# Cosmology in a non-standard statistical background

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We study the primordial neutron to baryon ratio in a cosmological expanding background within the context of non-extensive statistics, in a fully analytical approach. First order corrections to the weak interaction rates and energy densities in the early universe are obtained and their consequences on the Helium synthesis are analized. We find that, if the nucleosynthesis scenario should be in agreement with observation, this entails a very restrictive bound upon $q$, the non-extensive parameter. We also study some cosmological interesting situations such as the moment of pair annihilation, conserved comoving quantities and derive for the nonextensive statistics, temperature relationships, before and after the removal of pairs.

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## I. INTRODUCTION

An interesting generalization of the Boltzmann-Gibbs entropy form has been recently proposed by Tsallis [1], for a recent review see [2,3], for a full bibliography see [4]. This new entropy, that we discuss below, posseses the usual properties of positivity, concavity and irreversibility and generalize the additivity in a non-extensive way. In the later years, a lot of work has been done in order to show either that many relationships involving energy and entropy in the usual scheme holds in the new one or in order to search for suitable generalizations for them. We should mention, among others, the work of Curado and Tsallis [5], that shows that the whole mathematical structure of thermodynamics –Legendre transform based– becomes invariant under a change from extensive to non-extensive statistics (NES). This, in fact, is a property recently proved to be valid for any entropic functional form [6], making then important to have a partition function within the formalism. Boltzmann-Gibbs thermostatics constitutes very complete and powerful techniques in situations whenever thermodynamics extensivity holds. But this formalism provide divergent partition, energy and entropy functions whenever the physical system includes long range forces or long memory effects. In this regard, the connection with quantum mechanics and with information theory has been stablished [7] and further applications to self-gravitating objects [8] and astrophysics [9], to Lévy flights [10], turbulence of fluids [11] and to the problem of solar neutrinos [12] have also been studied. In general, systems which present a fractal like or unconventional structure in their space-time description or in their phase space evolution could develop hardly tractable mathematical problems within the standard formalism [3]. Very recently, Grigera [13] derived a molecular dynamic test for systems of long range interacting particles with potentials of the form $A/r^{12} - B/r^{\alpha}$. He found that the potential energy per particle do not converge for some cases of $\alpha$ and sizes of the system, whereas non-extensive quantities do. Also see Refs. [14,15].

Non-extensive thermostatistics was also used to study the cosmic blackbody radiation [16,17]. This was the setting in which powerful bounds upon the non-extensive parameter were obtained. We shall review some of these results in our final section. A new analysis of NES blackbody cosmological radiation showed that, independently of the degree of non-extensivity, the temperature of the universe varies as the inverse of the scale factor of a flat Friedmann-Robertson-Walker metric [18]. This result suggested cosmology of the early universe as a very profitable arena for searching observational consequences of NES. As far as we know, the first of such studies were presented in [19], where a stringent bound on non-extensivity, of comparable order of magnitude as those obtained via the cosmic blackbody radiation, was derived. There, it was analized how a first order variation in the non-extensive parameter would affect the amount of primordial Helium production, a quantity which can be compared with observational results, via a rather easy and straightforward approximate technique.

We now turn to the issue of early universe cosmology and particularly, to the computation of primordial neutron to baryon ratio in the framework of NES. We shall also analyze cosmological situations of great importance, such as pair annihilation, temperature relationships, and comoving quantities. We thus refine the previous study and enhance it by a sharpening of the model. Although we shall present a very brief review of the main formulae of NES; in first

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approach, this work should be understood as the search of deviations from the standard model of the early universe provided by a different statistical description.

What we want to use is a direct analytical computation of Helium production, following the work of Bernstein, Brown and Feimberg [20] for the standard model, to allow for an approximate study. It is a matter of fact, that even when nucleosynthesis processes may be as complicated as one can afford, some simple assumptions makes results agree, within a typical few percent, with those generated by a numerical code, thus allowing for the isolation of the essential physics. That is exactly what it is needed here in order to study how, for instance, a different formula for the number of particles influence the evolution of the early universe and how can this be used to extract testeable predictions. Nucleosynthesis has given a strong basis to do such a thing when alternatives theories of gravitation are concerned, see for instance [21–23], and also for particle physics [24–26] and we hope to show, the same is applicable for alternative statistics.

The rest of the work is organized as follows. Firstly, some useful concepts and formulae of NES are presented. Afterwards, the analytical computation of the neutron to baryon ratio is done, and finally we study the necessary approach to handle with neutron decay corrections. In the final section, we can use our previous results to estimate the mass fraction of primordial Helium produced in the context of NES. Although we shall not have at that moment every ingredient needed to do such a thing with great precision, the bound which we obtain hardly constrain the degree of non-extensivity in the very early universe and is already comparable with previous results. Finally, we state our conclusions and mention other recently obtained related bounds.

