

Brans–Dicke Boson Stars: Configurations and Stability through Cosmic History

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We make a detailed study of boson star configurations in Jordan–Brans–Dicke theory, studying both equilibrium properties and stability, and considering boson stars existing at different cosmic epochs. We show that boson stars can be stable at any time of cosmic history and that equilibrium stars are denser in the past. We analyze three different proposed mass functions for boson star systems, and obtain results independently of the definition adopted. We study how the configurations depend on the value of the Jordan–Brans–Dicke coupling constant, and the properties of the stars under extreme values of the gravitational asymptotic constant. This last point allows us to extract conclusions about the stability behaviour concerning the scalar field. Finally, other dynamical variables of interest, like the radius, are also calculated. In this regard, it is shown that the radius corresponding to the maximal boson star mass remains roughly the same during cosmological evolution.

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I. INTRODUCTION

Although boson stars [1–4] are so far entirely theoretical constructs, they give rise to one of the simplest possible stellar environments in which to study gravitational phenomena mathematically. One can find numerical solutions which are nonsingular and yet exhibit strong gravitational effects. Many of their properties bear close resemblance to those of neutron stars.

Boson stars were first conceived as Klein–Gordon geons — systems held together by gravitational forces and composed of classical fields. They are a gravitationally-bound macroscopic state made up of scalar bosons. As with neutron stars, the pressure support which leads to their existence is intrinsically quantum. For neutron stars, the pressure support derives from the Pauli exclusion principle, and for boson stars this is replaced by Heisenberg’s uncertainty principle. Assuming that the quantum state contains sufficient particles for gravitational effects to be important, and that particle interactions can be neglected, an estimate of the mass is readily obtained as follows. For a quantum state confined into a region of radius R , and with units given by $\hbar = c = 1$, the boson momentum is $p = 1/R$. If the star is moderately relativistic, $p \simeq m$, then $R \simeq 1/m$. If we equate R with the Schwarzschild radius $2M/m_{\text{Pl}}^2$ (recall that $G \equiv m_{\text{Pl}}^{-2}$), we find $M \simeq m_{\text{Pl}}^2/m$.

In practice, one assumes the existence of a classical scalar field with a given Lagrangian density, and adopts an ansatz for its time dependence which implicitly encodes the Heisenberg uncertainty. This time dependence is of course of a form which still permits a static metric. With these ingredients, one then solves Einstein’s equations, something which must be done numerically. When no self-interaction term is present in the Lagrangian density, the masses concur with the estimate above. How-

ever, if self-interaction is present it is typically the dominant contributor to pressure support, and leads instead to masses of order m_{Pl}^3/m^2 . If the boson mass is comparable to a nucleon mass, this order of magnitude is comparable to the Chandrasekhar mass, about $1M_{\odot}$ [5]. Thus, boson stars arise as possible candidates for non-baryonic dark matter, and are possibly detectable by microlensing experiments.

Boson stars have been widely studied in general relativity, where the basic model has been extended in various ways, such as including a $U(1)$ charge [6], allowing a mixture of boson and fermion components [7], or including a non-minimal coupling of the boson field to gravity [8]. These and others works are summarized in two reviews [3,4], and more recently in Ref. [9]. The possibility of direct observational detection of boson stars was studied recently in Ref. [10], where it was asked whether radiating baryonic matter moving *within* a boson star could be converted into an observational signal. Unfortunately, any direct detection looks a long way off.

Given the simplicity of the boson star, it is natural to examine boson star solutions in theories of gravity other than general relativity, to examine whether new phenomena arise. The most-studied class of such theories are the scalar–tensor theories of gravity [11], which include the Jordan–Brans–Dicke (JBD) theory as a special case. In these, Newton’s gravitational constant is replaced by a field ϕ known as the Brans–Dicke field, the strength of whose coupling to the metric is given by a function $\omega(\phi)$. If ω is a constant, this is the JBD theory [12], which is the simplest scenario one may have in this framework. General relativity is attained in the limit $1/\omega \rightarrow 0$. To ensure that the weak-field limit of this theory agrees with current observations, ω must exceed 500 at 95% confidence [11] from solar system timing experiments, i.e. experiments taking place in the current cosmic time. This

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limit is both stronger and less model-dependent than limits from nucleosynthesis [13]. Scalar–tensor theories have regained popularity through inflationary scenarios based upon them [14], and because a JBD model with $\omega = -1$ is the low-energy limit of superstring theory [15].

The first scalar–tensor models of boson stars were studied by Gunderson and Jensen [16], who concentrated on JBD theory with $\omega = 6$. This was generalized by Torres [17], both to other JBD couplings and to some particular scalar–tensor theories with non-constant $\omega(\phi)$ chosen to match all current observational constraints. This allowed a study of some models which, inside the structure of the star, have couplings deviating greatly from the large value required today. The conclusion is that boson star models can exist in any scalar–tensor gravity, with masses which are always smaller than the general relativistic case (for a given central scalar field density), irrespective of the coupling.

A vital point to consider is that when one finds cosmological solutions in scalar–tensor theories, the gravitational coupling is normally evolving. This has important implications for astrophysical objects, because it means that the asymptotic boundary condition for the ϕ field is in general a function of epoch. One can then ask, as originally done by Barrow in the context of black holes [18], how the structure of the astrophysical object is affected given that the asymptotic gravitational coupling continues to evolve after the object forms. Two possibilities exist; either the star can adjust its structure in a quasi-stationary manner to the asymptotic gravitational constant, or it might ‘remember’ the strength of gravity at the time it formed. Barrow called this latter possibility *gravitational memory*. In the former case, stellar evolution is driven entirely by gravitational effects, while in the latter case objects of the same mass could differ in other physical properties, such as their radius. Either possibility has fascinating consequences, which we have already explored in Ref. [19].

