Supersymmetry and Bosonization in three dimensions

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Abstract

We discuss on the possible existence of a supersymmetric invariance in purely fermionic planar systems and its relation to the fermion-boson mapping in three-dimensional quantum field theory. We consider, as a very simple example, the bosonization of free massive fermions and show that, under certain conditions on the masses, this model displays a supersymmetric-like invariance in the low energy regime. We construct the purely fermionic expression for the supercurrent and the non-linear supersymmetry transformation laws. We argue that the supersymmetry is absent in the limit of massless fermions where the bosonized theory is non-local.

It is a well-known fact that two-dimensional models admit non-linear realizations of the supersymmetry algebra that involve only fermions [1]. Indeed, in two dimensions one can hope to represent the full supersymmetry algebra solely in terms of either bosonic or fermionic fields by means of bosonization (fermionization) [2, 3]. This property of two-dimensional purely fermionic models was originally analyzed by Witten [4] and then investigated in detail by Aratyn and Damgaard [5]. Several interesting fermionic models were then

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investigated and shown to provide a non-linear realization of supersymmetry [6, 7].

In order to extend this result to higher dimensional models, one should be provided with a bosonization recipe valid for $d > 2$. It was not until very recently that a fermion-boson mapping has been established in $d = 3$ dimensions [8, 9, 10, 11]. The resulting bosonization rules for fermionic currents, either for free or interacting models, result to be the natural extension of the two-dimensional ones [10, 11]:

$$j_\mu = -i\bar{\Psi}\gamma_\mu\Psi \to \frac{1}{\sqrt{4\pi}}\epsilon_{\mu\nu\alpha}\partial_\nu A_\alpha = \frac{1}{\sqrt{4\pi}}\mathcal{H}_\mu \ , \tag{1}$$

where Ψ is a Dirac fermion, A_α is a vector field and $\mathcal{H}_\mu$ is the corresponding dual field strength. It is understood that we are working in three-dimensional euclidean space-time. The exact bosonic Lagrangian equivalent to a given fermionic one, however, cannot be obtained unless some approximations are taken into account.

In the present work we plan to address the problem of bosonization (fermionization) of three-dimensional supersymmetric models. The study of supersymmetric planar systems has recently shown to be of interest in the domain of cosmology [12]. It could also be relevant to the supermembrane theory, whose world volume action should be a three-dimensional supersymmetric field theory.

We consider in this letter, as a very simple toy model, the case of free massive fermions, as it displays all the interesting novel features of our construction: a non-linear realization of supersymmetry in terms of fermionic variables at low energy, which is broken in the limit of massless fermions. We will report separately on the uses of our technique to treat more involved interacting systems [13].

We start from a purely fermionic Abelian model consisting of two non-interacting massive Dirac and Majorana fermions. We carry out the bosonization of the Dirac fermion using path-integral methods, arriving to an effective theory of a Majorana spinor and a vector field whose kinetic action consists of a couple of highly involved functions of the Laplacian. We show that the resulting lagrangian displays a supersymmetric invariance in the low energy regime, provided the original fermionic masses satisfy a definite ratio. We compute the purely fermionic supersymmetry transformation laws taking advantage of the exact bosonization recipe for currents. When massless fermions are considered, instead, the resulting bosonized model is not in principle supersymmetric. Finally, we add external sources to our system and find a very simple relation between correlation functions of currents computed in the purely fermionic system and those calculated in the corresponding supersymmetric model.