## II. NONEXTENSIVE STATISTICS

Let us first recall some formulae of nonextensive statistic (NES), in Tsallis's approach. The formalism starts by postulating [1]

**Postulate 1**.- *The entropy of a system that can be found with probability $p_i$ in any of $W$ different microstates $i$ is given by*

$$S_q = k\frac{1}{(q-1)}\sum_{i=1}^{W}[p_i\ -\ p_i^q] = k\frac{1}{q-1}\left(1-\sum_{i=1}^{W}p_i^q\right), \tag{1}$$

*with $q$ a real parameter.* We have a different statistics for every possible $q-$value. In (1) we have used, of course, that

$$\sum_i p_i = 1. \tag{2}$$

In general,

$$S_q = k\frac{1-\mathrm{Tr}\rho^q}{q-1}. \tag{3}$$

**Postulate 2**.- *An experimental measurement of an observable $A$, whose expectation value in microstate $i$ is $a_i$, yields the $q$- expectation value* (generalized expectation value (GEV))

$$<\ A\ >_q = \sum_{i=1}^{W} p_i^q\, a_i = \mathrm{Tr}\rho^q\hat{A}, \tag{4}$$

*for the observable $A$.*

These two statements have the rank of *axioms*. As such, their validity is to be decided exclusively by the conclusions to which they lead, and ultimately by comparison with experiment. The entropy given in (1) is non-negative and reproduces the Boltzmann-Gibbs one ($S_1 = k\sum_i p_i \ln p_i$) in the limit $q\to 1$. In fact, note that (1) can be obtained from the Boltzmann-Gibbs expression by writing $\ln(x) = \lim_{q\to 1}(x^{q-1}-1)/(q-1)$ and then dropping the limit operator. It satisfies the pseudoadditivity property which states that if $A$ and $B$ are two independent systems ($p_{ij}^{(A+B)} = p_i^A p_j^B$) then,

$$\frac{S_q(A+B)}{k} = \frac{S_q(A)}{k} + \frac{S_q(B)}{k} + (1-q)\frac{S_q(A)}{k}\frac{S_q(B)}{k}, \tag{5}$$

which shows that $(1-q)$ is a measure of the degree of non-extensivity of the system. The optimization of $S_q$ given by (3) together with the constraints $\mathrm{Tr}\rho = 1$ and $\mathrm{Tr}\rho^q \hat{H} = U_q < \infty$ yield to the canonical ensemble equilibrium distribution [1,2],

$$\hat{\rho} = \frac{1}{Z_q} \left[ \hat{1} - (1-q)\beta \hat{H} \right]^{\frac{1}{1-q}} , \tag{6}$$

and to the generalized partition function,

$$Z_q = Tr \left[ \hat{1} - (1-q)\beta \hat{H} \right]^{\frac{1}{1-q}} . \tag{7}$$

Here, as usual, $\beta = 1/kT$ and $\hat{H}$ is the hamiltonian of the system. We shall focus in the $\beta(q-1) \to 0$ limit, in which a first order expansion allows analytical computations. The expression of a general mean value of an operator was computed in this limit by Tsallis, Sa Barreto and Loh [16]. When applied to particle number operators, the result is:

$$< \hat{n} >_q = < \hat{n} >_{BG} Z_{BG}^{q-1} \left[ 1 + (1-q)x \left[ \frac{< \hat{n}^2 >_{BG}}{< \hat{n} >_{BG}} + \frac{x}{2} \left( < \hat{n}^2 >_{BG} - \frac{< \hat{n}^3 >_{BG}}{< \hat{n} >_{BG}} \right) \right] \right] , \tag{8}$$

where $x$ stands for $\epsilon/kT$ ($\epsilon$ is the energy of a simple particle) and the symbol $BG$ means *to be computed in Boltzmann-Gibbs statistics*. With the standard values of $< \hat{n}^2 >_{BG}$ and $< \hat{n}^3 >_{BG}$, for fermions and bosons, we obtain the main corrections (the second terms in equation (8)), $C_{bosons,fermions}$, as:

$$C_{bosons} = \frac{1}{\exp{(x)} - 1} x \left( \frac{1 + \exp{(-x)}}{1 - \exp{(-x)}} - \frac{x}{2} \frac{1 + 3\exp{(-x)}}{(1 - \exp{(-x)})^2} \right) \tag{9}$$

$$C_{fermions} = \frac{1}{\exp(x) + 1} x \left( 1 + \frac{x}{2} \left( \frac{1}{1 + \exp{(x)}} - \frac{1 + \exp{(2x)} + 2\exp{(x)}}{(1 + \exp{(x)})^2} \right) \right) \tag{10}$$

When $x$ is such that $\exp{(x)} \gg 1$, then, $C_{bosons} \simeq C_{fermions} \simeq x(1 - x/2)\exp{(-x)}$; while for $x$ big enough we get $C_{bosons} \simeq C_{fermions} \simeq (-x^2/2)\exp{(-x)}$.

## III. NEUTRON-PROTON RATIO IN AN EXPANDING BACKGROUND

We turn now our attention to the description of the model for the evolution of the neutron abundance as the universe evolves. As stated in the introduction, we shall follow the leading ideas of Bernstein, Brown and Feimberg [20]. We shall denote by $\lambda_{pn}(T(t))$ the rate for the weak processes to convert protons into neutrons and by $\lambda_{np}(T(t))$ the rate for the reverse ones. $X(T(t))$ will be, as usual, the number of neutrons to the total number of baryons. For it, a kinetic equation may be built,

$$\frac{dX(t)}{dt} = \lambda_{pn}(T)(1 - X(t)) - \lambda_{np}(T)X(t). \tag{11}$$

The solution to the previous equation reads:

$$X(T) = \int_{t_0}^{t} dt' I(t,t') \lambda_{pn}(t') + X(t_0) I(t,t_0), \tag{12}$$

with the integrating factor given by,

$$I(t,t') = \exp\left( - \int_{t'}^{t} d\hat{t} \Lambda(\hat{t}) \right) \tag{13}$$

where,

$$\Lambda(t) = \lambda_{pn}(t) + \lambda_{np}(t). \tag{14}$$

Note that the form of this solution does not depend on the statistical mechanics used. However, changing the statistical mechanics entails modifications in the rates, and this will make the solution differs from the standard. Expression

