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December 1996

Abstract

We compute the exact effective string vacuum backgrounds of the level $k = 81/19$ $SU(2,1)/U(1)$ coset model. A compact $SU(2)$ isometry present in this seven dimensional solution allows to interpreting it after compactification as a four dimensional non-abelian $SU(2)$ charged instanton with a singular submanifold and an $SO(3) \times U(1)$ isometry. The semiclassical backgrounds, solutions of the type II strings, present similar characteristics.

*This work was partially supported by CONICET, Argentina
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1. Introductory remarks

The search of interesting vacua representing the effective arena in which a string moves has been during the last decade one of the most explored subjects in string theory [1]. The main reason behind this is to try to elucidate some natural mechanism of compactification from 26 (or 10 in the supersymmetric case) dimensions to the usual 4, or in any case to have consistently defined a four dimensional theory, with the hope of obtaining string models compatible with well-known low energy physics [2]. In this context the Kaluza-Klein (KK) mechanism naturelly arises. String solutions of this type comes in the form of exactly solvable two-dimensional sigma models known as gauged Wess-Zumino-Witten models (GWZWM's). In Reference [3] we studied this mechanism in an abelian case. In this letter we present a non trivial example of it based on the $SU(2,1)/U(1)$ coset model that gives rise to non abelian $SU(2)$ gauge fields.

Let us start remembering some relevant facts about non abelian KK dimensional reduction [4]. Let us assume that our fields live on a $d+N$ dimensional manifold of the form $M \times \Sigma$ where the "space-time" M has dimension d and the compact Σ is N dimensional, and let us restrict ourselves to product-like coordinate patches $\{(z^M) = (x^\mu, y^m),\ \mu = 1,\ldots,d\ ,\ m = 1,\ldots,N\}$.

Let G_i be a group of diffeomorfisms of Σ (and then, of $M \times \Sigma$) of dimension d_i

$$\begin{aligned} x^\mu &\rightarrow x'^\mu(x) = x^\mu \\ y^m &\rightarrow y'^m(y) = y^m + \epsilon^a\ u_a^m(y) + \mathcal{O}(\epsilon^2) \end{aligned} \tag{1}$$

generated by the vector fields

$$\begin{aligned} u_a &= u_a^m(y)\,\partial_m \quad , \quad a = 1,\ldots,d_i \\ [u_a, u_b] &= f^c{}_{ab}\, u_c \end{aligned} \tag{2}$$

where $f^c{}_{ab}$ are the structure constants of $\mathcal{G}_i$, $(X_a)^c{}_b = f^c{}_{ab}$ being the adjoint representation. We assume they are complete on $T_0^1(\Sigma|_y)$, the tangent space of Σ at y (and then $d_i \geq N$). Under this hypothesis, a G_i-invariant scalar field D is just a field independent of the coordinates (y^m). Let G_g be the group of diffeomorfisms of Σ of dimension d_g generated by the vector fields that are invariant under G_i, i.e. [1]

$$\begin{aligned} v_{\overline{a}} &= u_{\overline{a}}^m(y)\,\partial_m \quad , \quad \overline{a} = 1,\ldots,d_g \\ [v_{\overline{a}}, v_{\overline{b}}] &= \overline{f}^{\overline{c}}{}_{\overline{a}\overline{b}}\, v_{\overline{c}} \\ \mathcal{L}_{u_a}(v_{\overline{b}}) &= [u_a, v_{\overline{b}}] = 0 \end{aligned} \tag{3}$$

[1] That they generate the Lie algebra of a group is easily showed. $\mathcal{L}_{u_a}$ stands for the Lie derivative [5]. In usual notation the last equation in (3) reads

$$v_{\overline{a}}^m(y') = \partial_n y'^m(y)\ v_{\overline{a}}^n(y)$$

with y' given by (1).

where $\overline{f}^{\overline{c}}{}_{\overline{a}\overline{b}}$ are the structure constants of $\mathcal{G}_g$.

