedure that was carried out to explain the *P(x)*curve.**	4)
4- Classical and quantum results in DSE	**4.1- To draw the *P(x)*curve for different mass values** **4.2- To tell the difference between forming and detecting the interference pattern.**	2) 1.e)

Table 1: Concepts, key principles, explanations, and explicit actions of students related to the test items

Students' answers are analysed in the next section and examples for every kind of concept in the table are given:

1- Electron as a "Quantum System"

Nearly all the students knew the wave behavior of the electrons, but they did not accept that electrons are waves. The wave characteristic of the electrons would possibly be understood by the students as a new property additional to the particle view characteristic that electrons are striking the screen one by one. As S_{20}*(18)*[4] said *"it is not true that the interference pattern is due to the electrons being waves"*.



①
a) Debe considerarse a los ē como ondas, y asignar les una longitud de onda, pero no es verdad que el patrón de interferencia se deba a que ellos son ondas

B-

S_{20}(18)

The students did not abandon the particle viewpoint, however they did recognize the wave behavior of the electrons. Then, some students used the expression "quantum particles" about electrons, even though this word had never been used during the didactic sequence development. It would suggest that they kept the corpuscular idea adding "quantum" as an adjective. It is possible that the way they used to think, mention and imagine electrons would be an obstacle to understanding electrons as quantum systems.

However, a group of students agreed with the wave behavior of the electrons, eight mentioned "quantum system" talking about electrons, as we used to do in the class. They gave more or less detailed explanations about this new idea. In the first part of her reply S_{22}(17) wrote that it was not possible to say that electrons were neither particles or waves. She also said that one could think of them as something new, with special characteristics. S_{22} (17)'s reply is given below:

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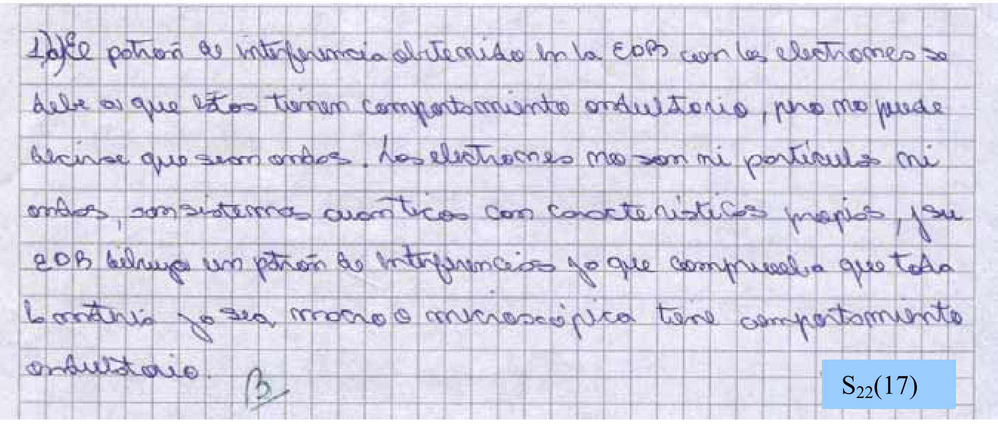
[4] Each student is identified with a nickname S_i and its age in brackets



1) d) El patrón de interferencia obtenido en la EDB con los electrones se debe a que éstos tienen comportamiento ondulatorio, pero no puede decirse que sean ondas. Los electrones no son ni partículas ni ondas, son sistemas cuánticos con características propias, y su EDB dibuja un patrón de interferencias lo que comprueba que toda la materia, lo sea, macro o microscópica tiene comportamiento ondulatorio.

B

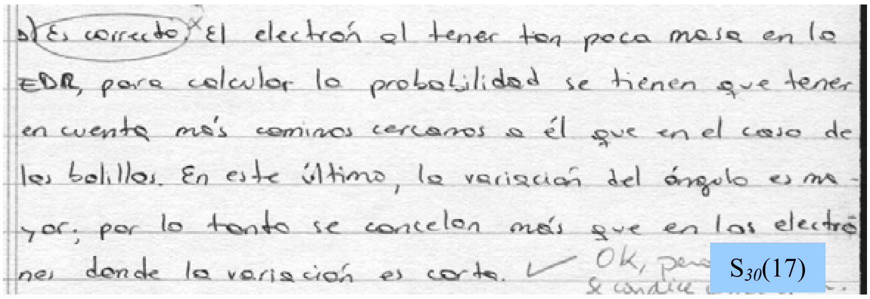
S22(17)

2- Using the SAA technique for the free electron

2.1 Describing the electron movement

By using the SAA technique and the wave behaviour of the electrons the students could infer that there is not only one specific *x(t)* to describe correctly[5] the electron motion. To calculate the probability a whole set of paths *x(t)* (paths nearest to the classical *x(t)*) must be taken into account. It did not make sense to think that it described the movement in the same way as classical mechanics. Although it could be understood that the electron motion is as if "it followed all these paths at the same time"; this idea was purposely not mentioned during the didactic sequence, because the path idea does not refer to a spacial reality, but to position-time functions connecting initial and final states. The SAA was showed as a technical calculation that takes into account a set of ways around the classical one to obtain the probability exchange from the initial to the final state. The question (1. d) evaluates if the students think that the electron takes all the paths at once or only one path – based on the "old idea" of path – ; or if they understand that it is impossible to know the free electron path, because there is not only one function *x*(*t*) that describes the electron's motion. Then, neither one nor a simultaneous set of paths is the right reply.

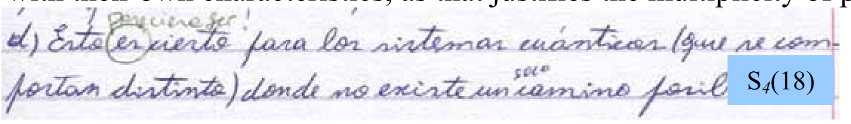
Only two students said that proposition (1.d) was true. In the other cases, the reply only mentioned the first part of the proposition, referring to the cancellation or addition of the associated amplitude of the vectors in the case of the free electron. Below, S_{30} (17) said that the proposition (1.d) was true, but she did not mention the interpretation of the SAA technique in the case of the free electron:



d) Es correcto. El electrón al tener tan poca masa en la EDR, para calcular la probabilidad se tienen que tener en cuenta más caminos cercanos a él que en el caso de las bolillas. En este último, la variación del ángulo es mayor; por lo tanto se cancelan más que en los electrones donde la variación es corta. ✓ Ok, pero

S30(17)

Student S_4(18) said that the proposition (1.d) was true and she added that electrons are quantum systems with their own characteristics, as that justifies the multiplicity of paths:



¿Parece ser!

d) Esto es cierto para los sistemas cuánticos (que se comportan distinto) donde no existe un solo camino posible

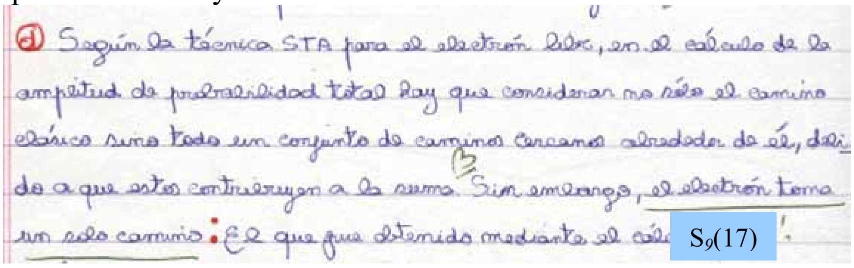
S4(18)

Twenty two students (22) said that the proposition was false. Four of them said that although

---

[5] If the function x(t) is known, it is possible to determine the position of the object in a specific time and vice- versa.

