LA PLATA-Th 98/10

Bogomol'nyi Bounds and the Supersymmetric Born-Infeld Theory

S. Gonorazky*, C. Núñez† ,
F.A. Schaposnik‡ and G. Silva†
[c]*Departamento de Física, Universidad Nacional de La Plata*
C.C. 67, 1900 La Plata, Argentina

Abstract

We study $N = 2$ supersymmetric Born-Infeld-Higgs theory in 3 dimensions and derive Bogomol'nyi relations in its bosonic sector. A peculiar coupling between the Higgs and the gauge field (with dynamics determined by the Born-Infeld action) is forced by supersymmetry. The resulting equations coincide with those arising in the Maxwell-Higgs model. Concerning Bogomol'nyi bounds for the vortex energy, they are derived from the $N = 2$ supersymmetry algebra.

*FOMEC-UNLP
†CONICET
‡Investigador CICBA

1 Introduction

The supersymmetric extension of the Born-Infeld theory [1]-[2] was studied in refs. [3]-[4] by means of superspace techniques. Remarkably, connections between Euler-Heisenberg effective Lagrangians derived from certain supersymmetric theories and the Born-Infeld Lagrangian were discovered [3],[5]. More recently, supersymmetric extensions of 10-dimensional Born-Infeld theory have been shown to play a central rôle in the dynamics of D-branes [6]-[12].

Closely related to the issue of supersymmetry completion of the Born-Infeld theory, the study of Bogomol'nyi relations and BPS solutions in this theory is the main object of the present work. To this end, we center the analysis in the study of a $N = 2$ supersymmetric Born-Infeld theory in $d = 3$ dimensions, which, when coupled to a Higgs field, has a bosonic sector which admits Bogomol'nyi equations [13]-[14] [1]. Interestingly enough, the Born-Infeld BPS equations coincides with those of the Maxwell theory and hence the exact vortex solutions found in this last case [16] also solve the more involved Born-Infeld theory.

As it is well-known, Bogomol'nyi relations can be found just by establishing an inequality between the energy and the topological charge [15] or by analysing the conditions under which a bosonic theory with topological solutions can be extended to a $N = 2$ supersymmetric theory in which the central charge coincides with the topological charge [17]. In this respect, it is very enlighting to derive, via the Noether method, the explicit supersymmetric algebra from which the origin and properties of BPS relations becomes transparent. This was done for the Maxwell-Higgs model in [19] and for the case of local supersymmetry in [20]-[21]. In the present work we proceed to a similar analysis with the case of a supersymmetric Born-Infeld-Higgs theory.

The plan of the paper is the following: in Section 2 we present the $N = 1$ supersymmetric Born-Infeld theory in $d = 4$ dimensions giving an explicit formula for the fermionic Lagrangian which will be necessary for constructing the SUSY charges. Then, in Section 3 we proceed to a dimensional reduction to $d = 3$ thus obtaining a $N = 2$ supersymmetric Born-Infeld theory with a bosonic sector obeying first order Bogomol'nyi equations. The

[1] Originally, Bogomol'nyi equations were discovered in a $d = 3$ model with a Maxwell Lagrangian determining the dynamics of the gauge field [15]- [16]. Already in [16] the connection with supersymmetry is signaled

$N = 2$ supersymmetric algebra is constructed in section 4 where Bogomol'nyi bounds are discussed. Finally in section 5 we present a discussion of our results. The explicit expressions for superfields in components are detailed in an Appendix.

2 Supersymmetric Born-Infeld theory

In this section we shall start by writing the $N = 1$ supersymmetric version of the Born-Infeld (BI) theory in four dimensional space-time. Then, by dimensional reduction, we shall obtain in the next section a 3-dimensional $N = 2$ supersymmetric Lagrangian which will then be coupled to a Higgs scalar.

The 4-dimensional Born-Infeld Lagrangian is

$$L_{BI} = \frac{\beta^2}{e^2}\left(1 - \sqrt{-\det\left(g_{\mu\nu} + \frac{1}{\beta}F_{\mu\nu}\right)}\right) \tag{1}$$

(The signature of the metric $g_{\mu\nu}$ is $(+,-,-,-)$).

Use of the identity

$$\det\left(g_{\mu\nu} + \frac{1}{\beta}F_{\mu\nu}\right) = -1 - \frac{1}{2\beta^2}F^{\mu\nu}F_{\mu\nu} + \frac{1}{16\beta^4}\left(F^{\mu\nu}\tilde{F}_{\mu\nu}\right)^2 \tag{2}$$

allows to write (1) in the form

$$L_{BI} = \frac{\beta^2}{e^2}\left(1 - \sqrt{1 + \frac{1}{2\beta^2}F^{\mu\nu}F_{\mu\nu} - \frac{1}{16\beta^4}\left(F^{\mu\nu}\tilde{F}_{\mu\nu}\right)^2}\right) \tag{3}$$

Here, $\tilde{F}_{\mu\nu} \equiv \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$

In order to construct the SUSY extension of the Born-Infeld Lagrangian, we shall follow [3]-[4]. Although the complete derivation of the bosonic part of the SUSY model has been presented in these references (see also [9]-[10]) we shall here give a detailed description of the supersymmetric construction since, for our purposes, knowledge of the explicit form of certain fermion and fermion-boson terms is necessary.

We start by writing the BI Lagrangian (1) in the form

$$L_{BI} = \frac{\beta^2}{e^2}\sum_{n=0}^{\infty} q_n\left(\frac{1}{2\beta^2}F^{\mu\nu}F_{\mu\nu} - \frac{1}{16\beta^4}\left(F^{\mu\nu}\tilde{F}_{\mu\nu}\right)^2\right)^{n+1} \tag{4}$$

where

$$\begin{aligned} q_0 &= -\frac{1}{2} \\ q_n &= \frac{(-1)^{n+1}}{4^n} \frac{(2n-1)!}{(n+1)!\,(n-1)!} \quad \text{for} \quad n \geq 1 \end{aligned} \tag{5}$$

Eq.(4) can be rewritten as

$$L_{BI} = \sum_{n=1}^{\infty} q_{n-1} \sum_{j=0}^{n} \binom{n}{j} \left(\frac{1}{2\beta^2} F^{\mu\nu} F_{\mu\nu} \right)^j \left(-\frac{1}{16} (\frac{1}{\beta^2} F^{\mu\nu} \tilde{F}_{\mu\nu})^2 \right)^{n-j} \tag{6}$$

The basic ingredient for the supersymmetric extension of the BI action is the curvature supermultiplet

$$W_\alpha = -\frac{1}{4} \bar{D}_{\dot{\beta}} \bar{D}^{\dot{\beta}} D_\alpha V \tag{7}$$

where V is the gauge vector superfield which in the Wess-Zumino gauge reads

$$V = -\theta \sigma^\mu \bar{\theta} A_\mu + i\theta\theta\bar{\theta}\bar{\lambda} - i\bar{\theta}\bar{\theta}\theta\lambda + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta} D \tag{8}$$

Here A_μ is a vector field, λ and $\bar{\lambda}$ are two-component spinors which can be combined to give a four-component Majorana fermion and D is an auxiliary field. The covariant derivatives D_α and $\bar{D}_{\dot{\alpha}}$ act on chiral variables

$$\begin{aligned} y^\mu &= x^\mu + i\theta\sigma^\mu\bar{\theta} \\ y^{\mu\dagger} &= x^\mu - i\theta\sigma^\mu\bar{\theta} \end{aligned} \tag{9}$$

where we use $\alpha, \beta, \ldots$ for spinor indices and $\mu, \nu, \ldots$ for Lorentz indices. As usual, $\sigma^\mu = (I, \vec{\sigma})$ with σ^i the Pauli matrices. Explicitly,

$$\begin{aligned} D_\alpha &= \frac{\partial}{\partial\theta^\alpha} + 2i \left(\sigma^\mu \bar{\theta} \right)_\alpha \frac{\partial}{\partial y^\mu} \\ \bar{D}_{\dot{\alpha}} &= -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} \end{aligned} \tag{10}$$

are the covariant derivatives acting on functions of $(y, \theta, \bar{\theta})$ and

$$\begin{aligned} D_\alpha &= \frac{\partial}{\partial\theta^\alpha} \\ \bar{D}_{\dot{\alpha}} &= -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - 2i \left(\theta\sigma^\mu \right)_{\dot{\alpha}} \frac{\partial}{\partial \bar{y}^\mu} \end{aligned} \tag{11}$$

are the corresponding covariant derivatives on functions of $(y^\dagger, \theta, \bar\theta)$.

