Competition between standard and exotic double beta decays

C. Barbero[†], F. Krmpotić[†], A. Mariano[†]

Departamento de Física, Facultad de Ciencias

Universidad Nacional de La Plata, C. C. 67, 1900 La Plata, Argentina.

Abstract

We evaluate the contributions of higher order terms in weak Hamiltonian to the standard two-neutrino double beta decay. It is shown that only the first-forbidden unique transitions can alter the two-electron energy spectrum. Yet, their effect is too small to screen the detection of exotic neutrinoless double beta decays, which are candidates for testing the physics beyond the standard model.

PACS: 13.15+q;14.80;21.60Jz;23.40

[†]Fellow of the CONICET from Argentina.

The double beta ($\beta\beta$) experiments furnish a unique window onto whatever new physics may replace the Standard Electroweak Model (SM). To be observable the new physics should: i) violate the electron-lepton-number (L_e) conservation, that is fulfilled in the SM, and/or ii) fit scalar particles (currently called Majorons), light enough to be produced in the $\beta\beta$ decay.

The quantity that is used to discern experimentally between the ordinary SM two-neutrino decays ($\beta\beta_{2\nu}$) and the exotic neutrinoless $\beta\beta$ events, both without ($\beta\beta_{0\nu}$) and with Majoron emissions ($\beta\beta_{M}$), is the electron energy spectrum $d\Gamma/d\epsilon$ of the decay rate Γ, as a function of the sum $\epsilon = \epsilon_1 + \epsilon_2$ of the energies of the two emitted electrons. For a transition from the initial state $|0_I\rangle$ in the (N, Z) nucleus to the final state $|0_F\rangle$ in the $(N-2, Z+2)$ nucleus (with energies E_I and E_F and spins and parities $J^\pi = 0^+$) the differential decay rate reads

$$d\Gamma(0^+_I \to 0^+_F) = \frac{4G^4\bar{g}^2}{15\pi^5}\left|\mathcal{M}(0^+_I \to 0^+_F)\right|^2 d\Omega,$$

where $G = (2.996 \pm 0.002)\times 10^{-12}$ is the Fermi coupling constant (in natural units), and $\bar{g}$, $\mathcal{M}$, and $d\Omega$ are, respectively, the effective coupling constant, the nuclear matrix element, and the differential phase space. With the spectrum shape is also measured the half-life $T = \ln 2/\Gamma$.

For instance, when no new light particles are created, the L_e-violating terms in the weak Lagrangian, that generate a Majorana mass for the electron, can be identified if they produce $\beta\beta_{0\nu}$ decays. Then,

$$d\Omega_{0\nu} = \frac{1}{64\pi^2}\delta(Q - \epsilon_1 - \epsilon_2)\prod_{k=1}^{2} p_k\epsilon_k F_0(\epsilon_k)d\epsilon_k,$$

and the energy spectrum is just a spark at the released energy $Q = E_I - E_F$. ($p_k = |\mathbf{p}_k|$ is the magnitude of the electron three-momentum, and $F_0(\epsilon)$ is the Fermi function that describes the distortion of the electron wave function due to the electric charge of the nucleus [1, 2, 3].)

The phase space for $\beta\beta_{2\nu}$ and $\beta\beta_M$ decays can be written as

$$d\Omega_i = \frac{1}{64\pi^2}(Q - \epsilon_1 - \epsilon_2)^{n_i}\prod_{k=1}^{2} p_k\epsilon_k F_0(\epsilon_k)d\epsilon_k, \qquad (1)$$

where the spectral index is: $n_{2\nu} = 5$ for $\beta\beta_{2\nu}$, and $n_M = 1, 3$ and 7 for $\beta\beta_M$, depending on whether one or two Majorons are emitted and on the leptonic charge ($L_e = 0, -1, -2$) they carry [4, 5, 6, 7, 8, 9]. Thus, both the $\beta\beta_{2\nu}$ and $\beta\beta_M$ decays exhibit continuous spectra in the interval $2 \leq \epsilon \leq Q$, and the $\beta\beta_{0\nu}$ and $\beta\beta_M$ processes, that are potentially capable to reveal the new physics, are clearly distinguishable from the SM $\beta\beta_{2\nu}$ decay. A plot of spectral shapes in the ^{76}Ge nucleus, for various choices for n_i, is shown in Fig. 1.