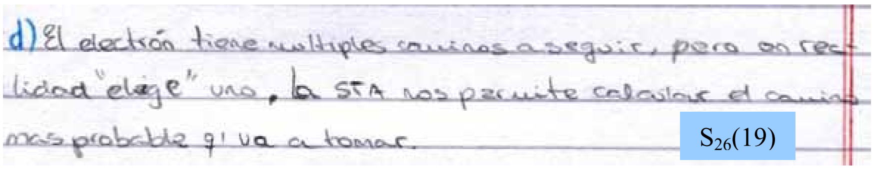
in the case of the electron a set of paths were taken into account, finally, the electron will follow only one. It seems that they maintained the classical particle idea of the electron. Student S_9(17) said that although the nearby paths were taken into account in the sum, finally, the calculation would decide the path followed by the electron:



d) Según la técnica STA para el electrón libre, en el cálculo de la
amplitud de probabilidad total hay que considerar no sólo el camino
clásico sino todo un conjunto de caminos cercanos alrededor de él, debi-
do a que estos contribuyen a la suma. Sin embargo, el electrón toma
un solo camino: El que fue obtenido mediante el cál

S_9(17)

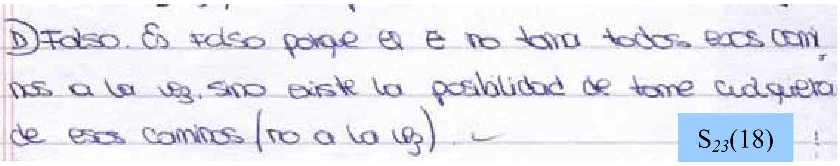
Adopting an animistic viewpoint S_{26}(19) said that she accepted the idea of probable paths, although "the electron itself will choose" because the SAA technique permits the calculation of the most probable path the electron will follow.



d) El electrón tiene multiples caminos a seguir, pero en rea-
lidad "elige" uno, la STA nos permite calcular el camino
mas probable q' va a tomar.

S_{26}(19)

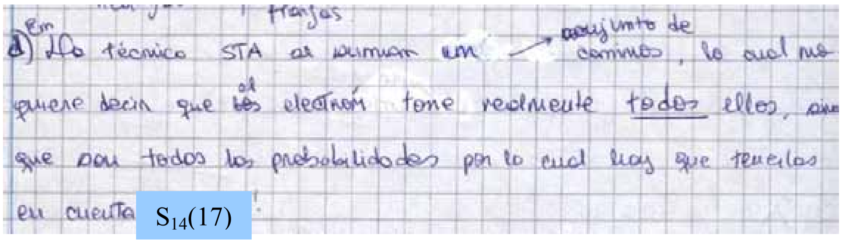
Twelve students mentioned that the SAA technique includes different probable paths for the electron, but they did not mentioned if whether or not it is possible to describe the electrons motion. For instance S_{23}(18):



D) Falso. Es falso porque el e no toma todos esos cami-
nos a la vez, sino existe la posibilidad de tome cualquiera
de esos caminos (no a la vez).

S_{23}(18)

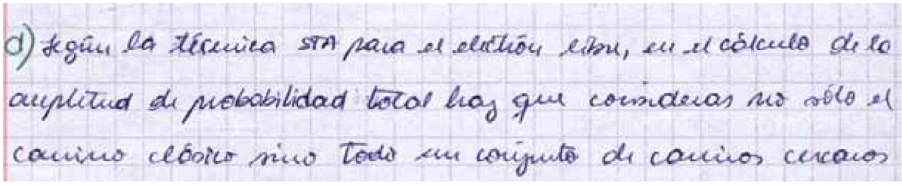
Referring to the instrumental feature of the SAA technique six students said that the SAA technique is useful only to calculate the probable path the electron will follow. Therefore it is impossible to know the path the electron will take. S_{14}(17) said "*taking into account different paths to calculate the probability does not imply making a decision for one path or another*". She did not mention the uncertain character of the motion:



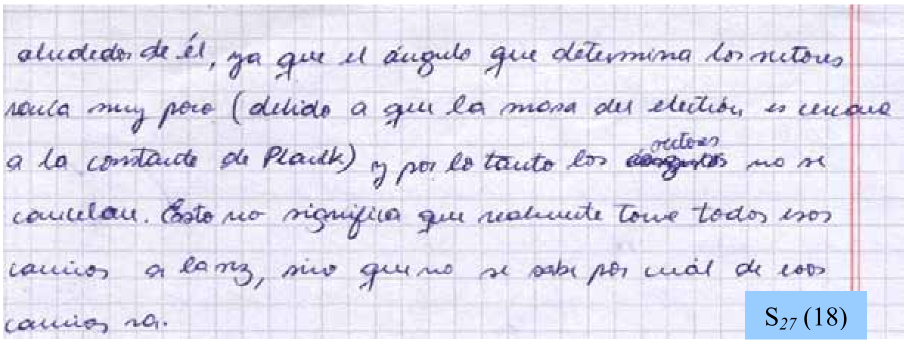
franjas
En conjunto de
d) La técnica STA se suman un → caminos, lo cual me
el
quiere decir que los electrón tome realmente todos ellos, sino
que son todas las probabilidades por lo cual hay que tenerlas
en cuenta

S_{14}(17)

Student S_{11}(17) spoke about the DSE, and said that the electron did not take all the paths.



d) Según la técnica STA para el electrón libre, en el cálculo de la
amplitud de probabilidad total hay que considerar no sólo el
camino clásico sino todo un conjunto de caminos cercanos



alrededor de él, ya que el ángulo que determina los vectores varía muy poco (debido a que la masa del electrón es cercana a la constante de Planck) y por lo tanto los ~~ángulos~~ vectores no se cancelan. Esto no significa que realmente tome todos esos caminos a la vez, sino que no se sabe por cuál de esos caminos va.

S_{27} (18)

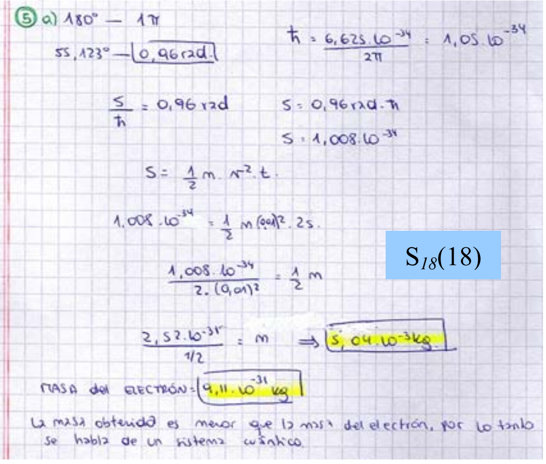
S_{27} (18)'s reply shows she understands very well the SAA technique application. She did not analyse how the paths nearest to the classical path are interpreted:

2.2 Applying the SAA technique for the case of the free electron

Twenty six (26) students mentioned the SAA technique when replying some questions in the test – sometimes in a literal way –, although it was not required. To resolve the last question in the test, it was necessary to know the SAA technique. The students had no difficulty reproducing the technique's steps, nor calculating the action S knowing the angle of the amplitude vector associated to the classical *x (t)*.