As it is well known, one can construct the $N = 1$ supersymmetric Maxwell Lagrangian in terms of the chiral superfield W_α and its hermitic conjugate $\bar W_{\dot\alpha}$ by considering

$$L_0 = \frac{1}{4}\left[\int d^2\theta W^2\left(y,\theta\right) + \int d^2\bar\theta \bar W^2\left(y^\dagger,\bar\theta\right)\right] \tag{12}$$

since the last F-components in W^2 and $\bar W^2$ contain the terms $D^2 - \frac{1}{2}\left(F^2 \pm iF\tilde F\right)$. Now, in order to get higher powers of F^2 and $F\tilde F$ necessary to construct the BI action, it has been shown [3] that one has to include powers of superfields X and Y defined as

$$X = \frac{1}{8}(D^2W^2 + \bar D^2\bar W^2) \tag{13}$$

$$Y = -\frac{i}{16}(D^2W^2 - \bar D^2\bar W^2) \tag{14}$$

Note that, as shown in the Appendix, one can write

$$X|_{\theta=\bar\theta=0} = -(\frac{1}{\beta^2}D^2 - \frac{1}{2\beta^2}F^{\mu\nu}F_{\mu\nu} - i\lambda\partial\!\!\!/\bar\lambda - i\bar\lambda\bar{\partial\!\!\!/}\lambda) \tag{15}$$

$$Y|_{\theta=\bar\theta=0} = \frac{1}{2}(\frac{1}{2\beta^2}F^{\mu\nu}\tilde F_{\mu\nu} + \lambda\partial\!\!\!/\bar\lambda - \bar\lambda\bar{\partial\!\!\!/}\lambda) \tag{16}$$

Hence, lowest components of X and Y include the invariants F^2 and $F\tilde F$.

Thus, we consider the following supersymmetric Lagrangian whose bosonic part, as we shall see, leads to the BI theory

$$L_{BI}^{SUSY} = \frac{\beta^2}{4e^2}\left[\frac{1}{\beta^2}\int d^2\theta W^2 + \frac{1}{\beta^2}\int d^2\bar\theta\bar W^2\right] + \sum_{r,s,t=0}^{\infty} a_{rst}\int d^4\theta\left(W^2\bar W^2\right)^r X^sY^t \tag{17}$$

(here a_{rst} are coefficients to be determined). Concerning the last term in (17), note that higher powers of F^2 and $F\tilde F$ are necessary for the construction of the BI Lagrangian, not only W^2 and $\bar W^2$ should be considered but also products and powers of these chiral superfields have to be introduced. Now, since the highest component of $W^2\bar W^2$ takes the form

$$W^2\bar W^2|_{\theta\theta\bar\theta\bar\theta} = \theta\theta\bar\theta\bar\theta\left((D^2 - \frac{1}{2}F_{\mu\nu}F^{\mu\nu})^2 + (\frac{1}{2}\tilde F_{\mu\nu}F^{\mu\nu})^2\right) \tag{18}$$

one can see that powers of this general superfield can be used to reproduce the expansion in (6).

As stated above, coefficients a_{rst} should be chosen so as to get the BI Lagrangian from the bosonic part of (17). Since the product X^sY^t contains derivatives of F and $\tilde{F}$ in its D-component, a_{0st} must vanish in order to eliminate unwelcome purely bosonic terms. Moreover, one can see that the simplest choice leading to the BI action is to take $a_{rst} = 0$ for $r > 1$. Then, in superfield notation, the SUSY BI Lagrangian that we shall consider is:

$$L_{BI}^{SUSY} = \frac{1}{4e^2}\left[\int d^2\theta W^2 + \int d^2\bar{\theta}\bar{W}^2\right] + \sum_{s,t=0}^{\infty} a_{1st}\int d^4\theta W^2\bar{W}^2 X^s Y^t \qquad (19)$$

One can see that this Lagrangian coincides with those proposed in [3]-[4].

It remains to determine coefficients a_{1st} so that the bosonic sector of the theory does coincide with the BI Lagrangian. We then concentrate in the purely bosonic terms of L_{BI}^{SUSY} and this we do by putting fermions to zero. At this stage, we shall impose

$$\begin{aligned} L_{BI}^{SUSY}\Big|_{BOS} &\equiv \tilde{L}_{BI} \\ &= \frac{\beta^2}{e^2}\left(1 - \sqrt{1 + \frac{1}{2\beta^2}F^{\mu\nu}F_{\mu\nu} - \frac{1}{16\beta^4}\left(F^{\mu\nu}\tilde{F}_{\mu\nu}\right)^2 - \frac{1}{\beta^2}D^2}\right) \end{aligned} \qquad (20)$$

Note that when the equation of motion for the D field (which in the present case gives $D = 0$) is used, the bosonic part of the supersymmetric Lagrangian coincides with the BI Lagrangian, i.e. $\tilde{L}_{BI}[D = 0] = L_{BI}$.

Coefficients a_{1st} can be now computed by imposing identity (20). From the bosonic components of $W^2\bar{W}^2$ (given in the Appendix) one finds a recurrence relation which connects the $a's$ coefficients in SUSY BI Lagrangian expansion with coefficients $q's$ (eq.(5)) in the expansion of the BI Lagrangian,

$$\begin{aligned} &a_{100} = \frac{1}{8\beta^2} \\ &a_{1\ n-2m\ 2m} = \frac{(-1)^m}{\beta^{2n+4}}\sum_{j=0}^{m} 4^{m-j}\binom{n+2-j}{j} q_{n+1-j} \\ &a_{1\ n\ 2m+1} = 0 \end{aligned} \qquad (21)$$

With the knowledge of coefficients $a's$, the supersymmetric Born-Infeld Lagrangian can be explicitly written in the form

$$L_{BI}^{SUSY} = \tilde{L}_{BI} + L_{fer} + L_{fb} \tag{22}$$

where L_{fer} contains self-interacting fermion terms while L_{fb} includes kinetic fermion and crossed boson-fermion terms. Both L_{fer} and L_{fb} can be calculated as expansions in increasing powers of fermionic and bosonic fields. For our purposes, namely the discussion of Bogomol'nyi relations through the supersymmetry algebra, only certain terms, quadratic in fermion fields, will be necessary.