However, which of the two scenarios is correct remains unknown, either for black holes or boson stars. Since boson star solutions are non-singular, they appear to offer better prospects for determination the actual behaviour. Consequently, it is important at this stage to have a complete description of boson star models at different eras of cosmic history, which may be used later either as an initial condition for, or to compare with the output of, a dynamical evolutionary code.

Recently, two other works have been presented concerning scalar–tensor gravity effects on equilibrium boson stars. In the first of them, Comer and Shinkai [20] studied zero and higher mode configurations for the Damour–Nordtvedt approach to scalar–tensor theories [21]. They also studied stability properties of boson stars both at the present time and in the past. They concluded that no stable boson stars exist before a certain cosmic time, due to all possible configurations possessing a positive binding energy. This result appears surprising, as the boson stars should have no particular awareness of the

present value of the gravitational coupling, and it would appear a great coincidence that the transition between instability and stability should occur at a recent cosmic epoch. In fact, their result has already been questioned by Whinnett [22], in a detailed discussion of the meaning of the boson star mass in scalar–tensor theories. Our results also indicate that boson stars may form and be stable at any cosmic epoch. Finally, the dynamical formation of boson stars was analyzed in Ref. [23], where a similar behaviour to that of general relativity was found.

In this paper, we aim to provide a comprehensive study of equilibrium configurations of boson stars, emphasizing their characteristics, such as mass and radii, at different moments of cosmic history. We shall also study, using catastrophe theory, their stability properties. As seen in Ref. [17], the features of JBD and general scalar–tensor boson stars do not differ much. Hence, we shall concentrate only on JBD boson stars, examining the dependence on ω . Finally, we shall test whether the Brans–Dicke scalar can induce any change in the stability properties even for extreme values of Newton’s constant.

The organization of the rest of this work is as follows. In the next section we briefly introduce the formalism, following Ref. [17]. The following section will analyze some recently-proposed mass functions for JBD boson stars, and justify our choice for this work. We shall also comment on the use of catastrophe theory in the study of stability properties. Finally, the results of our numerical simulations will be given in Sec. V and our conclusions will be stated in Sec. VI.

II. FORMALISM

First we derive the equations corresponding to a general scalar–tensor theory. The action for these generalized JBD theories is

$$S = \int \frac{\sqrt{-g}}{16\pi} dx^4 \left[\phi R - \frac{\omega(\phi)}{\phi} \partial_\mu \phi \partial^\mu \phi + 16\pi \mathcal{L}_m \right]. \quad (1)$$

Here $g_{\mu\nu}$ is the metric, R the scalar curvature, ϕ the Brans–Dicke field, and \mathcal{L}_m the Lagrangian of the matter content of the system.

We take this \mathcal{L}_m to be the Lagrangian density of a complex, massive, self-interacting scalar field ψ . This Lagrangian reads as:

$$\mathcal{L}_m = -\frac{1}{2} g^{\mu\nu} \partial_\mu \psi^* \partial_\nu \psi - \frac{1}{2} m^2 |\psi|^2 - \frac{1}{4} \lambda |\psi|^4. \quad (2)$$

The $U(1)$ symmetry leads to conservation of boson number. Varying the action with respect to $g^{\mu\nu}$ and ϕ we obtain the field equations:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi}{\phi} T_{\mu\nu} + \frac{\omega(\phi)}{\phi} \left(\phi_{,\mu} \phi_{,\nu} - \frac{1}{2} g_{\mu\nu} \phi^{,\alpha} \phi_{,\alpha} \right) + \frac{1}{\phi} (\phi_{,\mu;\nu} - g_{\mu\nu} \square \phi), \quad (3)$$

$$\square\phi = \frac{1}{2\omega + 3} \left[8\pi T - \frac{d\omega}{d\phi} \phi^{,\alpha} \phi_{,\alpha} \right], \quad (4)$$

where $T_{\mu\nu}$ is the energy–momentum tensor for the matter fields and T its trace. This energy–momentum tensor is given by

$$T_{\mu\nu} = \frac{1}{2} (\psi_{,\mu}^* \psi_{,\nu} + \psi_{,\mu} \psi_{,\nu}^*) - \frac{1}{2} g_{\mu\nu} (g^{\alpha\beta} \psi_{,\alpha}^* \psi_{,\beta} + m^2 |\psi|^2 + \frac{1}{2} \lambda |\psi|^4). \quad (5)$$

Commas and semicolons are derivatives and covariant derivatives, respectively. The covariant derivative of this tensor is null. That may be proved either from the field equations, recalling the Bianchi identities, or by intuitive arguments such as the minimal coupling between the field ϕ and the matter fields. This implies,

$$\psi^{;\mu}_{;\mu} - m^2 \psi - \lambda |\psi|^2 \psi^* = 0. \quad (6)$$

We now introduce the background metric, corresponding to a spherically-symmetric system which is the symmetry we impose upon the star. Then

$$ds^2 = -B(r) dt^2 + A(r) dr^2 + r^2 d\Omega^2. \quad (7)$$

We also demand a spherically-symmetric form for the scalar field describing the bosonic part and we adopt a form consistent with the static metric,

$$\psi(r, t) = \chi(r) \exp[-i\omega t]. \quad (8)$$

To write the equations of structure of the star, we use a rescaled radial coordinate, given by

$$x = mr. \quad (9)$$

From now on, a prime will denote a derivative with respect to the variable x . We also define dimensionless quantities by

$$\Omega = \frac{\omega}{m}, \quad \Phi = \frac{\phi}{m_{\text{Pl}}^2}, \quad \sigma = \sqrt{4\pi} \frac{\chi(r)}{m_{\text{Pl}}}, \quad \Lambda = \frac{\lambda}{4\pi} \frac{m_{\text{Pl}}^2}{m^2}, \quad (10)$$

