DIGITAL DS-CDMA DETECTION IN IMPULSIVE NOISE: BASE-BAND vs. BAND-PASS NONLINEAR PROCESSING

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Abstract— **In the context of *DS-CDMA* detection, a digital base-band model for the received signal after chip-matched filtering, is often adopted. Nonlinear treatment of these samples for robust detection in impulsive environments has been already proposed. In this paper we study the advantages of band-pass nonlinear detection schemes for *DS-CDMA* receivers, over those in base-band. We consider for both schemes i) an impulsive noise model with independent α-stable distributed sequences and ii) a hard-limiter as memoryless nonlinearity. Probability of error and asymptotic relative efficiency expressions are given, with simulations that validate them. Results shown that band-pass nonlinear detection is preferable.**

Keywords— **DS-CDMA, Nonlinear Detection, Impulsive Noise, α-stable Distribution, Hard Limiter.**

I. INTRODUCTION

The most widespread model for the channel noise in *DS-CDMA* systems is additive Gaussian, justified by the central limit theorem. However, the presence of man-made and atmospheric electromagnetic interferences in urban areas called for other noise models with impulsive behavior. One of the simplest models of impulsive channels uses ϵ-mixtures, as in Aazhang and Poor (1988). In the last decade, the family of α-stable distributions received great attention as an appropriate model for impulsive phenomena (Shao and Nikias, 1993). Due to the generalized central limit theorem (Grigoriu, 1995) they can be applied when the noise is thought as a superposition of infinitely many statistically independent sources. The index α parameterizes the level of impulsiveness ($0 < \alpha \leq 2$), including the Gaussian distribution for $\alpha = 2$ and the Cauchy distribution for $\alpha = 1$. These facts and other important mathematical properties, comparable to the gaussian distribution, make the α-stable model attractive. However, there are not closed expressions for their probability densities except for $\alpha = 1$ and $\alpha = 2$, and moments of order greater than α do not exist. Then, except for $\alpha = 2$, the model forces an infinite variance noise.

The performance of linear receivers, which are optimum in the gaussian case, degrades drastically when operating in impulsive environments. In the schemes we found in the literature of robust *DS-CDMA* receivers, the received signal is first chip-matched filtered (i.e. linearly processed). The base-band signal is then passed through a memoryless nonlinearity and finally correlated with the local replica of the spreading code to form the test statistic whose sign decides the estimated bit (Aazhang and Poor, 1988; Aazhang and Poor, 1989; Deliç and Hocanin, 2002; Chuah and Hinton, 2000). This scheme is justified with the concept of locally optimum detection (Spaulding, 1985). That is, for signal to noise ratio (*SNR*) tending to zero the Maximum Likelihood (*ML*) detector becomes a memoryless nonlinearity followed by the conventional correlator, when the signal and noise are at base-band.

However, the impulsive noise appears already in the samples of the received signal, at the front-end or band-pass stage, and propagates into the base-band signal. Moreover, the assumption of low *SNR* is more adequate at the band-pass samples, prior to any averaging. Then, we propose to place the nonlinearity directly at the band-pass stage. In this way, the occasionally large impulses are "suppressed" before they can corrupt other samples. We show results where there are considerable advantages of band-pass nonlinear detection structure over the base-band.

This paper is organized as follows. In section II we describe the model of received signal and the detection systems to consider. In section III the performance of these systems is calculated using an asymptotic analysis. The results of these calculations are compared with simulations in IV, and we conclude in section V.

II. SYSTEM MODEL AND DETECTION SCHEMES

We consider the digital detection of a *BPSK* signal, transmitted using *DS-CDMA* through a channel with additive white impulsive noise. There is a single user with perfect code and carrier synchronization. The samples of the received signal during a bit interval can be expressed as

$$r[n] = bAs[n]\cos(\Omega_c n + \phi) + v[n] \tag{1}$$

where $b \in \{+1, -1\}$ is the transmitted bit, A is the amplitude of the received signal, $s[n]$ are samples of the spreading code (with values +1 or -1) with K chips per bit, and

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$v[n]$ is a sequence of independent identically distributed (i.i.d.) symmetric α-stable random variables ($S\alpha S$) with dispersion parameter σ (Grigoriu, 1995).