Let us begin by considering a simple purely fermionic model in three-dimensional Euclidean space-time described by the following Lagrangian density:

$$\mathcal{L}_f = \frac{1}{2}\bar{\Psi}(\partial\!\!\!/ + m_\Psi)\Psi + \frac{1}{2}\bar{\lambda}(\partial\!\!\!/ + m_\lambda)\lambda \ , \tag{2}$$

where Ψ is a massive Dirac spinor and λ is a massive Majorana fermion. The corresponding partition function

$$Z_f = \int \mathcal{D}\bar{\Psi}\mathcal{D}\Psi\mathcal{D}\bar{\lambda}\mathcal{D}\lambda \exp\left[-\frac{1}{2}\int d^3x \left(\bar{\Psi}(\partial\!\!\!/ + m_\Psi)\Psi + \bar{\lambda}(\partial\!\!\!/ + m_\lambda)\lambda\right)\right] \ , \tag{3}$$

can be trivially factored as $Z_f = Z_\Psi Z_\lambda$, with

$$Z_\Psi = \int \mathcal{D}\bar{\Psi}\mathcal{D}\Psi \exp\left[-\frac{1}{2}\int d^3x\bar{\Psi}(\partial\!\!\!/ + m_\Psi)\Psi\right] \ , \tag{4}$$

$$Z_\lambda = \int \mathcal{D}\bar{\lambda}\mathcal{D}\lambda \exp\left[-\frac{1}{2}\int d^3x\bar{\lambda}(\partial\!\!\!/ + m_\lambda)\lambda\right] \ . \tag{5}$$

Let us perform the change on Dirac fermionic variables $\Psi = g(x)\Psi'$ and $\bar{\Psi} = \bar{\Psi}'g^{-1}(x)$, with $g(x) = \exp[i\theta(x)]$ an element of U(1). One can always define for Dirac fermions a path-integral measure invariant under such transformations. After the change of variables, the partition function Z_Ψ becomes

$$Z_\Psi = \int \mathcal{D}\bar{\Psi}\mathcal{D}\Psi \exp\left[-\frac{1}{2}\int d^3x\bar{\Psi}(\partial\!\!\!/ + i\,\partial\!\!\!/\theta + m_\Psi)\Psi\right] \tag{6}$$

(we have omitted primes in the new fermionic variables). In this formula, $\partial_\mu\theta$ can be simply seen as a flat connection. Thus, it can be replaced by a gauge field connection b_μ provided a flatness condition is imposed,

$$b_\mu = \partial_\mu\theta \quad \text{with} \quad \bar{\mathcal{F}}_\mu = \epsilon_{\mu\nu\gamma}\partial_\nu b_\gamma = 0 \ . \tag{7}$$

Now, following the treatment of Ref.[11], the partition function (6) becomes

$$Z_\Psi = \int \mathcal{D}A_\mu\mathcal{D}b_\mu det(D\!\!\!\!/\,[b] + m_\Psi) \exp\left[i\int d^3x A_\mu\bar{\mathcal{F}}_\mu\right] \ ,$$

where $D_\mu[b] = \partial_\mu + ib_\mu$ is the covariant derivative. It is the Lagrange multiplier A_μ that would become the bosonic equivalent of the original Dirac fermion Ψ. Of course, one has still to perform the b_μ integration to obtain explicitely the dynamics of this vector field. To this end, we should first compute the fermion determinant in three dimensions as exactly as we can. In the limit of very massive fermions, it is a well-known result that the fermion determinant leads to an effective action given by a Chern-Simons term [14, 15]. More recently, this calculation has been done for any value of the fermion mass by making a quadratic expansion in powers of the b_μ-field, with the following result [16, 17, 18]:

$$
-\log\det(\not{D}[b] + m_\Psi) \cong \frac{1}{4}\int d^3x \left(b_{\mu\nu}F(-\partial^2)b_{\mu\nu} + 2ib_\mu G(-\partial^2)\bar{\mathcal{F}}_\mu\right) \ , \qquad (8)
$$

where $b_{\mu\nu} = \partial_\mu b_\nu - \partial_\nu b_\mu$. Both contributions in Eq.(8) can be traced to come from the parity-conserving and parity-violating pieces of the vacuum-polarization tensor [15]. Functions $F(-\partial^2)$ and $G(-\partial^2)$ can be given through their momentum-space representations $\tilde{F}$ and $\tilde{G}$ as follows [16, 17, 18, 19]:

$$
\frac{4\pi k^2}{|m_\Psi|}\tilde{F}(k) = 1 - \frac{1 - \dfrac{k^2}{4m_\Psi^2}}{\left(\dfrac{k^2}{4m_\Psi^2}\right)^{1/2}} \arcsin\left(1 + \frac{4m_\Psi^2}{k^2}\right)^{-1/2} \qquad (9)
$$

which results to be regularization independent, and

$$
\tilde{G}(k) = \frac{q}{4\pi} + \frac{m_\Psi}{2\pi \mid k \mid} \arcsin\left(1 + \frac{4m_\Psi^2}{k^2}\right)^{-1/2} , \qquad (10)
$$

which is regularization dependent. Indeed, the parameter q can assume any integer value [20, 21].

The quadratic expansion of the fermion determinant must be taken with care. There is no small parameter in the theory that allows a sensible perturbative expansion of the path-integral. Therefore, generically, it has no physical sense to cut the whole series of b_μ-insertions into the fermion loop leading to the value of the determinant. In spite of this fact, it is important to mention that there are some cases in which this difficulty can be overcomed, one of them being the limit of very heavy fermions. If one is to analyse the low energy regime of the theory, imposed by letting $m_\Psi \to \infty$, one can take profit of the well-known fact [9] that the series expansion of the fermion determinant can be accomodated as an expansion in powers of the inverse fermion mass. The truncation of the late series is well-defined in the low-energy limit. Indeed, it leads to a local effective lagrangian dominated by those terms that have the lowest scaling dimension [9], which are precisely those appearing in (8), when evaluated in the appropriate limit.

In order to illustrate this argument, one can compute the leading (parity conserving) contribution in m_Ψ^{-1} of the fermion loop with four b_μ-insertions, which result to be proportional to $m_\Psi^{-5}(F_{\mu\nu}^2)^2$. This result can be also obtained following the derivative expansion approach [22], as was recently analysed in [23]. Taking into account these remarks, one must keep in mind in what follows that sensible results are those emerging in the low energy limit. It is worthwhile to mention, however, that there is another situation in which the quadratic approximation of the fermion determinant is physically relevant for any value of the mass (including the limit $m_\Psi \to 0$). This is the calculation of the current algebra first done in Ref.[19], for which the following terms in the expansion are irrelevant. In fact, the quadratic approximation was explicitely shown to be enough to reproduce the result of the current commutators that corresponds to that of the dual fermion theory. We will come back to this sort of discussion below.

Using the quadratic approximation to compute the fermionic determinant (8), it is possible to perform the exact integration of the b_μ field [18], the resulting partition function being:

$$Z_f = \int \mathcal{D}A_\mu \exp\left[-\int d^3x \left(\frac{1}{4}F_{\mu\nu}C_1(-\partial^2)F_{\mu\nu} - \frac{i}{2}A_\mu C_2(-\partial^2)\mathcal{H}_\mu\right)\right] Z_\lambda \ , \tag{11}$$

with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. Concerning C_1 and C_2, they are given through their momentum-space representations $\tilde{C}_1$ and $\tilde{C}_2$ as follows,

$$\tilde{C}_1(k) = \frac{|u|^2\tilde{F}}{k^2\tilde{F}^2 + \tilde{G}^2} \quad , \quad \tilde{C}_2(k) = \frac{|u|^2\tilde{G}}{k^2\tilde{F}^2 + \tilde{G}^2} \ , \tag{12}$$

where u is an arbitrary function of the momentum, arising in the diagonalization procedure. Current correlation functions are u-independent [19].