Let us consider now an arbitrary tensor field $t \in \tau_2^0(\Sigma)$

$$\begin{aligned} t &= t^{MN}(z)\, dz^M \otimes dz^N \\ &= t_{\mu\nu}\, dx^\mu \otimes dx^\nu + t_{mn}\, dy^m \otimes dy^n + t_{\mu m}\, dx^\mu \otimes dy^m + t_{m\mu}\, dy^m \otimes dx^\mu \end{aligned} \tag{4}$$

If we now ask for t to be G_i-invariant, then the set of equations

$$\mathcal{L}_{u_a}(t_{MN}) = 0 \tag{5}$$

should be fullfilled; the solution of (5) leads to the following general form for t

$$t = t_{\mu\nu}(x)\, dx^\mu \otimes dx^\nu + t_{mn}(x,y)\, dy^m \otimes dy^n + A^{\overline{a}} \otimes \alpha_{\overline{a}} + \alpha_{\overline{a}} \otimes B^{\overline{a}} \tag{6}$$

where according to (5)

$$t_{mn}(x,y) = \partial_m y'^p(y)\, \partial_n y'^q(y)\, t_{pq}(x',y') \tag{7}$$

The $\{\alpha_{\overline{a}}\}$ is a basis of G_i-invariant one-forms

$$\alpha_{\overline{a}} = \alpha_{\overline{a},m}(x,y)\, dy^m = \alpha_{\overline{a},m}(x',y')\, dy'^m \tag{8}$$

and $A^{\overline{a}} = A^{\overline{a}}_\mu(x)\, dx^\mu$, $B^{\overline{a}} = B^{\overline{a}}_\mu(x)\, dx^\mu$ are one-forms on M.

Going to cases of interest, if we consider a non degenerate G_i-invariant metric on $M \times \Sigma$, $t \equiv G$ symmetric, we can choose

$$\alpha_{\overline{a},m}(x,y) = G_{mn}(x,y)\, v^n_{\overline{a}}(y) \tag{9}$$

that clearly satisfy (8) and then (6) can be put on the form

$$G = G^{(d)}_{\mu\nu}(x)\, dx^\mu \otimes dx^\nu + G_{mn}(x,y)\, \left(dy^m + v^m_{\overline{a}}(y)\, A^{\overline{a}}\right) \otimes \left(dy^n + v^n_{\overline{a}}(y)\, A^{\overline{a}}\right) \tag{10}$$

where

$$\begin{aligned} G^{(d)}_{\mu\nu}(x) &= G_{\mu\nu}(x) - g_{\overline{a}\overline{b}}(x)\, A^{\overline{a}}_\mu(x)\, A^{\overline{b}}_\nu(x) \\ g_{\overline{a}\overline{b}}(x) &= G_{mn}(x,y)\, v^m_{\overline{a}}(y)\, v^n_{\overline{b}}(y) \end{aligned} \tag{11}$$

Due to G_i invariance the scalar product $g_{\overline{a}\overline{b}}$ must be y-independent; if $d_g = N$ (the case we will be interested in) we can introduce the dual forms to the $\{v_{\overline{a}}\}$

$$\begin{aligned} \omega^{\overline{a}} &= dy^m\, \omega^{\overline{a}}_m(y) \\ \delta^{\overline{b}}_{\overline{a}} &= v^m_{\overline{a}}(y)\, \omega^{\overline{b}}_m(y) \end{aligned} \tag{12}$$

Then (11) enforces $G_{mn}(x,y)$ to have the form

$$G_{mn}(x,y) = g_{\overline{a}\overline{b}}(x)\, \omega^{\overline{a}}_m(y)\, \omega^{\overline{b}}_n(y) \tag{13}$$

and G can be written as

$$G = G^{(d)}_{\mu\nu}(x)\; dx^\mu \otimes dx^\nu + g_{\overline{a}\overline{b}}(x)\;\left(\omega^{\overline{a}} + A^{\overline{a}}\right) \otimes \left(\omega^{\overline{b}} + A^{\overline{b}}\right) \tag{14}$$