In fact, as will become clear in section (IV) , only quadratic terms of the form $\lambda\partial_\mu\bar{\lambda}$ and $\bar{\lambda}\partial_\mu\lambda$ will give a contribution to the current algebra (higher order terms vanish when fermions are put to zero). Then, we shall only give the explicit form of those terms in L_{fb} which will be necessary in what follows (terms in L_{fer} do not contribute to the SUSY algebra). Denoting $L_{fb}[\lambda, \partial\bar{\lambda}]$ the sum of relevant terms (other terms of equal or higher order in fermionic fields can be calculated straightforwardly), we have

$$L_{fb} = L_{fb}^{I}[\lambda, \partial\bar{\lambda}] + L_{fb}^{II}[\bar{\lambda}, \partial\lambda] + \text{ other terms} \tag{23}$$

$$\begin{aligned} L_{fb}^{I}\left[\lambda, \partial\bar{\lambda}\right] &= -\frac{i}{2}\lambda\partial\!\!\!/\bar{\lambda} - i\sum_{s,t=0}^{\infty} a_{1st}\lambda\sigma^{\nu}\partial_{\mu}\bar{\lambda}(X_{BOS})^{s-1}(Y_{BOS})^{t-1} \\ &\left(-2iX_{BOS}Y_{BOS} + A^{*}(isY_{BOS} + \frac{t}{2}X_{BOS})\right)\left(A\delta^{\mu}_{\nu} + \frac{1}{2}\Omega^{*\mu\rho}\Omega_{\rho\nu}\right) \end{aligned} \tag{24}$$

$$L_{fb}^{II}[\bar{\lambda}, \partial\lambda] = L_{fb}^{I}\left[\lambda, \partial\bar{\lambda}\right]^{\dagger} \tag{25}$$

Here A and $\Omega^{*\mu\rho}\Omega_{\rho\nu}$, calculated in the Appendix, are given by

$$A = D^2 - \frac{1}{2}F^{\mu\nu}F_{\mu\nu} - \frac{i}{2}F^{\mu\nu}\tilde{F}_{\mu\nu} \tag{26}$$

$$\Omega^{*\nu\rho}\Omega_{\rho\mu} = \left(D^2 + \frac{1}{2}F_{\alpha\beta}F^{\alpha\beta}\right)\delta^{\nu}_{\mu} - 2D\eta^{\nu\rho}\tilde{F}_{\rho\mu} + 2F^{\nu\rho}F_{\rho\mu} \tag{27}$$

We end this section by rewriting L_{BI}^{SUSY} defined in eq.(22), which was worked out in terms of the two component fermions λ and $\bar{\lambda}$, using a four component fermion Λ,

$$\Lambda = \begin{pmatrix} \lambda_{\alpha} \\ \bar{\lambda}^{\dot{\alpha}} \end{pmatrix} \tag{28}$$

Concerning 4 dimensional Dirac matrices Γ^μ, we use

$$\Gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix} \tag{29}$$

Then, instead of eq.(23), we have for L_{fb}

$$\begin{aligned} L_{fb} \;\; = \;\; & -\frac{i}{2}\bar{\Lambda}\partial\!\!\!/\Lambda + \sum_{s,t=0}^{\infty} \beta^{2(s+2t+1)} a_{1st} X_{BOS}^{s-1} Y_{BOS}^{2t-1} \\ & \Big(i\bar{\Lambda}\partial\!\!\!/\Lambda Y_{BOS} \left[s(X_{BOS}^2 + 4Y_{BOS}^2) - X_{BOS} \left(Z_{BOS} - 2X_{BOS} \right) \right] \\ & + 2i\bar{\Lambda}\Gamma^\mu\partial^\nu\Lambda(D\tilde{F}_{\nu\mu} - F_{\nu\rho}F^\rho_{\ \mu}) \left[X_{BOS}Y_{BOS} + 2\left(2sY_{BOS}^2 + tX_{BOS}^2 \right) \right] \\ & + \bar{\Lambda}\Gamma^5\partial\!\!\!/\Lambda X_{BOS} \left[t(X_{BOS}^2 + 4Y_{BOS}^2) + 4Y_{BOS}^2 + Y_{BOS}Z_{BOS}(s-2t) \right] \\ & - 2\bar{\Lambda}\Gamma^5\Gamma^\mu\partial^\nu\Lambda(D\tilde{F}_{\nu\mu} - F_{\nu\rho}F^\rho_{\ \mu}) X_{BOS}Y_{BOS}(s-2t) \Big) \end{aligned} \tag{30}$$

where

$$\begin{aligned} X_{BOS} &= -D^2 + \frac{1}{2}F_{\mu\nu}F^{\mu\nu} \\ Y_{BOS} &= \frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu} \\ Z_{BOS} &= D^2 + \frac{1}{2}F_{\mu\nu}F^{\mu\nu} \end{aligned} \tag{31}$$

With this, the complete supersymmetric Born-Infeld Lagrangian (22)

$$L_{BI}^{SUSY} = \tilde{L}_{BI} + L_{fer} + L_{fb}$$

is invariant under the following $N = 1$ supersymmetry transformations

$$\begin{aligned} &\delta A_\mu = -i\bar{\epsilon}\Gamma_\mu\Lambda \qquad\qquad \delta\Lambda = i(-\Sigma^{\mu\nu}F_{\mu\nu} + \Gamma^5 D)\epsilon \\ &\delta D = i\bar{\epsilon}\Gamma^5\partial\!\!\!/\Lambda \end{aligned} \tag{32}$$

where $\Sigma_{\mu\nu} = \frac{i}{4}[\Gamma_\mu, \Gamma_\nu]$ and $\Gamma^5 = i\Gamma^1\Gamma^2\Gamma^3\Gamma^0$.

3 The Supersymmetric Born-Infeld Higgs model and Bogomol'nyi equations

In the previous section we have constructed the $d = 4$, $N = 1$ supersymmetric Born-Infeld theory. Now, since we are seeking for Bogomol'nyi relations for a spontaneously broken gauge theory, one has to consider, in addition to the Lagrangian already derived, a SUSY Higgs Lagrangian. This can be done by considering a chiral supermultiplet Φ coupled to the vector superfield (8) in the usual gauge invariant way. Also, one adds a Fayet-Iliopoulos term to break the gauge symmetry. We shall not give the details here but directly give the resulting SUSY Higgs Lagrangian.

The first part of this section is devoted to a dimensional reduction to $d = 3$ space-time thus obtaining a $N = 2$ supersymmetric theory. It is in this model that Bogomol'nyi relations for vortices arise. Then, we shall discuss Bogomol'nyi equations.

Now, as it is well known, enlargement of supersymmetry from $N = 1$ to $N = 2$ leads to a bosonic sector obeying first order Bogomol'nyi equations [18]-[19]. These equations are obtained at the end of this section while the supersymmetry algebra (leading to Bogomol'nyi bounds) is discussed in the next one.

The dimensional reduction proceeds as follows. We take $A_3 = N$ and N will play the role of an additional scalar field in the 3-dimensional model. Concerning the fermion Λ defined in eq.(28), its components can be accomodated into a couple of two-component 3-dimensional Majorana fermion χ and ρ Four dimensional Γ matrices are related to 2×2 Dirac matrices in three dimensions γ^i $(i = 0, 1, 2)$ as follows

$$\Gamma^i = \gamma^i \otimes \tau_3 \quad , \quad \Gamma^3 = 1 \otimes i\tau_2 \quad , \quad \Gamma^5 = 1 \otimes \tau_1$$

$$\Sigma^{ij} = \sigma^{ij} \otimes 1 \quad , \quad \Sigma^{i3} = -\Sigma^{3i} = \gamma^i \otimes \tau_1 \tag{33}$$

where $\sigma^{ij} = 1/2[\gamma^i, \gamma^j]$.

