



White-Dwarf Asteroseismology With the *Kepler* Space Telescope

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In the course of their evolution, white-dwarf stars go through at least one phase of variability in which the global pulsations they undergo allow astronomers to peer into their interiors, making it possible to shed light on their deep inner structure and evolutionary stage by means of asteroseismology. The study of pulsating white dwarfs has undergone substantial progress in the last decade, and this is largely thanks to the arrival of continuous observations of unprecedented quality from space, like those of the CoRoT, *Kepler*, and *TESS* missions. This, along with the advent of new detailed theoretical models and the development of improved asteroseismological techniques, has helped to unravel the internal chemical structure of many pulsating white dwarfs, and, at the same time, has posed new questions that challenge theoreticians. In particular, uninterrupted monitoring of white-dwarf stars for months has allowed discovering phenomena impossible to detect with ground-based observations, despite previous admirable efforts like the Whole Earth Telescope (WET). Here, we start by reviewing the essential properties of white-dwarf and pre-white dwarf stars and their pulsations, and then, we go through the different families of pulsating objects known to date. Finally, we review the most outstanding findings about pulsating white dwarfs and pre-white dwarfs made possible with the unprecedented-quality observations of the *Kepler* space telescope, although we envisage that future analyses of space data from this mission that still await examination, may reveal new secrets of these extremely interesting variable stars.

Keywords: stellar evolution, white dwarf stars, stellar interiors, stellar oscillations, asteroseismology

OPEN ACCESS

Edited by:

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Specialty section:

This article was submitted to
Stellar and Solar Physics,
a section of the journal
Frontiers in Astronomy and Space
Sciences

Received: 13 April 2020

Accepted: 18 June 2020

Published: 28 August 2020

Citation:

Córscico AH (2020) White-Dwarf
Asteroseismology With the *Kepler*
Space Telescope.
Front. Astron. Space Sci. 7:47.
doi: 10.3389/fspas.2020.00047

1. BASIC PROPERTIES OF WHITE-DWARF AND PRE-WHITE DWARF STARS

White-dwarf stars constitute the most common compact remnants of the evolution of stars with initial masses below $\sim 10 - 11M_{\odot}$ (Woosley and Heger, 2015), including our Sun. Comprehensive accounts referring to the origin and evolution of white dwarfs can be found in the review articles of Fontaine et al. (2001) and Althaus et al. (2010). White dwarfs are extremely old objects, with typical ages in the range $1 - 10$ Gyr ($1 \text{ Gyr} \equiv 10^9 \text{ yr}$). These stellar fossils are found in a range of masses of $0.15 \lesssim M_{\star}/M_{\odot} \lesssim 1.25$, with an average value of $M_{\star}/M_{\odot} \sim 0.60$ (Tremblay et al., 2016), although for the majority of white dwarfs the mass lies between $M_{\star}/M_{\odot} \sim 0.50$ and $M_{\star}/M_{\odot} \sim 0.70$. Since they are characterized by planetary dimensions ($R_{\star} \sim 0.01R_{\odot}$), the matter inside is highly concentrated, with average densities of white dwarfs being in the order of $\bar{\rho} \sim 10^6 \text{ g/cm}^3$. Consequently, the prevailing equation of the state inside white dwarfs is that of a

highly-degenerate Fermi gas (Chandrasekhar, 1939). Indeed, the hydrostatic equilibrium in the interior of a white dwarf is provided by the pressure of degenerate electrons counteracting the gravity force, thus avoiding the collapse of the star. In particular, electron degeneracy is responsible for a curious relationship between the stellar mass and radius: the more massive the white dwarf, the smaller its radius. The process of discovery of white dwarfs, starting from the observation of variations of the proper motion of Procyon and Sirius by Friedrich Wilhelm Bessel in 1844 (Bessel, 1844), was fascinating. The emergence of the quantum-mechanics theory toward the first quarter of the 20th century paved the way in explaining the existence of white dwarfs. An excellent historical account of the discovery of white dwarfs and the theoretical arguments that led to explaining the existence of these compact objects in nature is provided by Van Horn (2015).

Apart from the degenerate electrons, the interior of white dwarfs is composed by non-degenerate ions that store the thermal energy generated by previous nuclear burning—generally extinct in the white-dwarf stage. White dwarfs are found at a huge interval of effective temperatures ($4,000 \lesssim T_{\text{eff}} \lesssim 200,000$ K) thus spanning seven orders of magnitude in luminosity, $10^{-4} \lesssim \log(L_*/L_\odot) \lesssim 10^3$. Regarding their inner chemical composition, average-mass white dwarfs are characterized by cores likely made of ^{12}C and ^{16}O —the products of He burning—although ultra-massive white dwarfs ($M_*/M_\odot \gtrsim 1.0$) could contain cores composed of ^{16}O , ^{20}Ne , and ^{24}Mg as well, and the lowest-mass white dwarfs (low-mass and extremely low-mass white dwarfs, abbreviated as LM and ELM white dwarfs, respectively) could harbor cores made of ^4He . In particular, ELM white dwarfs ($M_*/M_\odot \lesssim 0.2$) must be formed in interacting binary systems, otherwise they would be much older than the Universe. A description of the different formation channels of white dwarfs, either single- or binary-star evolution, can be found in the review articles of Althaus et al. (2010) and Córscico et al. (2019). Unlike other stages of stellar evolution, the white-dwarf phase is characterized by a relatively simple process of gradual cooling of the star, shining at the expense of the heat stored in its core during the prior evolutionary history. This process was first described in a simplified way by Mestel (1952). The cooling rate of white dwarfs depends on many factors: surface temperature, stellar mass, core chemical composition, surface chemical composition, etc. The surface chemical composition of white dwarfs defines the spectral type. Most white dwarfs ($\sim 80\%$) have atmospheres rich in ^1H , and are called DA white dwarfs. About a $\sim 15\%$ of white dwarfs have atmospheres rich in ^4He , and are denominated by DB and DO white dwarfs. The remaining population of white dwarfs ($\sim 5\%$) show atmospheres rich in ^4He , ^{12}C , and ^{16}O (PG1159 stars)¹, ^{12}C and ^4He (DQ white dwarfs), and metals without ^1H or ^4He present (DZ white dwarfs). There are also some exotic objects, such as SDSS J0922+2928 and SDSS J1102+2054, that exhibit O in their atmospheres (Gänsicke et al., 2010), SDSS J1240+6710 that has a nearly pure ^{16}O atmosphere, diluted only

by traces of ^{20}Ne , ^{26}Mg , and ^{28}Si (Kepler et al., 2016a), and finally WD J0551+4135, that shows a mixed ^1H and ^{12}C atmosphere (Hollands et al., 2020). The origin of these unfamiliar objects is not yet known, but might involve stellar mergers. By virtue of the high densities and small radii of the white dwarfs, they are characterized by very high surface gravities, in the range of $\log g \sim 6 - 9$ [cm/s^2]. The extremely high gravities of white dwarfs force heavy elements to sink and light nuclear species to float on the white-dwarf surface. This effect, called gravitational settling, explains the purity of the atmospheres of DA and DB/DO white dwarfs. Any “impurity” observed in white dwarf atmospheres at intermediate temperatures—for instance, photospheric trace metals found in many white dwarfs—must be attributed either to accreted material (i.e., tidally disrupted planetesimals; see Koester et al., 2014), material floating on the surface due to radiative levitation, or material dredged up from the interior due to convection. The typical chemical structure of an average-mass DA white dwarf consists of a core composed of ^{12}C and ^{16}O —in unknown proportions—containing $\sim 99\%$ of the mass, surrounded by a ^4He layer with a mass $M_{\text{He}} \lesssim 10^{-2}M_*$, it is in turn enclosed by a thin ^1H envelope with a mass of $M_{\text{H}} \lesssim 10^{-4}M_*$. The thickness of the ^1H and ^4He mantles depends on the mass of the white dwarf. Despite the thinness of these layers, they play a fundamental role in the cooling rate (Wood, 1990).

The population of known white dwarfs has increased dramatically in recent years. Ground-based observations, principally with the spectral observations of the Sloan Digital Sky Survey (SDSS; York et al., 2000), have enlarged the number of known white dwarfs by a factor of 15 (Kleinman et al., 2013; Kepler et al., 2016b, 2019). Recently, Gentile Fusillo et al. (2019) presented a catalog of $\sim 260,000$ high-confidence white-dwarf candidates selected from *Gaia* DR2.

White-dwarf research has relevant applications to various areas of modern astrophysics. A primary application of the analysis of white-dwarf properties, either individual or collective—like the mass distribution, core chemical composition, and cooling times—is to place constraints on the evolution of low-mass stars, including third dredge up and mass loss on the Asymptotic Giant Branch (AGB), the efficiency of extra-mixing during core He burning, and nuclear reaction rates (Kunz et al., 2002; Metcalfe, 2003; Straniero et al., 2003; Salaris et al., 2009; Fields et al., 2016).

By virtue of the large number of white dwarfs that are currently known, and thanks to the extreme longevity of these stars (they are among the oldest objects of the Galaxy), they are a key part in our understanding of the formation and evolution of stars, evolution of planetary systems, and the history of our Galaxy itself. For instance, since the existing population of white dwarfs holds a detailed account of the early star formation in the Galaxy, then accurate white-dwarf luminosity functions can be employed to deduce the age, structure, and evolution of the Galactic disk and the nearest open and globular clusters, through what is called *cosmochronology* (Fontaine et al., 2001; Bedin et al., 2009, 2015; García-Berro et al., 2010; Campos et al., 2013, 2016; García-Berro and Oswalt, 2016; Kilic et al., 2017).

From another perspective, since the majority of low-mass stars will evolve into white dwarfs, then most of the host stars of

¹Strictly speaking, since these stars still have substantial nuclear burning, they should be considered pre-white dwarfs.

planetary systems will end their lives as white dwarfs. At present, many white dwarfs with planetary debris are currently being detected, and their study provides valuable information about the chemical composition of extra-solar planets (Gänsicke et al., 2012; Hollands et al., 2018). In particular, Gänsicke et al. (2019) have detected a white dwarf accreting material probably coming from a giant planet.