Before continuing we would like to make some remarks about two facts we find not properly clear in the literature [4]. The first one is that the Killing fields are the $\{u_a\}$, not the $\{v_{\overline{a}}\}$ that appear in (10); the gauge fields are connections on a G_g vector bundle, not on a G_i one. The second fact is that in standard KK considerations

$$G_{mn}(x,y) \equiv e^{2\,\phi(x)}\; g_{mn}(y) \tag{15}$$

for some G_i invariant g_{mn}, that is equivalent to have

$$g_{\overline{a}\overline{b}}(x) = \eta_{\overline{a}\overline{b}}\; e^{2\,\phi(x)} \tag{16}$$

for some constant $\eta_{\overline{a}\overline{b}}$, but in general there is no need for this to be so and we will remain with $\frac{N(N+1)}{2}$ scalars fields (moduli) on M. We will see later (see equation (51)) an example in which (16) is not verified.

Finally we find in a similar way that a G_i-invariant antisymmetric tensor B can be written as

$$B = \frac{1}{2}\; B_{\mu\nu}(x)\; dx^\mu \wedge dx^\nu + \frac{1}{2}\; b_{mn}(x,y)\; dy^m \wedge dy^n + b^{\overline{a}} \wedge \alpha_{\overline{a}} \tag{17}$$

with $b_{mn}(x,y)$ obeying (7) and $b^{\overline{a}} = b^{\overline{a}}_\mu(x)\; dx^\mu$ a one form on M.

2. The model

It is well-known that GWZWM's are two dimensional conformal field theories that explicitely realize the Goddard-Kent-Olive G/H coset construction [6] and give rise to a sigma model with specific backgrounds (G, B, D) corresponding to the metric, antisymmetric tensor and dilaton modes of the string. The field equations for them are only perturbatively known; by working out the $d+N$ dimensional objects (curvature, stress tensor, etc) related to the fields in (10,17) we can obtain the dimensionally reduced effective action in terms of the d dimensional fields (G, A, B, b, D). This action contains the bosonic part of $d = 10, N = 1$ SUGRA coupled to SUSY Yang-Mills theory, the last coupling being correctly reproduced by the dimensional reduction, and then reproducing the bosonic sector of the low energy heterotic and type I strings.

We do not describe all this here because we will not need it and mainly because we will follow a purely algebraic route that yields to the exact fields (the two dimensional functional method gives only the one-loop result).

Models of this type based on the $SU(2,1)/SU(2) \times U(1)$ and $SU(2,1)/SU(2)$ cosets were considered in [8] and [3] respectively. Here we will consider the gauging of a $U(1)$ non maximal subgroup that leads to a $8-1=7$ dimensional space-time, a $SU(2,1)/U(1)$ coset that in some sense is the natural generalization of the $SU(1,1)/U(1)$ Witten's black hole [9]. However as it was pointed out in [10], isometries should appear associated with the maximal group commuting with H, in our case $SU(2)$, that will allow to interpret the solution as a compactification to $d=4$ dimensions in the spirit of the KK dimensional reduction reviewed before.

From the conformal field theory point of view the model has a central charge

$$c(k)=\frac{8\,k}{k-3}-1=7\,\frac{k+\frac{3}{7}}{k-3} \tag{18}$$

Asking as usual the cancellation against the ghost contribution $c_{ghost}=-26$ gives a conformal value $k_c=\frac{81}{19}$. At difference of other coset models, there exists only one string theory corresponding to the perturbative phase of the model. In any case, a positive central charge requires $k>3$; we will assume for reasons to be clarified in Section 4 that $k>4$, i.e.

$$\lambda\equiv\frac{4}{k}<1 \tag{19}$$

Let us move now to the computations of the fields recalling some facts described at length in References [3,8]. An arbitrary element $g\in SU(2,1)$ may be locally parametrized as follows

$$\begin{aligned} g &= T(s\,\vec{n})\;e^{i\frac{\sqrt{3}}{2}\varphi\lambda_8}\;H(Y,1)\\ T(s\,\vec{n}) &= \begin{pmatrix}(1+s^2\,\vec{n}\,\vec{n}^\dagger)^{\frac{1}{2}} & s\,\vec{n}\\ s\,\vec{n}^\dagger & c\end{pmatrix}\quad,\quad \vec{n}=\begin{pmatrix}n^1\\ n^2\end{pmatrix}\\ H(Y,1) &= \begin{pmatrix}Y & \vec{0}\\ \vec{0}^t & 1\end{pmatrix}\quad,\quad Y\in SU(2) \end{aligned} \tag{20}$$