Six students tried to calculate the mass value, but they did not succeed. The rest of them were able to calculate some mass value and decide what kind of system it was. In these cases the answers can be summarized in three kinds:

a) Sixteen students (16) compared the mass value obtained with the mass of the electron. S_{18}(18) made a mistake in the last calculation and she obtained a mass value that was half of the electron mass. S_{18}(18) did not realize that she had obtained a mass value smaller than the electron. The initial parameters had been carefully selected in the sequence situations to avoid problems with mass values smaller than the electron. Although it could be physically possible, as in this case, that particles with a mass value smaller than the electron do not have an independent existence, it was decided by the researchers to consider the electron as a prototypical case of a quantum system.



⑤ a) 180° — 1π

55,123° — $\boxed{0,96\,rad}$

$\hbar = \frac{6,625 \cdot 10^{-34}}{2\pi} = 1,05 \cdot 10^{-34}$

$\frac{S}{\hbar} = 0,96\,rad$

$S = 0,96\,rad \cdot \hbar$

$S = 1,008 \cdot 10^{-34}$

$S = \frac{1}{2} m \cdot v^2 \cdot t$

$1,008 \cdot 10^{-34} = \frac{1}{2} m (0,01)^2 \cdot 2s$

$\frac{1,008 \cdot 10^{-34}}{2 \cdot (0,01)^2} = \frac{1}{2} m$

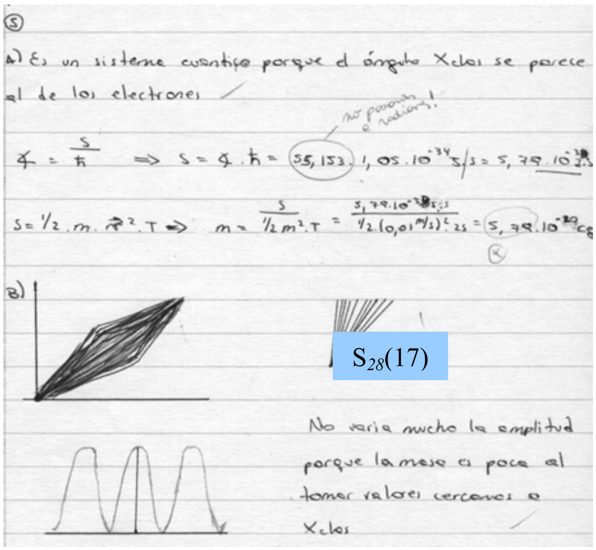
$\frac{2,52 \cdot 10^{-31}}{1/2} = m \Rightarrow \boxed{5,04 \cdot 10^{-3} kg}$

MASA del ELECTRÓN = $\boxed{9,11 \cdot 10^{-31}\,kg}$

La masa obtenida es menor que la masa del electrón, por lo tanto se habla de un sistema cuántico.

S_{18}(18)

b) Four students said that it was a quantum system because the given angle was similar to the vector angle associated to the classical path. For instance S_{28}(17) justified her conclusions as can be seen below:



⑤

a) Es un sistema cuántico porque el ángulo Xclas se parece al de los electrones

no pasan a radianes!

$\phi = \frac{S}{\hbar} \Rightarrow S = \phi \cdot \hbar = \boxed{55,153} \cdot 1,05 \cdot 10^{-34} \, J/s = 5,79 \cdot 10^{-3}$ [illegible]

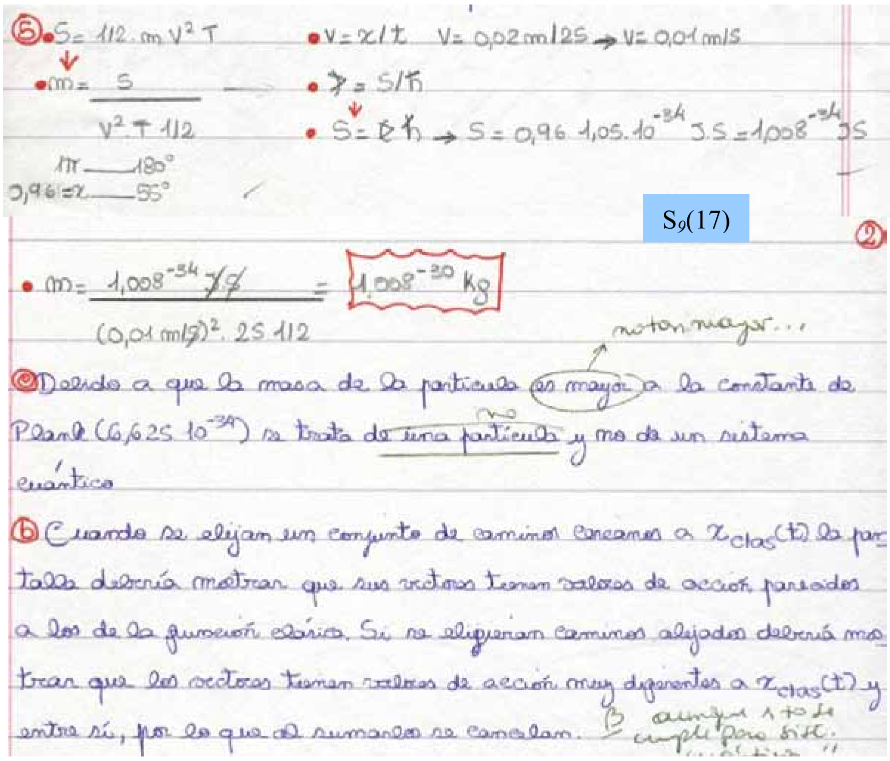
$S = 1/2 \cdot m \cdot v^2 \cdot T \Rightarrow m = \frac{S}{1/2 \, m^2 \cdot T} = \frac{5,79 \cdot 10^{-3} \, J.s}{1/2 \, (0,01 \, m/s)^2 \cdot 25} = 5,79 \cdot 10^{-29}$ cg

B)

No varía mucho la amplitud porque la masa es poca al tomar valores cercanos a Xclas

$S_{28}(17)$

c) Four students used Planck's constant to decide if it was a quantum system or not. $S_9(17)$ correctly calculated the mass and compared it to Planck's constant, instead of comparing it to the electron mass, then she said that it was not a quantum system.



⑤ • $S = 1/2 \cdot m \, V^2 \, T$

• $m = \frac{S}{V^2 \cdot T \cdot 1/2}$

• $V = x/t \quad V = 0,02 \, m/25 \rightarrow V = 0,01 \, m/s$

• $\phi = S/\hbar$

• $S = \phi \hbar \rightarrow S = 0,96 \cdot 1,05 \cdot 10^{-34} \, J.S = 1,008^{-34} \, JS$

1π —— 180°

$0,96 = x$ —— 55°

$S_9(17)$

②

• $m = \frac{1,008^{-34} \, \cancel{J}/\cancel{s}}{(0,01 \, m/s)^2 \cdot 25 \cdot 1/2} = \boxed{1,008^{-30} \, kg}$

notan mayor...