where $m_{\text{Pl}} \equiv G_0^{-1/2}$ is the present Planck mass. Note that our dimensionless variables are defined with respect to our observed Planck mass, regardless of whether or not that corresponds to the Planck mass at that time. Our observed gravitational coupling implies $\Phi = 1$.^{*} In order to consider the total amount of mass of the star within a radius x we change the function A in the metric to its Schwarzschild form,

$$A(x) = \left(1 - \frac{2M(x)}{x\Phi(\infty)} \right)^{-1}. \quad (11)$$

This expression *defines* $M(x)$. The issue of mass definitions in JBD boson stars will be examined more deeply in the following section. Note that a factor $\Phi(\infty)$ appears in Eq. (11). This is crucial to obtain the correct value of the mass, which from comparing this to the asymptotic form of the JBD–Schwarzschild solution is given by

$$M_{\text{star}} = M(\infty) \frac{m_{\text{Pl}}^2}{m}, \quad (12)$$

for a given value of m .[†]

With all these definitions, the equations of structure reduce to the following set:

$$\begin{aligned} \sigma'' + \sigma' \left(\frac{B'}{2B} - \frac{A'}{2A} + \frac{2}{x} \right) \\ + A \left[\left(\frac{\Omega^2}{B} - 1 \right) \sigma - \Lambda \sigma^3 \right] = 0, \end{aligned} \quad (13)$$

$$\begin{aligned} \Phi'' + \Phi' \left(\frac{B'}{2B} - \frac{A'}{2A} + \frac{2}{x} \right) + \frac{1}{2\omega + 3} \frac{d\omega}{d\Phi} \Phi'^2 \\ - \frac{2A}{2\omega + 3} \left[\left(\frac{\Omega^2}{B} - 2 \right) \sigma^2 - \frac{\sigma'^2}{A} - \Lambda \sigma^4 \right] = 0, \end{aligned} \quad (14)$$

$$\begin{aligned} \frac{B'}{xB} - \frac{A}{x^2} \left(1 - \frac{1}{A} \right) = \frac{A}{\Phi} \left[\left(\frac{\Omega^2}{B} - 1 \right) \sigma^2 + \frac{\sigma'^2}{A} - \frac{\Lambda}{2} \sigma^4 \right] \\ + \frac{\omega}{2} \left(\frac{\Phi'}{\Phi} \right)^2 + \left(\frac{\Phi''}{\Phi} - \frac{1}{2} \frac{\Phi' A'}{\Phi A} \right) + \frac{1}{2\omega + 3} \frac{d\omega}{d\Phi} \frac{\Phi'^2}{\Phi} \\ - \frac{A}{\Phi} \frac{2}{2\omega + 3} \left[\left(\frac{\Omega^2}{B} - 2 \right) \sigma^2 - \frac{\sigma'^2}{A} - \Lambda \sigma^4 \right], \end{aligned} \quad (15)$$

$$\begin{aligned} \frac{2BM'}{x^2\Phi(\infty)} = \frac{B}{\Phi} \left[\left(\frac{\Omega^2}{B} + 1 \right) \sigma^2 + \frac{\sigma'^2}{A} + \frac{\Lambda}{2} \sigma^4 \right] \\ + \frac{B}{\Phi} \frac{2}{2\omega + 3} \left[\left(\frac{\Omega^2}{B} - 2 \right) \sigma^2 - \frac{\sigma'^2}{A} - \Lambda \sigma^4 \right] \\ + \frac{\omega}{2} \frac{B}{A} \left(\frac{\Phi'}{\Phi} \right)^2 - \frac{B}{A(2\omega + 3)} \frac{d\omega}{d\Phi} \frac{\Phi'^2}{\Phi} - \frac{1}{2} \frac{\Phi'}{\Phi} \frac{B'}{A}. \end{aligned} \quad (16)$$

To solve these equations numerically, we use a fourth-order Runge–Kutta method, for which details may be found in Ref. [17]. In general relativity, the possible equilibrium solutions are entirely parametrized by the central value of the boson field, $\sigma(0)$. In JBD theory, one also

^{*}There is actually a post-Newtonian correction to this of order $1/\omega$ [11], which we shall not concern ourselves with.

[†]This corrects an error in Eq. (10) of Ref. [19]. That error was typographical only and did not affect any computations in that paper.

needs to specify the asymptotic strength of the gravitational coupling, Φ_∞ , or equivalently the value of Φ at the centre of the star.

The particle number, conserved due to the $U(1)$ symmetry of the ψ field, is

$$N_{\text{star}} = \frac{m_{\text{Pl}}^2}{m^2} \Omega \int_0^\infty \sigma^2 \sqrt{\frac{A}{B}} x^2 dx \equiv \frac{m_{\text{Pl}}^2}{m^2} N_\infty, \quad (17)$$

where the last equality defines N_∞ . If the particles comprising the star were widely separated, their mass would be mN_{star} . One can therefore define a binding energy, $\text{BE}_{\text{star}} = M_{\text{star}} - mN_{\text{star}}$, and a necessary, though not sufficient, condition for the star to be stable is that the binding energy be negative. Considerable care is however necessary in deciding how to define the mass which appears in this expression [22], and we discuss this at length in the next Section. It is normally convenient to consider a dimensionless binding energy, defined by

$$\text{BE} = M(\infty) - mN_\infty. \quad (18)$$

Finally, to get a feeling for the possible rate of variation of Φ , we consider the solution corresponding to homogeneous matter-dominated cosmologies, which is [24,25]:

$$\Phi(t) \propto t^{2/(4+3\omega)} \propto a^{1/(1+\omega)}. \quad (19)$$

At the current limit $\omega = 500$, the variation in Φ since matter–radiation equality at around $z_{\text{eq}} = 24\,000 \Omega_0 h^2$ is a couple of percent. During radiation domination Φ , and hence G , is constant.