The α-stable random variables lack closed expressions of their probability densities but they are described by their characteristic function (Grigoriu, 1995). In our case,

$$\phi_v(\omega) = e^{|\sigma\omega|^{\alpha}} \tag{2}$$

The standard *SNR* becomes useless in this context, since $S\alpha S$ random variables have infinite variance (except for $\alpha = 2$, the gaussian case). Thus, a strength measure, well-defined for the complete $S\alpha S$ family is used: the geometric power (Gonzalez, 1997) given by

$$S_0 = C_g^{(\frac{1}{\alpha}-1)}\sigma \tag{3}$$

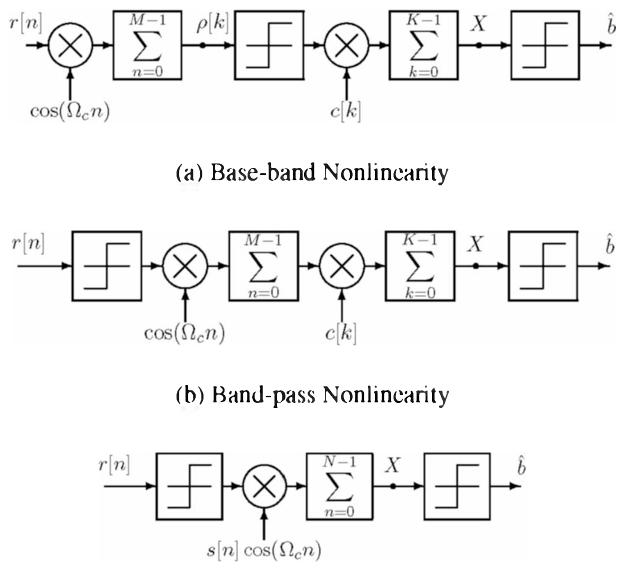
where $C_g \approx 1.78$ is the exponential of the Euler-Mascheroni constant. Then, to measure the relative strength between the received signal and the channel noise, we define the geometric signal to noise ratio (*GSNR*) half the one defined in Gonzalez (1997) to account for the sinusoidal signal,

$$GSNR = \frac{1}{4C_g}\left(\frac{A}{S_0}\right)^2 \tag{4}$$

In this way, the *GSNR* reduces to the standard *SNR* when $\alpha = 2$, since then the noise variance is $2\sigma^2$.

For simplicity we assume that there are exactly M samples per chip, therefore the number of samples per bit is $N = KM$, and if $c[k] \in \{+1, -1\}, k = 0, .., (K-1)$ is the PN sequence of the spreading code, then $s[n] = c[k], kM \leq n < (k+1)M$. We also consider that the nonlinearity is a hard-limiter as in Aazhang and Poor (1988). The model for the "traditional" detection scheme, with chip-matched filter followed by a memoryless nonlinearity and correlation, is shown in Fig. 1 a.

What we propose is to switch the nonlinearity in the digital receiver, as it is shown in Fig. 1 b. This scheme, which can be re-arranged as in Fig. 1 c, approximates the locally optimum detector of the received signal, i.e. a band-pass signal. In addition, it is even simpler to implement because it requires only 1 bit A/D converters in the sampling process and eliminates the need for multipliers, it utilizes only sign reversals. Due to its lower computational load it has been proposed as a suboptimal scheme for binary detection in gaussian noise (Beaulieu, 1985).



(a) Base-band Nonlinearity

(b) Band-pass Nonlinearity

(c) Band-pass Nonlinearity (simplified)

Figure 1: Schemes of Nonlinear Receivers.

III. PERFORMANCE OF DETECTION SCHEMES

We will calculate the bit error rate (*BER*) of both receiving schemes described in the previous section with the assumptions of low *GSNR* and large number of samples. Formally, we will consider that *GSNR* $\rightarrow 0$ and $N \rightarrow \infty$ such that *N.GSNR* remains constant. Validity of these assumptions will be established on next section by means of simulations.