In searching for exotic decays, one should be absolutely sure of that there is no effect coming from the standard $\beta\beta_{2\nu}$ decay that could lead to similar experimental consequences. In the above discussion, the usual allowed approximation (A) has been assumed for the $\beta\beta_{2\nu}$ decay. This implies to consider only the virtual states with spin and parity $J^\pi = 0^+, 1^+$. The relevance of the first-forbidden non-unique (F) virtual states $J^\pi = 0^-, 1^-$ has been examined recently [10]. It was found there that, although their contribution to the half-life is quite sizable (of the order of 30%), they have the same phase space as the A transitions, *i.e.,* that given by eq. (1) with $n_{2\nu} = 5$. Then one should go a step further and try the first-forbidden unique (FU) transitions to the virtual $J^\pi = 2^-$ states. It is well known that the spectrum shape of a single β transition of this type, provides for the emission of more high- and low-energy electrons, than are found in spectra that have the allowed shape [11, 12]. One might expect that the inclusion of the virtual $J^\pi = 2^-$ states would lead to similar consequences. Thus, we aim to confront the FU transitions in the $\beta\beta_{2\nu}$ decay with the exotic $\beta\beta_{0\nu}$ and $\beta\beta_{M}$ decays. This has not been hitherto done by workers in the field.

To start with, we write the $\beta\beta_{2\nu}$ decay rate

$$d\Gamma_{2\nu} = 2\pi \sum\!\!\!\!\!\!\int |R_{2\nu}|^2 \delta(\epsilon_1 + \epsilon_2 + \omega_1 + \omega_2 - Q) \prod_{k=1}^{2} d\mathbf{p}_k d\mathbf{q}_k, \tag{2}$$

where the symbol $\sum\!\!\!\!\!\!\int$ represents both the summation on lepton spins, and the integration on neutrino momenta and electron directions. The transition amplitude is:

$$R_{2\nu} = \frac{1}{2(2\pi)^6} \sum_N [1 - P(e_1 e_2)][1 - P(\nu_1\nu_2)] \frac{\langle 0_F^+ | H_W(e_2\nu_2) | N\rangle \langle N | H_W(e_1\nu_1) | 0_I^+\rangle}{E_N - E_I + \epsilon_1 + \omega_1}, \tag{3}$$

where $e_i \equiv (\epsilon_i, \mathbf{p}_i, s_{e_i})$, $\nu_i \equiv (\omega_i, \mathbf{q}_i, s_{\nu_i})$, $P(l_1 l_2)$ exchanges the quantum numbers of leptons l_1 and l_2, and N runs over all levels in the $(N-1, Z+1)$ nucleus. The weak Hamiltonian reads

$$H_W(e\nu) = \frac{G}{\sqrt{2}} \int d\mathbf{x} j_\mu(\mathbf{x}) J^{\mu\dagger}(\mathbf{x}) + h.c.,$$

where $j^\mu(\mathbf{x})$ is the usual left-handed leptonic current [1, 2, 3], and for the hadronic current

$$J^\mu(\mathbf{x}) = (\rho_V(\mathbf{x}) - \rho_A(\mathbf{x}), \mathbf{j}_V(\mathbf{x}) - \mathbf{j}_A(\mathbf{x})),$$

the following non-relativistic approximation will be used [13, 14]