With this, the dimensionally reduced $N = 2$, $d = 3$ action takes the form

$$S^{(3)} = S^{(3)}_{bos} + S^{(3)}_{fb} + S^{(3)}_{fer} \tag{34}$$

Here

$$\begin{aligned} S^{(3)}_{bos} \;\; = \;\; & -\frac{\beta^2}{e^2}\int d^3x \\ & \left(\sqrt{1 - \frac{1}{\beta^2}D^2 + \frac{1}{2\beta^2}F^{ij}F_{ij} - \frac{1}{\beta^2}\partial_i N \partial^i N - \frac{1}{\beta^4}(\varepsilon_{ijk}F^{ij}\partial^k N)^2} - 1\right) \end{aligned} \tag{35}$$

Concerning $S_{fb}^{(3)}$, it can be written in the form

$$\begin{aligned} S_{fb} &= -\frac{i}{2}\int d^3x\bar{\Sigma}\partial\!\!\!/\Sigma + i\sum_{s,t=0}^{\infty}\frac{a_{1st}}{\beta^{2s+4t+2}}\left(X^{(3)}\right)^{s-1}\left(Y^{(3)}\right)^{2t-1} \\ &\left\{\bar{\Sigma}\partial\!\!\!/\Sigma\left[s\left(\left(X^{(3)}\right)^2+\left(Y^{(3)}\right)^2\right)Y^{(3)}+\left(X^{(3)}-2Z^{(3)}\right)X^{(3)}Y^{(3)}+\right.\right. \\ &\left.\frac{1}{2}tX^{(3)}\left(\left(X^{(3)}\right)^2+\left(Y^{(3)}\right)^2\right)+\left(Y^{(3)}\right)^2X^{(3)}\right]-\bar{\Sigma}\gamma^i\partial^j\Sigma \\ &\left(X^{(3)}\eta_{ij}+2F_{ik}F_j^k\right)\left[X^{(3)}Y^{(3)}\ (F_{ik}F_j^k+\quad\frac{s}{2}-t)\right. \\ &\left.+\left(s\left(Y^{(3)}\right)^2+t\left(X^{(3)}\right)^2\right)\right] \\ &\left.+\bar{\Sigma}\partial^j\Sigma D\tilde{F}_j\left[\left(Y^{(3)}\right)^2+2t\left(X^{(3)}\right)^2-(1+2s-4t)\,X^{(3)}Y^{(3)}\right]\right\} \end{aligned} \tag{36}$$

where

$$X^{(3)} = \frac{1}{2}F_{ij}F^{ij} - D^2 - (\partial_i N)^2 \tag{37}$$

$$Y^{(3)} = \tilde{F}^i\partial_i N \tag{38}$$

$$Z^{(3)} = D^2 + \frac{1}{2}F_{ij}F^{ij} \tag{39}$$

and Σ is a Dirac fermion constructed from the two Majorana fermions χ and ρ,

$$\Sigma = \chi + i\rho \tag{40}$$

Finally, $S_{fer}^{(3)}$ is the dimensionally reduced purelly fermionic action whose explicit form is irrelevant for our main purpose, namely the evaluation of the supersymmetry algebra.

As announced, we shall add a $N = 2$, $d = 3$ Higgs action $S_H^{(3)}$ for the Higgs field whic takes the form [19]

$$\begin{aligned} S_H^{(3)} &= \int d^3x\left(\frac{1}{2}|D_i\phi|^2+\frac{i}{2}\bar{\psi}\,D\!\!\!\!/\,\psi+\frac{1}{2}|F|^2+\frac{i}{2}(\bar{\psi}\Sigma\phi-\bar{\Sigma}\psi\phi^*)+\right. \\ &\left.\frac{D}{2}(|\phi|^2-\xi^2)+\frac{1}{2}N(F\phi^*+F^*\phi-\bar{\psi}\psi)\right) \end{aligned} \tag{41}$$

ϕ is a complex charged scalar, ψ a Dirac spinor, N a real scalar and F a complex auxiliary field. The covariant derivative

$$D_i = \partial_i + iA_i \tag{42}$$

Using the equation of motion for the auxiliary field F, action (41) reads

$$\begin{aligned} S_H^{(3)} &= \int d^3x \left(\frac{1}{2}|D_i\phi|^2 + \frac{i}{2}\bar{\psi}\, \not{D}\psi + \frac{i}{2}(\bar{\psi}\Sigma\phi - \bar{\Sigma}\psi\phi^*) + \frac{D}{2}(|\phi|^2 - \xi^2) - \right. \\ & \left. \frac{1}{2}N^2|\phi|^2 - \frac{1}{2}N\bar{\psi}\psi \right) \end{aligned} \tag{43}$$

The complete $N = 2$, $d = 3$ supersymetric Born-Infeld-Higgs action is then given by

$$S_{SUSY}^{(3)} = S_{bos}^{(3)} + S_{fb}^{(3)} + S_{fer}^{(3)} + S_H^{(3)} \tag{44}$$

where the different actions have been defined through eqs.(34)-(36) and (43). Dimensions of parameters and fields in units of mass are: $[\beta] = m^2$, $[e] = m^{\frac{1}{2}}$, $[\xi] = m^{\frac{1}{2}}$, $[(A_\mu, N, \Sigma, D)] = (m, m, m^{\frac{3}{2}}, m^2)$ and $[(\phi, \psi, F))] = (m^{\frac{1}{2}}, m, m^{\frac{3}{2}})$.

Action (44) remains invariant under the following $N = 2$ supersymmetry transformations (with Dirac fermion parameter ϵ)

$$\begin{array}{lll} \delta\phi = \bar{\epsilon}\psi & \delta\psi = -(i\, \not{D}\phi + N\phi)\epsilon & \delta N = i\bar{\epsilon}\Sigma + \text{h.c.} \\ \delta A_i = \bar{\epsilon}\gamma_i\Sigma & \delta\Sigma = (\frac{1}{2}\varepsilon_{ijk}F^{ij}\gamma^k + D + i\not{\partial}N)\epsilon & \delta D = \frac{1}{2}\bar{\epsilon}\not{\partial}\Sigma - \text{h.c.} \end{array} \tag{45}$$

Since we have already used the equation of motion of the F field, eqs.(45) correspond to an on-shell invariance. In order to have an off-shell invariance one just has to use (41) instead of (43) and supplement (45) with the transformation law for F

$$\delta F = i\bar{\epsilon}\, \not{D}\psi + (i\bar{\epsilon}\Sigma\phi + \text{h.c.}) \tag{46}$$

The connection between supersymmetry and Bogomol'nyi equations is by now well-known. In the "normal" Maxwell-Higgs theory, imposing the supersymmetry variation of the gaugino to be zero gives one of the Bogomol'nyi equations (that for the gauge curvature) while the vanishing of the Higgsino supersymmetry variation leads to the second Bogomol'nyi equation, the one for the Higgs field. The same happens in the present case. Indeed, suppose we want to obtain the Bogomol'nyi equations for the bosonic theory defined by action (44) with all fermion fields put to zero. If we use the equation of motion for the auxiliary field D,

$$D = -\frac{e^2}{2}\frac{\phi^2-\xi^2}{\sqrt{1+\frac{e^4}{4\beta^2}(\phi^2-\xi^2)^2}} \times \sqrt{1+\frac{1}{2\beta^2}F^{ij}F_{ij}-\frac{1}{\beta^2}\partial_i N\partial^i N-\frac{1}{\beta^4}(\varepsilon_{ijk}F^{ij}\partial^k N)^2} \qquad (47)$$

the bosonic action becomes

$$S \equiv S^{(3)}_{bos} + S^{(3)}_H = \frac{\beta^2}{e^2} - \frac{\beta^2}{e^2}\int d^3x \sqrt{\left(1+\frac{1}{2\beta^2}F^{ij}F_{ij}-\frac{1}{\beta^2}\partial_i N\partial^i N-\frac{1}{\beta^4}(\varepsilon_{ijk}F^{ij}\partial^k N)^2\right)V[\phi]} \qquad (48)$$

where $V[\phi]$ is the resulting symmetry breaking potential

$$V[\phi] = 1+\frac{e^4}{4\beta^2}(\phi^2-\xi^2)^2 \qquad (49)$$

It is interesting to note that in our treatment, the symmetry breaking potential appears as a multiplicative factor inside the BI square root as a result of searching the supersymmetric extension of the bosonic theory. That is, $N=2$ *supersymmetry forces this functional form for the action* (the same happens if one remains in $d=4$ dimensions with the $N=1$ theory). In ref. [13] this functional form was selected from the infinitely many possibilities of adding to the BI theory a Higgs field and its symmetry breaking potential just by trying to obtain the usual (i.e. Maxwell+Higgs) Bogomol'nyi equations. Thus, supersymmetry explains the rationale of the choice associated with the Born-Infeld-Higgs model.