(12) is simplified by setting the initial value $t_0 = 0$. We expect that $\lambda_{np}$ and $\lambda_{pn}$ be very large at early times and high temperatures. Then, the integration factor $I(t, t_0)$ will be very small for times order $t \simeq 1/\Lambda(t_0)$ and therefore the term $X(t_0)I(t, t_0)$ can be omitted. Besides, we also expect that due to the large interaction rates, the integral in the first term will be not sensitive to the change of $t_0$ by 0. With this choice, we now have,

$$X(t) = \int_0^t dt' I(t,t')\lambda_{pn}. \tag{15}$$

Finally, we may note that

$$I(t,t') = \frac{1}{\Lambda(t')}\frac{d}{dt'}I(t,t'), \tag{16}$$

and integrate by parts to obtain

$$X(t) = \frac{\lambda_{pn}(t)}{\Lambda(t)} - \int_0^t dt' I(t,t')\frac{d}{dt'}\left(\frac{\lambda_{pn}(t')}{\Lambda(t')}\right). \tag{17}$$

### A. Rate Formulae

To explicitly compute expression (17), we need to know the functional form for the rates. Let us first consider the rate $\lambda_{np}(t)$. This is the sum of the rates of three individual processes,

$$\lambda_{np} = \lambda_{\nu+n\to p+e^+} + \lambda_{e^+ + n\to p+\hat{\nu}} + \lambda_{n\to p+e^- +\hat{\nu}} \tag{18}$$

which are given by [27],

$$\lambda_{\nu+n\to p+e^-} = A\int_0^\infty dp_\nu p_\nu^2 p_e E_e (1- < \hat{n}_e >) < \hat{n}_\nu > \tag{19}$$

$$\lambda_{e^+ + n\to p+\hat{\nu}} = A\int_0^\infty dp_e p_e^2 p_\nu E_\nu (1- < \hat{n}_\nu >) < \hat{n}_e > \tag{20}$$

$$\lambda_{n\to p+e^- +\hat{\nu}} = A\int_0^{p_0} dp_e p_e^2 p_\nu E_\nu (1- < \hat{n}_\nu >)(1- < \hat{n}_e >), \tag{21}$$

where $A$ is an overall effective constant –fixed by the experimental value of $\lambda_{n\to p+e^- +\hat{\nu}}$–, $p_{\nu,e}$ stands for the magnitudes of the neutrino and electron momentum and $E_{\nu,e}$ for their energies. The recoil energy of the nucleons may be neglected, and this enable us to write the energy conservation equation $E_\nu + m_n = E_e + m_p$ for (19) and $E_\nu + m_p = E_e + m_n$ for (20). These equations must be used in order to explicitly compute the integrals. In (21), $E_\nu = \Delta m - E_e > 0$, –with $\Delta m = m_n - m_p = 1.29 MeV$– and this gives the upper limit in the integration range. Finally, $n_{\nu,e}$ are distributions functions and $(1 - n_{\nu,e})$ are blocking Pauli factors. In the usual scheme, $n_{\nu,e}$ are given by Fermi distributions, whereas in our case, by the $< \hat{n}_{\nu,e} >_q = n_{\nu,e}(q)$ of the previous section. In general, the electron and neutrino temperatures, $T_e$ and $T_\nu$, may differ because at the end of the freezing out period, electrons and positrons annihilate, heating only the photons and mantaining with them thermal equilibrium. The magnitude of this difference will be discussed below. As we shall see, this difference will be of a few percent during the period of consideration and our first approximations, following Bernstein et. al., is to set all temperatures equal, $T = T_e = T_\nu = T_\gamma$. When working in the Boltzmann-Gibbs framework this unique assumption ensure that the rates for reverse reactions, such as $e^- + p \to n + \nu$, obey the principle of detailed balance (PDB), that we analize below in this general setting.

### B. Principle of Detailed Balance

Let us consider, in the Boltzmann-Gibbs (BG) statistics and as an example, the reverse rate for (19), that is,

$$\lambda_{e^-+p\to n+\nu} = A\int_0^\infty dp_e p_e^2 p_\nu E_\nu (1- < \hat{n}_\nu >) < \hat{n}_e > . \tag{22}$$

Using energy conservation and the fact that $p_e dp_e = E_e dE_e$ we have, neglecting the recoil energy of the nucleon, $dE_e = dE_\nu = dp_\nu$, and thus:

$$\lambda_{e^-+p\to n+\nu} = A\int_0^\infty dp_\nu p_\nu^2 p_e E_e (1- < \hat{n}_\nu >) < \hat{n}_e > . \tag{23}$$

As $< \hat{n}_{\nu,e} >= n_{\nu,e}$ are Fermi-Dirac distributions, it is easily seen that $(1-n_\nu) = n_\nu \exp(E_\nu/T)$ and that $n_e = (1-n_e)\exp(-E_e/T)$. Replacing this into (23) and recalling that $\Delta m = E_e - E_\nu$, we get

$$\lambda_{e^-+p\to n+\nu} = \exp(-\Delta m/T)\lambda_{n+\nu\to e^-+p} \tag{24}$$

which is the expression of the PDB.