$$\begin{aligned}
\rho_V(\mathbf{x}) &= g_V \sum_n \tau_n^+ \delta(\mathbf{x} - \mathbf{r}_n), \\
\rho_A(\mathbf{x}) &= \frac{g_A}{2M_N} \sum_n \tau_n^+ [\boldsymbol{\sigma}_n \cdot \mathbf{p}_n \delta(\mathbf{x} - \mathbf{r}_n) + \delta(\mathbf{x} - \mathbf{r}_n) \boldsymbol{\sigma}_n \cdot \mathbf{p}_n], \\
\mathbf{j}_V(\mathbf{x}) &= \frac{g_V}{2M_N} \sum_n \tau_n^+ [\mathbf{p}_n \delta(\mathbf{x} - \mathbf{r}_n) + \delta(\mathbf{x} - \mathbf{r}_n)\mathbf{p}_n + f_W \boldsymbol{\nabla} \times \boldsymbol{\sigma}_n \delta(\mathbf{x} - \mathbf{r}_n)], \\
\mathbf{j}_A(\mathbf{x}) &= g_A \sum_n \tau_n^+ \boldsymbol{\sigma}_n \delta(\mathbf{x} - \mathbf{r}_n),
\end{aligned}$$

where M_N is nucleon mass, and g_V, g_A and f_W are, respectively, the vector, axial-vector and weak-magnetism effective coupling constants. In the discussion of the $\beta\beta_{2\nu}$ decay we ignore both the weak-magnetism term, and the action of the velocity dependent terms on the lepton current. These terms cause the "second-forbidden" contributions, which do not alter the electron spectrum shape and will be discussed elsewhere [15]. Additionally, it will be assumed that the Coulomb energy of the electron at the nuclear radius is larger than its total energy, which leads to the ξ-approximation [10, 12]. Thus, for the purposes of the present study, and after some straightforward algebra, the weak Hamiltonian is rewritten in the form

$$H_W(e\nu) = -\frac{G}{2}\sum_{\pi J} \mathsf{W}^\pi_J \cdot \mathsf{L}_J(e\nu),$$

where W^+_J and W^-_J are, respectively, the allowed and forbidden nuclear operators, and

$$\mathsf{L}_J(e\nu) \;=\; sg(s_\nu)\sqrt{\frac{\epsilon+1}{2\epsilon}F_0(\epsilon)}\chi^\dagger(s_e)\left(1-\frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{\epsilon+1}\right)\ell_J(1-\boldsymbol{\sigma}\cdot\hat{\mathbf{q}})\chi(-s_\nu),$$

with $\chi(s)$ being the usual Pauli spinor. The leptonic operators ℓ_J are listed in Table 1, together with W^π_J.

Table 1: Operators ℓ_J and W^π_J for different multipoles J; $\bar{p} = p[F_1(\epsilon)/F_0(\epsilon)]^{1/2}$, $\mathbf{v} = \mathbf{p}/M_N$ and $\xi = \alpha Z/2R$.

J	ℓ_J	W^+_J	W^-_J
0	1	g_V	$-g_A(\boldsymbol{\sigma}\cdot\mathbf{v}+\xi i\boldsymbol{\sigma}\cdot\mathbf{r})$
1	$\boldsymbol{\sigma}$	$g_A\boldsymbol{\sigma}$	$-g_V\mathbf{v}-\xi[g_V i\mathbf{r}-g_A(\boldsymbol{\sigma}\times\mathbf{r})]$
2	$[\boldsymbol{\sigma}\otimes(\mathbf{q}+\bar{\mathbf{p}})]_2$	-	$ig_A(\boldsymbol{\sigma}\otimes\mathbf{r})_2/\sqrt{5}$