III. MASS DEFINITIONS

The definition of mass in scalar–tensor theories is a subtle one, which has recently been examined in detail by Whinnett [22]. When one leaves the security of general relativity, one first has to worry about which conformal frame one should work in, either the original Jordan frame as given in Eq. (1), or the Einstein frame obtained by carrying out a conformal transformation to make the gravitational sector match general relativity. Additionally, while in the Einstein frame all reasonable definitions coincide, in the Jordan frame they do not.

Whinnett studied three possible definitions. He found huge differences for $\omega = -1$, but the three definitions approach each other in the large ω limit, as one expects since they coincide in general relativity. These are to be compared with the *rest mass*, which is just the particle number multiplied by the particle mass. The definitions are:

- The Schwarzschild mass, given by

$$m(r) = 4\pi \int_0^r \rho r^2 dr, \quad (20)$$

where ρ is defined as the right-hand side of Einstein’s timelike equation. This corresponds to the ADM mass in the Jordan frame. It is the commonly-used definition of mass and in the limit $r \rightarrow \infty$ coincides with M_{star} defined in Eq. (12).

- The Keplerian mass, given by,

$$m_K(r) = r^2 \frac{B'}{2}. \quad (21)$$

- The Tensorial mass, given by,

$$m_T(r) = r^3 \frac{B'\phi + \phi'B}{2\phi r + r^2\phi'}. \quad (22)$$

The last two definitions are orbital masses. A non-self-gravitating test particle in a circular geodesic motion in the geometry of Eq. (7) moves with an angular velocity given by

$$\frac{d\varphi}{dt} = \sqrt{\frac{B'}{2r}}, \quad (23)$$

as measured by an observer at infinity [26]. Then, applying Kepler’s third law, the mass of the system can be obtained by making,

$$M(\infty) = \lim_{r \rightarrow \infty} \left[r^3 \left(\frac{d\varphi}{dt} \right)^2 \right]. \quad (24)$$

So, the Keplerian mass is *Kepler’s third law mass* in the Jordan frame, whereas the Tensorial mass is *Kepler’s third law mass* in the Einstein frame. In the Einstein frame all mass definitions coincide, so the Tensorial mass is also the Einstein frame ADM mass.

These definitions differ impressively for the $\omega = -1$ case, and, in general, for low values of ω [22]. Then, of course, it becomes very important to have a correct description of the stellar mass, because it will decide stability properties and binding energy behaviour. For the case $\omega = -1$, the Keplerian mass would lead to positive binding energy for all values of central density, suggesting that every solution is generically unstable. The Schwarzschild mass would instead lead to negative binding energies for every value of central density, suggesting that every solution is potentially stable, even for large values of $\sigma(0)$.[‡] This leads one to feel that neither of these two masses is likely to be the correct one to use in the binding energy calculation. Further, it is the Tensorial mass which

[‡]In fact, for small values of central density the Schwarzschild mass becomes negative for low ω . This might indicate that a classical wormhole can form, in much in the same way as the solution presented in Ref. [27].

peaks (as a function of central density) at the same location as the rest mass, an important property in the general relativity case [28] which is crucial in permitting the application of catastrophe theory to analyze the stability properties. This property presumably originates from the Tensorial mass being the Einstein frame ADM mass, though we have no mathematical proof at present. There is therefore a strong case [22] towards the adoption of the Tensorial mass as the real mass of the star, especially for the strong field cases of low ω values.

For the simulations we analyze in this work, we have computed both the Schwarzschild and the Tensorial mass. As expected, we find that for large ω values, which are the ones in which we are interested, the difference is negligible; every graph we plot is unchanged if we replace the Schwarzschild mass by the Tensorial mass. Hence, for reasons of numerical simplicity we actually compute the Schwarzschild mass, as it is directly obtained from the set of differential field equations.

IV. STABILITY ANALYSIS USING CATASTROPHE THEORY

Catastrophe theory provides a very direct route to the stability properties of boson stars [29]. The technique was described in a review of the stability of solitons [30], in which it was shown that the identification of conserved quantities of a physical system is sufficient for the determination of stable and unstable solitons. In the case of boson stars, we are dealing with nontopological solitons which are characterized by mass and particle number, the only conserved quantities of this theoretical model. For every central value of the scalar field, there is a unique value for the mass and particle number. By drawing the conserved quantities against each other, the so-called *bifurcation diagram* is created. If *cusps* are present in this diagram, one can immediately read off the stable and unstable states. Starting with small central densities where mass and particle number is also small, one assumes that these stars are stable (against small radial perturbations). If, as the central density is increased, one meets a cusp, the stability of the boson star changes from stable to unstable if the following states — the branch as a whole — have higher mass. This method is applied again at every succeeding cusp. Should it be that at some cusp the masses beyond the cusp are smaller, then the state changes from unstable to stable. The reason behind this method is that the cusp is a projection of a saddle point of a Whitney surface [29]. The curve leading to the cusp consists of projections of fold points; fold points and cusps are the singularities of the Whitney surface, just the points recognizable in the bifurcation diagram. The fold points are the projection of maxima and minima of Whitney's surface; maxima determine unstable solutions while minima govern stability.