Had we added in the partition function (3) an external source s_μ covariantly coupled to the fermionic current

$$Z_f[s] = \int \mathcal{D}\bar{\Psi}\mathcal{D}\Psi\mathcal{D}\bar{\lambda}\mathcal{D}\lambda \exp\left[-\frac{1}{2}\int d^3x \left(\bar{\Psi}(\not{D}[s] + m_\Psi)\Psi + \bar{\lambda}(\not{\partial} + m_\lambda)\lambda\right)\right] , \tag{13}$$

we would obtain –after the same procedure is applied–, the bosonized expression:

$$\begin{aligned} Z_f[s] &= \int \mathcal{D}A_\mu \exp\left[-\int d^3x \left(\frac{1}{4}F_{\mu\nu}C_1(-\partial^2)F_{\mu\nu} - \frac{i}{2}A_\mu C_2(-\partial^2)\mathcal{H}_\mu\right.\right. \\ &+ \left.\left. ius_\mu\mathcal{H}_\mu\right)\right] Z_\lambda \ . \end{aligned} \tag{14}$$

Note that, choosing the value $u = i/4\sqrt{\pi}$, the preceding equation makes evident the exactness of the bosonization recipe for currents (1) in correspondence with our previous discussions [19].

If we let the mass m_Ψ go to infinity, $\tilde{F}(k)$ and $\tilde{G}(k)$ take simpler expressions such that the leading order contributions to $\tilde{C}_1$ and $\tilde{C}_2$ are

$$\tilde{C}_1(k) \to \frac{1}{3\mid m_\Psi \mid} \quad \text{and} \quad \tilde{C}_2(k) \to -1 \ , \tag{15}$$

and the partition function then becomes

$$\begin{aligned} Z_{m_\Psi\to\infty} &= \int \mathcal{D}\bar{\lambda}\mathcal{D}\lambda\mathcal{D}A_\mu \exp\left[-\int d^3x \left(\frac{1}{12|m_\Psi|}F^2_{\mu\nu} + \frac{i}{2}A_\mu\mathcal{H}_\mu\right.\right. \\ &+ \left.\left. \frac{1}{2}\bar{\lambda}(\not{\partial} + m_\lambda)\lambda\right)\right] \ . \end{aligned} \tag{16}$$

After a rescaling of the bosonized field A_μ,

$$A_\mu = \sqrt{3m_\Psi}A'_\mu \ , \tag{17}$$

the partition function (16) simply reads

$$\begin{aligned} Z_{m_\Psi\to\infty} &= \int \mathcal{D}\bar{\lambda}\mathcal{D}\lambda\mathcal{D}A_\mu \exp\left[-\int d^3x \left(\frac{1}{4}F'_{\mu\nu}F'_{\mu\nu} + \frac{3im_\Psi}{4}\epsilon_{\mu\nu\gamma}F'_{\mu\nu}A'_\gamma\right.\right. \\ &+ \left.\left. \frac{1}{2}\bar{\lambda}(\not{\partial} + m_\lambda)\lambda\right)\right] \ , \end{aligned} \tag{18}$$

whose bosonic sector is given by the Maxwell-Chern-Simons theory [15]. The partition function (18) describes a free vector boson with a gauge invariant mass which is three times bigger than the mass of the Dirac fermion that was at its origin, $m_{bos} = 3m_\Psi$, and a free massive Majorana fermion with $m_{fer} = m_\lambda$. It would be possible to place these fields together into the same representation of $N = 1$ supersymmetry provided their mass is equal. In fact, choosing

$$m_\lambda = 3m_\Psi \ , \tag{19}$$

the partition function (18) is invariant under the supersymmetry transformations

$$\delta A'_\mu = \bar{\epsilon}\gamma_\mu\lambda \quad , \quad \delta\lambda = \frac{i}{2}\epsilon_{\mu\nu\gamma}\gamma_\mu F'_{\nu\gamma}\epsilon \ , \tag{20}$$

with ϵ a real spinor parameter. As this is a free theory the invariance of the action does not, by itself, establish supersymmetry. It is easy to check that the commutator of two transformations (20) produces a traslation on the fields. Unfortunately this does not say much about the possibility of obtaining the same kind of results once interactions are included: determining non-trivial aspects of this Supersymmetry certainly requires thorough investigations on the ground of interacting theories. In this respect, it is worth to mention that some very simple interacting systems can be handled whithin this approach arriving to analogous results [13].