In the next step we evaluate the transition amplitude and get

$$R_{2\nu} = \frac{G^2}{4(2\pi)^6}[1-P(\nu_1\nu_2)]\left[\mathsf{L}_0(e_1\nu_1)\cdot\mathsf{L}_0(e_2\nu_2)\left(\mathcal{M}^A_{2\nu}+\mathcal{M}^F_{2\nu}\right)-\mathsf{L}_2(e_1\nu_1)\cdot\mathsf{L}_2(e_2\nu_2)\mathcal{M}^{FU}_{2\nu}\right], \tag{4}$$

where $\mathcal{M}^A_{2\nu}$, $\mathcal{M}^F_{2\nu}$ and $\mathcal{M}^{FU}_{2\nu}$ are, respectively, the A, F and FU parts of the $\beta\beta_{2\nu}$ matrix element

$$\mathcal{M}_{2\nu}(0^+_I\rightarrow 0^+_F) = \sum_{\alpha J\pi}(-1)^J\frac{\langle 0^+_F||\mathsf{W}^\pi_J||J^\pi_\alpha\rangle\langle J^\pi_\alpha||\mathsf{W}^\pi_J||0^+_I\rangle}{E_{J^\pi_\alpha}-E_{0^+_I}+Q/2}.$$

Introducing (4) into (2) we obtain

$$d\Gamma_{2\nu} \;=\; \frac{4G^4}{15\pi^5}\left[|\mathcal{M}^A_{2\nu}+\mathcal{M}^F_{2\nu}|^2+f(\epsilon_1\epsilon_2)|\mathcal{M}^{FU}_{2\nu}|^2\right]d\Omega_{2\nu}, \tag{5}$$

Table 2: Kinematical factors $\mathcal{G}_{2\nu}$, and the nuclear matrix elements $\mathcal{M}_{2\nu}$ evaluated within the QRPA formalism.

Nucleus	$\mathcal{G}^{A}_{2\nu}$ $[yr^{-1}]$	$\mathcal{G}^{FU}_{2\nu}$ $[yr^{-1}]$	$\mathcal{M}^{A}_{2\nu}$	$\mathcal{M}^{F}_{2\nu}$	$\mathcal{M}^{FU}_{2\nu}$
^{76}Ge	$5.39\ 10^{-20}$	$2.10\ 10^{-19}$	0.050	-0.008	$1.0\ 10^{-5}$
^{82}Se	$1.80\ 10^{-18}$	$2.54\ 10^{-17}$	0.060	-0.009	$9.8\ 10^{-6}$
^{100}Mo	$3.91\ 10^{-18}$	$5.50\ 10^{-17}$	0.051	-0.014	$1.1\ 10^{-5}$
^{128}Te	$3.53\ 10^{-22}$	$8.77\ 10^{-23}$	0.059	-0.012	$1.5\ 10^{-5}$

where

$$f(\epsilon_1\epsilon_2) = \frac{25}{288}\sum_{\kappa=0}^{2} a_\kappa (Q-\epsilon_1-\epsilon_2)^{2\kappa},$$

and $a_0 = \bar{p}_1^2\bar{p}_2^2$, $a_1 = 16\bar{p}_1^2/35$ and $a_2 = 1/21$. The corresponding total decay rate is

$$\Gamma_{2\nu}(0^+_I \rightarrow 0^+_F) = \ln 2\left(\mathcal{G}_{2\nu}\,|\mathcal{M}^{A}_{2\nu} + \mathcal{M}^{F}_{2\nu}|^2 + \mathcal{G}'_{2\nu}|\mathcal{M}^{FU}_{2\nu}|^2\right),$$

where

$$\mathcal{G}_{2\nu} = \frac{4G^4}{15\pi^5 \ln 2}\int d\Omega_{2\nu},$$

is the usual 2ν kinematical factor [2] and $\mathcal{G}'_{2\nu}$ differs from $\mathcal{G}_{2\nu}$ by the additional factor $f(\epsilon_1\epsilon_2)$ in the integrand.

The spectrum shapes generated by the first and second terms in (5) are shown in Fig. 1. The FU one turns out to be even softer than that with the spectral index $n_M = 7$, which corresponds to the emission of two scalar particles with $L_e = -1$ [8]. At variance with the single β emission, the spectrum for the FU double beta process deviates from the allowed shape in the low-energy region, but not for $\epsilon \cong Q$. This is a direct consequence of the anti-symmetrization carried out in Eq. (3).