When NES is concerned, and correspondingly change from the Fermi-Dirac distributions to the $n(q)$ ones, the PDB is no longer valid. In this framework, it may be seen that

$$\exp(-\Delta m/T)(1-n_e(q))n_\nu(q) = (1-n_\nu(q))n_e(q) + (q-1)\phi, \tag{25}$$

where the correction is

$$\begin{aligned}\phi = (1-n_{e,BG})C_\nu\left(\exp\left(-\frac{\Delta m}{T}\right)+\exp\left(-\frac{E_e}{T}\right)\right) - \\ n_{\nu,BG}C_e\left(\exp\left(-\frac{\Delta m}{T}\right)+\exp\left(\frac{E_\nu}{T}\right)\right)\end{aligned} \tag{26}$$

and the $C$ factors are as in (10). In order to get a simplified picture, and following the BG case, we further approximate the model -exactly as was made in the standard case- by assuming that during the period of freezing, the temperature $T$ is low in comparison of the typical energies $E$ that contributes in the integrals for the rates. Hence, we may replace the Fermi-Dirac distributions by Boltzmann weights (i.e. $n_{\nu,e} \simeq \exp(-E/T)$) and consistently neglect the the Pauli blockings (i.e. $(1-n_{\nu,e}(q)) \simeq 1$). Even in the case of nonextensive statistics, Pauli Blockings corrections are $1-n_q = 1-n_{BG}(1-x^2\exp(-x)(q-1)/2)$, which are neglectable for $x \gg 1$ and a first order deviation from $q=1$. In addition, doing this allows for the analytical study that follows and which is the objective of the work. In this $x \gg 1$ regime, the one which we are going to use thorough, we obtain $C_e \simeq C_\nu \exp(-\Delta m/T)$, and we recover for the reverse rates the expression given in (24) of the PDB.

### C. Rates Computation

Before we finally compute the integrals for the rates we neglect the electron mass in equations (19) and (20) in comparison to the energies $E_{\nu,e}$. When this is made, the two rates become identical, exactly as they were in BG statistics. Then we obtain:

$$\lambda_{\nu+n\to p+e^-} = \lambda^{standard}_{\nu+n\to p+e^-} + \left(480T^5 + 2\times 84T^4Q + 18T^3Q^2\right)(1-q)A \tag{27}$$

where,

$$\lambda^{standard}_{\nu+n\to p+e^-} = \left(4!T^2 + 2\times 3!TQ + 2!Q^2\right) AT^3. \tag{28}$$

Concerning the rate of free decay of neutrons, we note that NES does not present any change within the context of previous approximations. That is due to the disappearence of all distributions functions from (21). In that equation we cannot neglect the mass of the electron and the standard result holds,

$$\frac{1}{\tau} = \lambda_{n\to p+e^-+\hat{\nu}} = 0.0157A\Delta m^5. \tag{29}$$

This enable to eliminate $A$ in favor of the measured quantity $\tau$,

$$A = \frac{a}{\tau}\frac{1}{4}\Delta m^5, \qquad a = 255. \tag{30}$$

All along in this section, we shall neglect the free neutron decay rate when computing the total $\lambda_{np}$. Thus, we have

$$\lambda_{np} = 2\lambda_{\nu+n\to p+e^-}\,. \tag{31}$$

Finally, by using the variable $y = \Delta m/T$, we get

$$\lambda_{np} = \frac{a}{\tau y^5}\left[\left(12+6y+y^2\right)+(1-q)\left(240+84y+9y^2\right)\right], \tag{32}$$

for the neutron-proton rate and

$$\lambda_{pn} = \exp\left(-y\right)\lambda_{np}, \tag{33}$$

for the reverse reaction.

### D. Evolution of the neutron abundance

In equation (17), we change now variables to the scaled temperature $y$ and obtain,

$$X(y) = \frac{\lambda_{pn}(y)}{\Lambda(y)} - \int_0^y dy' I(y,y')\frac{d}{dy'}\left(\frac{\lambda_{pn}(y')}{\Lambda(y')}\right). \tag{34}$$

The integrating factor now becomes,

$$I(y,y') = \exp\left(-\int_{y'}^{y} d\hat{y}\left(\frac{d\hat{t}}{d\hat{y}}\right)\Lambda(\hat{y})\right). \tag{35}$$

To evaluate the jacobian $d\hat{t}/d\hat{y}$, we need to recall that the scale factor of the universe, $R$, in a Friedmann-Robertson-Walker metric, behaves as $R \simeq 1/T$, independently of the statistics [18]. Therefore, $\dot{T}/T = -\dot{R}/R$, and the right hand side is given by Einstein equations:

$$\frac{\dot{R}}{R} = \left(\frac{8\pi G}{3}\rho\right)^{\frac{1}{2}}. \tag{36}$$