Let us now write, exploiting supersymmetry, the first order equations for the Born-Infeld-Higgs theory which are the analogous to the Bogomol'nyi equations for the "normal" Nielsen-Olesen model. To this end we consider the static case with $A_0 = N = 0$.

From the supersymmetry point of view, Bogomol'nyi equations follow from the analysis of the gaugino and Higgsino supersymmetry variations.

More precisely, one decomposes the (Dirac) parameter of the supersymmetry transformation into its chiral components $\epsilon_\pm$. Then, by imposing the vanishing of half of the supersymmetry variations, say those generated by ϵ_+ (ϵ_-) one gets the Bogomol'nyi equations in a soliton (anti-soliton) background. The other half supersymmetry is broken. In the present case this amounts, for a vortex with positive magnetic flux, to the conditions

$$\delta_{\epsilon_+}\Sigma = 0 \quad \rightarrow \quad \frac{1}{2}\varepsilon_{0ij}F^{ij} = -D \tag{50}$$

$$\delta_{\epsilon_+}\psi = 0 \quad \rightarrow \quad D_1\phi = iD_2\phi \tag{51}$$

$$\begin{aligned} \delta_{\epsilon_-}\Sigma &\neq 0 \\ \delta_{\epsilon_-}\psi &\neq 0 \end{aligned} \tag{52}$$

Using the explicit expresion given by (47), we can rewrite (50) in the form

$$\frac{1}{2}\varepsilon_{0ij}F^{ij} = \frac{e^2}{2}\frac{\phi^2-\xi^2}{\sqrt{1+\frac{e^4}{4\beta^2}(\phi^2-\xi^2)^2}}\sqrt{1+\frac{1}{2\beta^2}F^{ij}F_{ij}} \tag{53}$$

From this equations, we obtain a simple expression for the magnetic field which in fact coincides with that corresponding to the "normal" (i.e. with Maxwell dynamics) Bogomol'nyi equation

$$B \equiv (1/2)\varepsilon_{0ij}F^{ij} = \frac{e}{2}(|\phi|^2-\xi^2) \tag{54}$$

This equation, together with (51) are the Bogomol'nyi equations for the Born-Infeld-Higgs system. They coincide with those arising in the Maxwell-Higgs system, i.e., the original Bogomol'nyi equations [15]-[16] and hence they have the same exact solutions originally found in [16]

4 SUSY algebra

Given the 3 dimensional model defined by action (34), one can easily construct the associated conserved supercurrent and from it the supercharge commutators. The corresponding supercharges $\bar{Q}$ and Q can be written in the following form

$$\bar{Q}\epsilon \equiv \int d^2x \left(\frac{\partial L}{\partial(\partial_0\Sigma)}\delta\Sigma + \frac{\partial L}{\partial(\partial_0\psi)}\delta\psi\right) \tag{55}$$

$$Q \equiv \gamma^0 \bar{Q}^\dagger \tag{56}$$

After some work one gets

$$\bar{Q} = \frac{i}{2e^2}\int d^2x \Sigma^\dagger \left(1 + 2\sum_0^\infty a_{1s0}\beta^{-2(s+1)}(B^2 - D^2)^s((2s+3)D^2 - \right.$$
$$\left. B^2 - 2(s+1)\gamma^0 BD)\right)(\gamma^0 B + D) + \frac{i}{2}\int d^2x \psi^\dagger \not{D}\phi \tag{57}$$

$$Q = -\frac{i}{2e^2}\int d^2x (B + \gamma^0 D)\left(1 + 2\sum_0^\infty a_{1s0}\beta^{-2(s+1)}(B^2 - D^2)^s\right.$$
$$\left.((2s+3)D^2 - B^2 - 2(s+1)\gamma^0 BD)\right)\Sigma - \frac{i}{2}\int d^2x (\not{D}\phi)^* \psi \tag{58}$$

As in the previous section, we have considered Nielsen-Olesen vortices by putting $A_0 = N = 0$ after differentiation (We also restrict ourselves to the static case). More important, we have only included terms linear in the fermionic fields, this because we are interested in extracting, from the SUSY charge algebra, just the pure bosonic term from which the (bosonic) Bogomol'nyi equations will be derived. That is why, after computing the algebra, all fermion fields should be put to zero (Non-linear fermionic terms in the charges necessarily give fermionic contributions to the algebra which vanish when fermions are put to zero).

Our purpose is to compute the Born-Infeld SUSY charge algebra and, from it, to explicitly obtain the Bogomol'nyi bounds in terms of energy and central charge. Since the expansion of the Born-Infeld Lagrangian in powers of $1/\beta^2$ leads to Maxwell, Euler-Heisenberg, ... Lagrangians, it will be instructive to show how the algebra reproduces, in a $1/\beta^2$ expansion, the corresponding Maxwell, Euler-Heisenberg, ... SUSY algebra and then present the arguments leading to the complete result. Indeed, to zero order in $1/\beta^2$ one gets for the SUSY charges, which we denote to this order as $\bar{Q}^{(0)}$ and $Q^{(0)}$,

$$\bar{Q}^{(0)} = \frac{i}{2e^2}\int d^2x \Sigma^\dagger(\gamma^0 B + D) + \frac{i}{2}\int d^2x \psi^\dagger \not{D}\phi \tag{59}$$

With this and $Q^{(0)}$ which can be computed from eq.(56), one can compute the SUSY algebra which takes the form

$$\{Q^{(0)}, \bar{Q}^{(0)}\} = \not{P} + Z \tag{60}$$

with P_μ the 4-momentum and Z the central charge. Using the explicit forms for $\bar{Q}^{(0)}$ and $Q^{(0)}$ obtained above one can compute the Poisson bracket corresponding to the l.h.s. in (60) and, comparing with the r.h.s. in this last equation one can identify

$$P^0 = \mathrm{tr}(\gamma^0\{Q^{(0)}, \bar{Q}^{(0)}\}) = \int d^2x \left(\frac{1}{2e^2}B^2 + \frac{1}{2}|D_i\phi|^2 + \frac{e^2}{8}(\phi^2 - \xi^2)^2\right) \quad (61)$$

$$Z = \mathrm{tr}(\{Q^{(0)}, \bar{Q}^{(0)}\}) = \int d^2x \left(B(\phi^2 - \xi^2) + \varepsilon_{ij}D_i\phi(D_j\phi)^*\right) = \xi^2 \oint A_\mu dx^\mu \quad (62)$$

As it is well-known [17], hermiticity of anticommutator (60) leads to a Bogomol'nyi bound which in the present case corresponds to the Maxwell-Higgs theory,

$$P^0 = E \geq |Z| \quad (63)$$

or

$$E \geq \xi^2 2\pi n \quad (64)$$

where n is the number of flux lines measured by Z [19].