The two dimensional complex vector $\vec{n}$ is unimodular what allows to define an associated $SU(2)$ matrix in the following way

$$N\equiv\begin{pmatrix}n^{1*} & n^{2*}\\ -n^2 & n^1\end{pmatrix}=e^{i\frac{\alpha}{2}\sigma_3}\;e^{i\frac{\beta}{2}\sigma_2}\;e^{i\frac{\gamma}{2}\sigma_3} \tag{21}$$

where we have introduce Euler angles (α,β,γ) and Pauli matrices $\{\sigma_i\}$.

On the other hand, it is easy show that

$$g=e^{-i\delta\lambda_8}\;T(s\,e^{i\sqrt{3}\,\delta}\vec{n})\;e^{i\frac{\sqrt{3}}{2}\varphi\lambda_8}\;H(Y,1)\;e^{i\delta\lambda_8} \tag{22}$$

for any δ, i.e. it does not depends on δ. In terms of the matrix N,

$$e^{i\sqrt{3}\delta}\;\vec{n}\Leftrightarrow e^{-i\sqrt{3}\,\delta\sigma_3}\;N \tag{23}$$

Therefore if we are to consider a theory gauge invariant under vector transformations

$$g\to h\;g\;h^{-1} \tag{24}$$

with h generated by λ_8, equations (20-23) tell us that the Euler angle α in (21) should be gauged away. Then we will remain with the seven gauge invariant variables $0 \leq \beta \leq \frac{\pi}{2}$, $0 \leq \gamma < 2\pi$ parametrizing a two sphere S^2, together with $0 \leq \varphi < 2\pi, s \equiv \sinh r \geq 0$ ($c = \cosh r$) and some S^3 variables parametrizing Y that we will not need to explicit.

3. Currents and the exact solution

Here we present the exact solution for the metric and dilaton fields guessed from the änsatz in References [11]. To this end we introduce some notation and collect some useful relations. We refer the indices of the currents to the generators given by $\{(\vec{\lambda})_i = \lambda_i, i = 1,2,3;\ \lambda_1^{\pm} = \frac{1}{2}(\lambda_4 \pm i\lambda_5);\ \lambda_2^{\pm} = \frac{1}{2}(\lambda_6 \pm i\lambda_7);\ \lambda_8\}$, where $\{\lambda_a\}$ are the Gell-Mann matrices. If $X = x_0 1 + i\ \vec{x} \cdot \vec{\sigma}$ is an arbitrary $SU(2)$ element (${x_0}^2 = 1 - \vec{x}^2$) the adjoint representation is given by the 3×3 matrix

$$R(X)_{ij} \equiv \frac{1}{2}\, tr(\sigma_i X \sigma_j X^\dagger) = (2{x_0}^2 - 1)\ \delta_{ij} + 2\ x_i\ x_j + 2\ x_0\ \epsilon_{ijk}\ x_k \tag{25}$$

and the left and right $SU(2)$ (thought as linear operators in equations (27,28,30,33)) vector fields together with their dual forms are [2]

$$\begin{aligned} \hat{\xi}_i^L &= x_0\ \partial_i - \epsilon_{ijk}\ x_j\ \partial_k = -\hat{\xi}_i^R|_{-\vec{x}} \\ \omega_L^i &= x_0\ dx_i + \frac{x_i}{x^0}\ x_j\ dx_j - \epsilon_{ijk}\ x_j\ dx_k = -\omega_R^i|_{-\vec{x}} \\ \delta_i^j &= \hat{\xi}_i^L(\omega_L^j) = \hat{\xi}_i^R(\omega_R^j) \end{aligned} \tag{26}$$