The hard limiter with saturation levels ± 1, ensures unit variance output. Then, considering M fixed, in both schemes the central limit theorem allows us to argue that the test statistic X (see Fig. 1) is Gaussian. Using the symmetry of the noise distribution and assuming equiprobable transmitted bits the probability of bit error is

$$P = Pr\{\hat{b} = -1 | b = 1\} = Q\left(\sqrt{\frac{E^2\{X|b=1\}}{Var\{X\}}}\right) \tag{5}$$

where $Q(x) = \int_x^{\infty} \frac{e^{-y^2/2}}{\sqrt{2\pi}} dy$ is the gaussian Q-function. In addition, since A tends to zero, the same does $E\{X|b = 1\}$, and we can approximate (Aazhang and Poor, 1989)

$$P \approx Q\left(\sqrt{\frac{A^2(\frac{dE\{X|b=1\}}{dA})^2}{Var\{X\}}}\right) \approx Q\left(\sqrt{LA^2\mu'^2\overline{\varsigma^2}\nu}\right) \tag{6}$$

with $\mu' = \frac{d\mu}{dA}$, where μ is the location parameter of the distribution of the samples at the input of the hard-limiter when $b = 1$, L is the number of samples of finite variance added to form X (M or K depending of the case), $\overline{\varsigma^2}$ is the mean value of these variances, and ν is the detection efficiency of the hard-limiter equal to $4f_n^2(0)$, where f_n is the probability density function of the noise at the input of the hard-limiter (Aazhang and Poor, 1989). Then, replacing in (6) we get

$$P = Q\left(\sqrt{4LA^2\mu'^2\overline{\varsigma^2}\,f_n^2(0)}\right) \tag{7}$$

In particular, for the case of the $S\alpha S$ distribution with dispersion σ_n it is found that

$$f_n(0) = \frac{f_\alpha(0)}{\sigma_n} = \frac{1}{\sigma_n}\frac{\Gamma(\frac{1}{\alpha})}{\pi\alpha} \tag{8}$$

where we have called $f_\alpha(\cdot)$ the probability density function of $S\alpha S$ distribution with dispersion 1, and $\Gamma(\cdot)$ is the gamma function.

A. Detection with Base-Band Nonlinearity

In the scheme of Fig. 1 a, the base-band sequence $\rho[k]$ is a linear combination of $r[n]$, which is an independent α-stable sequence. Therefore, $\rho[k]$ is an α-stable sequence (Grigoriu, 1995), with location and dispersion parameter given by

$$\mu_{\rho[k]} = bAc[k] \sum_{n=kM}^{(k+1)M-1} \cos^2(\Omega_c n + \phi) \approx \frac{bAMc[k]}{2} \quad (9)$$

$$\sigma^\alpha_{\rho[k]} = \sigma^\alpha \sum_{n=kM}^{(k+1)M-1} |\cos(\Omega_c n + \phi)|^\alpha \approx \sigma^\alpha M C_\alpha \quad (10)$$

where

$$C_\alpha = \frac{\Gamma(\frac{\alpha+1}{2})}{\Gamma(\frac{\alpha}{2}+1)\sqrt{\pi}} \quad (11)$$

and we have approximated the sums by M times the mean value of their terms (assuming that there are many carrier cycles during a chip interval that are asynchronously sampled).

Therefore, using (7) and (8) with $L = K$, $\mu'^2 = M^2/4$, $\overline{\varsigma^2} = 1$ and $\sigma_n = \sigma_{\rho[k]}$, and using the definition of *GSNR* of (4), we find

$$P_{bb} = Q\left(\sqrt{\frac{4}{(C_\alpha)^{\frac{2}{\alpha}}}\left(\frac{C_g}{M}\right)^{(\frac{2}{\alpha}-1)} f_\alpha^2(0)\, N\, GSNR}\right) \quad (12)$$

B. Detection with Band-Pass Nonlinearity

Considering Fig. 1 c) it is clear that we can apply (7) with $S\alpha S$ noise distribution of σ dispersion, $L = N$, $\mu'^2 = 1$, and $\overline{\varsigma^2} = \frac{1}{2}$ because the output of the hard-limiter is multiplied by sinusoidal values. Then, using (4) and (8), we get

$$P_{pb} = Q\left(\sqrt{8\,(C_g)^{(\frac{2}{\alpha}-1)} f_\alpha^2(0)\, N\, GSNR}\right) \quad (13)$$

C. Asymptotic Relative Performance

If we compare expressions (12) and (13) it is easy to find the gain in the performance of the receiver with band-pass nonlinearity with respect the receiver with a base-band one. The ratio of the arguments of the square roots in (12) and (13) can be seen as their Asymptotic Relative Efficiency (Capon, 1961),

$$ARE_{pb,bb} = 2(C_\alpha)^{\frac{2}{\alpha}} M^{(\frac{2}{\alpha}-1)} \quad (14)$$

It is interesting to note that if $\alpha = 2$ then $ARE_{pb,bb} = 1$ and both schemes have the same degradation of $\frac{\pi}{2}$ or $1.96dB$, with respect to the linear receiver -optimum in this case (Beaulieu, 1985). For the rest of the α-stable cases $(0 < \alpha < 2)$ the gain of using the hard-limiter at band-pass increases with M (the number of averaged samples by the chip-matched filter), and the faster when the more impulsive is the noise.