Numerical results for the kinematical factors and the nuclear matrix elements, for several experimentally interested nuclei, are displayed in Table 2. The last were evaluated within the pn-QRPA model, following the procedure adopted in our previous works [10, 16]. As noted first by Williams and Haxton [17], there is destructive interference between the matrix elements $\mathcal{M}^{A}_{2\nu}$ and $\mathcal{M}^{F}_{2\nu}$, and the neat effect of the latter is to decrease $\Gamma_{2\nu}$ in $\sim 30\%$. The contribution of the matrix elements $\mathcal{M}^{FU}_{2\nu}$ is relatively small, and in the most favorable case (^{100}Mo)

$$\frac{\mathcal{G}^{FU}_{2\nu}|\mathcal{M}^{FU}_{2\nu}|^2}{\mathcal{G}^{A}_{2\nu}\,|\mathcal{M}^{A}_{2\nu} + \mathcal{M}^{F}_{2\nu}|^2} \sim 10^{-6}.$$

Note that there is no interference term between the A and FU matrix elements. This is because, in doing the spin summations and angular integration in Eq. (2), the contribution of the lepton matrix elements $\mathsf{L}_2(e_1\nu_1)\cdot\mathsf{L}_2(e_2\nu_2)\mathsf{L}^*_0(e_1\nu_i)\cdot\mathsf{L}^*_0(e_2\nu_j)$ turns out to be identically null for $i,j = 1,2$ or $2,1$.

Among the new Majoron models the most hopeful one to be observed experimentally [18] is the charged Majoron (CM) model, designed by Burgess and Cline [6] and by Carone [7] (with $L_e = -2$ and $n_{CM} = 3$). At variance with the ordinary Majoron (OM) model, in the CM model the $\beta\beta$ decay proceeds via the relativistic corrections in $J^\mu(\mathbf{x})$, and the nuclear matrix element is of the form $\mathcal{M}_{CM} = \mathcal{M}^+_{CM} - \mathcal{M}^-_{CM}$, with $\mathcal{M}^\pm_{CM}$ being the contributions of two heavy Dirac neutrinos with masses $M_\pm$. In ref. [9] it has been pointed out that: (i) the most favorable situation for CM emission occurs for is that $M_+ \rightarrow \infty$, and (ii) there is a strong destructive interference between $\mathcal{M}^+_{CM}$ and $\mathcal{M}^-_{CM}$ when $M_+ \cong M_-$. In the second case the decay rates Γ_{CM} are of similar order of magnitude as those furnished by the FU transitions. This is illustrated in Table 3, where are shown the QRPA results for Γ_{CM} and Γ_{FU}, with the following parametrization for the CM model: $\bar{g}_{CM} = \theta^2/2$, $M_\pm = M\sqrt{1 \pm \theta}$, $M = 100$ MeV and $\theta = 0.1$ [9]. (Note that in the CM model it is not possible to make a clear disunion between $\mathcal{M}_{CM}$ and $\bar{g}_{CM}$.) In the same table are also listed the experimental limits for the anomalous events, and the decrease of the 2ν decay rates by the effect of $\mathcal{M}^F_{2\nu}$.

Table 3: Experimental limits for the anomalous events and the calculated decay rates (in units of yr^{-1}).

Nucleus	Γ_{exp}	Γ_{CM}	$-\Gamma_F$	Γ_{FU}
^{76}Ge	$< 4.2\ 10^{-23}$	$5.3\ 10^{-30}$	$2.7\ 10^{-23}$	$1.5\ 10^{-29}$
^{82}Se	$< 4.3\ 10^{-22}$	$7.3\ 10^{-29}$	$1.2\ 10^{-21}$	$1.7\ 10^{-27}$
^{100}Mo	$< 2.1\ 10^{-21}$	$1.7\ 10^{-28}$	$3.3\ 10^{-21}$	$5.0\ 10^{-27}$
^{128}Te	$< 9.0\ 10^{-26}$	$1.8\ 10^{-31}$	$4.9\ 10^{-25}$	$1.4\ 10^{-32}$