In a different context, white dwarfs are found in binary systems. This allows exploring the interactions among stars, and in particular, to study type Ia supernovae progenitors (Maoz et al., 2014). Indeed, since the mechanical properties of a white dwarf are described by a Fermi gas of degenerate electrons, there exist a limit mass—the Chandrasekhar mass, $\sim 1.4M_{\odot}$ —beyond which the structure of a white dwarf becomes unstable. Therefore, a $^{12}\text{C}/^{16}\text{O}$ -core white dwarf in a binary system that receives mass from its companion, can approach the Chandrasekhar mass and explode as a supernova.

Finally, another very important aspect of white dwarfs is that, due to the high density prevailing in their interiors, they are extremely useful as cosmic laboratories to study dense plasma physics and solid state physics (crystallization; Montgomery and Winget, 1999; Montgomery et al., 1999; Winget et al., 2009; Tremblay et al., 2019), and “exotic physics” (axions, neutrino magnetic dipole moment, variation of fundamental constants, etc; Isern et al., 1992, 2008, 2018; Córscico et al., 2001, 2012b, 2013, 2014; Miller Bertolami, 2014; Miller Bertolami et al., 2014).

2. PULSATION PROPERTIES OF WHITE DWARFS AND PRE-WHITE DWARFS AND THE CLASSIFICATION OF THE DIFFERENT TYPES OF VARIABLES

An important characteristic of white dwarfs is that, along their evolution, all of them go through at least one phase of pulsational instability that converts white dwarfs into variable stars. As such, variable white dwarfs can be studied through *asteroseismology*, which allows for key information about the internal structure of pulsating stars to be extracted, through the study of their normal modes. In a sense, the global pulsations are something like “windows” that allow astronomers to “see” inside the stars (Cox, 1980; Unno et al., 1989; Aerts et al., 2010; Catelan and Smith, 2015). In the case of white dwarfs, the properties derived through asteroseismology for pulsating objects are likely valid for non-pulsating white dwarfs as well. On the other hand, this is a unique tool that makes it possible to delve inside the white dwarfs. Finally, and due to the fact that the equilibrium structures of white dwarfs are simple—by virtue of its degenerate nature—their pulsation properties are consequently simple and relatively easy to model to a meaningful precision. In particular, they can be treated within the context of the linear theory of stellar pulsations. These unique qualities explain why white-dwarf asteroseismology has, in recent years, become one of the most powerful tools to learn about their origin, evolution, and internal structure, complementing the traditional techniques of spectroscopy, photometry, and astrometry (Fontaine and Brassard, 2008; Winget and Kepler,

2008; Althaus et al., 2010; Córscico et al., 2019). As a matter of fact, white-dwarf asteroseismology has been a cornerstone of the field of asteroseismology (Brown and Gilliland, 1994). Below, we briefly describe the pulsational properties of white dwarfs along with their immediate precursors, the pre-white dwarfs, and the different classes of these compact pulsators known at the time of writing this review (March 2020).

Pulsations in white dwarfs manifest as brightness fluctuations in the optical, the ultraviolet (UV), and infrared (IR) regions of the electromagnetic spectrum, with amplitudes between 0.001 mag and 0.4 mag. The variations are thought to be produced by changes in the surface temperature due to non-radial g modes (McGraw, 1979; Robinson et al., 1982) with low harmonic degree ($\ell = 1, 2$) and low and intermediate radial order (k). In particular, Robinson et al. (1982) showed that during pulsations, the changes of the stellar radius are fairly small ($\Delta R_{\star} \sim 10^{-5}R_{\star}$), and that the variations in surface temperature ($\Delta T_{\text{eff}} \sim 200$ K) are the true cause of the variability of pulsating white dwarfs. Kepler (1984) investigated the effects of r -mode non-radial pulsations² on the lightcurves and line profiles of a slowly rotating star, and concluded that the observed variations of the prototype H-rich pulsating white dwarf, G117–B15A, are not caused by r -mode pulsations, but are consistent with g -mode pulsations. By virtue of the high electron degeneracy in the core of white dwarfs, the g -mode critical frequency—the “buoyancy” or Brunt-Väisälä frequency—is very low at the core regions. This forces g modes to probe, in particular, the regions of the envelope (see, for instance Brassard et al., 1991), although some low radial-order modes are generally sensitive to the central regions. In the case of hot pre-white dwarfs like PG1159 stars, degeneracy is not as important, and g modes can propagate throughout the star, including the core regions (Kawaler et al., 1985; Córscico and Althaus, 2006). The first pulsating white dwarf ever detected, HL Tau 76, was discovered unexpectedly by Landolt (1968). Since then, an increasing number of pulsating white dwarfs have been detected, either through observations from Earth—mainly for candidates identified from SDSS—and recently as a result of uninterrupted observations of space missions, such as *Kepler/K2* (Borucki et al., 2010; Howell et al., 2014). Prior to the era of observations from space, an important international collaborative tool called the Whole Earth Telescope (WET; Nather et al., 1990) was successfully used to observe pulsating white dwarfs and pre-white dwarfs for long intervals of time, without interruption. As a matter of fact, observations from this world-wide network of telescopes resulted in the most precise light curves ever obtained for any pulsating star at that time (see, for instance, Winget et al., 1991, 1994). The properties of the light curves of pulsating white dwarfs and pre-white dwarfs are varied, ranging from very simple and with low-amplitude variations (containing a single period) to complex and with high-amplitude fluctuations (usually harboring several periods). The first ones are usually associated with pulsating white dwarfs located at the hot edge of the specific instability domain, and the last ones are found generally for stars populating the cool boundary of that instability

² r modes are toroidal modes in presence of rotation (Papaloizou and Pringle, 1978).

TABLE 1 | Basic characteristics of the different types of pulsating white dwarfs and pre-white dwarfs, sorted by decreasing effective temperature.

Class	Year of disc. (#)	T_{eff} ($\times 1,000$ K)	$\log g$ (C.G.S.)	Period range (s)	Amplitudes (mag)	Main surface composition
GW Vir (PNNV)	1984 (10)	100 – 180	5.5 – 7	420 – 6,000	0.01 – 0.15	He, C, O
GW Vir (DOV)	1979 (9)	80 – 100	7.3 – 7.7	300 – 2,600	0.02 – 0.1	He, C, O
V777 Her (DBV)	1982 (27)	22.4 – 32	7.5 – 8.3	120 – 1,080	0.05 – 0.3	He (H)
GW Lib	1998 (20)	10.5 – 16	8.35 – 8.7	100 – 1,900	0.007 – 0.07	H, He
ZZ Cet (DAV)	1968 (260)	10.4 – 12.4	7.5 – 9.1	100 – 1,400	0.01 – 0.3	H
pre-ELMV	2013 (7)	8 – 13	4 – 5	300 – 1,400	0.001 – 0.05	He, H
ELMV	2012 (11)	7.8 – 10	6 – 6.8	100 – 6,300	0.002 – 0.044	H

region—as the convection zone deepens, longer-period modes are excited. In the case of the complex lightcurves, that usually exhibit features of non-linearity, it is common to find harmonics and linear combinations of eigenfrequencies that are not related to real pulsation modes. Rather, they are likely connected with physical phenomena inherent to the outermost regions of the white dwarf, probably related to the outer convection zone. In the case of pulsating pre-white dwarfs, the effective temperatures are so high that these stars probably lack an extensive external convective zone. This may be the explanation for the fact that non-linearities are not detected in any pulsating hot pre-white dwarf star.

Currently, the number of known pulsating white dwarfs and pre-white dwarfs is about 350 (Córscico et al., 2019). They are distributed in six confirmed types, namely ZZ Ceti (or DAV) stars, GW Lib stars, V777 Her (or DBV) stars, GW Vir (or pulsating PG1159) stars, ELMV stars, and pre-ELMV stars. There are two additional claimed classes of pulsating white dwarfs, the hot DAV stars ($T_{\text{eff}} \sim 30,000$ K, $7.3 \lesssim \log g \lesssim 7.8$; Kurtz et al., 2008, 2013) and the DQV stars (Montgomery et al., 2008). Pulsational instabilities in hot DAVs were predicted on theoretical grounds by Shibahashi (2005, 2007); the actual existence of pulsations in these stars must be confirmed by additional observations. On the other hand, DQVs are hot DQ (C- and He-rich atmospheres) white dwarfs with surface parameters $19,000$ K $\lesssim T_{\text{eff}} \lesssim 22,000$ K and $8 \lesssim \log g \lesssim 9$. At present, there is a growing consensus that the variability of these objects could be due to other effects than global pulsations. The main characteristics of the confirmed classes of pulsating white dwarfs are listed in Table 1 and the location in the $\log T_{\text{eff}} - \log g$ is depicted in Figure 1 (details can be found in Córscico et al., 2019). In Table 1, the second column indicates the discovery year of the first star of each type and the number of detected objects at the time of writing this review, the third column corresponds to the range of effective temperatures at which they are found (instability domain), the fourth column shows the range of surface gravity, the fifth column indicates the range of periods detected, the sixth column is the range of amplitudes of the fluctuations in the light curves, and the seventh column shows the surface chemical composition. Pulsation periods usually span the interval $\sim 100 - 1,400$ s; however PNNVs and ELMVs display longer periods, up to $\sim 6,300$ s. In Figure 2 we show the periods detected in the different families of pulsating