We now consider the next order in the $1/\beta^2$ expansion, namely the Euler-Heisenberg theory. In that case, instead of (59) we have

$$\bar{Q}^{(1)} = \bar{Q}^{(0)} + \frac{i}{2e^2}\int d^2x\Sigma^\dagger \left(\frac{1}{4\beta^2}(3D^2 - B^2) - \frac{1}{2\beta^2}\gamma^0 BD\right)(\gamma^0 B + D) \quad (65)$$

The SUSY charges anticommutator leads in this case to

$$\begin{aligned} P^0 = E \;\; &= \;\; \int d^2x \left(\frac{1}{2e^2}B^2 + \frac{1}{2}|D_i\phi|^2 + \frac{e^2}{8}(\phi^2 - \xi^2)^2 - \right. \\ &\left. \frac{1}{8\beta^2 e^2}\left(B^2 - \frac{e^4}{4}(\phi^2 - \xi^2)^2\right)^2\right) \end{aligned} \quad (66)$$

Z is still given by (62) and eq.(64) also holds in this case.

The next order leads to the following results

$$\begin{aligned} \bar{Q}^{(2)} \;\; &= \;\; \bar{Q}^{(1)} + \frac{i}{2e^2\beta^4}\int d^2x\Sigma^\dagger \frac{1}{2}(B^2 - D^2)\left((5D^2 - B^2) - \frac{1}{4}\gamma^0 BD\right) \\ &(\gamma^0 B + D) \end{aligned} \quad (67)$$

$$P^0 = E = \int d^2x \frac{1}{2e^2}\left(B^2 + \frac{e^4}{4}(\phi^2-\xi^2)^2\right)\left(\frac{1}{2}|D_i\phi|^2 - \frac{1}{4\beta^2}\left(B^2 - \frac{e^4}{4}(\phi^2-\xi^2)^2\right) + \frac{1}{16\beta^4 e^2}\left(B^2 - \frac{e^4}{4}(\phi^2-\xi^2)^2\right)\right) \left(B^2 + \frac{e^4}{4}(\phi^2-\xi^2)^2\right)^2 \tag{68}$$

Again, Z is given by eq.(62), the same expression as in the Maxwell and Euler-Heisenberg case. In fact, this coincidence is not accidental and one can understand it as follows. If one were to obtain the Bogomol'nyi bound not from supersymmetry but as originally done by Bogomol'nyi, one should look at the purely bosonic Born-Infeld-Higgs theory and write the energy as a sum of squares plus a surface term. This surface term is responsible for the appearence of the topological charge as the bound for the energy. Now, the surface term is not modified by the fact that one deals with a BI and not a Maxwell gauge field Lagrangian. Moreover, the Bogomol'nyi equations do coincide for these two theories. Viewed from the supersymmetry side, the bound for the energy is provided by the central charge which again does not depend on the form of gauge field kinetic energy term.

Coming back to the complete SUSY algebra, let us write the charge $\bar{Q}$ given by eq.(57) in the form:

$$\bar{Q} = \frac{i}{2e^2}\int d^2x \Sigma^\dagger (1 + f + \gamma^0 g)(\gamma^0 B + D) + \frac{i}{2}\int d^2x \psi^\dagger \not{D}\phi \tag{69}$$

with

$$f = -1 + \frac{B^2+D^2}{B^2-D^2}M - \frac{2BD}{(B^2-D^2)^2}N \tag{70}$$

$$g = \frac{B^2+D^2}{B^2-D^2}\left(N - \frac{2BD}{(B^2-D^2)^2}M\right) \tag{71}$$

with

$$M = 2\left(\frac{\beta^2+B^2}{r} - \beta^2\right) \tag{72}$$

$$N = \frac{2BD}{r} \tag{73}$$

$$r = \sqrt{1 + \frac{1}{\beta^2}(B^2+D^2)} \tag{74}$$

With this, we can compute the SUSY algebra

$$\{Q, \bar{Q}\} = \not{P} + Z \tag{75}$$

and identify the energy and central charge in terms of f and g,

$$E = \frac{1}{2e^2} \int d^2x \left((B^2 + D^2)(f+1) + 2gBD \right) + \frac{1}{2} \int d^2x |D_i\phi|^2 \tag{76}$$

$$Z = \frac{1}{2e^2} \int d^2x \left((B^2 + D^2)g + 2(f+1)BD \right) + \frac{1}{2} \int d^2x \varepsilon_{ij} D_i\phi^* D_j\phi \tag{77}$$

Now, using the equation of motion for the auxiliary field D (eq.(47) one can see that the r.h.s. in eqs.(76)-(77) take the form

$$E = \sqrt{\left((1 + \frac{B^2}{\beta^2})(1 + \frac{e^4}{4\beta^2}(\phi^2 - \xi^2)^2) \right)} - \frac{\beta^2}{e^2} + \frac{1}{2}|D_i\phi|^2 \tag{78}$$

$$Z = \int d^2x \left(B(\phi^2 - \xi^2) + \varepsilon_{ij} D_i\phi D_j\phi^* \right) = \xi^2 \oint A_\mu dx^\mu = \xi^2 n \tag{79}$$

and then squaring (75) one again gets the Bogomol'nyi bound

$$E \geq |Z| \tag{80}$$

this showing the consistency of our supersymmetric construction. We stress that the results summarized in eqs.(69)-(80) correspond to the exact supersymmetric Born-Infeld model and not just some approximation in powers of $1/\beta^2$.

5 Summary and Discussion

Studying the $N = 2$ supersymmetric completion of the Born-Infeld-Higgs model in $d = 3$ dimensions, we have found the Bogomol'nyi relations for the bosonic theory. The interest in $d = 3$ dimensions arises from the fact that in such space-time dimensions vortex solutions to Bogomol'nyi equations are known to exist. Remarkably, we have found that the same set of equations (and hence of solutions) hold when a Born-Infeld Lagrangian determines the dynamics of the gauge field.

Originally, supersymmetric extensions of the Born-Infeld theory were constructed using the superfield formalism [3]-[4] and only the bosonic sector was explicitly written in component fields. Since one of our goals was to derive Bogomol'nyi relations from the supersymmetry $N = 2$ algebra, we needed the explicit form of the fermionic Lagrangian, at least, up to quadratic terms in the fermion fields which in turn lead to linear terms in the Noether current which give the sole non-vanishing contributions to the algebra in the bosonic background sector.

Our analysis shows that supersymmetry forces a particular functional form of the bosonic action in which the Higgs potential enters in the Born-Infeld square root (see eq.(48) in such a way as to ensure that the same Bogomol'nyi relations hold both for the Maxwell and the Born-Infeld theory.

As it was to be expected, the central charge of the $N = 2$ SUSY algebra coincides with the topological charge (the number of vortex magnetic flux units) of the model this ensuring that the Bogomol'nyi bound is not modified when one has a Born-Infeld theory. This was explicitly shown by constructing the SUSY algebra and deriving from Bogomol'nyi inequality in the usual way.

Acknowledgments: We would like to thank A. Lugo for helpful comments and discussions. F.A.S. is partially supported by CICBA, CONICET and Fundación Antorchas, Argentina and a Commission of the European Communities contract No:C11*-CT93-0315.