We have shown, using the path-integral bosonization framework, the equivalence at low energy between the purely fermionic model and the Maxwell-Chern-Simons supersymmetric one, provided the fermion masses satisfy a definite ratio. To our knowledge, this kind of equivalence has not been studied before in planar systems neither in higher dimensional models. It is worthwhile to note that this limiting case could also be considered when an external source is coupled to the fermionic current. Consequently, from eq.(13), the correlation function of these currents in the low energy limit, after bosonization, are simply given by

$$< j_\mu(x) j_\nu(y) >= \frac{e^2}{4\pi} < \mathcal{H}_\mu(x)\mathcal{H}_\nu(y) >_{SUSY} \ , \tag{21}$$

where the subscript $SUSY$ means that the correlation function at the r.h.s. of (21) is computed in the supersymmetric theory with the partition function given in (18). Thus, the purely fermionic free correlator can be calculated as the correlation function of the dual electromagnetic field strength in the $N = 1$ supersymmetric Maxwell-Chern-Simons theory.

In order to further study the supersymmetry of the purely fermionic model, let us compute the Noether supercurrent corresponding to the Lagrangian in eq.(18). It is given by the following expression

$$\mathcal{J}_\mu = i\mathcal{H}'_\nu \gamma_\nu \gamma_\mu \lambda \ , \tag{22}$$

whose vacuum expectation value can be written by adding sources as

$$< \mathcal{J}_\mu^\gamma >= -i \left[\frac{16\pi}{3m_\Psi}\right]^{1/2} (\gamma_\nu \gamma_\mu)^\gamma{}_\beta \frac{\delta^2 \log Z[\eta, \bar{\eta}, s]}{\delta s_\nu \delta \bar{\eta}_\beta}|_{\eta,\bar{\eta},s=0} \ , \tag{23}$$

where $Z[\eta, \bar{\eta}, s]$ is the partition function (18) coupled to external sources given by

$$\begin{aligned} Z[\eta, \bar{\eta}, s] &= \int \mathcal{D}\bar{\lambda}\mathcal{D}\lambda\mathcal{D}A_\mu \exp\left[-\int d^3x \left(\frac{1}{4}F'_{\mu\nu}F'_{\mu\nu} + \frac{3im_\Psi}{4}\epsilon_{\mu\nu\gamma}F'_{\mu\nu}A'_\gamma\right.\right. \\ &- \left.\left.\left(\frac{3m_\Psi}{16\pi}\right)^{1/2} s_\mu \mathcal{H}'_\mu + \frac{1}{2}\bar{\lambda}(\not{\partial} + m_\lambda)\lambda + \bar{\eta}\lambda + \bar{\lambda}\eta\right)\right] \ . \end{aligned} \tag{24}$$

We can fermionize back the low energy action (24), following the procedure we have just discussed, to end with

$$\begin{aligned} Z[\eta, \bar{\eta}, s] &= \int \mathcal{D}\bar{\Psi}\mathcal{D}\Psi\mathcal{D}\bar{\lambda}\mathcal{D}\lambda \exp\left[-\frac{1}{2}\int d^3x \left(\bar{\Psi}(\not{D}[s] + m_\Psi)\Psi\right.\right. \\ &+ \left.\left.\bar{\lambda}(\not{\partial} + m_\lambda)\lambda + \bar{\eta}\lambda + \bar{\lambda}\eta\right)\right] \ , \end{aligned} \tag{25}$$

this leading to the purely fermionic expression for the vacuum expectation value of the supercurrent

$$< \mathcal{J}_\mu^\gamma >= \alpha^{-1/2} < \bar{\Psi}\gamma_\nu \Psi(\gamma_\nu\gamma_\mu\lambda)^\gamma > \ , \tag{26}$$

with $\alpha = 3m_\Psi/4\pi$. The expression (26) gives, in the low energy limit, the vacuum expectation value of the supercurrent that corresponds to the supersymmetric invariance of the purely fermionic Lagrangian (2) subjected to the condition (19).