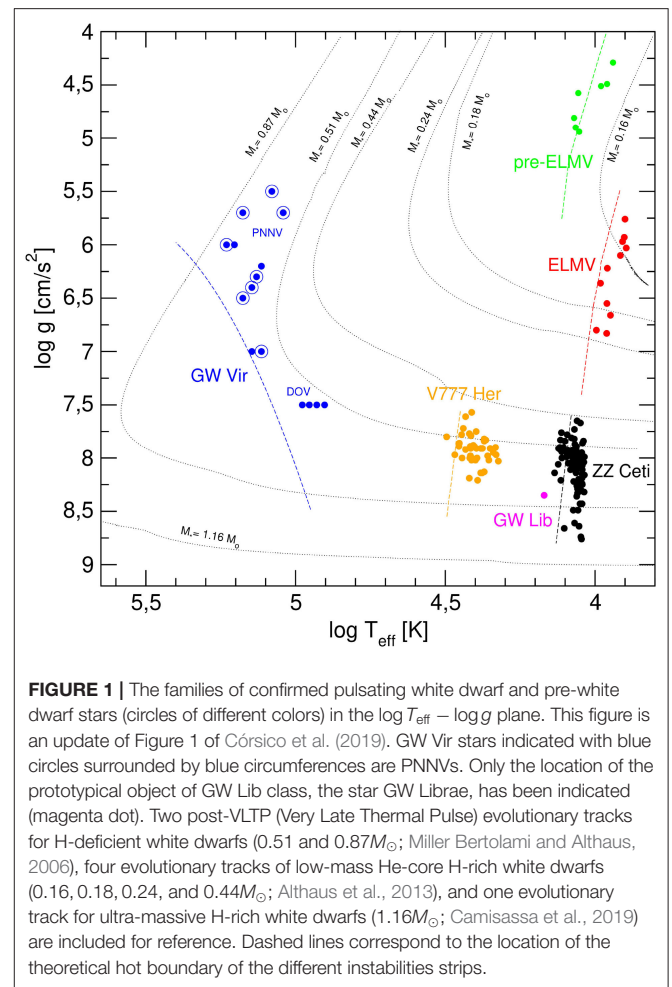
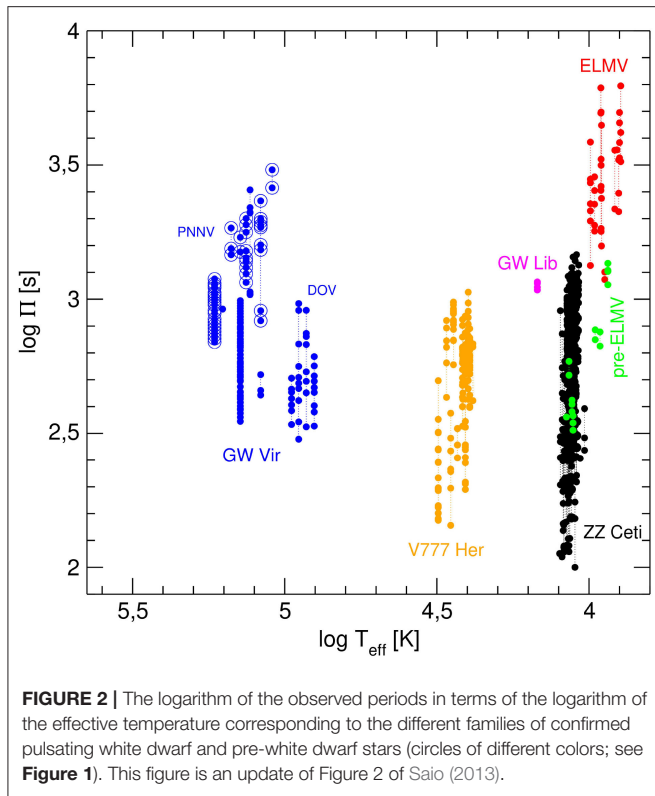


FIGURE 1 | The families of confirmed pulsating white dwarf and pre-white dwarf stars (circles of different colors) in the $\log T_{\text{eff}} - \log g$ plane. This figure is an update of Figure 1 of Córscico et al. (2019). GW Vir stars indicated with blue circles surrounded by blue circumferences are PNNVs. Only the location of the prototypical object of GW Lib class, the star GW Librae, has been indicated (magenta dot). Two post-VLTP (Very Late Thermal Pulse) evolutionary tracks for H-deficient white dwarfs (0.51 and $0.87M_{\odot}$; Miller Bertolami and Althaus, 2006), four evolutionary tracks of low-mass He-core H-rich white dwarfs (0.16 , 0.18 , 0.24 , and $0.44M_{\odot}$; Althaus et al., 2013), and one evolutionary track for ultra-massive H-rich white dwarfs ($1.16M_{\odot}$; Camisassa et al., 2019) are included for reference. Dashed lines correspond to the location of the theoretical hot boundary of the different instabilities strips.

white dwarfs and pre-white dwarfs. Curiously, and due to the compactness of these objects, the periods of g modes in white dwarfs have magnitudes comparable to the values of the periods of p modes exhibited by non-degenerate pulsating stars, like the Sun.

Pulsations in white dwarfs are self-excited, in contrast to the stochastic pulsations, which are forced oscillations driven by turbulent convection in the Sun, the solar-like, and red giant



pulsating stars. The cause of the pulsations in white dwarfs appears to be a mechanism that begins to act when the star cools down to an effective temperature at which the dominant nuclear species becomes partially ionized near the stellar surface (that is, when there is a coexistence of neutral and ionized atoms of the same chemical element; “partial ionization zone”). At the partial ionization zones, the opacity increases at the compression phase of the oscillations, and this excites global pulsations with periods that are of the same order of magnitude as the thermal timescale at the driving region. The $\kappa - \gamma$ mechanism has been invoked to explain at least the onset of pulsations in all the categories of pulsating white dwarfs (see, for instance, Winget et al., 1982; Starrfield et al., 1984, for the case of the DAV and GW Vir stars, respectively). For DAVs and DBVs, the validity of the $\kappa - \gamma$ mechanism has been questioned. Instead, it has been proposed that the so-called “convective driving” mechanism (Brickhill, 1991; Goldreich and Wu, 1999), that becomes efficient when the base of the outer convective zone sinks, is the responsible mode of driving. For GW Vir stars, which do not have surface convection zones as a result of their very-high effective temperatures, the $\kappa - \gamma$ mechanism seems to be enough to destabilize high-order g -mode pulsations³. The proposed mechanisms are able to predict with decent precision the T_{eff} of the blue (hot) edge

³There exist theoretical evidence of another mechanism that could excite global pulsations in white dwarfs and pre-white dwarfs, that is, the ϵ mechanism due to nuclear burning. This could be responsible for short-period g -mode pulsations in GW Vir stars (Córscico et al., 2009), ELMVs (Córscico and Althaus, 2014), ZZ Ceti stars coming from low-metallicity progenitors (Camisassa et al., 2016), and in

of the instability strips. However, in general, all theories fail in terms of the predicted effective temperature of the red (cool) boundary, where white dwarfs quench their pulsations. This is discouraging, since even detailed calculations that take into account the interaction between convection and pulsations (Van Grootel et al., 2012) fail to reproduce the location of the red edges in the cases of DAVs and DBVs. Recently, Luan and Goldreich (2018) proposed a theoretical framework that accounts for the existence of the red edge of the DAV instability strip.

Therefore, the physical processes that excite and quench pulsations in white dwarfs are not fully understood. This happens, in general, for all the classes of pulsating stars, aside from white dwarfs. However, this does not prevent us from extracting information from the pulsation periods through adiabatic asteroseismological analyses alone. In the case of white dwarfs, these analyses are based on the forward approximation, in which the measured periods are compared with the periods calculated on a huge set of white-dwarf models to obtain a representative asteroseismological solution of the star under study. The theoretical models can be the result of detailed computations that take into account the complete evolutionary history of the white-dwarf progenitor, the approach adopted by the La Plata Group⁴ (see e.g., Córscico et al., 2008, 2012a), or instead, they can consist of static white-dwarf models with parameterized chemical composition profiles (see, for instance Giammichele et al., 2018). A variant of this last method is the use of evolutionary models with parameterized chemical profiles (see Bischoff-Kim et al., 2019). These approaches are complementary, among them (Córscico et al., 2019). In some cases, a large set of consecutive periods can be detected, and it is possible to make asteroseismological inferences of the stellar mass from the mean period of spacing. This is the case of several DBV and GW Vir stars.

3. PULSATING WHITE DWARFS MONITORED BY THE *KEPLER* SPACECRAFT

The primary goal of the *Kepler* spacecraft was to detect transiting planets, discovering by the end of the mission a total of more than 4,000 exoplanets (Thompson et al., 2018). As a byproduct of the planetary hunt, high-quality photometric data of variable stars was collected, allowing a rapid blooming of asteroseismology of many classes of pulsating stars, in particular solar-like and red giant pulsators. The initial survey of pulsating stars undertaken by the *Kepler* Asteroseismic Science Consortium (KASC) contained a total of 113 compact pulsator candidates (including subdwarfs and white dwarfs) which were checked for variability using *Kepler* short-cadence exposures. Unfortunately, none of the 17 white dwarfs contained in the sample turned out to be variable. In order not to miss the exceptional capacity of the *Kepler* mission, other candidate objects (not included in the

very hot DA white dwarfs (Maeda and Shibahashi, 2014; Calcaferro et al., 2017). However, the predicted pulsations have not been detected, so far, in any case.

⁴<http://evolvegroup.fcaglp.unlp.edu.ar/>

TABLE 2 | All the published pulsating white-dwarf stars observed with the *Kepler* spacecraft (*Kepler* and *K2* missions).