IV. RESULTS AND SIMULATIONS

In order to clearly show the performance of the analyzed detection schemes and validate the previous analytic results, we consider a *DS-CDMA* system with spreading sequences of length $K = 31$. The received signal is sampled ten times per chip ($M = 10$). We take $\alpha = 1.9$ as a slightly impulsive environment, $\alpha = 1.5$ as a moderately impulsive environment, and $\alpha = 1$ (Cauchy) as a severe impulsive environment. The case $\alpha = 2$ is not considered here, since results of detection in Gaussian noise are well known, see Beaulieu (1985) for example.

To illustrate the need for a nonlinear treatment of the received signal in impulsive noise, we include in the results the performance of a linear receiver. In this receiver, similar to Fig. 1 c) without the hard-limiter on the left, the test statistic X is α-stable, similar to $\rho[k]$ in Fig. 1 a). Thus the calculations follow easily with N in place of M. Then, its probability of error can be obtained in terms of the complement of cumulative distribution of a $S\alpha S$ random variable of unit dispersion, that can be calculated numerically. The result is

$$P_{lin} = Q_\alpha\left(\sqrt{\frac{1}{(C_\alpha)^{\frac{2}{\alpha}}}\left(\frac{C_g}{N}\right)^{(\frac{2}{\alpha}-1)} N\, GSNR}\right) \quad (15)$$

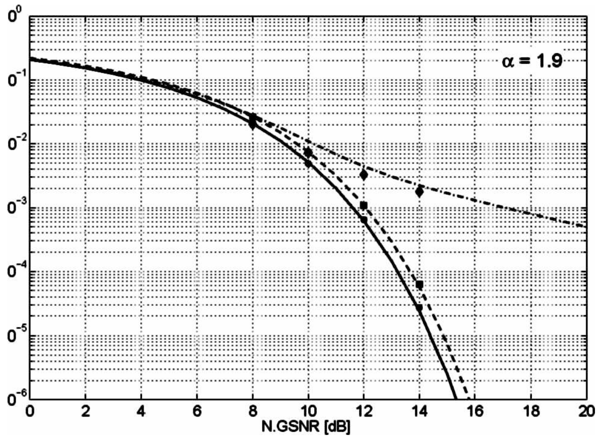
Note that for $\alpha = 2$, $Q_2(x) = Q(x/\sqrt{2})$, since unit dispersion means variance 2 in this case. In addition, since $C_2 = 1/2$ and $GSNR = SNR$, the probability of error reduces to the well-known result $P_{lin} = Q(\sqrt{N.SNR})$.

The results obtained using (12), (13) and (15) for the three selected values of α are presented in Fig. 2. It can be clearly seen how the receiver with band-pass nonlinearity outperforms the others when the impulsiveness of the channel becomes more important. Moreover, the linear receiver becomes practically useless when $\alpha < 2$. The Asymptotic Relative Efficiency, in (14), for this three cases is approximately 0.5, 3 and 8 dB respectively.

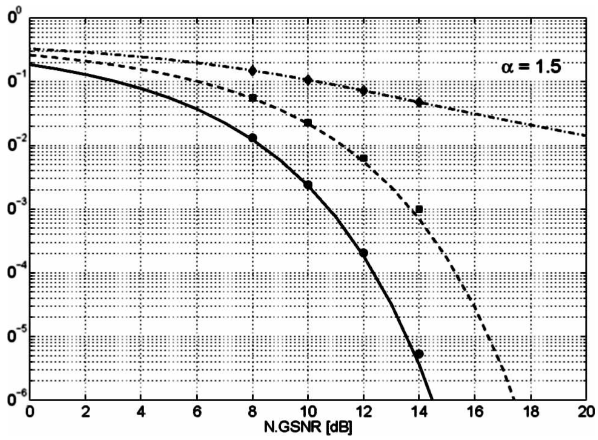
In the same figures we present the results of simulation that show the good accuracy of the previous calculations. Only in the case $\alpha = 1$ and *GSNR*= 10dB there appears that the asymptotic assumption is not as accurate. In the simulation process we used enough runs to ensure that two standard deviations of the estimates are less than a 10% of the estimated value. The small discrepancies observed in the case of the linear receptor, in Fig. 2 a and 2 c, can be attributed to numeric errors in the calculation of $Q_\alpha(\cdot)$.