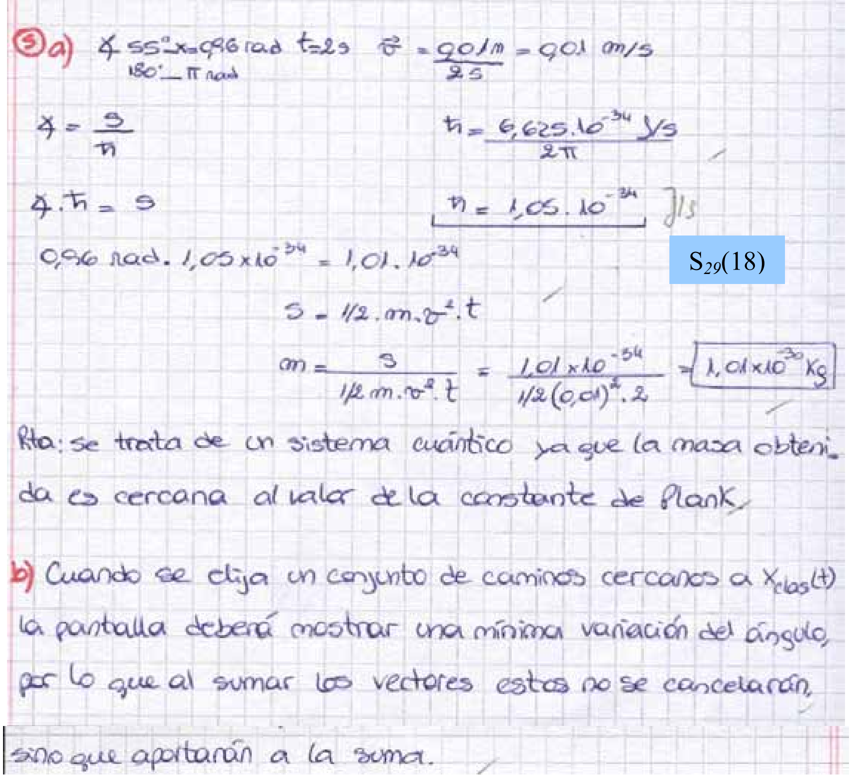
ⓒ Debido a que la masa de la partícula es mayor a la constante de Planck (6,625 10^{-34}) se trata de una partícula y no de un sistema cuántico

no

ⓑ Cuando se elijan un conjunto de caminos cercanos a $x_{clas}(t)$ la partícula debería mostrar que sus vectores tienen valores de acción parecidos a los de la función clásica. Si se eligieran caminos alejados debería mostrar que los vectores tienen valores de acción muy diferentes a $x_{clas}(t)$ y entre sí, por lo que al sumarlos se cancelan.

B aunque esto se cumple para sist.

$S_{29}(18)$'s answer had similar calculations to $S_9(17)$, but a different interpretation was made. $S_9(17)$ compared the value of the mass to Planck's constant and answered that it was a quantum system.



③ a) $\measuredangle\ 55° \times = 0{,}96\ rad$ $t = 2s$ $\vec{v} = \frac{0{,}01\ m}{2s} = 0{,}01\ m/s$

$180° \_\ \pi\ rad$

$\measuredangle = \frac{S}{\hbar}$ $\hbar = \frac{6{,}625 \cdot 10^{-34}\ Js}{2\pi}$

$\measuredangle \cdot \hbar = S$ $\underline{\hbar = 1{,}05 \cdot 10^{-34}}\ J/s$

$0{,}96\ rad \cdot 1{,}05 \times 10^{-34} = 1{,}01 \cdot 10^{-34}$

$S = 1/2 \cdot m \cdot v^2 \cdot t$

$m = \frac{S}{1/2\, m \cdot v^2 \cdot t} = \frac{1{,}01 \times 10^{-34}}{1/2(0{,}01)^2 \cdot 2} \rightarrow \boxed{1{,}01 \times 10^{-30}\ Kg}$

Rta: se trata de un sistema cuántico ya que la masa obtenida es cercana al valor de la constante de Plank.

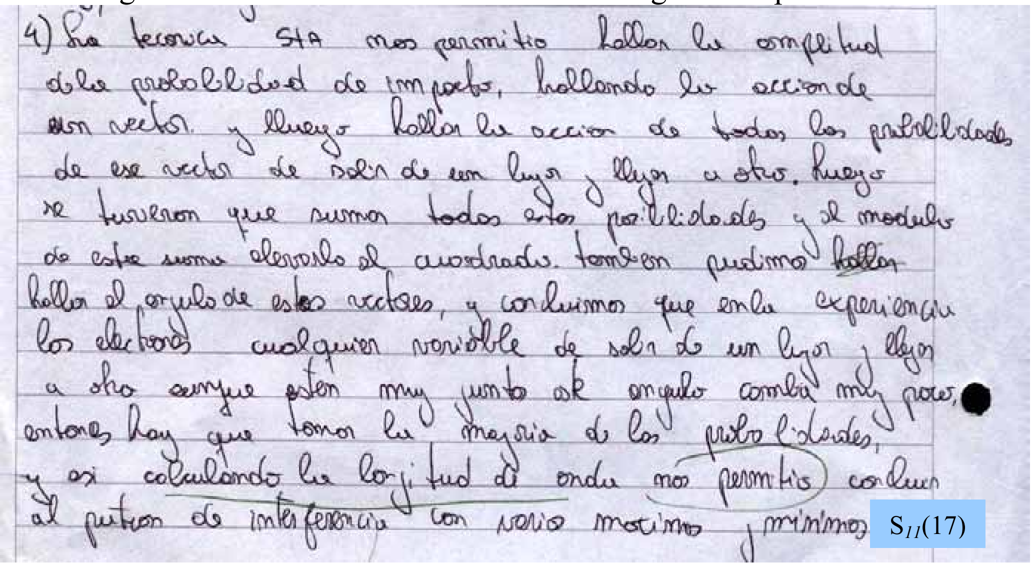
b) Cuando se elija un conjunto de caminos cercanos a $X_{clas}(t)$ la pantalla deberá mostrar una mínima variación del ángulo, por lo que al sumar los vectores estos no se cancelarán, sino que aportarán a la suma.

S$_{29}$(18)

3- Applying the SAA technique to the DSE

The fourth question in the test is about justifying why the *P(x)* expression obtained by applying the SAA technique to the DSE with electrons explains the interference pattern. The answer required was that the cosign square expression of *P(x)* agreed with the maximum and minimum of the curve obtained with the software (although without modulation). All students gave some kind of answer, that that can be classified as follows:

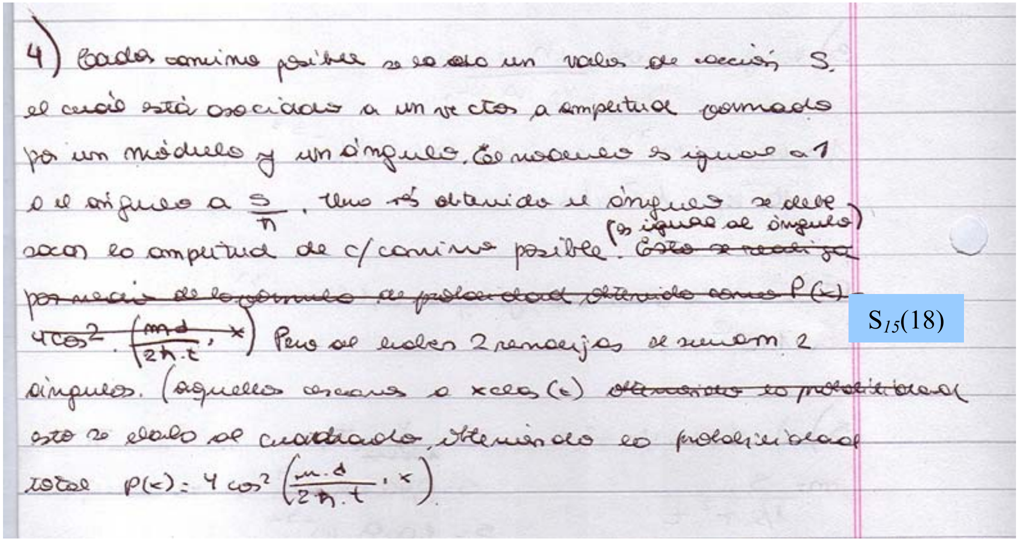
a) Six students could not explain the aspect of the *P(x)* curve for the electrons. They only described the general procedures of the SAA technique without applying it to the DSE with electrons.
b) Eight students described the procedures of the SAA technique and their results related to vectors. But, they were unable to explain the relationship between the SAA and the interference pattern resulting in the DSE. S$_{11}$(17) described the procedures of the DSE using electrons, but he did not write the *P(x)*expression, then he was unable to explain the maximum and minimum pattern. His answer is interesting because he used the associated wavelength concept.



4) La tecnica StA nos permitio hallar la amplitud de la probabilidad de impacto, hallando la accion de un vector y luego hallar la accion de todas las probabilidades de ese vector de salir de un lugar y llegar a otro. luego se tuvieron que sumar todas estas posibilidades y el modulo de esta suma elevarlo al cuadrado. tambien pudimos ~~hallar~~ hallar el angulo de estos vectores, y concluimos que en la experiencia los electrones cualquier variable de salir de un lugar y llegar a otro aunque esten muy junto el angulo cambia muy poco, entonces hay que tomar la mayoria de las probabilidades, y asi calculando la longitud de onda nos permitio concluir al patron de interferencia con varios maximos y minimos

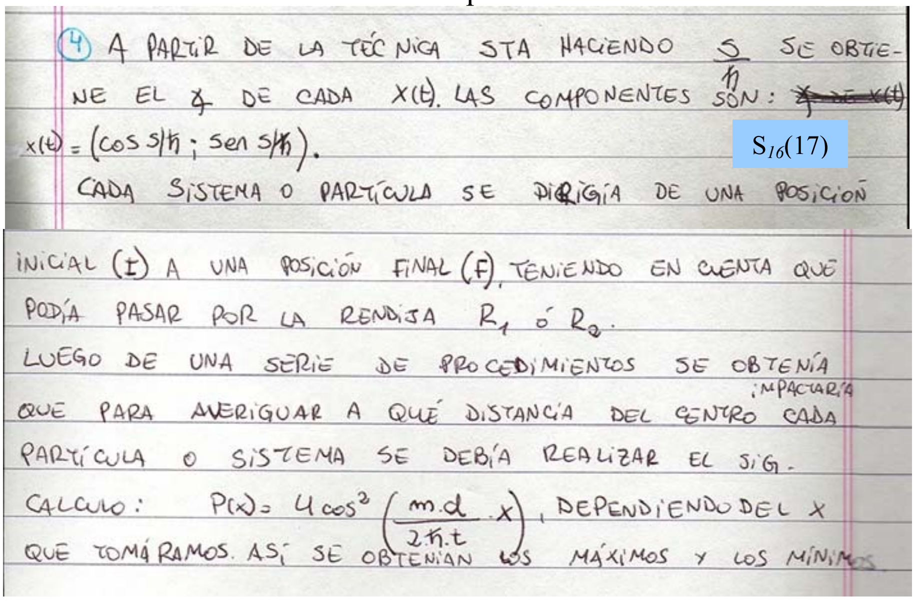
S$_{11}$(17)

c) More than half of the students (16) described the technique SAA application in the DSE. They wrote the resultant *P(x)* expression explaining the maximum and minimum. Shown below, is S_{15}(18)'s reply:



4) Cada camino posible se le asocia un valor de acción S,
el cual está asociado a un vector amplitud formado
por un módulo y un ángulo. El módulo es igual a 1
y el ángulo a $\frac{S}{\hbar}$. Una vez obtenido el ángulo se debe
(es igual al ángulo)
sacar la amplitud de c/camino posible. ~~Esto se realiza~~
~~por medio de la fórmula de probabilidad obtenida como P(x) =~~
~~$4\cos^2 \cdot \left(\frac{m.d}{2\hbar.t} \cdot x\right)$~~ Pero al haber 2 rendijas se suman 2
ángulos. (aquellos cercanos a x cero (c)) ~~obteniendo la probabilidad~~
esto se eleva al cuadrado, obteniendo la probabilidad
total $P(x) = 4\cos^2\left(\frac{m.d}{2\hbar.t} \cdot x\right)$.

S_{16}(17) said that the *P(x)* expression allowed the maximum and minimum obtained in the DSE when it was simulated with electrons to be explained:

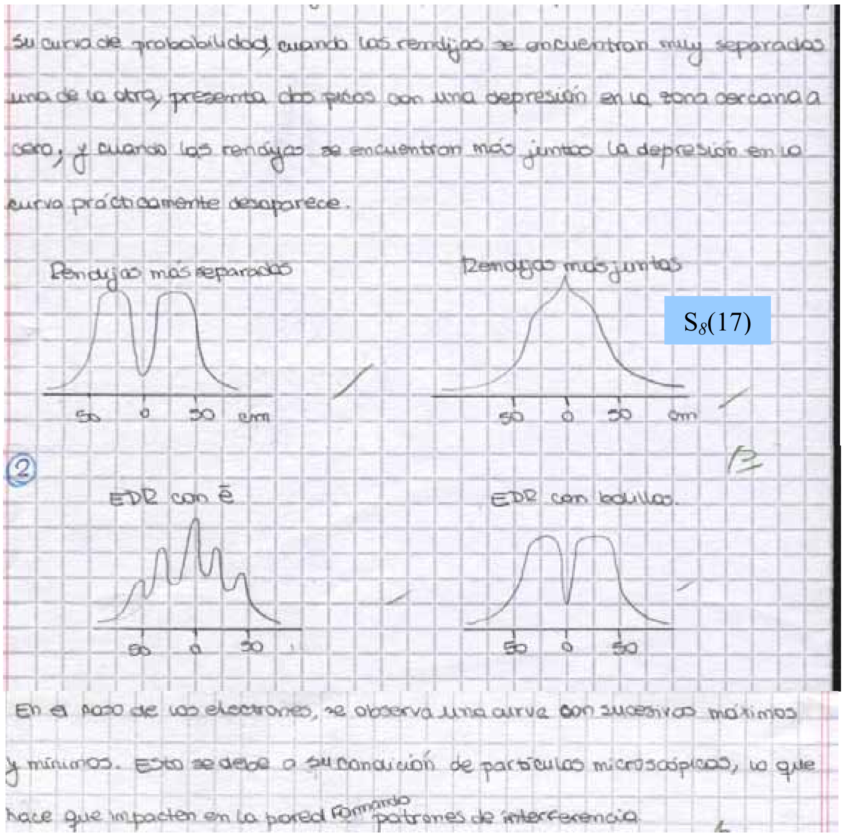


④ A PARTIR DE LA TÉCNICA STA HACIENDO $\frac{S}{\hbar}$ SE OBTIE-
NE EL ∡ DE CADA X(t). LAS COMPONENTES SON: ~~X DE X(t)~~
$x(t) = (\cos S/\hbar \,;\, \text{sen}\, S/\hbar)$.
CADA SISTEMA O PARTÍCULA SE DIRIGÍA DE UNA POSICIÓN
INICIAL (I) A UNA POSICIÓN FINAL (F), TENIENDO EN CUENTA QUE
PODÍA PASAR POR LA RENDIJA R_1 Ó R_2.
LUEGO DE UNA SERIE DE PROCEDIMIENTOS SE OBTENÍA
QUE PARA AVERIGUAR A QUÉ DISTANCIA DEL CENTRO IMPACTARÍA CADA
PARTÍCULA O SISTEMA SE DEBÍA REALIZAR EL SIG.
CÁLCULO: $P(x) = 4\cos^2\left(\frac{m.d}{2\hbar.t} . x\right)$, DEPENDIENDO DEL X
QUE TOMÁRAMOS. ASÍ SE OBTENÍAN LOS MÁXIMOS Y LOS MÍNIMOS.

4- Classical and quantum results in the DSE

4.1 Drawing *P(x)*for different mass values

The second question in the test asked the students to sketch the *P(x)* graph for electrons and small balls in the DSE, based on a given protons' curve. Taking into account the mass of the electrons and the small balls, the graphs had to show the different distances between maximums and minimums. Twenty students drew the two curves of *P(x)* correctly and they used the probability idea. But, they did not mention the different maximum separations between the curves drawn and the given protons' curve. S_8(17) drew both curves – small balls and electrons –. She analysed the changes in the small balls' curve taking into account the separation of the slits. Nevertheless in the electrons' case she correctly drew the experimental curve, and in the small balls' case she drew the experimental curve with little detail.



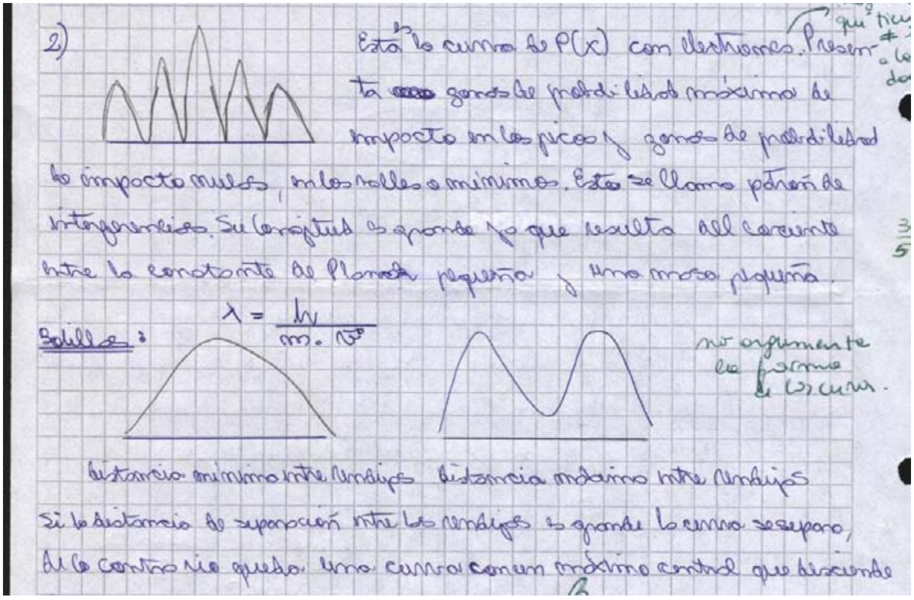
Su curva de probabilidad, cuando las rendijas se encuentran muy separadas una de la otra, presenta dos picos con una depresión en la zona cercana a cero, y cuando las rendijas se encuentran más juntas la depresión en la curva prácticamente desaparece.

②

En el caso de los electrones, se observa una curva con sucesivos máximos y mínimos. Esto se debe a su condición de partículas microscópicas, lo que hace que impacten en la pared formando patrones de interferencia.

$S_8(17)$

Four students drew the *P(x)* curves with electrons and small balls and they justified the graph form using the associated wavelength in every case. Below, is presented $S_{22}(17)$'s answer:



2) Esta es la curva de P(x) con electrones. Presenta ~~[illegible]~~ zonas de probabilidad máxima de impacto en los picos y zonas de probabilidad de impacto nulas, en los valles o mínimos. Esto se llama patrón de interferencia. Su longitud es grande ya que resulta del cociente entre la constante de Planck pequeña y una masa pequeña.

$\lambda = \frac{h}{m \cdot v}$

Bolillas:

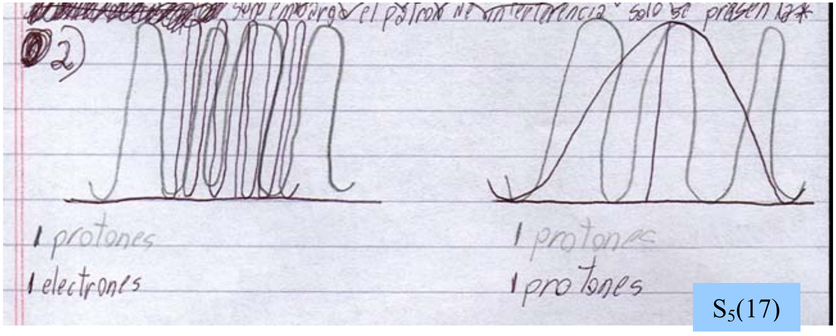
distancia mínima entre rendijas distancia máxima entre rendijas

Si la distancia de separación entre las rendijas es grande la curva se separa, al lo contrario queda una curva con un máximo central que desciende

no argumenta la forma de la curva.

3/5

Six students drew graphs taking into account the relationship between the maximums and the associated wavelength. $S_5(17)$ drew the graph, but he did not justify his drawing:

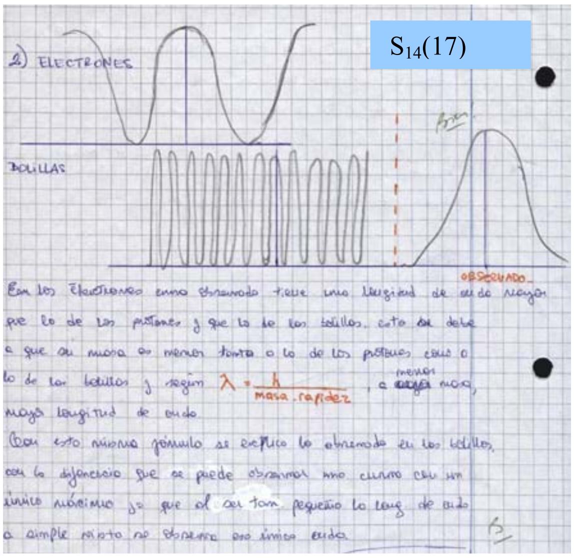


~~[illegible]~~ superponga el patrón de interferencia solo se presentan*

2)

$S_5(17)$

Student S_{14}(17) explicitly related the form of the graphs with the associated wavelength, as well as drawing it:



En los Electrones como observado tiene una longitud de onda mayor que la de los protones y que la de los bolillas. esto se debe a que su masa es menor tanto a la de los protones como a la de los bolillas y según $\lambda = \frac{h}{masa . rapidez}$, a ~~mayor~~ menor masa, mayor longitud de onda

Con esta misma fórmula se explica lo obtenido en los bolillas, con la diferencia que se puede observar una curva con un único máximo ya que al ser tan pequeña la long. de onda a simple vista no observamos un único onda.