If eq. (26) were valid at the operator level we would easily be able to show that it is a divergenceless current. However, being the current a composite operator, one must be very careful with potential Schwinger-like terms coming from the time ordered products. Using the previous expression for the current, and its $\mu = 0$ component, the supercharge, one can obtain the following fermionic infinitesimal transformation laws:

$$\delta_\epsilon \lambda = \alpha^{-1/2} \bar{\Psi} \gamma_\nu \Psi \gamma_\nu \epsilon \ , \tag{27}$$

$$\delta_\epsilon \Psi = \alpha^{-1/2} (\bar{\lambda} \epsilon \Psi + \bar{\lambda} \gamma_\mu \epsilon \gamma_\mu \Psi) \ . \tag{28}$$

Transformations (27) and (28) provide a non-linear realization of the supersymmetric invariance that is present in the low energy regime of the purely fermionic model given in eq.(2). It is inmediate to see that the rule (27), is nothing but the straight fermionization of the photino transformation law given in Eq.(20). Concerning Eq.(28), it is obtained by acting with the supercharge constructed from (22) over the Dirac spinor Ψ, not as in $1+1$-dimensional models where it is interestingly computed from the chiral anomaly [5] (a procedure which is, of course, impossible in our case).

The 'duality' between the purely fermionic model (2) and the $N = 1$ supersymmetric Maxwell-Chern-Simons one (18) –after condition (19) is imposed–, was established in the path-integral framework by considering the low energy limit of the fermionic determinant. It is interesting to explore whether this kind of correspondence is still present in the high energy regime (thought of as the limit of massless fermions, $m_\Psi \to 0$). As we extensively discussed before, this limit is somehow problematic in the sense that the quadratic approximation to the fermionic determinant could have no sense. However, we will briefly analyse this case, taking into account that the resulting bosonized action of massless fermions obtained by this method –in the quadratic approximation– was shown to give a sensible physical answer [24]. Also, the quadratic approximation that we followed has been shown to be enough in order to reproduce the current algebra of the fermionic system in terms of bosonic variables [19]. Let us then consider in what follows the case where m_Ψ is almost vanishing. In this case we have the following behaviour for $\tilde{F}$ and $\tilde{G}$:

$$\tilde{F}(k) \to \frac{1}{16 \mid k \mid} \quad \text{and} \quad \tilde{G}(k) \to \frac{q}{4\pi} \equiv \frac{1}{16} \cot \varphi \ . \tag{29}$$

Inserting these expressions into Eqs.(11) and (12), we are led to:

$$\begin{aligned} Z_{m_\Psi \to 0} &= \int \mathcal{D}A_\mu \exp\Big[-\frac{\sin^2\varphi}{\pi} \int \Big(F_{\mu\nu} \frac{1}{\sqrt{-\partial^2}} F_{\mu\nu} \\ &- 2i \cot\varphi \, \epsilon_{\mu\nu\gamma} A_\mu F_{\nu\gamma} \big) \, d^3x \Big] \, Z_\lambda \ , \end{aligned} \tag{30}$$

a non-local theory which is by no means supersymmetric. The fermionized supersymmetric invariance of the free massive fermion system seems to be lost at high energies. It is worth to comment at this point that non-local terms as that of Eq.(30) can be traced up as coming from a four-dimensional model, after a dimensional reduction procedure is applied [24, 25].

Let us end this letter by stressing its main results. We have presented a very simple purely fermionic model and showed that it is supersymmetric in the low energy regime, provided the fermion masses satisfy a definite ratio. We were able to construct the non-linear supersymmetry transformations corresponding to this invariance in terms of purely fermionic variables. In the limit of massless fermions, instead, the supersymmetry seems to be absent and the bosonized theory results to be non-local. It would be of interest to study the way in which the transition between both phases of the theory occurs.

