

Precision cosmology as a test for statistics

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We compute the shift in the epoch of matter-radiation equality due to the possible existence of a different statistical (non-extensive) background. The shift is mainly caused by a different neutrino-photon temperature ratio. We then consider the prospects to use future large galaxy surveys and cosmic microwave background measurements to constrain the degree of non-extensivity of the universe.

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A. Introduction

The possible need of non-extensive thermostatics in cosmology, gravitation and astrophysics has been discussed by several authors, for instance see Refs. [1]. Indeed, the question of to what extent the universe as a whole is an extensive system is well justified. It is known that whenever long range interactions are present, standard (Boltzmann-Gibbs) statistical sums and integrals may diverge (see Ref. [2] for a review). In any case, to check the validity of a theory we must locate it in a more general framework, a group of test theories, which in general will have a free parameter. Then, one has to isolate which of the theories of the group are able to reproduce experimental or observational facts. Although this is a common practice in cosmological models based on alternative theories of gravity, it is not so usual concerning alternative statistics. The problem in these cases is try to determine if different statistical theories may also yield the same observational results.

The group of test theories will be, for our purposes, that introduced by Tsallis in [3]. By use of them we expect to check the validity of the Boltzmann-Gibbs scenario. Let us recall the starting points of nonextensive statistic (NES), in Tsallis' approach. The formalism begins by postulating [3]:

Postulate 1.- *The entropy of a system that can be found with probability p_i in any of W different microstates i is given by*

$$S_q = k \frac{1}{(q-1)} \sum_{i=1}^W [p_i - p_i^q] = k \frac{1}{q-1} \left(1 - \sum_{i=1}^W p_i^q \right), \quad (1)$$

with q a real parameter. We have a different statistics for every possible q -value. In (1) we have used, of course, that

$$\sum_i p_i = 1. \quad (2)$$

In general,

$$S_q = k \frac{1 - \text{Tr} \rho^q}{q-1}. \quad (3)$$

Postulate 2.- *An experimental measurement of an observable A , whose expectation value in microstate i is a_i , yields the q -expectation value (generalized expectation value (GEV))*

$$\langle A \rangle_q = \sum_{i=1}^W p_i^q a_i = \text{Tr} \rho^q \hat{A}, \quad (4)$$

for the observable A .

These two statements have the rank of *axioms*. As such, their validity is to be decided exclusively by the conclusions to which they lead, and ultimately by comparison with observations. We shall not present any further the formalism in this work, referring the reader to Refs. [2-5]. A more detailed presentation can also be found in [6]. Instead we shall use only the results we need for our calculations. This non-extensive formal approach was applied in many different systems including Newtonian gravity [7,8], Levy type diffusion [9], the problem of solar neutrinos [10] and others. Concerning the universe as a whole, we have previously shown that this group of test theories will affect primordial nucleosynthesis yields and the evolution of the neutron to baryon ratio in a computable way. Later, this could be used to set bounds upon the free parameter they have and thus the upon the degree of non-extensivity present in the early universe [6,11,12]. In addition, a generalization of the Planck radiation law was derived and applied to the cosmic microwave background data [13,14]. In all these works, limits upon the non-extensive (free) parameter were imposed. The typical order of magnitude of the bounds is 10^{-4} , what means that possible Boltzmann-Gibbs' deviation may be about one part in ten thousands.

In this letter we shall consider the matter-radiation equality epoch. This moment of cosmic history is important for every cosmological model because this transition changes the growth rate of density perturbations. Whereas perturbations inside the horizon are frozen during a radiation dominated universe, perturbations on all

scales can grow (and structure can form) during the matter era [15]. Then, the size of the horizon at the time of the matter-radiation equality is an important parameter of the model (indeed, it is the only one for CDM models) and it is to be read from the spectrum of density perturbations [15]. In the following years, new satellites will measure cosmic microwave anisotropies and new large galaxy surveys will be available. In particular, the Sloan Digital Sky Survey (SDSS) will acquire 10^6 redshifts [16,17]. This will make cosmology to enter in an era of high precision and about ten cosmological parameters will be accurately determined [18]. In what follows, we expect to show that this set of data can also be used as a test of the underlying statistical theory. To do so, we shall compute the shift in the matter-radiation equality that would be caused by non-extensivity; in general, by the use of a different statistical background. As this changes the shape of the power spectrum of density perturbations, something that can be measured from galaxy surveys [16,19], a new bound upon the statistical description will arise. In addition, we shall comment on the prospect for the accurate determination of the cosmic neutrino background and the effective number of neutrino species. This will also allow us to extract conclusions on which statistical model may be the right description of the universe.