They generate the left and right transformations

$$\begin{aligned} \hat{\xi}_i^L(X) &= i\ \sigma_i\ X \\ \hat{\xi}_i^R(X) &= i\ X\ \sigma_i \end{aligned} \tag{27}$$

and satisfy the commutation relations

$$\begin{aligned} [\hat{\xi}_i^L, \hat{\xi}_j^L] &= 2\ \epsilon_{ijk}\ \hat{\xi}_k^L \\ [\hat{\xi}_i^R, \hat{\xi}_j^R] &= -2\ \epsilon_{ijk}\ \hat{\xi}_k^R \\ [\hat{\xi}_i^L, \hat{\xi}_j^R] &= 0 \end{aligned} \tag{28}$$

Now let us move to the computations. We define the left currents as linear operators on the group manifold G by

$$\hat{L}_a g = -\lambda_a\ g\ , \quad g \in G \tag{29}$$

In the parametrization (20) the computations yield ($(\check{e}_i)_j = \delta_{ij}$)

[2] We assume henceforth, when no explicitely specified, to refer them to Y - variables.

$$
\begin{aligned}
\hat{\vec{L}} &= i \left(\hat{\vec{\xi}}^L|_Y - \hat{\vec{\xi}}^R|_N \right) \\
\hat{L}^+_\alpha &= -\frac{1}{2} N_{1\alpha} \, (\partial_r - i \, \frac{s}{c} \, \partial_\varphi) + \vec{A}^+_\alpha \cdot \hat{\vec{\xi}}^L|_Y + \vec{B}^+_\alpha \cdot \hat{\vec{\xi}}^L|_N = (\hat{L}^-_\alpha)^* \\
\hat{L}_8 &= i \, \frac{2}{\sqrt{3}} \left(\partial_\varphi - \frac{3}{2} \, \hat{\xi}^L_3|_N \right)
\end{aligned}
\tag{30}
$$

where

$$
\begin{aligned}
\vec{A}^+_\alpha &= \frac{i}{2sc} \, R(N)^t \, \left(N_{1\alpha} \, \frac{s^2}{2} \, \check{e}_3 + N_{2\alpha} \, c \, (c-1) \, (\check{e}_1 - i \, \check{e}_2) \right) \\
\vec{B}^+_\alpha &= \frac{-i}{2sc} \left(N_{1\alpha} \, (2c^2 - 1) \, \check{e}_3 + N_{2\alpha} \, c^2 \, (\check{e}_1 - i \, \check{e}_2) \right)
\end{aligned}
\tag{31}
$$

Similarly we define the right currents by

$$
\hat{R}_a g = g \, \lambda_a \, , \quad g \in G \tag{32}
$$

and compute them to get ($u \equiv e^{i\frac{3}{2}\varphi}$)

$$
\begin{aligned}
\hat{\vec{R}} &= -i \, \hat{\vec{\xi}}^R|_Y \\
\hat{R}^+_\alpha &= \frac{u}{2} \, (NY)_{1\alpha} \, (\partial_r + i \, \frac{s}{c} \, \partial_\varphi) + \vec{A}^+_\alpha \cdot \hat{\vec{\xi}}^L|_Y + \vec{B}^+_\alpha \cdot \hat{\vec{\xi}}^L|_N = (\hat{R}^-_\alpha)^* \\
\hat{R}_8 &= -i \, \frac{2}{\sqrt{3}} \, \partial_\varphi
\end{aligned}
\tag{33}
$$

where

$$
\begin{aligned}
\vec{A}^+_\alpha &= \frac{i \, u}{2sc} \, R(N)^t \, \left((NY)_{1\alpha} \, \frac{s^2}{2} \, \check{e}_3 + (NY)_{2\alpha} \, c \, (c-1) \, (\check{e}_1 - i\check{e}_2) \right) \\
\vec{B}^+_\alpha &= \frac{i \, u}{2sc} \, ((NY)_{1\alpha} \, \check{e}_3 + (NY)_{2\alpha} \, c \, (\check{e}_1 - i\check{e}_2))
\end{aligned}
\tag{34}
$$

By construction both set of currents satisfy the corresponding λ_a-algebra. Now we introduce the Casimir operators ($g_{ab} = tr\lambda_a\lambda_b$)

$$
\Delta^L_G = g^{ab} \hat{L}_a \hat{L}_b \tag{35}
$$

and the Virasoro-Sugawara laplacian associated with the coset $G/H = SU(2,1)/U(1)$

$$
\hat{L}^L_0 = \frac{1}{k-3} \, \Delta^L_G - \frac{1}{k} \, \Delta^L_H \tag{36}
$$

with analog construction in the right sector.