Here $\rho$ is the energy density for relativistic particles in NES. This may be easily computed, if the distributions functions are known, by

$$\rho = \frac{g}{2\pi^2}\int_0^\infty E^3 n(q) dE, \tag{37}$$

where $g$ is the degeneracy factor. The result is

$$\rho = \rho_{bosons} + \rho_{fermions} = \frac{\pi^2}{30}gT^4 + \frac{1}{2\pi^2}\left(40.02\, g_b + 34.70\, g_f\right) T^4(q-1), \tag{38}$$

where $g = g_b + 7/8 g_f$. At high enough temperatures the energy density of the universe is essentially dominated by $e^-$, $e^+$, $\nu$ and $\gamma$'s. Interactions among these particles keep all them nearly the same temperature. Accordingly, we set, $g_b = 2$, $g_f = 2+2+2\times 3 = 10$ and $g = 43/4$. With these values we get,

$$\rho = \frac{\pi^2}{30}gT^4 + \frac{1}{2\pi^2}21.63\, T^4\,(q-1), \tag{39}$$

With this in mind, and using $\dot{T}/T = -\dot{a}/a$, we obtain

$$\frac{dT}{dt} = -\left[\left(\frac{4\pi^3 Gg}{45}\right) + \left(\frac{4G}{3\pi}\right)21.63(q-1)\right]^{\frac{1}{2}} T^3 \tag{40}$$

which yields, to first order in $(q-1)$ to,

$$\frac{dt}{dy} = \frac{y}{\Delta m^2}\left(\frac{45}{4\pi^3 Gg}\right)^{\frac{1}{2}}\left[1 - \frac{1}{2}\left(\frac{45}{3\pi^4}\right)\frac{21.63}{g}(q-1)\right]. \tag{41}$$

We shall call the constants,

$$b = \left(\frac{45}{4\pi^3 Gg}\right)^{\frac{1}{2}}\frac{1}{\tau\Delta m^2}a \tag{42}$$

and

$$c = b\left(\frac{45}{6\pi^4}\right)\frac{21.63}{g}. \tag{43}$$

With all above, the integrating factor becomes,

$$I(y,y') = \exp\left(K(y) - K(y')\right), \tag{44}$$

where,

$$K(y) = -b\int dy\left[\left(\frac{12}{y^4}+\frac{6}{y^3}+\frac{1}{y^2}\right)(1+c(1-q)) + \left(\frac{240}{y^4}+\frac{84}{y^3}+\frac{9}{y^2}\right)(1-q)\right]\times (1+\exp(-y)). \tag{45}$$

This integrates to give,

$$K(y) = b\left(\frac{4}{y^3}+\frac{3}{y^2}+\frac{1}{y}+\left(\frac{4}{y^3}+\frac{1}{y^2}\right)\exp(-y)\right)(1+c(1-q)) + b\left(\frac{80}{y^3}+\frac{42}{y^2}+\frac{9}{y}+\left(\frac{80}{y^3}+\frac{2}{y^2}+\frac{7}{y}\right)\exp(-y) - 7Ei(1,y)\right)(1-q). \tag{46}$$

where $Ei$ stands for the exponential integral [1]. Introducing,

$$X_{eq} = \frac{\lambda_{pn}(y)}{\Lambda(y)} = \frac{1}{1+\exp(y)}, \tag{47}$$

the neutron abundance ratio reads:

$$X(y) = X_{eq} + \int_o^y dy' \exp(y')X_{eq}(y')^2\exp\left[K(y) - K(y')\right]. \tag{48}$$

The exact previous result may be expanded again to first order in $(q-1)$. This gives,

$$X(y) = X_{standard} + (1-q)\exp(bA(y))(bB(y) + bcA(y))\times \int_o^y dy' \exp(y')X_{eq}(y')^2\exp(bA(y')) - (1-q)\exp(bA(y))\int_o^y dy'\exp(y')X_{eq}(y')^2\exp(bA(y')(bB(y') + bcA(y')), \tag{49}$$

where,

$$X_{standard} = X_{eq}(y) + \exp(bA(y))\int_o^y dy'\exp(y')X_{eq}(y')^2\exp(bA(y')), \tag{50}$$

and the functions $A$ and $B$ are:

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[1] The exponential integral is defined by $Ei(n,x) = \int_1^\infty \frac{\exp(-xt)}{t^n}dt$ for $Re(x) > 0$.

$$A(y) = \left(\frac{4}{y^3} + \frac{3}{y^2} + \frac{1}{y} + \left(\frac{4}{y^3} + \frac{1}{y^2}\right)\exp\left(-y\right)\right) \tag{51}$$

$$B(y) = \left(\frac{80}{y^3} + \frac{42}{y^2} + \frac{9}{y} + \left(\frac{80}{y^3} + \frac{2}{y^2} + \frac{7}{y}\right)\exp\left(-y\right) - 7Ei(1, y)\right). \tag{52}$$

We can numerically compute the integrals that appears here. Using explicit values for the constants $b = 0.252$ and $c = 0.00244$ with the mean life of the neutron given by $\tau = 889.8s$ we get the curves of Fig. 1. There, it is shown the standard value of the neutron to baryon ratio and the correction due to NES. In Fig. 2. are shown different explicit computations for a range of the $q$ parameter. Finally, it may be seen that $X(y)$ asymptotes to

$$X(y = \infty) = 0.15 + (q - 1)1.15, \tag{53}$$

for very low temperatures.