B. Non-extensive corrections and matter-radiation equality

The photon energy density is dominated by the cosmic microwave background. Its temperature is measured extremely well [20],

$$T_{\gamma,0} = (2.728 \pm 0.004)\text{K} \quad (5)$$

and this gives, for the standard model,

$$\rho_{\gamma,0} = 4.66 \times 10^{-34} \text{ g cm}^{-3}. \quad (6)$$

However, if we are to take into account possible non-extensive effects, the energy density will in general be given by,

$$\rho_q = \frac{\pi^2}{30} g T^4 + \frac{1}{2\pi} (40.02 g_b + 34.70 g_f) T^4 (q-1), \quad (7)$$

where $g_{b,f}$ are degrees of freedom of bosons and fermions, $g = g_b + 7/8 g_f$ and q is a real free parameter measuring the amount of non-extensivity in the system [6,11]. When $q = 1$, all results reduce themselves to the standard ones.

The present radiation density also has a contribution from relic neutrinos that, due to the impossibility of detecting them directly, must be fixed theoretically. The relationship between neutrino and photon temperature was computed in Ref. [6] for a non-extensive framework and is given by,

$$T_\nu = T_\gamma \left(\frac{4}{11} \right)^{1/3} (1 + (q-1)0.013). \quad (8)$$

As we shall see below, equation (8) is an important relationship, with a number of observational consequences. If N_ν is the number of neutrinos families (hereafter considered as three), then

$$\rho_\nu = \frac{7}{8} \frac{\pi^2}{15} T_\nu^4 N_\nu \left[1 + \frac{15}{\pi^4} \frac{8}{7} 34.70 (q-1) \right], \quad (9)$$

which, adding to ρ_γ obtained from (7) leads to,

$$\rho_{rad} = \frac{\pi^2}{15} T_\gamma^4 [1 + 0.681 + 10.33(q-1)] \quad (10)$$

for the total radiation density. The first term in the bracket is the usual contribution of photons, the second stands for the usual contribution of neutrinos while the third sums up all non-extensive corrections.

The Friedmann equation for a zero curvature Friedmann–Robertson–Walker universe is

$$\left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho, \quad (11)$$

where a is the cosmological scale factor and ρ is the energy density, which has contributions of non-relativistic matter and radiation. If these fluids are non-interacting, the conservation equation $T^{\mu\nu}_{;\nu} = 0$ holds for them separately and one gets,

$$\rho_m = \rho_{m,0} \left(\frac{a_0}{a} \right)^3, \quad \rho_{rad} = \rho_{r,0} \left(\frac{a_0}{a} \right)^4. \quad (12)$$

Here, $\rho_{m,0}$ stands for the present matter density (which is fixed if we assume spatial flatness) and $\rho_{rad,0}$ for the analogous radiation density. The redshift in the matter-radiation equality is given by,

$$1 + z_{eq} = \frac{a_0}{a_{eq}} = \frac{\rho_{m,0}}{\rho_{rad,0}}, \quad (13)$$

Then,

$$1 + z_{eq} = \frac{3H_0^2}{8\pi G} \frac{1}{\rho_{rad,0}} = 24000 h^2 \left[1 - \frac{10.33}{1.68} (q-1) \right], \quad (14)$$

where H_0 is the present value of the Hubble constant and h is the Hubble constant in units of $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$. Without using the full solution for the matter-radiation era, existing here because of the validity of General Relativity, it is possible to estimate the Hubble radius at equality by assuming that the matter solution holds all the way backwards in time. At equality,

$$\frac{a_{eq} H_{eq}}{a_0 H_0} = \sqrt{2} \sqrt{1 + z_{eq}} = 219 h [1 + (1-q)3.08], \quad (15)$$

where it was used a matter dominated solution for the scale factor in a flat Friedmann-Robertson-Walker universe, $a(t) \propto t^{2/3}$, equivalently, the dependence of ρ_m given in (12). The $\sqrt{2}$ factor appears because at equality, the right hand side of equation (11) has two equal energy densities. Again, the first term in the bracket is a complete standard result while the second one is the correction due to the change in the statistical model.