Star	Class	T_{eff} (K)	$\log g$	M_* (M_{\odot})	P_{rot} (h)	Mission	References
KIC 4357037	DAV	12,650	8.01	0.62	22.0	<i>Kepler</i>	Hermes et al., 2017a
KIC 4552982 [‡]	DAV	10,860	8.16	0.71	18.4	<i>Kepler</i>	Bell et al., 2015
KIC 7594781	DAV	11,730	8.11	0.67	26.8	<i>Kepler</i>	Hermes et al., 2017a
KIC 10132702	DAV	11,940	8.12	0.68	11.2	<i>Kepler</i>	Hermes et al., 2017a
KIC 11911480	DAV	11,580	7.96	0.58	74.7	<i>Kepler</i>	Greiss et al., 2014
EPIC 60017836	DAV	10,980	8.00	0.57	6.9	<i>K2</i>	Hermes et al., 2014
EPIC 201355934	DAV	11,770	7.97	0.59	...	<i>K2</i>	Hermes et al., 2017a
EPIC 201719578	DAV	11,070	7.94	0.57	26.8	<i>K2</i>	Hermes et al., 2017a
EPIC 201730811	DAV	12,480	7.96	0.68	2.6	<i>K2</i>	Hermes et al., 2015a
EPIC 201802933	DAV	12,330	8.11	0.68	31.3	<i>K2</i>	Hermes et al., 2017a
EPIC 201806008 [‡]	DAV	10,910	8.02	0.61	31.3	<i>K2</i>	Hermes et al., 2017a
EPIC 206212611	DAV	10,830	8.00	0.60	...	<i>K2</i>	Hermes et al., 2017a
EPIC 210377280	DAV	11,590	7.94	0.57	...	<i>K2</i>	Bell et al., 2017b
EPIC 210397465	DAV	11,200	7.71	0.45	49.1	<i>K2</i>	Hermes et al., 2017a
EPIC 211596649	DAV	11,600	7.91	0.56	81.8	<i>K2</i>	Hermes et al., 2017a
EPIC 211629697 [‡]	DAV	10,600	7.77	0.48	64.0	<i>K2</i>	Bell et al., 2016
EPIC 211891315 [§]	DAV	11,310	8.03	<i>K2</i>	Bell, 2017
EPIC 211914185	DAV	13,590	8.43	0.88	1.1	<i>K2</i>	Hermes et al., 2017a
EPIC 211916160	DAV	11,510	7.96	0.58	...	<i>K2</i>	Hermes et al., 2017a
EPIC 211926430	DAV	11,420	7.98	0.59	25.4	<i>K2</i>	Hermes et al., 2017a
EPIC 220204626	DAV	11,620	8.17	0.71	24.3	<i>K2</i>	Hermes et al., 2017a
EPIC 220258806	DAV	12,800	8.09	0.66	30.0	<i>K2</i>	Hermes et al., 2017a
EPIC 220274129	DAV	11,810	8.03	0.62	12.7	<i>K2</i>	Bell et al., 2017b
EPIC 220329764 [‡]	DAV	11,180	8.03	0.62	...	<i>K2</i>	Bell, 2017
EPIC 220347759	DAV	12,770	8.08	0.66	31.7	<i>K2</i>	Hermes et al., 2017a
EPIC 220453225 [‡]	DAV	11,220	8.04	0.62	...	<i>K2</i>	Hermes et al., 2017a
EPIC 228682478	DAV	12,070	8.18	0.72	109.1	<i>K2</i>	Hermes et al., 2017a
EPIC 228952212 [‡]	DAV	11,080	7.95	0.58	...	<i>K2</i>	Bell, 2017
EPIC 229227292 [‡]	DAV	11,210	8.03	0.62	29.4	<i>K2</i>	Bell et al., 2016
EPIC 229228364 [‡]	DAV	11,030	8.03	0.62	...	<i>K2</i>	Hermes et al., 2017a
EPIC 229228478	DAV	12,500	7.93	0.57	...	<i>K2</i>	Hermes et al., 2017a
EPIC 229228480	DAV	12,450	8.18	0.72	...	<i>K2</i>	Hermes et al., 2017a
KIC 8626021	DBV	29,700	7.890	0.56	43.0	<i>Kepler</i>	Østensen et al., 2011a
PG 0112+104	DBV	31,040	7.800	0.58	10.2	<i>K2</i>	Hermes et al., 2017b

The values of T_{eff} , $\log g$, and M_* are those published in Córscico et al. (2019), except in the case of EPIC 211891315, EPIC 220329764, EPIC 220453225, and EPIC 228952212, for which the values are those of Bell (2017). The last column corresponds to the paper in which the pulsations were analyzed.

[‡]Outbursting ZZ Ceti stars. [§]Probable outbursting ZZ Ceti star.

initial survey; Østensen et al., 2011b) began to be monitored with *Kepler*. Fortunately, this search resulted in the first observations of pulsating white dwarfs by this space mission (Hermes et al., 2011; Østensen et al., 2011a; Greiss et al., 2014, 2015, 2016). Once it finished the nominal 4-years mission, the *Kepler* spacecraft lost two of its four reaction wheels (2013 May). The mission was then redirected to monitoring new fields along the ecliptic plane during uninterrupted time-intervals of ~ 90 days (Howell et al., 2014). This extended *Kepler* mission was renamed as *K2* and operated from 2014 until the spacecraft ran out of fuel in 2018. The *K2* mission provided an exceptional chance to extend space-based monitoring for a lot of new white dwarfs. This extended mission was cleverly exploited by astronomers,

and, at the end, many pulsating white dwarfs were discovered and studied.

In the following sections, we summarize the main findings achieved in white-dwarf asteroseismology thanks to the observations of *Kepler* and *K2*. In particular, we focus on several detailed asteroseismological studies performed on pulsating white dwarfs observed by this space mission, and also on selected outstanding discoveries that have been the result of uninterrupted observations from space. As a result of the *Kepler* and *K2* observations, 2 DBV and 29 DAV stars have been intensively studied, and the results of those analyses have been published. We have to add to this list three more DAV stars analyzed and published in a PhD Thesis (Bell, 2017). One of the

DBV stars, KIC 8626021, has been the focus of intense modeling by several independent groups of asteroseismologists, thanks to which, its internal structure is known with unprecedented precision. In **Table 2** we show the main characteristics of all the published pulsating white dwarfs observed with the *Kepler* spacecraft (in both the *Kepler* and *K2* missions). The first column corresponds to the name of the star, the second column is the class of pulsating white dwarf, the third and fourth columns correspond to the effective temperature and surface gravity values, the fifth column is the stellar mass, the sixth column corresponds to the rotation period determined using the observed splittings of frequencies, the seventh column gives the phase of the space mission (*Kepler* or *K2*), and the last column shows the reference to the paper in which the pulsational analysis has been presented.

The findings reported from here to the end of the article are associated with results already published (up to early 2020). However, we emphasize that there is still a large amount of reduced data from the *Kepler* and *K2* missions that have not yet been analyzed, and which could translate into future new discoveries about pulsating white-dwarf and pre-white-dwarf stars.

3.1. A V777 Her Star in the Nominal *Kepler* Mission Field: KIC 8626021

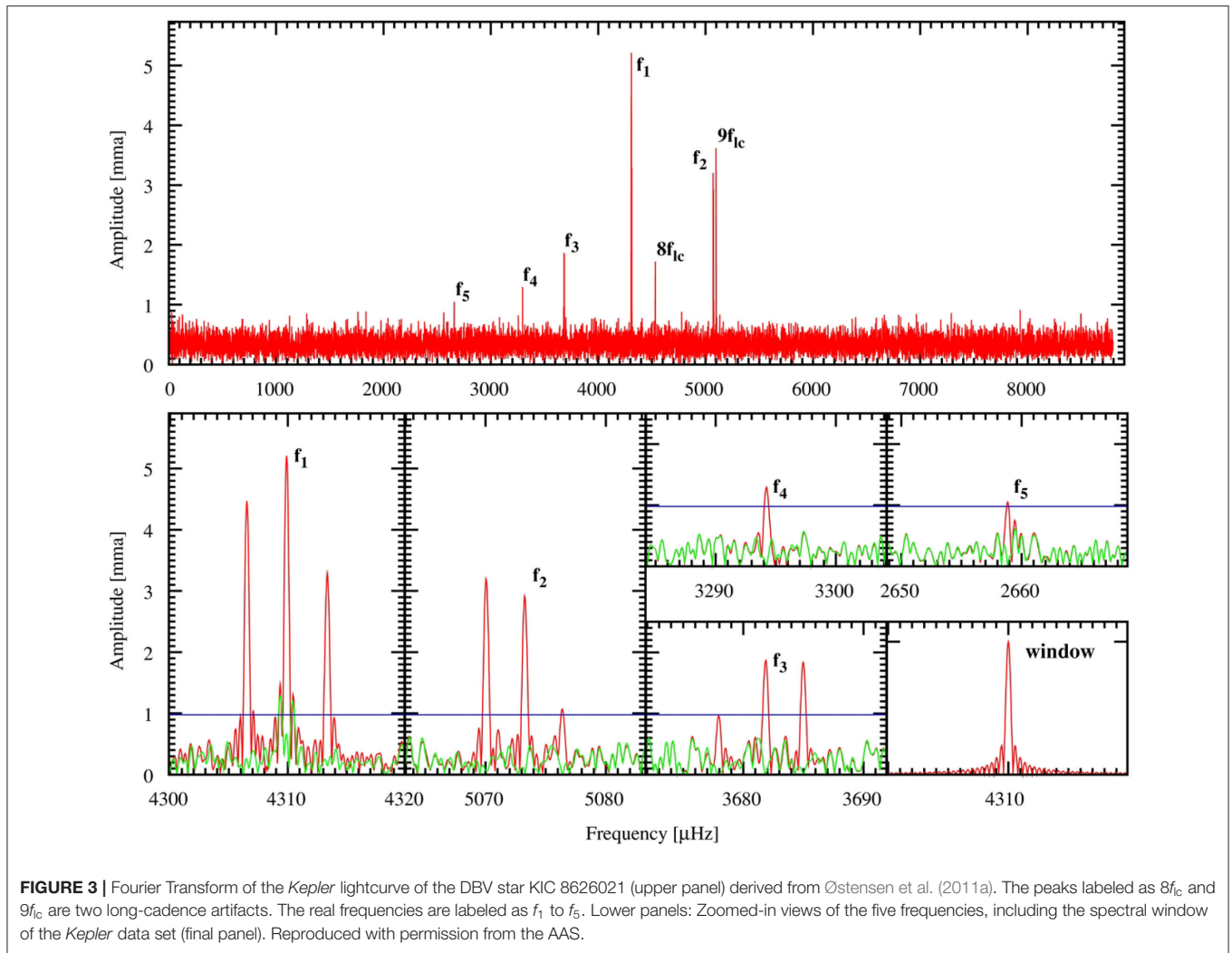
The DB white dwarf star GALEX J192904.6+444708 ($T_{\text{eff}} = 24,900 \pm 750$ K and $\log g = 7.91 \pm 0.07$), identified with the name KIC 8626021 in the *Kepler* input catalog (also known as WD J1929+4447), was photometrically observed by Østensen et al. (2011a) to check for variability from the Earth. This star was part of a number of targets that had not been included in the original sample of 17 white dwarfs to be observed by *Kepler*. A 2-hr photometry run on this star clearly revealed the presence of variability with a period of 232 s and an additional period around 270 s. With ground-based photometry, the dataset was too short to perform any meaningful analysis on, but it clearly demonstrated the pulsational nature of KIC 8626021. After that, Østensen et al. (2011a) analyzed 1 month of short-cadence observations of this pulsating star by *Kepler*. The Fourier transform of the lightcurve is shown in the upper panel of **Figure 3**. The detected frequencies are depicted in the sub-panels, three of which show clear signals of triplet structures due to rotation. The periods detected appear to be associated to a sequence of $\ell = 1$ g modes with low radial order values. The period spacing is close to 36 s, typical of DBV stars. On the basis of the frequency splitting ($\delta\nu \sim 3.3$ μHz) it is possible to infer an estimate of the rotational period of $P_{\text{rot}} \sim 1.7$ days. By using the grids of model atmospheres of Koester (2010), Østensen et al. (2011a) determined $T_{\text{eff}} = 24,900 \pm 750$ K and $\log g = 7.91 \pm 0.07$, placing the star in the middle of the DBV instability strip.