### E. Possible dependence on the number of neutrino types

We briefly assess here the possible dependence on the number of neutrino types. To do this we note that such a variation would introduce changes in the coefficients $b$ and $c$ quoted above. For three neutrino families, $g = 43/4$ and the addition of one more family change it in an amount $\delta g = 7/4$. Then, the standard case coefficient $b$ is affected as,

$$\frac{\delta b}{b} = -\frac{1}{2}\frac{\delta g}{g} \tag{54}$$

and the non-extensive related as,

$$\frac{\delta c}{c} = -\frac{3}{2}\frac{\delta g}{g}. \tag{55}$$

Introducing these new values in (46) and making all numerical computations again, it is possible to obtain the corrections due to a different number of neutrino families. In the sake of conciness, we shall explicitly skip these computations. Instead we note that, while in the standard case, the addition of one more neutrino family would involve an increment of the energy density of the universe and a speed up in its expansion, in the general case, the correction will depend on the sign of the factor $(q - 1)$.

## IV. TOWARDS COMPUTING NEUTRON DECAY CORRECTIONS

We have already solved the evolution of the neutron abundance neglecting the neutron decay. As in the standard formalism, it is useful to change notation by using an overbar for the result just obtained, $X(y) \to \bar{X}(y)$. Including the effects of the free neutron decay in the rate equation we would get,

$$X(t) = \exp\left(-t/\tau\right)\bar{X}(t), \tag{56}$$

because $\bar{X}$ does not vary much during the period in which neutrons decay. In the capture time $t = t_c$, when the temperature drops somewhat below the deuteron binding energy ($\epsilon_D = 2.62\text{MeV}$), the neutrons are captured in deuterons. Then, deuterons collide and almost all neutrons present at $t = t_c$ are converted into $^4He$. Inserting this time in (56) and using the asymptotic value of $\bar{X}(y)$ just derived would yields, thus, half of the mass fraction of helium produced in the early universe. In order to obtain a precise value of $t = t_c$ or $t_{c,q}$ for the nonextensive result, we have to analize the set of the basic reactions:

$$n + p \Longleftrightarrow D + \gamma \tag{57}$$

$$D + D \Longleftrightarrow p + T \tag{58}$$

$$T + D \Longleftrightarrow {}^4He + n. \tag{59}$$

Doing this in the standard case [20] yields a value of $t_c$ such as the correction $\exp\left(-t_c/\tau\right) \simeq 0.8$. Although the analysis of (57,58,59) in a NES framework is beyond the scope of the present work, we shall establish here the basis on which this could be done in the future. For this, we shall study the process of annihilation of pairs $e^-, e^+$.

### A. Conserved comoving quantities

We know that, when the temperature of the universe is low compared with the electron mass, electrons and positrons annihilate heating the photons and, morover, changing the effective quantity of degrees of freedom. Then, one of the assumptions of the previous section, i.e. $T = T_e = T_\nu = T_\gamma$, is no longer valid and $T_\gamma$ will be different from the decoupled neutrino temperature. We shall need the connection between these temperatures. To compute it, we begin by noting that, at these times, the number of neutrinos in a comoving volume is fixed, independently of the statistics. Effectively, after the moment in which the collision rate for neutrinos is lower than the expansion of the universe, $\dot{R}/R$, they cannot be neither created nor destroyed. As the number of neutrinos is given by,

$$n_\nu(q) = \frac{g}{2\pi^2}\int_0^\infty E^2 < \hat{n} >_q = \frac{3}{4\pi^2}\zeta(3)g_\nu T^3 + (q-1)\frac{g_\nu}{2\pi^2}5.50T^3 \tag{60}$$

the relationship,

$$n_\nu(t)R(t)^3 = const., \tag{61}$$

or, equivalently,

$$T_\nu(t)R(t) = const., \tag{62}$$

continues holding.

¿From the full set of Einstein field equations, it may be established that

$$d(\rho(q)R^3) = -p(q)d(R^3), \tag{63}$$

this may also be written as,

$$\frac{d}{dT}\left[(\rho(q)+p(q))\,R^3\right] = (R^3)\frac{dp(q)}{dT}. \tag{64}$$

But we may compute $p(q)$, as was done with the energy density, obtaining $p(q) = \rho(q)/3$ for the relativistic gas. Using this and (64), we get

$$\frac{d}{dT}\left[\frac{\rho(q)+p(q)}{T}R^3\right] = 0, \tag{65}$$

as can be cheked by direct differentiation. This means, that in general case, $(\rho(q)+p(q))/T$ is a conserved quantity in a comoving volume. When $q = 1$ the previous expression stands for the entropy of the system.

### B. Pair annihilation

We may now consider that pair annihilation occurs at a temperature $T$, and then indicate with symbols $-$ and $+$, quantities before and after $T$. From all above, we have

$$\frac{\rho(q)_- + p(q)_-}{T_-} = \frac{\rho(q)_+ + p(q)_+}{T_+}. \tag{66}$$

Here $\rho(q)_{-,+}$ is as in (38) with the corresponding values of $g_{-,+}$ and $g_{b,f,-,+}$. Due to the removal of pairs, we have that $g_+ < g_-$. We may now expand (66), to first order in $(q-1)$, obtaning the relationship,

$$T_+^3 = T_-^3\left(\frac{g_-}{g_+}\right)\left[1+(q-1)\delta\right], \tag{67}$$

where,

$$\delta = (40.02g_{b,-} + 34.70g_{f,-})\frac{30}{2\pi^4 g_-} - (40.02g_{b,+} + 34.70g_{f,+})\frac{30}{2\pi^4 g_+}. \tag{68}$$

Until the moment of $e^-, e^+$ annihilation, the gas composed by $e^-, e^+$and $\gamma$ follows a law of identical form that (62), as such temperatures, being $T_e = T_\gamma$. But after the annihilation, where only $\gamma' s$ remain, we have

$$T_\gamma = T_\nu\left(\frac{11}{4}\right)^{\frac{1}{3}}\left[1+(1-q)0.013\right], \tag{69}$$

where the corresponding values $g_- = g(e^-, e^+, \gamma) = 11/2$ and $g_+ = g(\gamma) = 2$ were used.