C. Galaxy surveys and parameter estimation

Now the analysis proceeds in much the same way as was done in Ref. [21] for Brans-Dicke gravitation, which can be seen for details. The shift in the matter-radiation equality will lead to a shift in the maximum of the power spectrum of horizontal nature. Such a shift was considered by Tegmark [16] who introduced a phenomenological parameter η and considered how accurately it can be measured by SDSS. Tegmark gives two expected estimates on $\Delta\eta/\eta$, (see panel 3 and 4 of Fig. 1 of Ref. [16]). The first one corresponds to the case in which all parameters are considered fixed –or measured by other experiments– except η . The second one, to the case in which all parameters are to be extracted from SDSS alone. For the scale currently going non-linear ($k \sim 0.1h\text{Mpc}^{-1}$), $\Delta\eta/\eta \sim 0.02$ and $\Delta\eta/\eta \sim 0.1$ respectively [16]. The accuracy of the measurements on $\Delta\eta/\eta$ depends on the scale considered. The scale taken here –and also in [21]– avoid one to worry about non-linearity or bias effects. These estimations can even be improved considering data of other satellites, like MAP or Planck, as explained in [16].

In the normal General Relativity situation, the focus is pointed towards the determination of $\Omega_m h$, where Ω_m is the matter density parameter. Changing $\Omega_m h$ yields a horizontal shift to the power spectrum. In our situation, using $\Delta\eta/\eta \sim 0.02$ and equation (15) we can read limits upon q to be of order 10^{-3} which is comparable to the nucleosynthesis bound obtained in [11,12] and slightly less restrictive than the more in depth analysis of [6] and the bounds given in [13] and [14]. However, it comes now from a different arena, being the properties of galaxy surveys and the neutrino-photon temperature relationship its main inputs.

D. Detectability of the cosmic neutrino background

In the standard Boltzmann-Gibbs statistics, the ratio of the energy density of neutrinos to that of photons is, as we have noted before,

$$\left(\frac{\rho_\nu}{\rho_\gamma}\right)_{BG} = \frac{7}{8} \left(\frac{4}{11}\right)^{4/3} N_\nu = 0.681. \quad (16)$$

However, recent studies have pointed out that the assumption that neutrinos decoupled completely before

$e^+ - e^-$ annihilation is not entirely correct [22,23]. If the neutrinos share in the heating somewhat, their energy density is larger than the above quoted value. This can be modeled as if the number of neutrinos families N_ν is effectively bigger than 3. Together with QED corrections to the energy density of e^\pm and γ 's [24], the increase in the effective number N_ν is,

$$\delta N_\nu = 0.04 \text{ to } 0.05. \quad (17)$$

This leads to an increase of $\sim 1\%$ in the neutrino energy density.

In the non-extensive context, we may think of formula (9) as an effective variation in N_ν , given by

$$\delta N_\nu = \frac{15}{\pi^4} \frac{8}{7} 34.70(q-1) = 6.10(q-1). \quad (18)$$

Then, if a direct detection of the cosmic neutrino background would be available, a straightforward comparison with (18) may also yield a bound upon $(q-1)$.

The precision detection of the neutrino energy density was considered by Lopez et al. [25]. They analyzed the possible consequences that a small increase in ρ_ν would have in the cosmic microwave background. New measurements planned for the forthcoming satellites MAP and Planck lead them to show that the sensitivity will be so great that the neutrino energy density will be detectable by itself. As in the work by Tegmark, estimates come in two different ways. If all other parameters (for instance, baryon density Ω_B , Hubble constant H_0 , slope of the primordial perturbations n , etc) are held fixed (as given by other experiments), the expected sensitivity in N_ν is given by Fig. 3 of Ref. [25]. Going up to multipole moment $l \sim 1000$ and using only temperature anisotropy data, the expected sensitivity will be ~ 0.01 [25]. If the standard result $N_\nu = 3$ is confirmed, one can then obtain –using (18)– a bound upon $(q-1)$ of order $\times 10^{-3}$. This could even be improved using polarization data in the analysis, as explained in Ref. [25].

E. Concluding remarks

We have presented the case for obtaining new cosmological bounds upon the degree of non-extensivity of the universe. These new bounds will come from accurate cosmological measurements of the microwave background and galaxy surveys. To allow for this bounds to be obtained, we have computed the shift in the epoch of matter-radiation equality due to a different statistical background. Afterwards, comparing with the expected measurements of $\Delta\eta/\eta$, we have shown how this can be used to constrain the possible non-extensive parameter. We have also analyzed the implications that a precision detection of the cosmic neutrino background may have concerning the fixing of the statistical description and we have shown that another bound may arise from there. The order of magnitude of these two bounds is expected