In order to clarify our discussion we have considered in this letter a free fermionic model. It is however possible to add certain interaction terms without spoiling the basic conclusions to which we have arrived here [13]. The possible generalization of these results to higher-dimensional systems deserve further investigations. We hope to report on these issues elsewhere.

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References

[1] P. van Nieuwenhuizen, in *Supersymmetry, Supergravity and Superstrings*, Eds. B. de Wit (World Scientific, Singapore, 1986), and references therein.

[2] S. Coleman, Phys. Rev. **D11**, 2088 (1975).

[3] S. Mandelstam, Phys. Rev. **D11**, 3026 (1975).

[4] E. Witten, Nucl. Phys. **B142**, 285 (1978).

[5] H. Aratyn and P.H. Damgaard, Nucl. Phys. **B241**, 253 (1984).

[6] P. di Vecchia, V.G. Knizhnik, J.L. Petersen and P. Rossi, Nucl. Phys. **B252**, 701 (1985).

[7] F.A. Schaposnik and M.L. Trobo, Phys. Rev. **D37**, 1663 (1988) and references therein.

[8] C.P. Burgess and F. Quevedo, Nucl. Phys. **B421**, 373 (1994); Phys. Lett. **B329**, 457 (1994).

[9] C.P. Burgess, C.A. Lütken and F. Quevedo, Phys. Lett. **B336**, 18 (1994).

[10] E. Fradkin and F.A. Schaposnik, Phys. Lett. **B338**, 253 (1994).

[11] F.A. Schaposnik, Phys. Lett. **B356**, 39 (1995).

[12] See, for example, E. Witten, Int. J. Mod. Phys. **A10**, 1247 (1995); K. Becker, M. Becker and A. Strominger, Phys. Rev. **D51**, 6603 (1995); J.D. Edelstein, C. Núñez and F.A. Schaposnik, Phys. Lett. **B375**, 163 (1996); J.D. Edelstein, Phys. Lett. **B390**, 101 (1997).

[13] J.D. Edelstein and C. Núñez, in preparation.

[14] See, for example, A.N. Redlich, Phys. Rev. Lett. **52**, 18 (1984); Phys. Rev. **D29**, 236 (1984); R.D. Pisarski and S. Rao, Phys. Rev. **D32**, 2081 (1985) and references therein.

[15] S. Deser, R. Jackiw and S. Templeton, Phys. Rev. Lett. **48**, 975 (1982); Ann. of Phys. (N.Y.) **140**, 372 (1982).

[16] K.D. Rothe, Phys. Rev. **D48**, 1871 (1993).

[17] I.J.R. Aitchison, C.D. Fosco and J.A. Zuk, Phys. Rev. **D48**, 5895 (1993).

[18] D.G. Barci, C.D. Fosco and L.E. Oxman, Phys. Lett. **B375**, 267 (1996).

[19] J.C. Le Guillou, C. Núñez and F.A. Schaposnik, Ann. Phys. (NY) **251**, 426 (1996).

[20] J. Fröhlich and T. Kerler, Nucl. Phys. **B354**, 369 (1991).

[21] R.E. Gamboa Saravi, G.L. Rossini and F.A. Schaposnik, Int. J. Mod. Phys. **A11**, 2643 (1996).

[22] I.J.R. Aitchison and C.M. Fraser, Phys. Rev. **D31** (1985) 2605; B.L. Hu and D.J. O'Connor, Phys. Rev. **D30** (1984) 743.

[23] I.J.R. Aitchison, C.D. Fosco and F.D. Mazzitelli, Phys. Rev. **D54** (1996) 4059.

[24] E.C. Marino, Phys. Lett. **B263**, 63 (1991); Nucl. Phys. **B408**, 551 (1993).

[25] J.D. Edelstein, J.J. Giambiagi, C. Núñez and F.A. Schaposnik, Mod. Phys. Lett. **A11**, 1037 (1996).