V. CONCLUSIONS

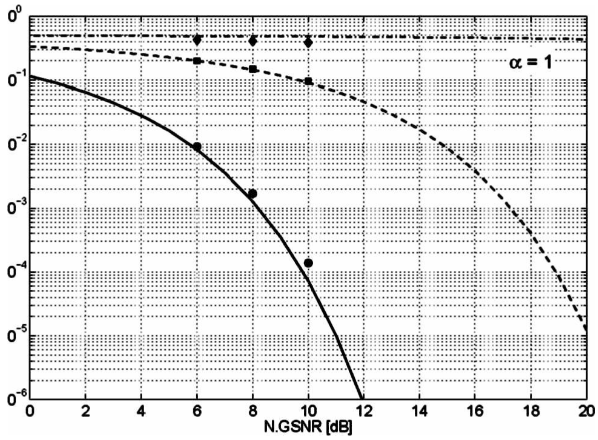
We have shown the advantages of band-pass nonlinear detection structures for the *DS-CDMA* receivers that operate in impulsive environments, over those that have it at base-band. The case of digital detection with independent α-stable distributed sequences as a model for the impulsive noise and hard-limiting has been analyzed in detail, obtaining expressions for the probability of error for both



(a) Probability of error with $\alpha = 1.9$



(b) Probability of error with $\alpha = 1.5$



(c) Probability of error with $\alpha = 1.0$

Figure 2: Performance of analyzed receivers. Analytically calculated: Linear (– · –), Base-Band NL (– – –) , and Pass-Band NL (———). Simulated: Linear (♦), Base-band NL (▪), and Pass-Band NL (•).

schemes. These results have been validated with simulations, that confirm that substantial gains can be achieved if the impulsiveness of the environment is important.

It has to be noted that the condition of independent noise samples is a strong assumption. It is frequently adopted in the literature, but its validity for real-word situations remains to be analyzed in detail. It suffices to think on how the RF front-end affects the statistics of the impulsive noise before it is sampled at the intermediate frequency stage in the receiver. This analysis may be difficult because the multivariate α-stable distribution is non-parametric (Shao and Nikias, 1993). Current work is being done to relax the independence assumption. The effect of a multiuser environment will also have to be studied.

REFERENCES

Aazhang, B. and H.V. Poor, "Performance of SS/CDMA Communications in Impulsive Channels - Part II: Hard-Limiting Correlation Receivers, *IEEE Trans. Comm.*, **36**, 88-91 (1988).

Aazhang, B. and H.V. Poor, "An Analysis of Nonlinear Direct-Sequence Correlators, *IEEE Trans. Comm.*, **37**, 723-731 (1989).

Beaulieu, N.C., "On the performance of three Suboptimum detection schemes for binary signaling, *IEEE Trans. Comm.*, **33**, 241-245 (1985).

Capon, J., "On the asymptotic Efficiency of Locally Optimum Detector, *IRE Trans. Inf. Theory*, **7**, 67–72 (1961).

Chuah, T.C., B.S. Sharif and O.R. Hinton, "Nonlinear decorrelator for multiuser detection in non-gaussian impulsive environments, *IEE Electronics Letters*, **36**, 920–922 (2000).

Deliç, H. and A.H. Hocanin, "Robust Dectection in DS-CDMA, *IEEE Trans. Vehicular Tech.*, **51**, 155–169 (2002).

Gonzalez, J.G., *Robust techniques for wireless communications in non-Gaussian environments*, PhD Thesis, Department of Electrical Computer Engineering, University of Delaware, U.S.A. (1997).

Grigoriu, M., *Applied non-gaussian Processes*, Prentice Hall PTR, New Jersey (1995).

Shao, M. and C.L. Nikias, "Signal Processing with Fractional Lower Order Moments: Stable Processed and their Applications, *Proc. IEEE*, **81**, 986–1010 (1993).

Spaulding, A.D., "Locally optimum and suboptimum detector performance in a non-gaussian interference enviornment, *IEEE Trans. Comm.*, **33**, 509–517 (1985).

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