Regarding the contributions of the first-forbidden transitions to the $\beta\beta_{2\nu}$, several conclusions can be drawn:

- The non-unique virtual states $J^\pi = 0^-, 1^-$ contribute significantly for the half-life, and should be considered in any rigorous evaluation of the decay rates. Still, as they do not modify the 2ν energy spectrum, they are not distinguishable experimentally from the allowed transitions to the $J^\pi = 0^+, 1^+$ states.

- The unique virtual states $J^\pi = 2^-$ are capable to produce distortion of the two-electron spectrum, but their effect is very small. Thus, it is very unlikely that the modification of the 2ν energy spectrum by the FU transitions could be observed in planned or ongoing experiments.

- The effect of $\mathcal{M}^{FU}_{2\nu}$ is so minute in comparison with that of $\mathcal{M}^F_{2\nu}$ because: 1) the operator $i(\boldsymbol{\sigma} \otimes \mathbf{r})_2$ is not enhanced by the Coulomb field as are its r-dependent partners $i\boldsymbol{\sigma} \cdot \mathbf{r}$, $i\mathbf{r}$ and $(\boldsymbol{\sigma} \times \mathbf{r})$ (see Table 1), and 2) there is no interference term between $\mathcal{M}^{FU}_{2\nu}$ and $\mathcal{M}^A_{2\nu}$, as happens with $\mathcal{M}^F_{2\nu}$ and $\mathcal{M}^A_{2\nu}$. The first of these two motives was known from the simple β decay, but the second one is rather an unexpected result of the present study. Otherwise, the contribution of the first-forbidden unique transitions would be quite more significant.

In summary, the higher order effects in the standard physics modify the $\beta\beta_{2\nu}$ spectrum shape but only very tinily and at low energy, where most backgrounds tend to dominate. Ergo, they do not interfere with the detection of exotic $\beta\beta_{0\nu}$ and $\beta\beta_M$ decays. The emission rate for the recently discovered Majoron models [6, 7, 8] is very strongly conditioned by the model parameters. More, it could be so small as that arising from the first-forbidden unique transitions, and thus out of reach of current experiments. So, models for emission of scalar particles in the $\beta\beta$-decay, more robust than the ones tailored so far, would be extremely welcome.

References

[1] W. C. Haxton and G. Stephenson, *Prog. in Part. and Nucl. Phys.* **12** (1984) 409.

[2] M. Doi, T. Kotani and E. Takasugi, *Prog. Theor. Phys. Suppl.* **83**, (1985) 1.

[3] T. Tomoda, *Rep. Prog. Phys.* **54** (1991) 53.

[4] G.B. Gelmini and M. Roncadelli, *Phys. Lett.* **B99** (1981) 25.

[5] H.M. Georgi, S.L. Glashow and S. Nussinov, *Nucl. Phys.* **B193** (1981) 297.

[6] C.P. Burgess and J.M. Cline, *Phys. Lett.* **B2** (1993) 141; *ibid Phys. Rev.* **D49** (1994) 5925.

[7] C.D. Carone, Phys. Lett. **B308** (1993) 85.

[8] P. Bamert, C.P. Burgess, R.N. Mohapatra, *Nucl. Phys.* **B449** (1995) 25.

[9] C. Barbero, J.M. Cline, F. Krmpotić, D. Tadić, *Phys. Lett.* **B371** (1996) 78; *ibid Phys. Lett.* **B392** (1997) 419.

[10] C. Barbero, F. Krmpotić and A. Mariano, *Phys. Lett.* **B345** (1995) 192.

[11] R.D. Evans, *The Atomic Nucleus* (McGraw-Hill book company, inc. Yew York, 1955).