The method of catastrophe theory has also been applied in the context of neutron stars [31], Einstein–Yang–

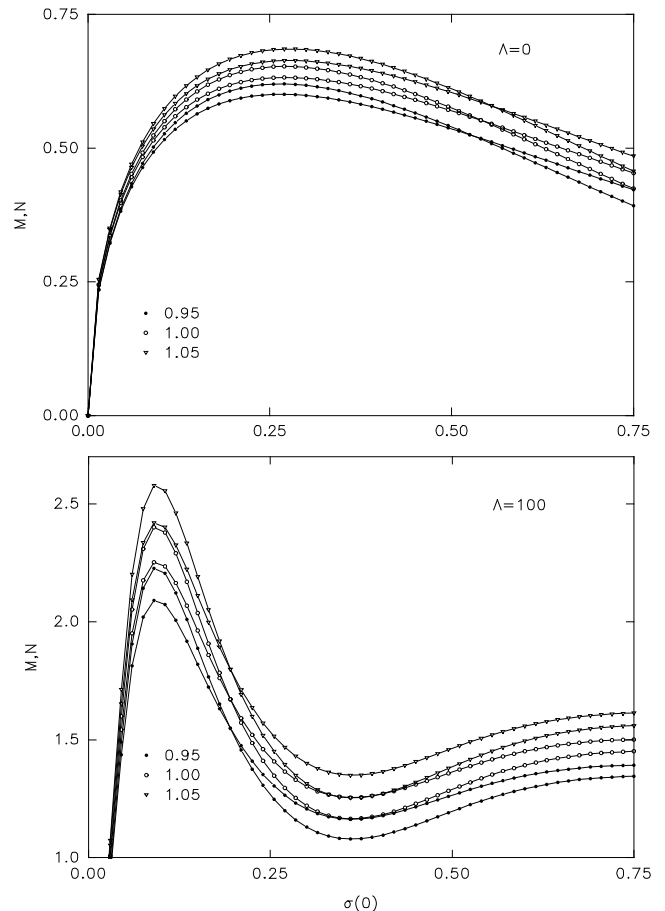


FIG. 1. Boson star equilibrium configurations for different values of $\Phi(\infty)$ and self-interaction. Of the two lines for each case, the one with the higher peak is the particle number. $\sigma(0)$ is in the interval $(0, 0.75)$ and there are 50 models per curve.

Mills black holes [32], and inflationary theory [33]. More recently, it has been introduced for neutron and boson stars in scalar–tensor theories, and in particular in JBD theory [20,34]. From these theories, one can learn how the properties of static solutions change if the asymptotic value of Φ changes. In the following, we show that there is no stability change at all within a JBD theory as Φ_∞ is changed; if a star is stable, then it is for every value of G . The binding energy can change its sign for stars which are unstable, but not for stable ones.

V. NUMERICAL RESULTS

First, we plot the equilibrium configuration diagrams for different values of the central density and asymptotic gravitational constant. In Fig. 1a ($\Lambda = 0$), we have 50 models with central density in the interval $(0, 0.75)$, with no self-interaction. Fig. 1b shows the same, but with $\Lambda = 100$. We recognize that at fixed central density $\sigma(0)$, the mass and particle number increase from earlier times ($\Phi(\infty) = 0.95$) to later times ($\Phi(\infty) = 1.05$). If we

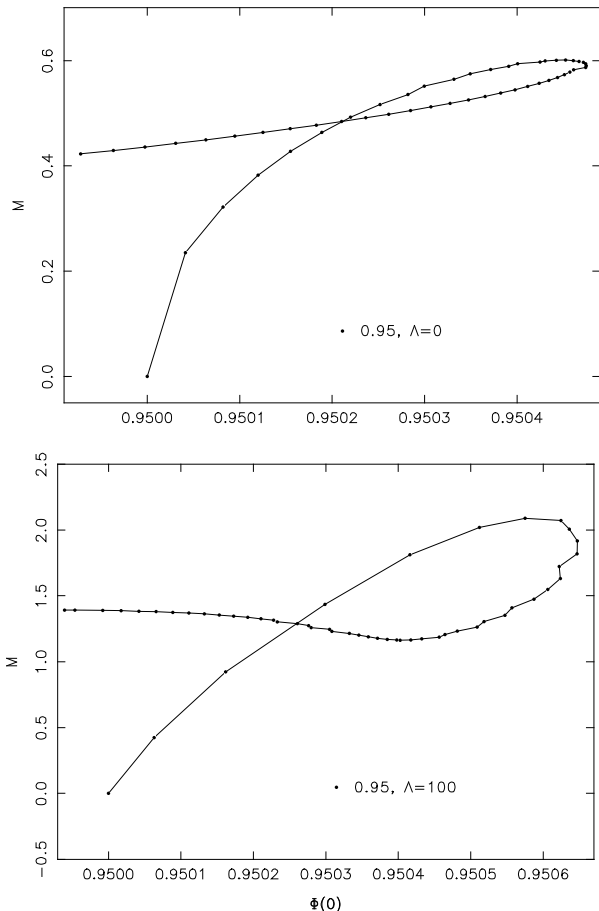


FIG. 2. Typical curves for boson star masses as a function of $\Phi(0)$, with $\Phi(\infty) = 0.95$. Note the narrow x -axis range.

draw the mass against the central value of the JBD field $\Phi(0)$, we find a loop, see Fig. 2. The curve starts at the flat spacetime solution ($\Phi = \text{constant}$ everywhere and zero mass), reaches the maximum at the same value of central scalar field as it reached the maximum of Fig. 1, cf. [20], makes a turn, and eventually reaches smaller $\Phi(0)$ values. Stable stars are characterized by $\Phi(0) > \Phi(\infty)$, i.e. $G(0) < G(\infty)$ where G is a function of r . Unstable stars can have $G(0)$ greater than or less than $G(\infty)$. There are two solutions for $\Phi(0) = \Phi(\infty)$: first, the flat spacetime solution and, secondly, an unstable boson star. The same characteristic curve is to be found for different values of the asymptotic G .