4.2 Making a distinction between performing and detecting the interference pattern

When the class group work finished, students and teacher agreed that explanations given by Quantum Mechanics must coincide with results obtained by Classical Mechanics both in the microscopic case and in the small balls. Also, the students analysed the simulations using small balls and they discussed the graphs of theoretical and experimental curves.

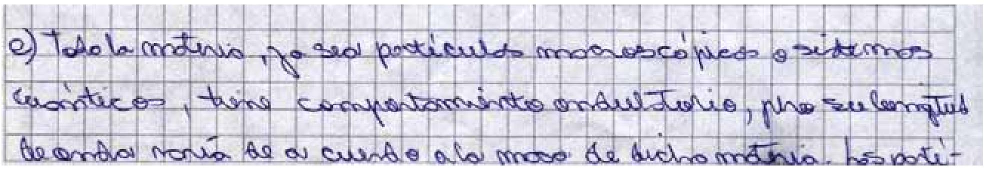
If students were able to answer to the test question (1.e) and if they wrote about performing and detecting the interference pattern, it could be said that they understood the quantum-classic transition and the macroscopic situation as a limit case of the quantum. Only two students did not answer, six students only referred to graphs they had seen in the software outputs, perhaps, because they saw the software as an explanatory mechanism.

Twenty two students wrote an explanation, and their answers could be classified as below:

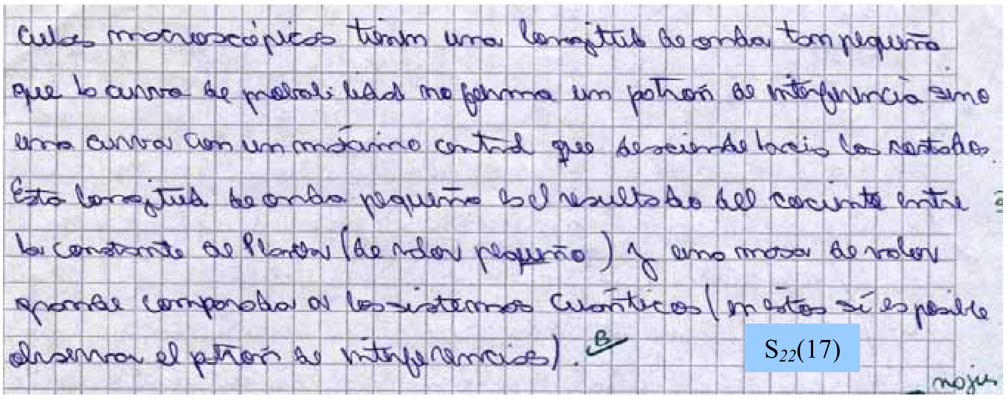
Rejecting the given proposition, without reflecting about theoretical and experimental results six students said that only if the wavelength is big enough the interference pattern will be formed, as in the case of the small mass particles or in the electrons. They did not understand the difference between the detection of the experimental interference pattern and the fact that it is formed in the same way even though it can not be perceived. As one of them (*17*) said *"It's false. The interference pattern was only formed with microscopic particles, because it has a smaller frequency compared to macroscopic particles"*

a) Differentiating between the interference pattern formation and its experimental detection.

Eighteen students said that although the pattern is formed, it can not always be seen or perceived with the experimental machines. S_{22}(17) offered an explanation based on the quotient between Planck's constant and the mass, as was discussed during the class.



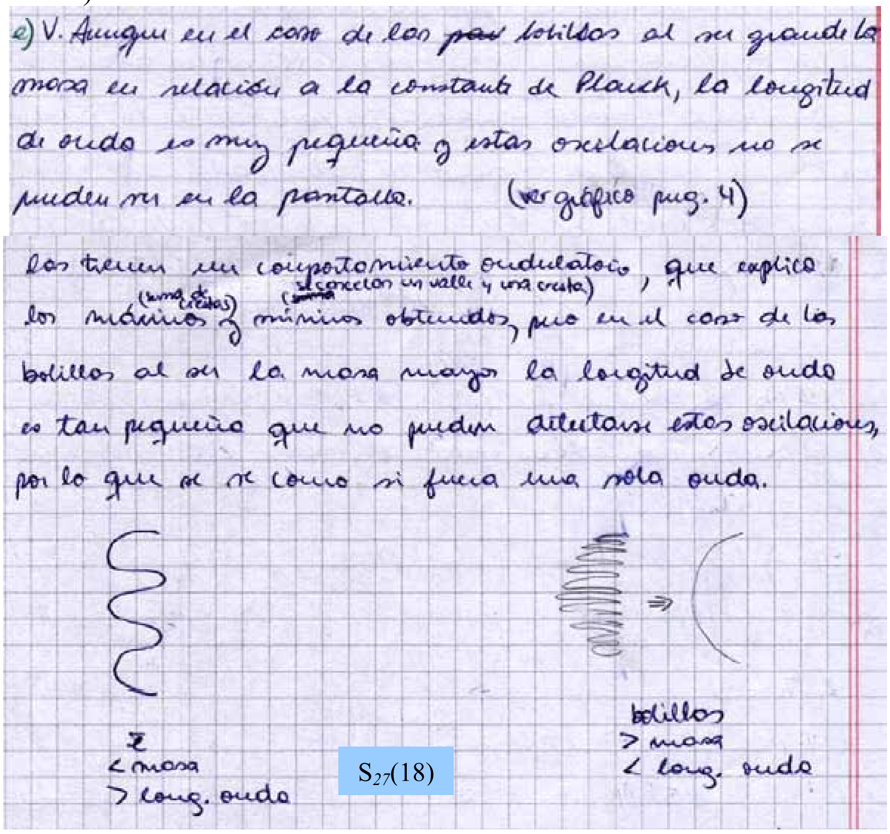
e) Toda la materia, ya sea partículas macroscópicas o sistemas cuánticos, tiene comportamiento ondulatorio, pero su longitud de onda varía de acuerdo a la masa de dicha materia. Las partí-



culas macroscópicos tienen una longitud de onda tan pequeña que la curva de probabilidad no forma un patrón de interferencia sino una curva con un máximo central que desciende hacia los costados. Esta longitud de onda pequeña es el resultado del cociente entre la constante de Planck (de valor pequeño) y una masa de valor grande comparada a los sistemas cuánticos (en estos sí es posible observar el patrón de interferencias). B

S22(17)

S_{27}(18) mentioned the difference between the phenomenon and its detection, she drew the graph of the *P(x)*curve for electrons and small balls. She drew the two curves *P*(*x*), on one hand the theoretical curve predicted by the SAA technique, and on the other hand the experimentally obtained (conditioned):

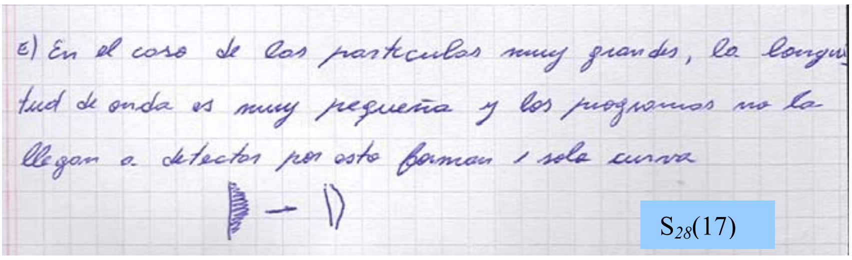


e) V. Aunque en el caso de los ~~par~~ bolillos al ser grande la masa en relación a la constante de Planck, la longitud de onda es muy pequeña y estas oscilaciones no se pueden ver en la pantalla. (ver gráfico pág. 4)

Los tienen un comportamiento ondulatorio, que explica los máximos (suma de crestas) y mínimos (se anulan un valle y una cresta) obtenidos, pero en el caso de los bolillos al ser la masa mayor la longitud de onda es tan pequeña que no pueden detectarse estas oscilaciones, por lo que se ve como si fuera una sola onda.