Finally we consider gauge invariant functions, i.e.

$$
(\hat{L}_8 + \hat{R}_8) f(g) = -i \, 2 \, \partial_\alpha f(g) = 0 \quad , \quad g \in SU(2,1) \tag{37}
$$

which are the α - independent ones as it should be as remarked above, and on this subspace we define the metric and dilaton fields to be those that obey the "hamiltonian" equation [3]

$$\begin{aligned}
\hat{H} f(g) &\equiv \frac{1}{k-3}\, \chi^{-1}\partial_\mu(\, \chi\, G^{\mu\nu}\, \partial_\nu) f(g) \\
\hat{H} &\equiv \hat{L}_0^L + \hat{L}_0^R = \frac{1}{k-3}\left(\vec{\hat{L}}^2 + 2\{\hat{L}_\alpha^+, \hat{L}_\alpha^-\} + \frac{3}{4}\lambda\, {L_8}^2\right) \\
\chi &\equiv e^D\, |\det\ G|^{\frac{1}{2}}
\end{aligned} \tag{38}$$

By carrying out the computations we read from these equations the exact backgrounds; it is useful to introduce the standard spherical versors on S^2

$$\begin{aligned}
\check{r} &= \sin\beta\, \cos\gamma\, \check{e}_1 + \sin\beta\, \sin\gamma\, \check{e}_2 + \cos\beta\, \check{e}_3 \\
\check{\beta} &= \cos\beta\, \cos\gamma\, \check{e}_1 + \cos\beta\, \sin\gamma\, \check{e}_2 - \sin\beta\, \check{e}_3 \\
\check{\gamma} &= -\sin\gamma\, \check{e}_1 + \cos\gamma\, \check{e}_2
\end{aligned} \tag{39}$$

as well as the 3×3 matrix Q and the function a,

$$\begin{aligned}
Q &= 1 - \left(1 - \left(1 + \frac{\lambda}{4}\, a\right)^{-\frac{1}{2}}\right)\, \check{r}\, \check{r}^t \\
a(r)^{-1} &= 1 - \lambda\, \frac{c^2}{s^2}
\end{aligned} \tag{40}$$

Then a convenient "seibenbein" is given by

$$\begin{aligned}
\vec{e} &= Q^{-1}\, \vec{\hat{\xi}^L} \\
e_4 &= \frac{1}{\sqrt{a}}\, \frac{s}{c}\, \left(\partial_\varphi + \frac{a}{2}\, \check{r}\cdot \vec{\hat{\xi}^L}\right) \\
e_5 &= \partial_r \\
e_6 &= \frac{2}{s}\, \left(\partial_\beta + \frac{c-1}{2}\, \check{\gamma}\cdot \vec{\hat{\xi}^L}\right) \\
e_7 &= \frac{2}{s\, \sin\beta}\, \left(\partial_\gamma - \frac{c-1}{2} \sin\beta\ \check{\beta}\cdot \vec{\hat{\xi}^L}\right)
\end{aligned} \tag{41}$$

whose dual basis is

$$\begin{aligned}
\vec{\omega} &= Q\, \left(\vec{\omega_L} + \vec{A}\right) \\
\omega^4 &= \sqrt{a}\, \frac{c}{s}\, d\varphi \\
\omega^5 &= dr \\
\omega^6 &= \frac{s}{2}\, d\beta \\
\omega^7 &= \frac{s}{2}\, \sin\beta\, d\gamma
\end{aligned} \tag{42}$$

[3] The computations in the left and right sectors lead to the same result.

where the one-forms $\vec{A}$ are given below in equation (50). Then the seven dimensional metric results

$$G = \eta^{ab}\ \omega_a \otimes \omega_b \tag{43}$$

where η_{ab} is minkowskian with signature $(- - - + + + +)$ (the actual signature depends on r and the value of k, however see next section).