Shortly after the discovery of pulsations in KIC 8626021, the first detailed asteroseismological analysis of the star was performed by Bischoff-Kim and Østensen (2011). They presented an analysis based on the five-mode pulsation spectrum detected by Østensen et al. (2011a) with *Kepler* data. The observed pulsational characteristics of the star and the asteroseismic

analysis strongly suggested that KIC 8626021 was actually hotter than indicated by model-atmosphere fits to the low signal-to-noise electromagnetic spectrum of the object. Indeed, Bischoff-Kim and Østensen (2011) employed a large set of evolutionary models of DB white dwarfs with parameterized chemical profiles, and took three different avenues to determine the effective temperature of KIC 8626021: (i) by means of an inspection of the observed pulsation spectrum, noting that only short-period g modes were present; (ii) employing the average separation between consecutive periods; and (iii) by carrying out asteroseismological period-to-period fits of the pulsation spectrum. All three approaches pointed to an effective temperature of $T_{\text{eff}} \sim 29,200$ K, in disagreement with the spectroscopic value derived by Østensen et al. (2011a) ($T_{\text{eff}} = 24,900$ K). If true, since hot DBVs are thought to lose a large portion of their internal energy through the emission of plasmon neutrinos (Winget et al., 2004), this star could be an extremely important target to place limits on the plasmon-neutrino emission rate. However, the measurement of a rate of period change requires extremely stable (over many years) oscillation modes, a condition that is not perfectly met by KIC 8626021 (see at the end of this section).

As a sanity check for this important result, an independent asteroseismological analysis was crucial at that time. This did not take long. Córscico et al. (2012a) carried out a second asteroseismological analysis, which was based on a set of fully evolutionary/pulsational DB white-dwarf models constructed with the LPCODE evolutionary code (Althaus et al., 2005) and the LP-PUL pulsation code (Córscico and Althaus, 2006). By considering the mean period spacing of KIC 8626021, Córscico et al. (2012a) found that the star should be substantially more massive than suggested by spectroscopy. From period-to-period fits these authors found an asteroseismological model characterized by an effective temperature much higher than the spectroscopic estimate, in agreement with the results of Bischoff-Kim and Østensen (2011). So, this analysis was the second piece of evidence that KIC 8626021 should be located near the blue edge of the DBV instability strip. A very interesting and exciting point is that the DB white-dwarf models used by both groups were completely different, particularly regarding the composition profiles (see **Figure 4**), but nonetheless, similar conclusions were reached regarding the effective temperature of the star.

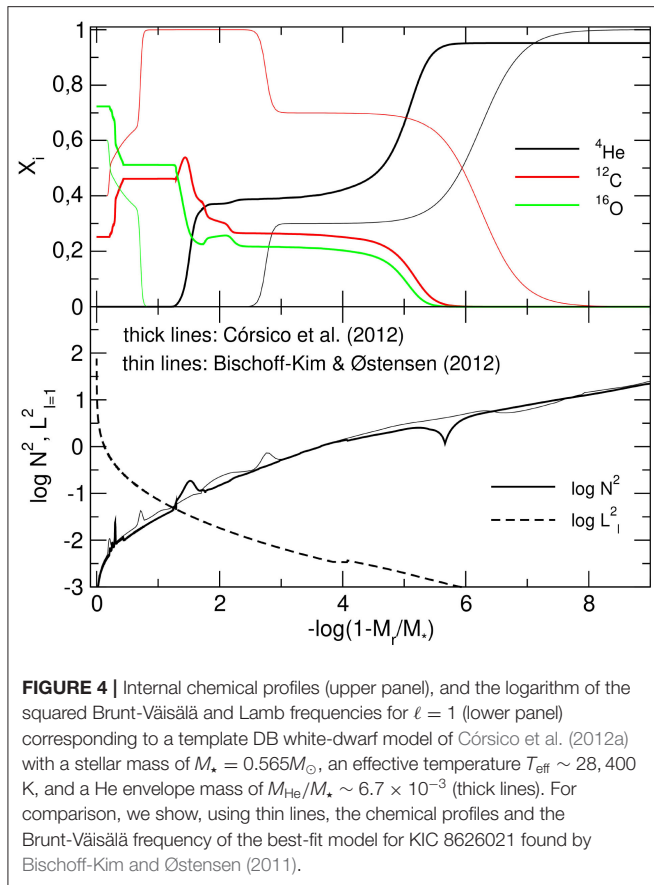
A subsequent asteroseismological analysis of KIC 8626021 was carried out by Bischoff-Kim et al. (2014) on the basis of an augmented set of observed pulsation periods and DB white-dwarf evolutionary models with parameterized chemical profiles, like in Bischoff-Kim and Østensen (2011). The analysis of Bischoff-Kim et al. (2014) was better constrained by the employment of a set of seven independent periods for this star—instead of the five original periods—resulting from 2-years long observations of *Kepler*. By exploiting the presence of the various triplets and doublets, the authors were able to constrain the value of ℓ and m , something that simplified the period-to-period fit and reduced the assumptions at the outset. The resulting asteroseismological model for KIC 8626021 is characterized by a thin pure-He envelope [$\log(1 - M_{\text{He}}/M_{\star}) = -7.90$] and a high effective temperature ($T_{\text{eff}} = 29,650$ K). This last result confirmed that



the star is near the blue edge of the DBV instability strip, in line with the previous analyses. Furthermore, a thin pure-helium layer constituted evidence that the hotter the DB model, the thinner the pure-He layer, supporting the theory that as a DB cools and He diffuses outward, the pure-He layer becomes thicker (see Bischoff-Kim et al., 2014, and references therein).

The most detailed asteroseismological analysis on KIC 8626021 was presented by Giammichele et al. (2018). These authors considered a set of eight periods from the analysis of Zong et al. (2016b). The additional period comes from the alternative interpretation that the structure in the $3.677 - 3.686 \mu\text{Hz}$ range of the spectrum, identified previously as a triplet of frequencies, is actually a doublet, and then the third component is actually an independent mode. Giammichele et al. (2018) presented a new method for parameterizing the chemical profiles in the core of static white-dwarf models, based on Akima splines. They derived an asteroseismological model characterized by a ^{16}O content and an extent of its core that clearly goes beyond the limits predicted by standard DB white dwarf models resulting from fully evolutionary computations. The asteroseismological

model is characterized by $T_{\text{eff}} = 29,968 \pm 200 \text{ K}$ and $\log g = 7.917 \pm 0.009$, which closely match the independent measurements obtained from spectroscopy by the same authors ($T_{\text{eff}} = 29,360 \pm 780 \text{ K}$ and $\log g = 7.89 \pm 0.05$) using the DB white-dwarf model atmospheres of Bergeron et al. (2011). One point to note is that these results confirm the predictions of previous studies that this star is a hot DBV and is close to the blue edge of the V777 Her instability strip. On the other hand, with the new spectroscopic determination of T_{eff} , the discrepancy between asteroseismology and spectroscopy is eliminated. But the most relevant conclusion of this work is that the star seems to have a ^{16}O core (and thus a ^{16}O content) much larger than what standard evolutionary calculations predict. The total ^{16}O content of the white-dwarf core reaches $78.0 \pm 4.2\%$, large in excess when compared with the expected value of around 64% for a standard evolutionary DB white-dwarf model of the same stellar mass. This result constitutes a challenge for the theory of white-dwarf formation and has aroused the interest of several research groups to try to explain how an object of these characteristics could be formed. For example, De Gerónimo et al. (2019) find



that, within our current understanding of white-dwarf formation and evolution, it is difficult (if not impossible) to replicate the most relevant features of the chemical structure of KIC 08626021 derived with asteroseismology. Timmes et al. (2018) have drawn attention to the impact that neutrino emission—a physical ingredient ignored in the DB white-dwarf modeling on which the Giammichele et al. (2018) analysis is based—should have on the pulsation periods of KIC 08626021. In order to assess the effect of neutrino emission on the asteroseismological solution of Giammichele et al. (2018), Charpinet et al. (2019) have redone the analysis on this star by incorporating the effects of neutrino cooling, and basically find the same asteroseismological model for KIC 08626021 than in Giammichele et al. (2018). A summary of all the asteroseismological analyses carried out until now for KIC 08626021 are given in Table 3.

We close this section by noting an interesting phenomenon discovered thanks to the continuous observations of KIC 08626021 with *Kepler*. Zong et al. (2016b) have detected amplitude and frequency modulations of the components of the triplets of frequencies due to a rotation present in the pulsation spectrum of KIC 08626021. Similar secular changes of oscillation frequencies and amplitudes have been measured with the *Kepler* mission in the subdwarf B star KIC 10139564 by Zong et al. (2016a). Since these frequency and amplitude changes occur with timescales that are several orders of magnitude shorter than

the cooling timescale of a DB white dwarf, these modulations of frequencies and amplitudes have nothing to do with any evolutionary effect, such as, e.g., neutrino cooling. Also, it is not expected that these modulations are provoked by orbiting companions around the star, since different timescales for the different triplets are involved. A clue to clarify this behavior could be a the possible interaction between the components of the rotationally split triplets, that is, the presence of non-linear resonant mode coupling (see, for details, Buchler et al., 1997). If true, this star could constitute an excellent opportunity to study non-linear effects in pulsating white dwarfs. Note that, unfortunately, the presence of frequency modulations can make the measurement of the evolutionary (cooling) rate of period change of KIC 08626021 unrealizable, thus hampering any possibility of placing constraints on the plasmon-neutrino emission rate.