to be $|q-1| \leq 10^{-3}$ and can be compared with other previously obtained cosmological constraints [11,13]. However it is important to stress here that many phenomena can cause the same effect. For instance, in the galaxy survey oriented scheme, a change in Ω_m or h , a different number of massless species or a change in the theory of gravity to a scalar-tensor one have all the same output on $a_{eq}H_{eq}$. In the case of the correction to the effective number of neutrino species N_ν , it may well be the case that the 1% correction arising from the extra heating and QED effects be hidden by corrections arising from a value of q different from 1. To determine this, the former corrections need to be computed directly in the non-extensive framework, without assuming Boltzmann-Gibbs statistics as an initial hypothesis. It is disturbing to note that whereas there is only one way of being *standard* there are many different and arbitrary ways to deviate from that situation. Even more disturbing is to see that different deviations may produce the same observational output. Despite of these caveats, observational facts like the one discussed here, as well as those concerning nucleosynthesis, are in the way of determining if the usual statistics is the right description of the universe. Although it of course seems a rather safe assumption, and in particular values of $|q-1| > 10^{-3}$ are completely discarded, there is certainly the need for more work to get a definite answer.

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- [6] D. F. Torres and H. Vucetich, to appear in *Physica A*, (1998); *astro-ph 9807043*.
- [7] A. R. Plastino and A. Plastino, *Phys. Lett.* **A174**, 384 (1993).
- [8] A. R. Plastino and A. Plastino, *Phys. Lett.* **A193**, 251 (1994).
- [9] P. A. Alemany and D. H. Zanette, *Phys. Rev.* **E49**, 956 (1994).
- [10] G. Kaniadakis, A. Lavagno and P. Quarati, *Phys. Lett.* **B369**, 308 (1996).
- [11] D. F. Torres, H. Vucetich and A. Plastino, *Phys. Rev. Lett.* **79**, 1588 (1997); **E80**, 3889 (1998).
- [12] U. Tirnakli and D. F. Torres, *Quantal distribution functions in non-extensive statistics and an early universe test revisited*, Ege University preprint (1998).
- [13] C. Tsallis, F. Sa Barreto and E. D. Loh, *Phys. Rev.* **E52**, 1447 (1995).
- [14] A. R. Plastino, A. Plastino and H. Vucetich, *Phys. Lett.* **A207**, 42 (1995).
- [15] P. Coles and F. Lucchin, *Cosmology* (John Wiley & Sons, 1995).
- [16] M. Tegmark, *Phys. Rev. Lett.* **79**, 3806 (1997).
- [17] M. Strauss, *astro-ph/9610033*.
- [18] G. Jungman et al., *Phys. Rev. Lett.* **76**, 1007 (1996).
- [19] D. M. Goldberg and M. A. Strauss, *astro-ph/9707209*; M. Tegmark, A. Hamilton, M. Strauss, M. voegeley and A. Szalay, *astro-ph/9708020*.
- [20] D. J. Fixsen et al., *Astrophys. J.* **473**, 576 (1996).
- [21] A. R. Liddle, A. Mazumdar and J. D. Barrow, *astro-ph/9802133*.
- [22] D. Dicus et al., *Phys. Rev.* **D26**, 2694 (1982).
- [23] N. Y. Gnedin and O. Y. Gnedin, *astro-ph/9712199*.
- [24] A. F. Heckler, *Phys. Rev.* **D49**, 611 (1994).
- [25] R. E. Lopez et al., *astro-ph/9803095*.

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- [1] D. Pavón, *Gen. Rel. Grav.* **19**, 375 (1987); P. T. Lansberg, *J. Stat. Phys.* **35**, 159 (1984); J. Binney and S. Tremaine, *Galactic Dynamics* (Princeton University Press, Cambridge, 1985), p.267; P. A. Alemany, *Phys. Lett.* **A235**, 452 (1997); V. H. Hamity and D. E. Baraco, *Phys. Rev. Lett.* **76**, 25 (1996).
 - [2] C. Tsallis, *Physica A* **221**, 227 (1995); *Fractals* **6**, 539 (1995).
 - [3] C. Tsallis, *J. Stat. Phys.* **52**, 479 (1988).
 - [4] E. M. F. Curado and C. Tsallis, *J. Phys.* **A24**, L69 (1991); Corrigenda: **A24**, 3187 (1991) and **A25**, 1019 (1992).
 - [5] U. Tirnakli, F. Büyükkılıç and D. Demirhan, *Physica A* **240**, 657 (1997); F. Büyükkılıç and D. Demirhan, *Phys. Lett.* **A181**, 24 (1993); F. Büyükkılıç, D. Demirhan and A. Gulec, *Phys. Lett.* **A197**, 209 (1995).