S27(18)

S_{28}(17) replied briefly adding at the end a diagram representing the situation:



e) En el caso de las partículas muy grandes, la longitud de onda es muy pequeña y los programas no la llegan a detectar por esto forman 1 sola curva

S28(17)

S_{28}(17) could have thought that classical mechanics laws are a limit case of quantum mechanics when the objects are macroscopic because she said: "in quantum physics, if the mass increases the results are the same as in classical physics."

Conclusion

As it has been mentioned before, this paper only analyses the students' answers in the final test. This test is just one of the tools that allows the evaluation of how a single situation and the whole sequence work. It was a big effort to carry out the didactic sequence as well as to design and analyse it beforehand. Then the teacher and the students had to work hard in the class. The class group collaborated very much and accepted the proposal and the challenge in every situation. The implementation was carried out in the scheduled time and the students performed the proposed tasks, in spite of the difficulties faced.

The analysis of the written test seems to indicate that:

- The students consider that the electrons have a special and characteristic behavior that allows us to think about them as quantum systems.
- Most students are not able to accept the impossibility of knowing which would be the function that describes the electron motion. After the sequence, such as the test results showed, they still thought: *"finally, the electron must take some path or other"*
- The students agreed that the SAA technique is a suitable mechanism to explain the interference pattern in the Double-Slit Experiment, in other ways inexplicable.
- The students understood that the wave behaviour allows to associate a wavelength as much to the macroscopic particles as to the microscopic ones.
- The students related the shape and detection interference pattern in the macroscopic and microscopic particles cases.

The classical concept of path in space was an obstacle to understanding the path concept established in this didactic sequence. This restricted the interpretation and its consequences when all contributions of different paths $x(t)$ were considered, calculating the probability. This difficulty could be minimized if in the first physics courses the association between the physical path and the image of the single and deterministic path of the instrumental and functional viewpoint is avoided. Quantum mechanics teaching requires emphasising the idea that physics does not involve "reality", but builds abstract models, and within them our old and crystallized images are inappropriate.

It has been a very complex process reducing and managing the knowledge of physics in this conceptual field to make it teachable in secondary school. To decide which concepts and principles could be studied is complicated, and how a Proposed Conceptual Structure for Teaching (PCST) could be designed, carried out and adjusted. We consider the (PCST) outlined here as just a beginning to discussing, modifying and talking to physicists, researchers in physics teaching and teachers. Without consultation with these three groups of actors, it would be impossible to bring alive knowledge that will bridge the gap between school and the scientific community.

At the moment we are analysing in depth all the protocols in their entirety about the six class groups and the synthesizing activities where the teacher and the students are interacting. We want to describe the teacher and student's actions, and the didactic subjects that appeared in the situations effectively developed in the classes.

The sequence is viable and we have repeated it twice, with adjustments and improvements to the proposed situations. But, there are lots of questions still to be answered: such as which the first obstacles for the conceptualization of the quantum mechanics concepts are. What we could do in the scholar genesis of physical concepts to help the conceptualizations. Which kind of interactions between the teacher and students are better to support the cognitive effort required by the sequence. How the emotional aspects are impacting the sequence development. These are only some of the challenges that await us.

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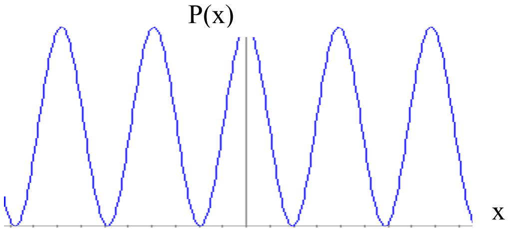
APPENDIX: Test

1- Analyse if the propositions below are true or false. Justify your answers.

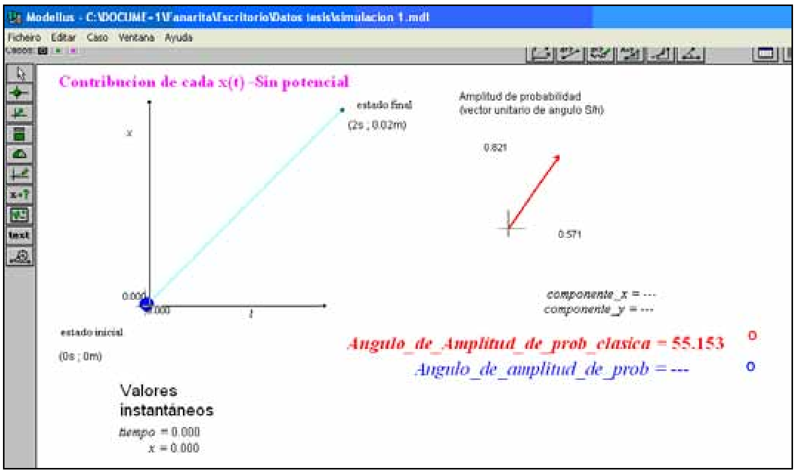
a) The Interference Pattern obtained in the DSE with electrons must be that they are waves
b) The matter in the SAA technique is that probabilities are added.
c) The final appearance of the impacts produced by the small balls on the screen in the DSE are always two columns like a projection of the slits.
d) As the SAA technique establishes for free electrons, when probability amplitude is calculated it must take into account, not only the classical path but all the nearby paths around it. This means that in reality the electron takes all the paths simultaneously.
e) The wavelength associated to the particles, always produces an interference pattern.

2- The next figure shows approximately the *P(x)* curve resultant from DSE experiment with both slits opened, using protons like projectiles. Draw a graph corresponding approximately to the *P(x)*curve when the experiment is realized with electrons and small balls, justifying in every case.



3- Supposing that, if the experiment is realized using particles with mass around 10^{-10} kg, an interference pattern has been detected on the screen during the DSE, just when the wavelength associated to the projectiles is 10^{-15} m. Will an interference pattern be seen on the screen?
4- Why does the SAA technique allow us to justify the interference pattern obtained with electrons in the DSE? Discuss the procedures taken.

5- If a simulation using Modellus is carried out, it will be represented on the screen as below:



componente_x = ---
componente_y = ---
Angulo_de_Amplitud_de_prob_clasica = 55.153
Angulo_de_amplitud_de_prob = ---
tiempo = 0.000
x = 0.000

a) ¿Is it a quantum system or a particle?
b) ¿What would the screen show when a set of paths near to the classical path $x_{clas}(t)$ are selected ?