Figs. 1 and 2 give us, in form of $(\sigma(0), \Phi(0))$, the complete information about the initial characteristics of a boson star at a certain ‘time’, characterized by the constant $\Phi(\infty)$.

For the investigation of stability, rather than the bifurcation diagram (M, N) we use the analogous figure of binding energy against the particle number, Fig. 3. It shows us that the stars with small central densities have negative binding energies. In Fig. 3b ($\Lambda = 100$) we see two cusps: the first one corresponds to the maximum of Fig. 1 and the second to the minimum. For $\Lambda = 0$ we

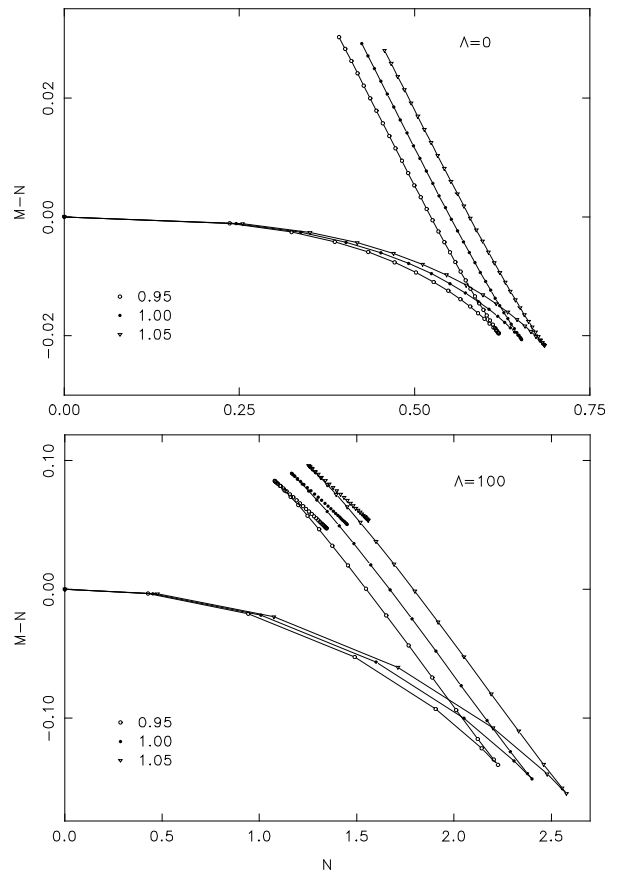


FIG. 3. The boson star binding energy as a function of the number of particles. The figure depicts several curves for different $\Phi(\infty)$ and Λ values, the coupling is $\omega = 400$.

did not go to high enough central densities to see the second cusp. The first cusp has negative binding energy and the other has positive binding energy. Stars with central densities from zero to the first cusp belong to projections of minima within a Whitney surface, i.e. they are stable. Beyond the cusp, one radial perturbation mode is becoming unstable, and at the second cusp a second mode becomes unstable.

To study the influence of the changing gravitational coupling on the exact position of the cusp we made a high resolution study around the position of the cusp for the $\Phi(\infty) = 1$ model. As can be seen from Fig. 1, it is at $\sigma(0) \simeq 0.27$. We then did simulations in the interval $\sigma(0) \in [0.265, 0.275]$, with eleven models in that range, shown in Fig. 4. For $\Phi(\infty) = 1.05$ the mass and the number of particles as a function of $\sigma(0)$ are increasing functions. That means all the models are in the stable branch. For $\Phi(\infty) = 0.95$ instead, mass and number of particles are decreasing functions, locating all models in the unstable branch. In the case of our present gravitational coupling, the cusp appears within the interval. Thus, going from the future to the past ($\Phi(\infty) = 1.05$ to $\Phi(\infty) = 0.95$) models with a given central density move

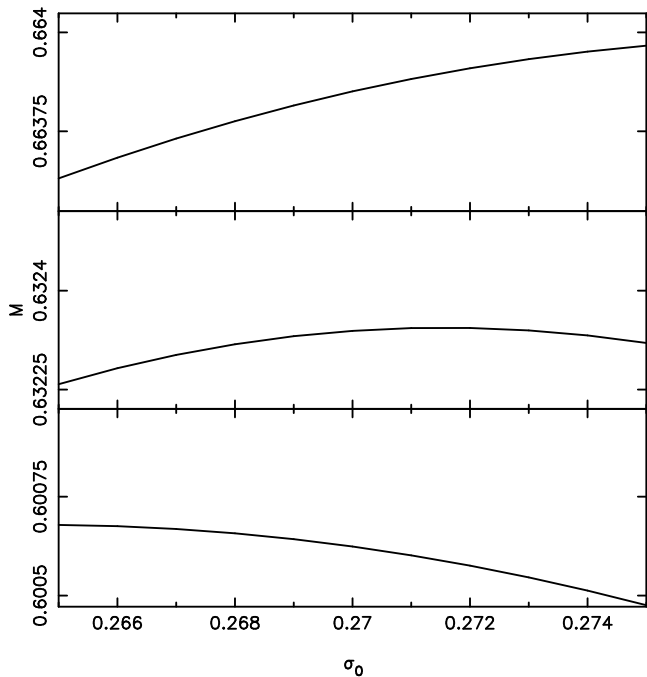


FIG. 4. Boson star masses for a reduced interval of values of $\sigma(0)$ around the cusp at three different ‘times’ $\Phi(\infty) = 1.05$ (top), $\Phi(\infty) = 1.0$ (middle), and $\Phi(\infty) = 0.95$ (bottom) [$\omega = 400$].

from the stable to the unstable branch. This agrees with the analytic prediction that in the general relativity limit one should find $\sigma_{\max}^2 \propto \Phi(\infty)$. The movement of the cusp is much the same as Comer and Shinkai reported in [20], except for one important point. They found no cusp at all for times well before the present, meaning that they did not find *any* stable star in the past. On the contrary, we have found that the cusp moves backwards in $\sigma(0)$, but it is still there, see Fig. 3. We believe that their conclusion derives from the use of a wrong mass definition.