The dilaton field on the other hand is given by

$$D(r) = D_0 + \ln |\ s^2\ \sqrt{1 + \frac{\lambda}{4}\ a^{-1}}\ | \tag{44}$$

We spend some words here about the antisymmetric tensor. The method does not permit to obtain its exact value (see however [14]), but it is possible to get the one loop result from the standard integrating out of the gauge fields mentioned at the beginning of Section 2. We have carried out that calculation (verifying of course the $\lambda = 0$ limit of (43,44)) and for completeness we quote the result

$$H^{(1l)} \equiv dB^{(1l)} = d(\vec{A}^{(1l)} \wedge \vec{\omega}_L) - \frac{1}{2}\ \sin\beta\ d\beta \wedge d\gamma \wedge d\varphi + 2\ \omega_L^1 \wedge \omega_L^2 \wedge \omega_L^3 \tag{45}$$

By comparing it with (17) we identify

$$\vec{b} = \vec{A} \quad , \tag{46}$$

equality that we conjecture to hold exactly.

4. The four dimensional interpretation

From the parametrization (20) it follows that the right global transformation

$$g \to g\ H(X,1) \quad \Leftrightarrow \quad Y \to Y\ X \quad , \quad X \in SU(2) \tag{47}$$

should be an invariance of the model, in particular an isometry. And in fact this is manifest in that the Y dependence of the backgrounds comes through the right invariant one forms ω_L^i

$$\omega_L^i = \frac{1}{2\,i}\ tr(\sigma_i\ dY\,Y^{-1}) = \frac{1}{2\,i}\ tr\left(\sigma_i\ d(YX)\,(YX)^{-1}\right) \tag{48}$$

Then in the language of Section 1, we identify $G_i \equiv SU(2)_{right}$ and according to (27, 28, 33) the generators of this transformation are the $u_a \sim \hat{\xi}_i^R$ vector fields. And therefore from (28) the identification of $G_g \equiv SU(2)_{left}$ and the $v_{\overline{a}} \sim \hat{\xi}_i^L$ right invariant vector fields (together with $\omega^{\overline{a}} \sim \omega_L^i$) is straightforward.

Taking into account these facts we can identify the four dimensional manifold $M \sim \Re \times S^1 \times S^2$ and the metric and (one loop) antisymmetric tensor as

$$
\begin{aligned}
G^{(4)} &= a\,\frac{c^2}{s^2}\,d^2\varphi + d^2r + \frac{s^2}{4}\,(d^2\beta + \sin^2\beta\,d^2\gamma) \\
B^{(4)} &= \frac{1}{2}\,\cos\beta\,d\gamma\wedge d\varphi
\end{aligned} \tag{49}
$$

The dilaton is always given by (44) and the $SU(2)$ gauge fields are

$$
\vec{A} = \vec{b} = -\frac{a}{2}\,\check{r}\,d\varphi - \frac{c-1}{2}\,\check{\gamma}\,d\beta + \frac{c-1}{2}\,\sin\beta\,\check{\beta}\,d\gamma \tag{50}
$$

On the other hand the fields $g_{mn}(x,y)$ and $b_{mn}(x,y)$ on $\Sigma \equiv S^3$ are read straight from (14, 17), in particular

$$
g(x) = -Q(x)^2 = -1 + \left(1 + \frac{4}{\lambda}\,a^{-1}\right)^{-1}\,\check{r}\,\check{r}^t \tag{51}
$$

The four dimensional backgrounds present a manifest and not at all obvious $SO(3)\times U(1)$ isometry. Furthermore is not asymptotically flat but asymptotic to a constant curvature geometry, an usual feature in bosonic string models due to the presence of the cosmological constant term [12, 3, 8]. This can be seen for example from the four dimensional Ricci scalar

$$
\frac{1}{6}\,\mathcal{R}^{(4)} = 1 - \frac{a^2}{s^4}\,\left(\frac{s^2}{a^2} - \lambda\right) \tag{52}
$$