### C. Neutron capture time

To finally compute the capture time $t_c$, we use that

$$\frac{1}{T_\nu}\frac{dT_\nu}{dt} = -\left(\frac{8\pi G}{3}\rho(q)\right)^{\frac{1}{2}}, \tag{70}$$

from where

$$t = \int_{T_\nu}^{\infty} \frac{dT'_\nu}{T'_\nu}\left(\frac{3}{8\pi G\rho(q)}\right)^{\frac{1}{2}} \tag{71}$$

follows. Here $\rho(q)$ adquires its low energy form

$$\rho_0 = \left[g_\nu \frac{\pi^2}{30}T_\nu^4 + (q-1)34.70T_\nu^4\frac{g_\nu}{2\pi^2}\right] + \left[g_\gamma \frac{\pi^2}{30}T_\gamma^4 + (q-1)40.02T_\gamma^4\frac{g_\gamma}{2\pi^2}\right], \tag{72}$$

where $g_\nu = 3$ and $g_\gamma = 2$. Recalling the relationship (69) we get, to first order in $(q-1)$,

$$\rho_0 = g_{eff}\frac{\pi^2}{30}T_\nu^4 + (q-1)T_\nu^4 14.84 \tag{73}$$

where $g_{eff} = g_\nu + g_\gamma(11/4)^{4/3}$.

In the neighbourhood of the time of the neutron capture , an approximate evaluation of (71) is in order. Using the expression for $\rho_0$ just derived, we get

$$t = \left(\frac{45}{16\pi^3 g_{eff}G}\right)^{\frac{1}{2}}\frac{1}{T_\nu^2}(1+(1-q)1.74) + t_0, \tag{74}$$

where $t_0$ is an additional integration constant and the corrective number 1.74 arise from the first order evaluation of $\rho(q)^{\frac{1}{2}}$. Recalling then the relationship between the temperatures, we may express

$$t = \left(\frac{45}{16\pi^3 g_{eff}G}\right)^{\frac{1}{2}}\frac{1}{T_\gamma^2}(1+(1-q)1.76) + t_0, \tag{75}$$

We may neglect $t_0$ in a first approximation or obtain it from a perturbation analysis of (71), [20].

Now, a comprehensive analysis of the reactions (57,58,59) would yield a value for $T_\gamma$, for which almost all nucleons are converted into helium . Using then (75), the evaluation of $t_c$ follows and thus, the mass fraction of primordial helium. In passing through NES, we have to note that a fundamental tool in this analysis, i.e. Saha formulae, is modified in a non-trivial way. This entails modifications in the abundance fractions that may in turn change their ratios, which are the magnitudes on which the set depends. So, although phenomenological fits for the rates may be considered, we acknowledge here that further research must be done to analytically solve for the reactions, in the case this be ultimately possible. This analysis is far out from the present work and we shall not discuss this problem in deeper detail.

## V. CONCLUSIONS AND COMPARISON WITH OTHERS RELATED RESULTS

We have presented a comprehensive study of the primordial neutron-baryon ratio in the context of non-extensive statistics. This work, thus, extends previous advances on the helium nucleosynthesis production problem [19] and is the result of a detailed analysis of the weak interaction rate and the energy densities corrections at first order in the non-extensive parameter $(q-1)$. We have also studied some important cosmological scenarios such as the possible existence of conserved comoving quantities, temperatures relationships and pair annihilation, together with pointing out where further research is needed. These other results will be used elsewhere to put new stringent bounds on the non-extensivity of the universe by using high precision satellite cosmological measurements of the cosmic microwave background and new galaxy surveys.

Even without getting a precise analytical value for the Helium synthesis, it is possible to obtain a first insight in the problem by considering it as exactly twice of the neutron-baryon asymptotic value, weighted by the free neutron decay:

$$Y_p^{theoretical} \sim 2X(t) \tag{76}$$

where $X(t)$ is given by equation (56). But as mentioned above, in order to put an exact bound we would need the value of $t_{c,q}$. However, even considering that $T_\gamma$ is the same as in the standard formalism ($T_\gamma \simeq 0.086$MeV) we could use (75) to get the correction in $t_c$. Using afterwards the first order expansion of (56) (which contains equation (53)) we would get the stringent bound:

$$|1-q| < 2.6 \times 10^{-4}. \tag{77}$$

In order to get this bound, we have imposed that the non-extensive corrections (coming from equations (75) and (53) and the theoretical value of $Y_p$ as explained above) be less than the observational error in the primordial Helium production, which is doubled so as to neglect the difference between the central values of the analytical and observational magnitudes [24], i.e.:

$$Y_p^{observational} = 0.23 \pm 0.02. \tag{78}$$

This abundance is a very safe estimate of the primordial Helium production, which in fact is a very complex observable magnitude, see for instance the recent review by Schramm and Turner [30] for details. Besides, it encodes other possible uncertainties such as the value of the cosmological constant or the exact number of neutrino species.