In addition, we have calculated solutions with constant central scalar field values at different ‘times’ $\Phi(\infty)$, also taking into account very small values of $\Phi(\infty)$ which are unphysical, see Fig. 5. This figure represents a bifurcation diagram with respect to $\Phi(\infty)$. It is evident that no cusp is present, so no stability change occurs.

Because a boson star has no clearly-defined surface, but rather an infinite exponentially-decreasing atmosphere, several radius definitions are in use. We apply here the common one, the radius which encloses 95% of the total mass. Fig. 6 represents the mass against the radius. The diagram shows that solutions with small central densities have large radii. Then, with growing central densities, the mass increases while the radius decreases. The maximum in this diagram is the most centrally-dense stable star solution. The radius of the maximal mass boson star remains roughly the same, but the mass corresponding to a given central density grows with time, producing a denser star. The increase of the self-interaction constant Λ gives larger radii as one expects from a repul-

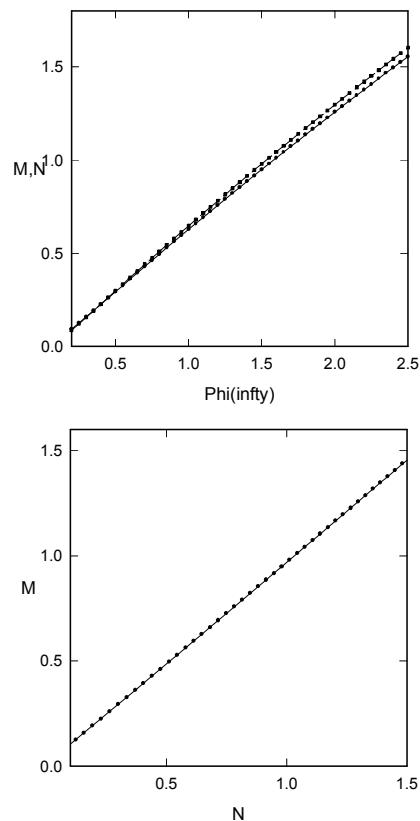


FIG. 5. The behaviour of masses and particle numbers for extreme values of the gravitational asymptotic constant. The model taken has $\Lambda = 0$, $\sigma(0) = 0.1$ and $\omega = 400$.

sive force. Compare this with similar results for neutron stars (Figure 7 in Ref. [28]) and for general relativistic boson stars (Figure 3 in Ref. [31]).

So far, we have recognized that the boson stars are denser the larger $\Phi(\infty)$ is. The reason can be understood as a deeper gravitational potential, expressed by an increase in the difference of $\Phi(0)$ and $\Phi(\infty)$, see Fig. 7.

For a fixed value of $\sigma(0)$, the behaviour of the binding energy, the radius, and $\delta\Phi$ (the difference between the central and the asymptotic value of the Brans–Dicke scalar) are all plotted in Fig. 8.

In Fig. 9, we show the dependence of equilibrium configurations on ω . To do this we plot the binding energy behaviour in the interval $\sigma(0) \in (0, 0.3)$ for two values in the asymptotic effective gravitational constant. The value of ω is in the range (50, 50000). The upper curves in both diagrams correspond to $\omega = 10000$ and 50000 and match each other exactly. The upper panel shows models with $\Phi(\infty) = 0.98$, while the other has our observed gravitational strength. This shows that, when ω tends to infinity, a general relativity like solution — with a different value for Newton’s constant — is obtained. Recall that even with the strong limit on ω valid today, we could have an evolving $\omega(\phi)$ which is much smaller in the past, and so small values of the coupling parameter may also be meaningful.

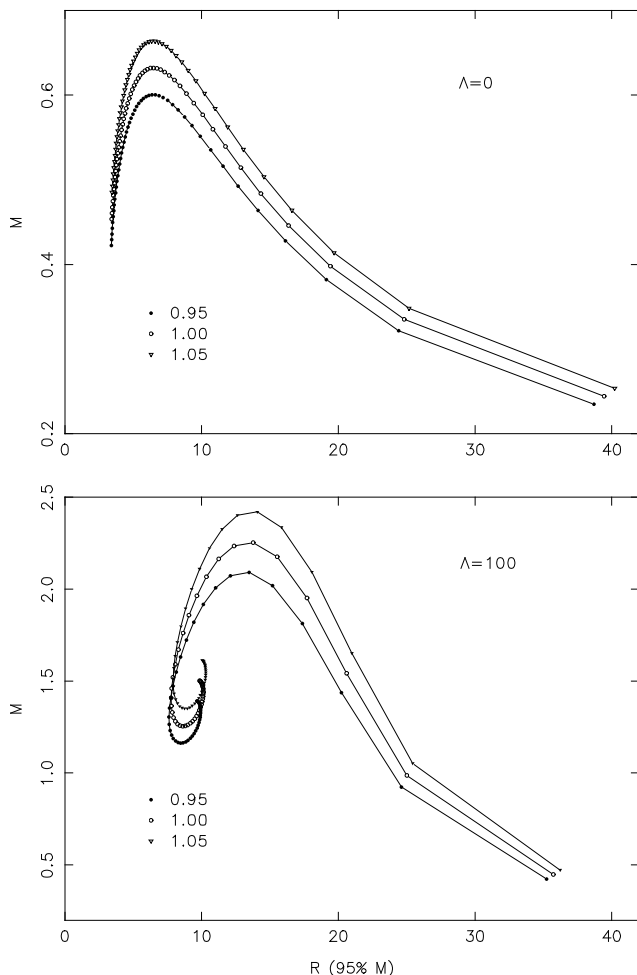


FIG. 6. The radius of equilibrium boson star configurations for different values of the self-interaction and central density in the range $(0, 0.75)$.