Appendix: Superfields in components

We start from the standard form for the chiral superfield W_α (see for example ref. [22])

$$W_\alpha\left(y,\theta,\bar{\theta}\right) = -i\lambda_\alpha + \theta_\alpha D - \frac{i}{2}\left(\sigma^m\bar{\sigma}^n\theta\right)_\alpha F_{mn} + \theta\theta\left(\sigma^m\partial_m\bar{\lambda}\right)_\alpha \tag{81}$$

where λ, $\bar{\lambda}$, D, F and $\tilde{F}$ are functions of the variable $y^m = x^m + i\theta\sigma^m\bar{\theta}$ and x^m is the usual 4-vector position (It will be convenient to write all the superfields in terms of the variable x instead of y.). The covariant derivatives are defined in eqs.(10) and (11).

Components of $W^\alpha W_\alpha$ and $\bar{W}_{\dot{\alpha}}\bar{W}^{\dot{\alpha}}$

$$W^2\left(x\right)\Big|_0 = -\lambda\lambda \tag{82}$$

$$W^2\left(x\right)\Big|_\theta = -2i\theta\lambda D + \theta\sigma^\mu\bar{\sigma}^\nu\lambda F_{\mu\nu} \tag{83}$$

$$W^2\left(x\right)\Big|_{\theta\theta} = \theta\theta\left(-2i\lambda\partial\!\!\!/\bar{\lambda} + A\right) \tag{84}$$

$$W^2\left(x\right)\Big|_{\theta\bar{\theta}} = -i\theta\sigma^\mu\bar{\theta}\partial_\mu\left(\lambda\lambda\right) \tag{85}$$

$$W^2\left(x\right)\Big|_{\theta\theta\bar{\theta}} = -\theta\theta\partial_\mu\left(\Omega^{\mu\nu}\eta_{\nu\rho}\lambda\sigma^\rho\bar{\theta}\right) \tag{86}$$

$$W^2\left(x\right)\Big|_{\theta\theta\bar{\theta}\bar{\theta}} = \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\Box\left(\lambda\lambda\right) \tag{87}$$

with

$$\begin{aligned} A &= D^2 - \frac{1}{2}F^{\mu\nu}F_{\mu\nu} - \frac{i}{2}F^{\mu\nu}\tilde{F}_{\mu\nu} \\ A^* &= D^2 - \frac{1}{2}F^{\mu\nu}F_{\mu\nu} + \frac{i}{2}F^{\mu\nu}\tilde{F}_{\mu\nu} \end{aligned}$$

$$\Omega^{\mu\nu} = D\eta^{\mu\nu} + iF^{\mu\nu} - \tilde{F}^{\mu\nu} \tag{88}$$

where σ^μ and $\bar{\sigma}^\nu$ are the Pauli matrices, defined as in ref. [22]. Everywhere, except if explicitly stated, $D, F_{\mu\nu}$, $\tilde{F}_{\mu\nu}$ and λ depend on x.

The components of $\bar{W}^2$ can be obtained from the former expressions by calculating the adjoint of the matrix elements.

Components of $W^2\bar{W}^2(x)$

$$W^2\bar{W}^2\left(x\right)\Big|_0 = \lambda\lambda\bar{\lambda}\bar{\lambda} \tag{89}$$

$$W^2\bar{W}^2\left(x\right)\Big|_\theta = 2i\bar{\lambda}\bar{\lambda}\left(\theta\lambda D - \frac{i}{2}\theta\sigma^\mu\bar{\sigma}^\nu\lambda F_{\mu\nu}\right) \tag{90}$$

$$W^2\bar{W}^2\left(x\right)\Big|_{\theta\theta} = -\theta\theta\bar{\lambda}\bar{\lambda}\left[-2i\lambda\partial\!\!\!/\bar{\lambda} + A\right] \tag{91}$$

$$\begin{aligned} W^2\bar{W}^2\left(x\right)\Big|_{\theta\bar{\theta}} &= -i\theta\sigma^m\bar{\theta}\left(\lambda\lambda\right)\overleftrightarrow{\partial_m}\left(\bar{\lambda}\bar{\lambda}\right) + 4\left(\theta\lambda D - \frac{i}{2}\theta\sigma^\mu\bar{\sigma}^\nu\lambda F_{\mu\nu}\right) \\ &\quad \left(\bar{\theta}\bar{\lambda}D + \frac{i}{2}\bar{\theta}\bar{\sigma}^\rho\sigma^\sigma\lambda F_{\rho\sigma}\right) \end{aligned} \tag{92}$$

$$\begin{aligned} W^2\bar{W}^2\left(x\right)\Big|_{\theta\theta\bar{\theta}} &= \theta\theta\left\{\left(2i\bar{\theta}\bar{\lambda}D - \bar{\lambda}\bar{\sigma}^\mu\sigma^\nu\bar{\theta}F_{\mu\nu}\right)\left(-2i\lambda\partial\!\!\!/\bar{\lambda} + A\right) + \right. \\ &\quad \left.\left(\bar{\theta}\bar{\sigma}^m\lambda\Omega^{pq}\eta_{qm}\right)\overleftrightarrow{\partial_p}\left(\bar{\lambda}\bar{\lambda}\right)\right\} \end{aligned} \tag{93}$$

with

$$\begin{aligned} W^2\bar{W}^2\Big|_{\theta\theta\bar{\theta}\bar{\theta}} &= \theta\theta\bar{\theta}\bar{\theta}\left\{-\frac{1}{4}\left(\lambda\lambda\Box\bar{\lambda}\bar{\lambda} + \bar{\lambda}\bar{\lambda}\Box\lambda\lambda\right) + \frac{1}{2}\partial_\mu\left(\lambda\lambda\right)\partial^\mu\left(\bar{\lambda}\bar{\lambda}\right)\right. \\ &\quad -4\left(\lambda\partial\!\!\!/\bar{\lambda}\right)\left(\bar{\lambda}\bar{\partial\!\!\!/}\lambda\right) - 2iA^*\lambda\partial\!\!\!/\bar{\lambda} - i\Omega^{*\nu\rho}\Omega_{\rho\mu}\lambda\sigma^\mu\partial_\nu\bar{\lambda} - \\ &\quad 2iA\bar{\lambda}\bar{\partial}\lambda - i\Omega^{\nu\rho}\Omega^*_{\rho\mu}\bar{\lambda}\bar{\sigma}^\mu\partial_\nu\lambda \\ &\quad \left.-i\partial_\nu\left(\Omega^{*\nu\rho}\right)\Omega_{\rho\mu}\lambda\sigma^\mu\bar{\lambda} - i\partial_\nu\left(\Omega^{\nu\rho}\right)\Omega^*_{\rho\mu}\bar{\lambda}\bar{\sigma}^\mu\lambda + AA^*\right\} \end{aligned}$$

where

$$A\overleftrightarrow{\partial}B = A\partial B - \left(\partial A\right)B$$

$$\Omega^{*\nu\rho}\Omega_{\rho\mu} = \left(D^2 + \frac{1}{2}F_{\alpha\beta}F^{\alpha\beta}\right)\delta^\nu_\mu - 2D\eta^{\nu\rho}\tilde{F}_{\rho\mu} + 2F^{\nu\rho}F_{\rho\mu} \tag{94}$$

and

$$Im\left(\partial_\nu\left(\Omega^{*\nu\rho}\right)\Omega_{\rho\mu}\right) = -D\partial_\nu F^{\nu\mu} + \partial_\nu\left(F^{\nu\alpha}\right)\tilde{F}_{\alpha\mu} + \tilde{F}^{\nu\alpha}\partial_\nu F_{\alpha\mu} + \frac{1}{2}\partial_\mu(F_{\alpha\beta})\tilde{F}^{\alpha\beta} \tag{95}$$