On the other the one loop solution presents a true singularity at $r = 0$ that remains at higher orders. However at the value of the radius $r = r_0 > 0$ defined by

$$
s_0 = \sinh r_0 = \left(\frac{\lambda}{1-\lambda}\right)^{\frac{1}{2}} \tag{53}
$$

the curvature also exploits and another true singularity of purely quantum origin appears; this means that if the quantum theory has a sense (fact not addressed here) we certainly cannot analytically continue the solution beyond r_0 and then the singularity becomes "shifted". Then the signature of the solution is also preserved.. In "Schwarzchild" like radial coordinate

$$
0 < R(r) = c_0\,\ln\left(\frac{s}{s_0} + \sqrt{\frac{s^2}{{s_0}^2} - 1}\right) < \infty \tag{54}
$$

it looks like an instanton in compactified euclidean time φ with a singularity at the origin $R = 0$

$$
\begin{aligned}
G^{(4)} &= F(R)^{-2}\, d^2\varphi + F(R)^2\, d^2R + \frac{{s_0}^2}{4}\, \cosh^2 \frac{R}{c_0}\, \left(d^2\beta + \sin^2\beta\, d^2\gamma\right) \\
F(R)^2 &= \lambda\,(1-\lambda)\, \frac{\sinh^2 \frac{R}{c_0}}{1 + \lambda\, \sinh^2 \frac{R}{c_0}}
\end{aligned} \tag{55}
$$

Furthermore, we can adscribe to it non abelian "hair". If we introduce as usual the $SU(2)$ connection and its strenght by

$$
\begin{aligned}
A &= i\, \vec{\sigma} \cdot \vec{A} \\
F &= dA + A \wedge A = i\, \vec{\sigma} \cdot \vec{F}
\end{aligned} \tag{56}
$$

then the non abelian magnetic charges are defined by

$$
Q \equiv -\frac{1}{4\,\pi} \int_{S^2} F \tag{57}
$$

where the two sphere is taken to be in the asymptotic region. The computation shows up a non zero charge associated to the "radial" generator $\vec{\sigma} \cdot \check{r}$

$$
Q = i\, \frac{1}{2\, {g_s}^2}\, \vec{\sigma} \cdot \check{r} \tag{58}
$$

where g_s is the string coupling constant at infinity.

5. Conclusions

We have obtained a highly non trivial instanton of the (unknown) exact classical effective action of the bosonic string theory. As showed in [11] the one loop results are (up to a trivial rescaling) solutions of the type II superstring, and from the remarks in section 4 they are qualitatively similar to the exact ones. It has an essential singular submanifold and couples to $SU(2)$ Yang-Mills fields with non trivial charge. On the other hand an obvious as well as unexpected isometry $SO(3) \times U(1)$ is present in the four dimensional fields, equations (49). However this is not an isometry of the whole (seven dimensional) solution, as it is seen e.g. from the form of the gauge fields, equation (50). [4] This fact is even more manifest in the equivalent backgrounds related to (43-45) by T-duality [13]. For example the one loop [5] four dimensional metric reads

$$
\tilde{G}^{(4)} = \frac{16\, s^2\, c^2}{(1 + 3\, c^2)^2}\, \left(d\varphi - \frac{1}{2}\, \cos\beta\, d\gamma\right)^2 + d^2r + \frac{s^2}{4}\, \left(d^2\beta + \sin^2\beta\, d^2\gamma\right) \tag{59}
$$

[4] Maybe it is worth to remark that the $SU(2)$ isometry of the model that allows the compactified interpretation has to do with the Y variables on the compact space S^3, but nothing to do with the four dimensional fields that in general should not present isometries.

[5] We cannot compute the exact dual backgrounds because we do not know the exact antisymmetric tensor.

Another feature of the one loop dual solution we believe interesting is that the dilaton field

$$\tilde{D} = \tilde{D}_0 + \ln(1 + \frac{3}{4}\, s^2) \tag{60}$$

goes to a constant (instead of being linear) in the asymptotic region $r \gg 1$, as most asymptotically flat solutions does, but we do not know if this behaviour survives at higher orders.

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