VI. CONCLUDING REMARKS

In this paper, we have thoroughly analyzed static boson star configurations in the framework of the Jordan–Brans–Dicke theory of gravitation. We studied their equilibrium and stability properties in the present as well as for other cosmic times, in the past or in the future. Stable boson stars may exist at any epoch, with stability depending on the value of central density. Together with this, a number of new physical features have been displayed concerning the radius–mass relation, the behaviour of the difference between the central and asymptotic value of Φ , the dependence on the structure upon the coupling parameter and other properties. This configurations can be used either to compare with the output of a numerical evolution code, or as the input into one. We expect that such a study will shed light on which scenario of the gravitational memory phenomenon might occur in practice. Whichever it might be, it is very likely that the same phenomena could also occur for fermionic stars, such as white dwarfs. In this sense, the results

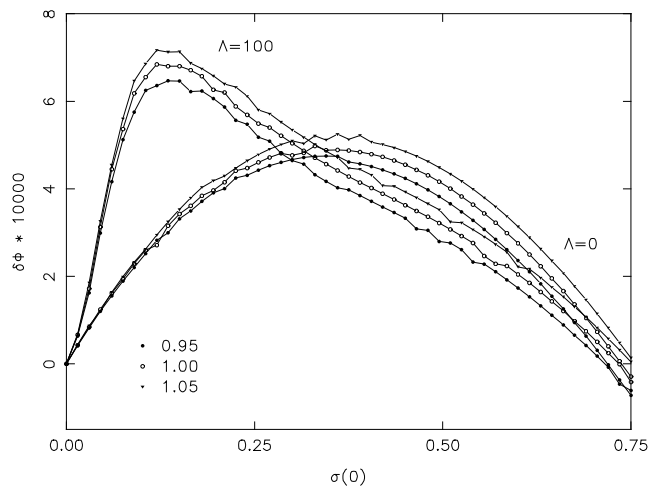


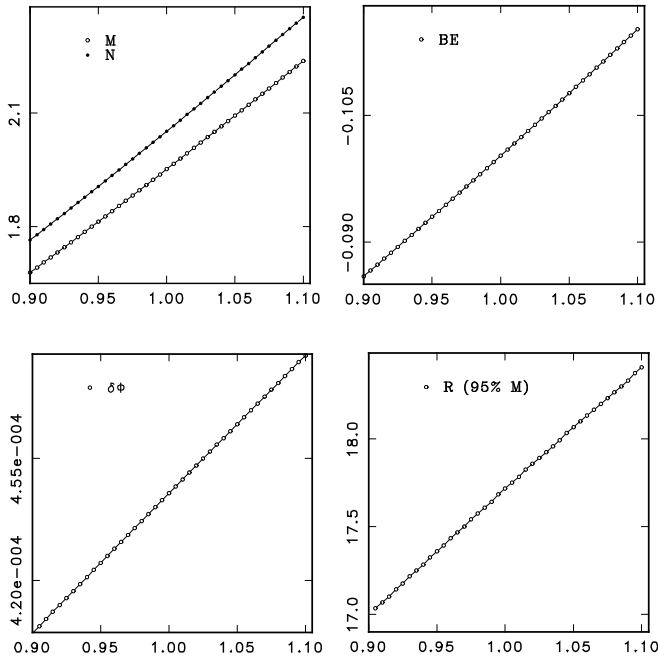
FIG. 7. The behaviour of the difference between the central and the asymptotic value of the Brans–Dicke scalar as a function of $\sigma(0)$ for different values of the effective gravitational constant. Note the highly expanded y -axis.

obtained in this paper can be regarded as of a general nature. Astrophysics should be unambiguously sensitive to the underlying theory of gravity, especially on cosmological times scales. It is in this framework, perhaps, where a crucial test of gravity could arise.

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BD Theory: $\omega=400$, $\sigma(0)=0.06$, $\Lambda=100$

FIG. 8. Boson star features for a given central density, as a function of Φ_∞ .

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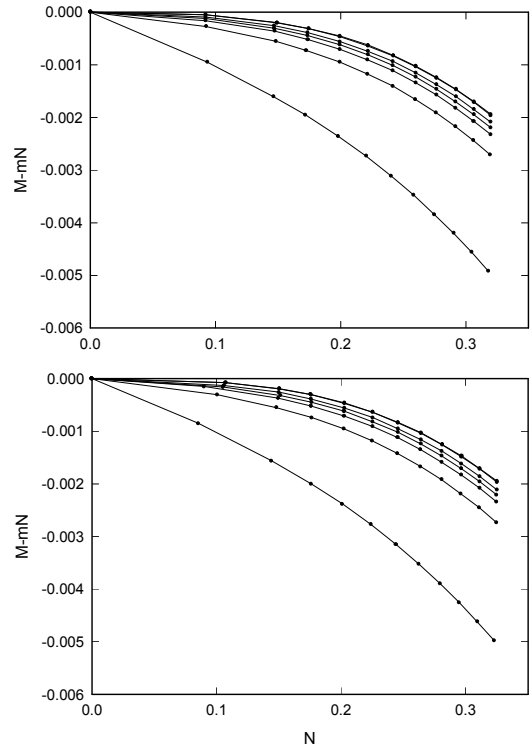


FIG. 9. Boson star binding energy as a function of the number of particles, for different values of ω ranging from 50 to 50000. The upper panel shows models with $\Phi(\infty) = 0.98$, the lower one, with $\Phi(\infty) = 1$. The value of $\sigma(0)$ lies in the range $(0, 0.3)$.

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