3.2. A V777 Her Star in the Field of the K2 Mission: PG 0112+104

Hermes et al. (2017b) presented a pulsational analysis of the already known DBV star PG 0112+104. With $T_{\text{eff}} \gtrsim 30,000$ K, this star is the hottest V777 Her star known, that defines the blue edge of the DBV instability strip. The star was analyzed on the basis of 78.7 days of nearly uninterrupted photometry from the *Kepler* space telescope—specifically the campaign eight of the K2 extended mission. Hermes et al. (2017b) discovered nine additional periods apart from the two periods that were already known from ground-based observations. The pulsation spectrum of this star includes clear patterns of rotational splittings from consecutive sequences of $\ell = 1$ and $\ell = 2$ modes. In addition, a surface rotational period of 10.17 h has been measured using an apparent spot modulation. To summarize, this hot star is a promising candidate to derive the dependence of rotational angular velocity with depth, that is, the differential rotation of the star, through asteroseismology. The estimation of the surface rotation independently from the spot would constitute a strong test for the asteroseismological analysis. On the other hand, as PG 0112+104 is such a hot DBV star, it constitutes an excellent laboratory for studying plasmon neutrino production if the rate of period change for any mode is measured. Since the pulsation frequencies in PG 0112+104 are extremely stable in phase, the secular change of the periods could be detected and measured with additional—ground- or space-based—observations.

3.3. ZZ Ceti Stars in the Nominal *Kepler* Mission Field

Hermes et al. (2011) reported the discovery of the first identified DAV, WD J1916+3938 (Kepler ID 4552982), in the original field of the *Kepler* mission. This ZZ Ceti star was first identified through ground-based, time-series photometry. A follow-up spectroscopic analysis indicated that it is a DA white dwarf with $T_{\text{eff}} = 11,129 \pm 115$ K and $\log g = 8.34 \pm 0.06$, situating it near the hot boundary of the ZZ Ceti instability domain. The object showed up to 0.5 % amplitude variability at several periods between 800 and 1,450 s (Bell et al., 2015). This star was submitted for *Kepler* short-cadence observations. These observations led

TABLE 3 | The main characteristics of KIC 08626021.

Quantity	ØEA11	BKØ11	CEA12	BKEA14	GEA18	CHEA19
T_{eff} (K)	24,950 ± 750	29,200	27,263	29,650	29,968	30,114
M_* (M_{\odot})	0.56 ± 0.03	0.570	0.664	0.550	0.570	0.562
$\log g$ (cm/s ²)	7.91 ± 0.07	—	8.099	—	7.917	7.905
$\log(L_*/L_{\odot})$	—	—	−1.14	—	−0.86	—
$\log(R_*/R_{\odot})$	—	—	−1.93	—	−1.86	−1.86
X_{O} (center)	—	0.60 – 0.65	0.65	0.55	0.86	0.84
$\log(1 - M_{\text{He}}/M_*)$	—	−6.30	−5.95	−7.90	−7.63	−7.83
$\log(1 - M_{\text{env}}/M_*)$	—	−2.80	−1.63	−3.10	−3.23	−3.48

The second column corresponds to the spectroscopic results by ØEA11 (Østensen et al., 2011a), the third, fourth, fifth, sixth, and seventh columns present the results from the asteroseismological studies of BKØ11 (Bischoff-Kim and Østensen, 2011), CEA12 (Córscico et al., 2012a), BKEA14 (Bischoff-Kim et al., 2014), GEA18 (Giannichele et al., 2018), and CHEA19 (Charpinet et al., 2019), respectively. The last two rows correspond to the coordinate of the basis of the pure-He envelope, and the basis of the mixed zone containing ⁴He, ¹²C, and ¹⁶O (see Figure 4).

to the unexpected discovery of a new phenomenon: outburst-like events in DAV stars with effective temperatures near the cool edge of the instability strip (Bell et al., 2015, see section 3.5 for details).

The second DAV in the *Kepler* field, KIC 11911480, was discovered by Greiss et al. (2014) using the *Kepler*-INT Survey (KIS; Greiss et al., 2012) to select white-dwarf candidates with color-colors diagrams. The variable nature of KIC 11911480 was confirmed using ground-based time series photometry. This star is close to the blue edge of the ZZ Ceti instability strip ($T_{\text{eff}} = 12,160 \pm 250$ K and $\log g = 7.94 \pm 0.10$). The star was scrutinized for 6 months with the short-cadence mode of the *Kepler* telescope, and a total of six independent pulsation periods in the range 172.9 – 324.5 s—typical of the hot ZZ Ceti stars—were detected. A preliminary analysis indicated that the star is rotating with a period of ~ 3.5 days. Greiss et al. (2015) and Greiss et al. (2016) reported the pulsation properties of three additional DAV stars observed with *Kepler*: KIC 10132702 ($T_{\text{eff}} = 11048 \pm 217$ K, $\log g = 8.07 \pm 0.08$), KIC 04357037 ($T_{\text{eff}} = 11898 \pm 200$ K, $\log g = 8.03 \pm 0.08$), and KIC 07594781 ($T_{\text{eff}} = 12217 \pm 1700$ K, $\log g = 7.54 \pm 0.22$). Using the frequency spacing in the triplets and doublets, these authors estimated the rotation periods of these stars, obtaining values in the range of (0.9 – 3.2) days.

3.4. ZZ Ceti Stars Monitored With the *K2* Mission

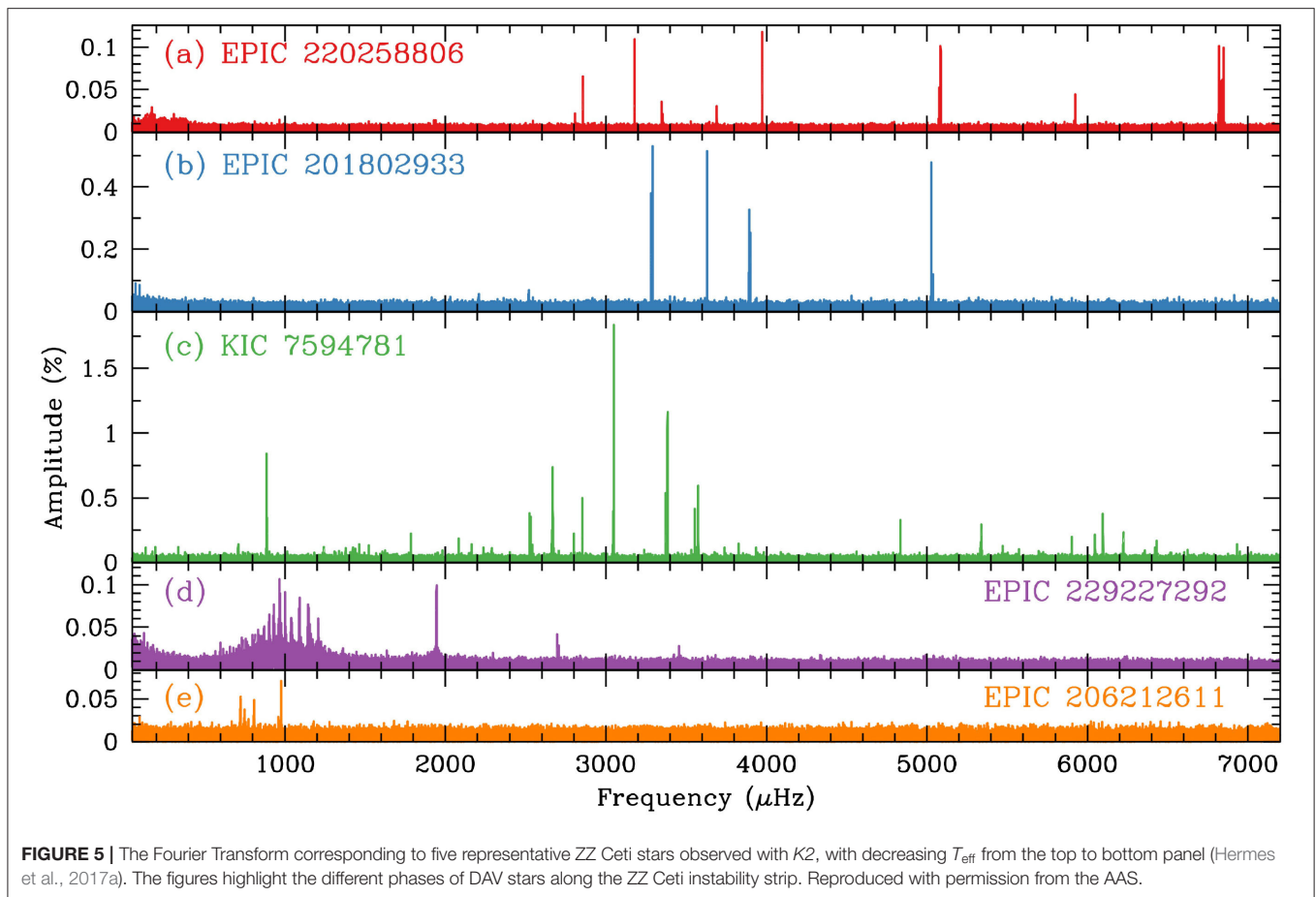
The first DAV star observed with the *K2* mission was GD 1212 (Hermes et al., 2014). This star, which was already known to be pulsating (Gianninas et al., 2006), is a cool DAV with $T_{\text{eff}} = 10,970 \pm 170$ K and $\log g = 8.03 \pm 0.05$, corresponding to a mass of $0.62 \pm 0.03 M_{\odot}$. *K2* short-cadence observations revealed at least 19 independent pulsation modes, ranging from 828.2 to 1220.8 s, and at least 17 non-linear combination frequencies. This star also exhibits amplitude and frequency variations on timescales less than a week, reminiscent to the behavior detected by Zong et al. (2016b) in the DBV star KIC 08626021 (see at the end of section 3.1). This phenomenon makes a precise determination of the pulsation periods of GD 1,212 more complicated. Since

this star is a cool ZZ Ceti, it would be expected to be characterized by high amplitudes. However, the independent modes detected, as well as the frequency combinations, show low amplitudes.

Another DAV star analyzed with the *K2* mission is SDSS J113655.17+040952.6 (hereafter SDSS J1136+0409), the first known pulsating DA white-dwarf in a post-common envelope binary system with a main-sequence (dM) companion star (Pyrzas et al., 2015). The ZZ Ceti component has $T_{\text{eff}} = 12,330 \pm 260$ K and $M_* = 0.601 \pm 0.036 M_{\odot}$. Nearly 78 days of *K2* observations allowed Hermes et al. (2015a) to detect seven independent pulsation modes, three of which are rotationally split multiplets compatible with a rotation period of 2.49 ± 0.53 h.