(we will see later that we do not need the real part of this expression)

Components of $X(x) \equiv \frac{1}{8}\left(D^2W^2(x) + \bar{D}^2\bar{W}^2(x)\right)$

$$X|_0 = \left(i\left(\lambda\bar{\partial\!\!\!/}\bar{\lambda} + \bar{\lambda}\bar{\partial\!\!\!/}\lambda\right) - D^2 + \frac{1}{2}F^2\right) \tag{96}$$

$$X|_\theta = -\eta^{qt}\left(\theta\sigma^p\right)_{\dot{\alpha}}\partial_q\left(\bar{\lambda}^{\dot{\alpha}}\Omega^*_{tp}\right) \tag{97}$$

$$X|_{\bar{\theta}} = -\eta^{qt}\left(\bar{\theta}\bar{\sigma}^p\right)^\alpha\partial_q\left(\lambda_\alpha\Omega_{tp}\right) \tag{98}$$

$$X|_{\theta\bar{\theta}} = -i\theta\sigma^p\bar{\theta}\partial_p\left(i\left(\lambda\bar{\partial\!\!\!/}\bar{\lambda} + \bar{\lambda}\bar{\partial\!\!\!/}\lambda\right) - D^2 + \frac{1}{2}F^2\right) \tag{99}$$

$$X|_{\theta\theta} = -\frac{1}{2}\theta\theta\Box\left(\bar{\lambda}\bar{\lambda}\right) \tag{100}$$

$$X|_{\bar{\theta}\bar{\theta}} = -\frac{1}{2}\bar{\theta}\bar{\theta}\Box\left(\lambda\lambda\right) \tag{101}$$

$$X|_{\theta\theta\bar{\theta}} = -\frac{1}{2}i\theta\theta\left[\Box\left(\theta^\alpha a_{\alpha\beta}\lambda^\beta\right) - \partial_m\partial^t\left(\theta\sigma^m\bar{\sigma}^n\lambda\Omega_{tn}\right)\right] \tag{102}$$

$$X|_{\theta\bar{\theta}\bar{\theta}} = -\frac{1}{2}i\bar{\theta}\bar{\theta}\left[\Box\left(\theta^\alpha a_{\alpha\beta}\lambda^\beta\right) - \partial_m\partial^t\left(\theta\sigma^m\bar{\sigma}^n\lambda\Omega_{tn}\right)\right] \tag{103}$$

$$X|_{\theta\theta\bar{\theta}\bar{\theta}} = -\frac{1}{4}\Box\left(i\left(\lambda\bar{\partial\!\!\!/}\bar{\lambda} + \bar{\lambda}\bar{\partial\!\!\!/}\lambda\right) - D^2 + \frac{1}{2}F^2\right) \tag{104}$$

Components of $Y(x) \equiv -\frac{i}{16}\left(D^2W^2(x) - \bar{D}^2\bar{W}^2(x)\right)$

$$Y|_0 = \frac{1}{2}\left(\left(\lambda\bar{\partial\!\!\!/}\bar{\lambda} - \bar{\lambda}\bar{\partial\!\!\!/}\lambda\right) + \frac{1}{2}F\tilde{F}\right) \tag{105}$$

$$Y|_\theta = \frac{i}{2}\eta^{qt}\left(\theta\sigma^p\right)_{\dot{\alpha}}\partial_q\left(\bar{\lambda}^{\dot{\alpha}}\Omega^*_{tp}\right) \tag{106}$$

$$Y|_{\bar{\theta}} = -\frac{i}{2}\eta^{qt}\left(\bar{\theta}\bar{\sigma}^p\right)^\alpha\partial_q\left(\lambda_\alpha\Omega_{tp}\right) \tag{107}$$

$$Y|_{\theta\bar{\theta}} = -\frac{i}{2}\theta\sigma^p\bar{\theta}\partial_p\left(\left(\lambda\partial\!\!\!/\bar{\lambda} - \bar{\lambda}\bar{\partial\!\!\!/}\lambda\right) + \frac{1}{2}F\tilde{F}\right) \tag{108}$$

$$Y|_{\theta\theta} = -\frac{i}{4}\theta\theta\Box\left(\bar{\lambda}\bar{\lambda}\right) \tag{109}$$

$$Y|_{\bar{\theta}\bar{\theta}} = \frac{i}{4}\bar{\theta}\bar{\theta}\Box\left(\lambda\lambda\right) \tag{110}$$

$$Y|_{\theta\theta\bar{\theta}} = -\frac{1}{4}\bar{\theta}\bar{\theta}\left[\Box\left(\theta^\alpha a_{\alpha\beta}\lambda^\beta\right) - \partial_m\partial^t\left(\theta\sigma^m\bar{\sigma}^n\lambda\Omega_{tn}\right)\right] \tag{111}$$

$$Y|_{\theta\theta\bar{\theta}\bar{\theta}} = -\frac{1}{8}\Box\left(\left(\lambda\partial\!\!\!/\bar{\lambda} - \bar{\lambda}\bar{\partial\!\!\!/}\lambda\right) + \frac{1}{2}F\tilde{F}\right) \tag{112}$$

References

[1] M. Born, Proc. R. Soc. London **A143** (1934) 410.

[2] M. Born and M. Infeld, Proc. R. Soc. London **A144** (1934) 425.

[3] S. Deser and R. Puzalowski, *J. Phys.* **A13** (1980) 2501.

[4] S. Ceccotti and S. Ferrara, *Phys. Lett.* **B187** (1987) 335.

[5] T. Hagiwara, Nucl. Phys. **B 189** (1981) 135

[6] E.S. Fradkin and A.A. Tseytlin, Phys. Lett. **163B** (1985) 123; A.A. Tseytlin, Nucl. Phys. **B276** (1986) 391.

[7] R.G. Leigh, Mod. Phys. Lett. **A4** (1989) 2767 and 2073.

[8] J.Polchinski, *Tasi Lectures on D-Branes*, hep-th/9611050.

[9] M. Aganagic, C. Popescu and J.H. Schwarz, Phys. Lett. **B393** (1997) 311; Nucl. Phys. **B 495** (1997) 99.

[10] J. Bagger and A. Galperin, Phys. Rev. **D 55** (1997) 1091.

[11] C.G. Callan and J.M. Maldacena, Nucl. Phys. **B 513** (1998) 198.

[12] G. Gibbons, Nucl. Phys. **B 514** (1998) 603.

[13] K. Shiraishi and S. Hirenzaki, Int. Jour. of Mod. Phys. **A6** (1991) 2635.

[14] E. Moreno, C. Núñez and F.A. Schaposnik, Phys. Rev. **D** (1998) in press, LA-PLATA-TH-04-98, e-Print Archive: hep-th/9802175.

[15] E.B. Bogomol'nyi, Sov. Jour. Nucl. Phys. **24** (1976) 449.

[16] H. de Vega and F.A. Schaposnik, Phys. Rev. Lett. **56** (1986) 2564. Phys. Rev. **D34** (1986) 3206.

[17] E. Witten and D. Olive, Phys. Lett. **B 78** (1978) 77.

[18] C. Lee, K. Lee and E.J. Weinberg, Phys. Lett. **B243** (1990) 105.

[19] J.D. Edelstein, C. Núñez and F.A. Schaposnik, Phys. Lett. **B329** (1994) 39

[20] K. Becker, M. Becker and A. Strominger, Phys. Rev. **D51** (1985) 6603.

[21] J.D. Edelstein, C. Núñez and F.A. Schaposnik, Nucl. Phys. **B 458** (1996) 165.

[22] J.Lykken, *Introduction to Supersymmetry*, hep-th 9612114.