[12] H. Beherens and W. Büring, *Electron Radial Wave Functions and Nuclear Beta Decay* (Clarendon, Oxford, England , 1982).

[13] A. Bohr and B.R. Mottelson, *Nuclear Structure* (Benjamin, New York, 1969) vol. 1, p. 348.

[14] J.D. Walecka, *Theoretical Nuclear and Subnuclear Physics* (Oxford University Press, New York, 1995)

[15] C. Barbero, F. Krmpotić and A. Mariano, (in preparation).

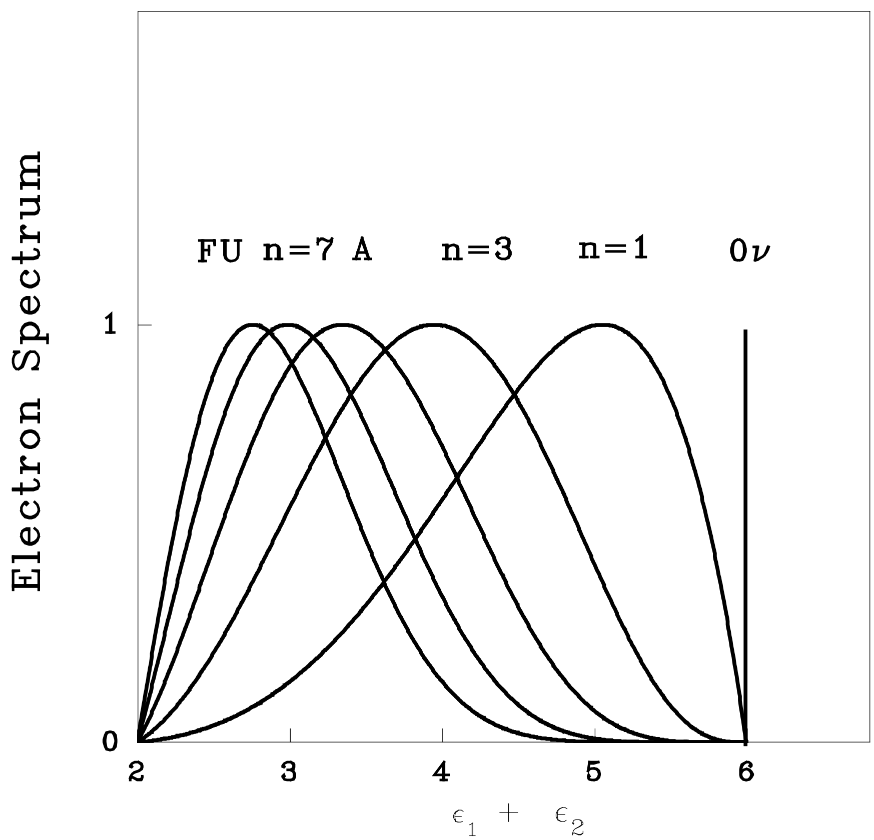
[16] F. Krmpotić and S. Shelly Sharma, *Nucl. Phys.* **A572** (1994) 329.

[17] A. Williams and W.C. Haxton, in *Intersections between Particle and Nuclear Physics*, ed. G.M. Bunce (AIP Conf. Proc. No. 176, 1988) p. 924.

[18] M. Hirsch, H.V. Klapdor-Kleingrothaus, S.G. Kovalenko, and H. Päs *Phys. Lett.* **B372** (1996) 8.

Figure Captions

Fig. 1. Electron energy spectrum for the nucleus ^{76}Ge, as a function of the sum of energies of the two emitted electrons, for: the standard 2ν allowed (A) and first-forbidden unique (FU) transitions, and the exotic neutrinoless decays, with Majoron emission ($n = 1, 3, 7$) and without (0ν). All five curves have been arbitrarily assigned the same maximal values for purposes of comparison.



Electron Spectrum
FU n=7 A
n=3
n=1
0ν
1
0
2
3
4
5
6
ϵ1 + ϵ2