Usually, observations of pulsating white dwarfs with the *Kepler* space telescope have to be performed using the short-cadence mode of observations (1-min exposures) to sufficiently over-sample typical white-dwarf pulsation periods (3–20 min) for straightforward frequency measurement. However, Bell et al. (2017b) have demonstrated that it is possible to combine long-cadence *K2* data (30 min exposures) with high-speed follow-up ground-based observations, to derive accurate oscillation frequencies to a precision of $\sim 0.01 \mu\text{Hz}$. Using this approach, Bell et al. (2017b) have discovered two new ZZ Ceti variables from *K2* long-cadence data: EPIC 210377280 and EPIC 220274129. For EPIC 220274129, Bell et al. (2017b) inferred a stellar rotation period of 12.7 ± 1.3 h.

The most complete and detailed study of ZZ Ceti stars observed with the *Kepler* space telescope has been presented by Hermes et al. (2017a). This monumental work, based on the observations of 27 ZZ Ceti stars collected with the nominal *Kepler* mission and mostly with the extended *K2* mission, presents, in detail, the occurrence of outbursts in ZZ Ceti stars near the red edge of the DAV instability strip (see section 3.5), the discovery of a dichotomy of the widths of peaks in the Fourier spectrum (section 3.6), and a possible relationship between the rotation periods derived from frequency splittings for 20 of the 27 DAVs and their stellar mass (see section 3.7). In addition, Hermes et al. (2017a) have been able to characterize the DAV instability strip as observed by *Kepler* and *K2*. Indeed,



they show how the different trends regarding the length of the excited periods and the amplitude of the modes evolve as the effective temperature decreases along the instability strip. In particular, Hermes et al. (2017a) find that the excited periods are short and with low amplitude near the blue edge of the instability domain, but then the excited periods grow (modes of larger radial orders are excited) and the amplitude also increases with decreasing T_{eff} . Toward effective temperatures near the cool boundary of the instability strip, some ZZ Ceti show outbursts. Finally, at the red edge, the excited modes of DAVs are the longest-period pulsations with relatively small amplitude. **Figure 5**, which displays the Fourier spectrum of ZZ Ceti stars at different effective temperatures, dramatically illustrates the described trend.

3.5. Outbursting ZZ Ceti Stars

The *Kepler* spacecraft observations of ZZ Ceti stars led to the discovery of a energetic phenomenon never observed before from the ground in this type of quiet pulsating stars: outburst-like events (Bell et al., 2017a). So far, this constitutes one of the most transcendent discoveries (perhaps the most transcendent) in relation to pulsations in white dwarfs observed with *Kepler*. An important aspect of these events is that they repeat chaotically, with no fixed period of recurrence. The first outbursting DAV,

KIC4552982⁵, which is located near the red edge of the ZZ Ceti instability strip ($T_{\text{eff}} = 11,129 \pm 115$ K, $\log = 8.34 \pm 0.06$), was observed during ~ 20 months with the nominal *Kepler* mission, yielding the longest pseudo-continuous light curve of a ZZ Ceti ever obtained (Bell et al., 2015). These observations allowed the detection of 20 pulsation modes with periodicities commonly observed in ZZ Ceti stars, along with 178 increments of brightness typical of outburst phenomena, with peaks of up to 17% above the quiescent level. The outbursts involve very energetic events ($E \sim 10^{33}$ erg), with a mean recurrence period of about 2.7 days and 4–25 h of duration (Bell et al., 2015). The second outbursting star, the already known cool DAV PG 1149+057 ($T_{\text{eff}} = 11,060 \pm 170$ K, $\log = 8.06 \pm 0.05$), was analyzed by Hermes et al. (2015b) with nearly continuous *K2* observations for more than 78.8 days. This star shows outbursts with a typical recurrence time of ~ 8 days and 15 h of duration, and very large flux enhancements of up to 45% above the quiescent level. For this star, the outbursts have a measurable impact on the spectrum of g modes (in amplitude and frequency), demonstrating that *outbursts are an intrinsic phenomenon of the (otherwise isolated) star*. In **Figure 6** we show

⁵This star was the first ZZ Ceti identified in the original *Kepler* mission field (Hermes et al., 2011, ; see section 3.3).

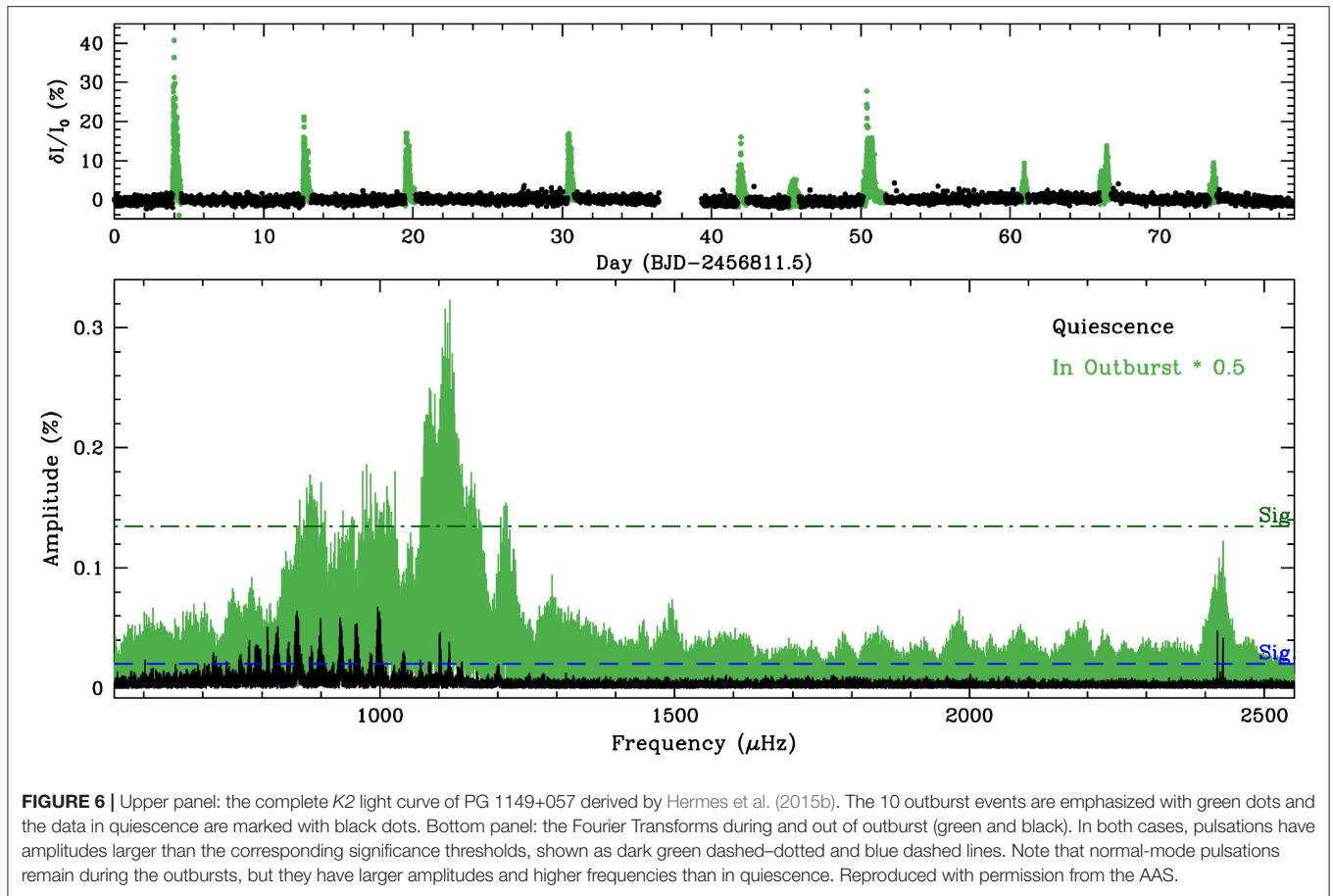


FIGURE 6 | Upper panel: the complete *K2* light curve of PG 1149+057 derived by Hermes et al. (2015b). The 10 outburst events are emphasized with green dots and the data in quiescence are marked with black dots. Bottom panel: the Fourier Transforms during and out of outburst (green and black). In both cases, pulsations have amplitudes larger than the corresponding significance thresholds, shown as dark green dashed–dotted and blue dashed lines. Note that normal-mode pulsations remain during the outbursts, but they have larger amplitudes and higher frequencies than in quiescence. Reproduced with permission from the AAS.

the complete *K2* light curve of PG 1149+057 (upper panel) and the Fourier Transforms for the case during and out of outburst (lower panels). At the time of writing this article, a total of eight outbursting (isolated) DAV stars have been discovered (Bell et al., 2016, 2017a), all of them populating the red edge of the ZZ Ceti instability strip (see Figure 12 of Córscico et al., 2019). This strongly suggests that outbursts could be related to the origin of the cool edge of the DAV instability domain, that is, the cessation of the *g*-mode pulsations (Bell et al., 2017a). A possible explanation for both the occurrence of outbursts and the origin of the cool edge of the ZZ Ceti instability strip, has been proposed by Luan and Goldreich (2018). It is connected with parametric instability through mode coupling of white-dwarf pulsations (Dziembowski, 1982; Wu and Goldreich, 2001; Luan and Goldreich, 2018).

3.6. Dichotomy of Mode-Line Widths in ZZ Ceti Stars

Another major observational result from the *Kepler* space telescope is the discovery by Hermes et al. (2017a) of a clear dichotomy of oscillation mode-line widths in the power spectrum of 27 DAVs observed from space. Indeed, *g* modes characterized by periods longer than roughly 800 s are generally incoherent (large line widths) over the length of observations, while *g* modes with periods \lesssim 800 s are observed to be much more stable

in phase and amplitude (smaller line width)⁶. We depict in Figure 7 (extracted from Hermes et al., 2017a) the half-width at half-maximum (HWHM) of all significant peaks (excluding any non-linear combination frequencies) in the power spectra of 27 DAVs observed through *Kepler* and *K2*. The dichotomy of mode-line widths could be intimately linked to the oscillation of the outer convection zone of a DA white dwarf during pulsations. Specifically, the oscillation out of and in the base of the convection zone would affect the radial eigenfunction of certain *g* modes—those that have the outer turning point of oscillation located precisely at the base of the convection zone (Montgomery et al., 2020). This mechanism may be relevant for the limitation of pulsation amplitudes in pulsating white dwarfs for modes with periods above a threshold period.