The bound (77) is comparable with that already obtained in Ref. [19]. Although it should be recalled that such derivation was different from this and the method followed skip the uncertainties of the neutron capture time here analyzed. In addition, the weak interaction rates were only approximately treated in [19] while here they were studied in full detail.

In a recent letter [28], it was suggested that the universe as a whole could behave with a non-extensive parameter roughly equal to 0.54. Such a value seems to be too high as to resist early universe tests as the one presented here.

Therefore, this bound, obtained in the simplest situation so as to admit an analytical solution, pertains to those imposed in the context of early cosmological models, and from this point of view, enlarge the problems that non-extensivity would have as a correct description of the physical universe thorough all eras of cosmic evolution. Very recently, other works have also been devoted to study possible deviation from the Boltzmann-Gibbs statistics in cosmological and other situations. Among them, and apart from Ref. [19] already commented, we should note the following ones:

- Plastino, Plastino and Vucetich [17] studied the value of the Stefan-Boltzmann constant in the new framework. A comparison with the experimental value leads to $|q-1| \leq 0.67 \times 10^{-4}$. They also used data from the FIRAS experiment on the COBE satellite concening the microwave background radiation, from there they obtain $|q-1| \leq 5.3 \times 10^{-4}$.

- Tsallis, Sa Barreto and Loh [16] also generalized the Planck radiation law and used satellite data from COBE to constrain the non-extensive parameter. They found $|q-1| < 3.6 \times 10^{-5}$.

- Tirnakli, Büyükkiliç and Demirhan [29,31] reworked the generalized quantal distributions functions and also obtain a similar bound: $|q-1| \leq 0.41 \times 10^{-4}$.

These works, among others, suggest that non-extensivity is highly constrained concerning the physics of the early universe.

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FIG. 1. Behaviour of the primordial neutron to baryon ratio in standard Boltzmann-Gibbs statistics (curve (1)) and correction due to non-extensivity weighted by $(q-1)$, (curve (2)). The complete behaviour of the neutron-baryon ratio in NES is obtained making the sum of curve (1) plus $(q-1)\times$ curve (2).

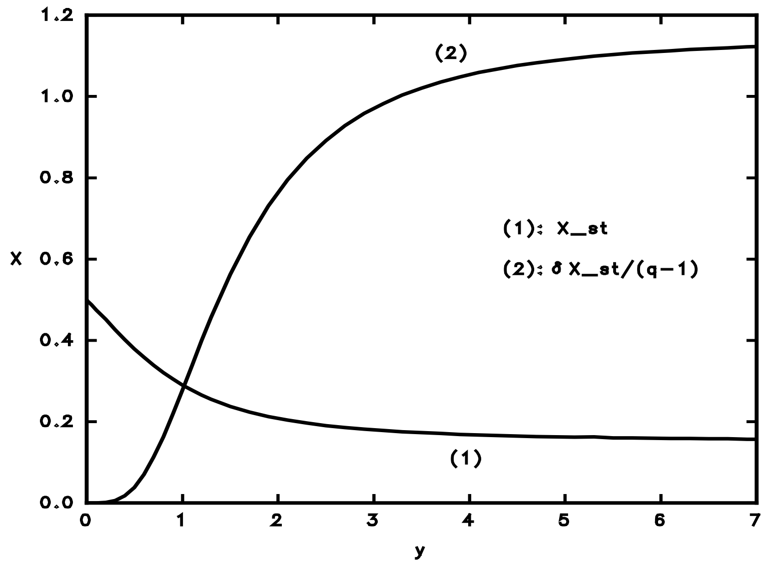


Fig. 1.

FIG. 2. Neutron to baryon ratio relationship for different non-extensive parameter $q$.

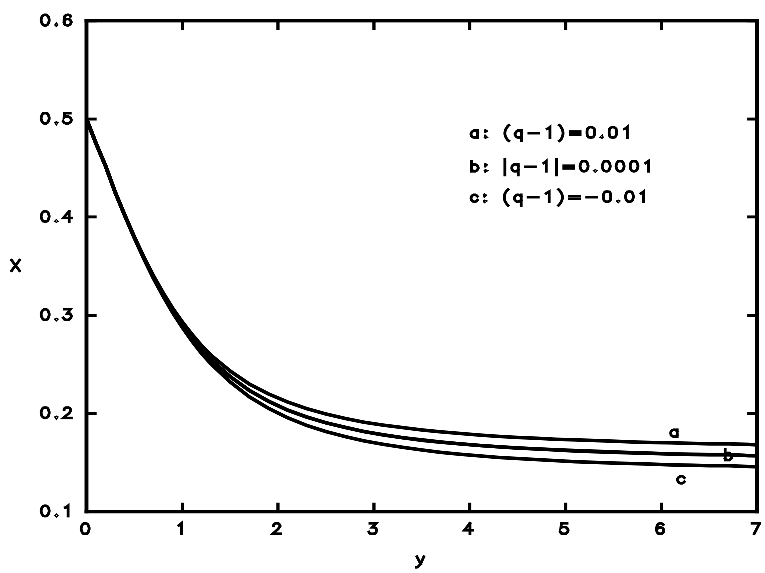


Fig. 2.


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