3.7. White-Dwarf Rotation Rates From Asteroseismology of ZZ Ceti Stars

Hermes et al. (2017a) have also derived the rotation rates of 20 isolated DAVs observed with *Kepler* and *K2* using rotational splittings, doubling the number of pulsating white dwarfs with

⁶This phenomenon cannot be associated to stochastic mode driving, which for white dwarfs would excite pulsations with very short periods, of the order of \sim 1 s (Saio, 2013).

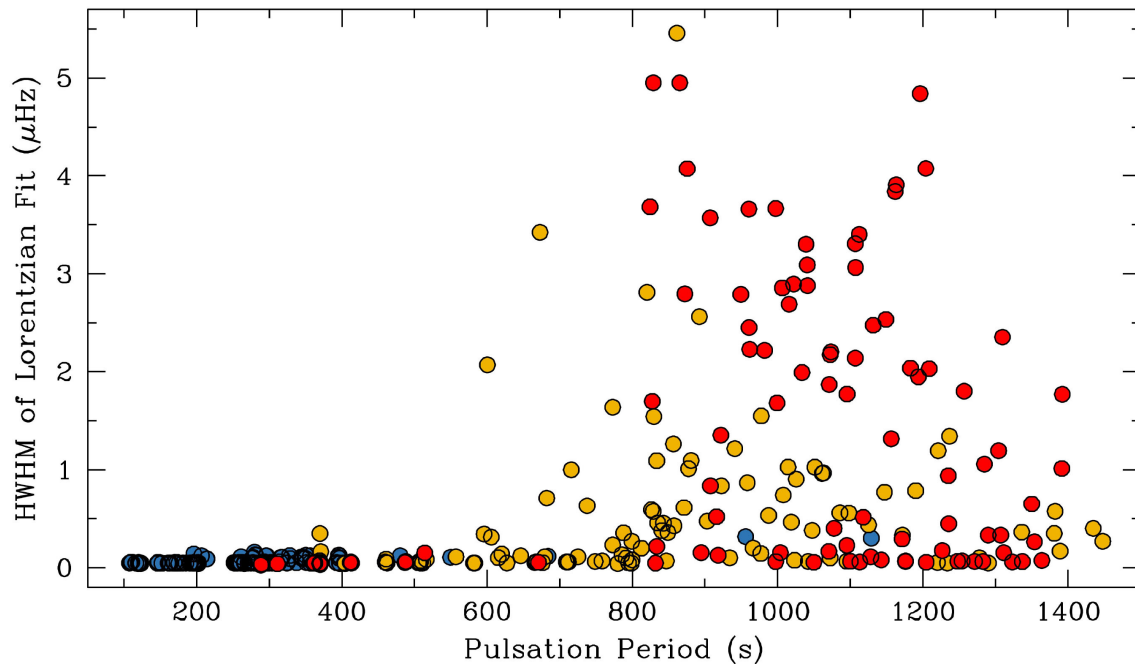


FIGURE 7 | Half-width at half-maximum (HWHM) of Lorentzian functions fit to all significant peaks (corresponding to genuine eigenmodes) in the power spectra of 27 DAVs observed through K2 (Hermes et al., 2017a). Blue circles are for objects with weighted mean period (WMP) < 600 s, gold with WMP > 600 s, and red are those with outbursts. A clear increase in HWHM at ~ 800 s is present. Reproduced with permission from the AAS.

measured rotation rates via asteroseismology⁷ (see **Table 2** of this article and **Table 10** of Córscico et al., 2019). The results indicate that the average rotation period is $\sim 30 - 40$ h, although ZZ Ceti stars that rotate faster (with periods of $\sim 1 - 2$ h) have also been found. There is some evidence of the existence of a correlation between high mass and fast rotation, an idea reinforced by the fastest rotation ever measured in an isolated white dwarf⁸, of 1.13 h, for the massive ZZ Ceti star SDSSJ0837+1856 ($M_{\star} = 0.87M_{\odot}$) by Hermes et al. (2017c), although more high-mass white dwarfs are necessary to confirm this trend. Hermes et al. (2017a) find that most isolated descendants of $1.7 - 3.0M_{\odot}$ ZAMS progenitors rotate at ~ 1.5 days, instead of minutes, which would be expected if the angular momentum of a $3.0M_{\odot}$ MS star with an initial rotation period of 10 h, were fully conserved (Kawaler, 2004). This fact strongly indicates that most internal angular momentum must be lost on the first-ascent giant branch.

3.8. Asteroseismology of *Kepler* ZZ Ceti Stars With Fully Evolutionary Models

In section 3.1 we described, in detail, the various asteroseismological analyses conducted to elucidate the internal structure of the *Kepler* DBV star KIC 8626021. Here, we briefly describe the asteroseismological analysis

on ZZ Ceti observed with *Kepler* carried out by Romero et al. (2017). This analysis was based on evolutionary and pulsational white-dwarf models computed with the LPCODE and LP-PUL codes, adopting the La Plata Group's approach of white-dwarf asteroseismology (see at the end of section 2). Specifically, the first four published ZZ Ceti stars observed with the *Kepler* mission, that is, EPIC 60017836 (GD 1212), EPIC 201730811 (SDSS J113655.17+040952.6), KIC 11911480, and KIC 4552982, were studied. Currently, this remains the only detailed asteroseismological study of DAVs observed with the *Kepler* space telescope. A period-to-period fit analysis of the four target stars was carried out by employing a grid of full evolutionary models representative of C/O-core DA white-dwarf stars constructed with detailed and updated input physics. These models have consistent chemical profiles for both the core and the envelope for various stellar masses, particularly suitable for asteroseismological fits of ZZ Ceti stars. The chemical profiles of the models were computed considering the complete evolution of the progenitor stars from the ZAMS through the thermally pulsing and mass-loss phases on the asymptotic giant branch (AGB).

Apart from the observed periods, the analysis by Romero et al. (2017) incorporated the amplitude of the modes, rotational splitting multiplets, and period spacing data, as well as photometry and spectroscopy information. For each analyzed star, an asteroseismological model that closely reproduces observed properties, such as periods, stellar mass, and effective temperature was presented. In the particular case of KIC 11911480 and EPIC 201730811, the asteroseismological masses

⁷Note that rotation rates can sometimes also be measured from long-cadence data, as in Bell et al. (2017b).

⁸Except for the magnetic DA white dwarf RE J0317-853, with an apparent period of ~ 725 s (Barstow et al., 1995), and SDSSJ125230.93?023417.72, which is the fastest-rotating apparently isolated white dwarf yet discovered, that rotates with a period of 317.278 ± 0.013 s (Reding et al., 2020).

are similar to each other, although the H envelope for EPIC 201730811 (that is part of a binary system and likely went through a common envelope phase) is 10 times thinner than that for KIC 11911480. This result confirms what was discussed in Hermes et al. (2015a) on the basis of a preliminary asteroseismological analysis, that the H-layer mass of EPIC 201730811 is roughly $10^{-5}M_{\star}$. In the case of KIC 4552982, which is a red-edge ZZ Ceti star, the asteroseismological model has a very thin H envelope mass ($M_{\text{H}}/M_{\odot} = 4.7 \times 10^{-9}$), which could be related to the outburst nature of this star, as reported by Bell et al. (2015). Whether this is a common characteristic between all the outbursting DAVs or not is a very interesting topic that deserves further exploration. Finally, in the case of EPIC 60017836 (another red-edge DAV), an asteroseismological model was obtained with a stellar mass compatible with the atmospheric parameters from photometry combined with parallax and spectroscopy.

4. CONCLUSIONS

The *Kepler* space telescope—both the nominal *Kepler* mission and the extended mission *K2*—has been very successful in its search for extrasolar planets and has also meant a revolution in the area of asteroseismology for many classes of pulsating stars. In the particular case of pulsating white dwarfs, the numbers speak for themselves. In total, 2,166 white dwarfs were observed by *Kepler/K2*. There were 81 DAVs observed by *Kepler/K2*, with 75 of them obtaining short cadence data. Of them, analyses of 32 DAVs have been published (29 stars analyzed in journals and three objects analyzed in a PhD Thesis; see Table 2). Thanks to *Kepler/K2*, the number of ZZ Ceti stars that have a rotation period measured with asteroseismology has increased by a factor of 2. Regarding DBVs, seven stars were observed, although the analysis of only two of them have been published to date. Finally, two unique GW Vir stars were observed, including the prototypical star PG 1159–035, but no results have been published yet.

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The story of the *Kepler* mission and the pulsating white dwarf stars does not end here. There is an immense amount of data waiting to be analyzed, and this will likely lead to new discoveries in the near future. In turn, this will drive new asteroseismological analyses, in addition to those already published. Altogether, the *Kepler* space telescope has had a strong impact (and will continue to have) on the area of white-dwarf asteroseismology. This first step will be multiplied by current and future space missions, such as the Transiting Exoplanet Survey Satellite (*TESS*; Ricker et al., 2015), which is already in full operation and is providing the first results on pulsating white dwarfs (Bell et al., 2019; Althaus et al., 2020; Bognár et al., 2020), and other space missions that will become operational in the coming years, such as Cheops (Moya et al., 2018) and Plato (Piotto, 2018).

AUTHOR CONTRIBUTIONS

The author confirms being the sole contributor of this work and has approved it for publication.

FUNDING

Part of this work was supported by AGENCIA through the Programa de Modernización Tecnológica BID 1728/OC-AR, by the PIP 112-200801-00940 grant from CONICET, and by the grant G149 from the University of La Plata.

ACKNOWLEDGMENTS

I thank the referees for their valuable suggestions that improved the content and presentation of the paper. I warmly thank Leandro G. Althaus, Keaton J. Bell, and J. J. Hermes for being so kind to read the manuscript and suggest relevant changes that greatly improved its content. This research has made intensive use of the NASA Astrophysics Data System.

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Conflict of Interest: The author declares that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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