

# Monopole and Dyon Solutions in AdS space

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## Abstract

We consider monopole and dyon classical solutions of the Yang-Mills-Higgs system coupled to gravity in asymptotically anti-de Sitter space. We discuss both singular and regular solutions to the second order equations of motion showing that singular Wu-Yang like dyons can be found, the resulting metric being of the Reissner-Nördstrom type (with cosmological constant). Concerning regular solutions, we analyze the conditions under which they can be constructed discussing, for vanishing coupling constant, the main distinctive features related to the anti-de Sitter asymptotic condition; in particular, we find in this case that the v.e.v. of the Higgs scalar,  $|\vec{H}(\infty)|$ , should be quantized in units of the natural mass scale  $1/er_0$  (related to the cosmological constant) according to  $|\vec{H}(\infty)|^2 = m(m+1)(er_0)^{-2}$ , with  $m \in Z$ .

## 1 Introduction

Soon after the discovery of the magnetic monopole and dyon solutions [1]-[4] in spontaneously broken  $SU(2)$  gauge theory with Higgs fields in the adjoint representation, different kinds of self-gravitating monopoles were discussed [5]-[7]. Subsequently, the gravitational properties of these solutions and the relation between monopoles and black holes were thoroughly investigated [8]-[11]; more recently, numerical studies clarifying gravitational instabilities and other peculiar features of the solutions have been presented [12]-[14] (see [15]-[16] for a more complete list of references). All these investigations correspond to asymptotically flat spaces. Less is known when space time is modified to include a cosmological constant  $\Lambda$ , in particular for  $\Lambda < 0$ , i.e. the case in which space

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is asymptotically anti-de Sitter space. Recently, black hole solutions [17] and soliton solutions [18] have been found in this case for the Einstein-Yang-Mills system (no Higgs field).

It is the purpose of this work to study monopole and dyon solutions for Yang-Mills-Higgs theory coupled to gravity in asymptotically anti-de Sitter (AdS) space.

In Section 2 we present the model, the spherically symmetric ansatz for the gauge field, the Higgs triplet and the metric, leading to radial equations of motion. Then, in Section 3 we present a dyonic AdS black hole solution (analogous to the one constructed in [5]-[6] in asymptotically flat space). Following different perturbative approaches, we analyse in Section 4 regular monopole solutions and discuss its main properties. Finally, we summarize our results in Section 5.

## 2 The model, the Ansatz and the equations of motion

We adopt the conventions in [19]-[20] for metric curvature, gauge fields and else

$$F \equiv dA + A \wedge A \quad (2.1)$$

where  $A$  is a one-form connection.

A Higgs field ( $H^i$ ) transforms in the representation  $R$  of the semisimple gauge group  $G$  which has Lie algebra  $\mathfrak{G}$  generated by  $X_a$  with commutation relations and metric (that rises and lows indices)

$$\begin{aligned} [X_a, X_b] &= f^c{}_{ab} X_c \\ \kappa_{ab} &\equiv \langle X_a, X_b \rangle \end{aligned} \quad (2.2)$$

$D_\mu$  stands for general covariant derivative and  $g_{ij}$  is a gauge invariant metric.

The action we consider is given by

$$\begin{aligned} S &= S_G + S_{YM} + S_H = \int d^D x \sqrt{|G|} (L_G + L_{YM} + L_H) \\ L_G &= \frac{1}{\alpha_0} \left( \frac{1}{2} R - \Lambda \right) \\ L_{YM} &= \frac{1}{4e^2} F_{\mu\nu}^a F_a^{\mu\nu} \\ L_H &= -\frac{1}{2} g_{ij} G^{\mu\nu} D_\mu H^i D_\nu H^j - V(H) \end{aligned} \quad (2.3)$$

where  $V(H)$  is the Higgs potential,  $\alpha_0 \equiv 8\pi G_D$  with  $G_D$  is the Newton constant in  $D$  dimensions (to be taken as  $D = 4$  in what follows),  $e$  the gauge coupling and  $\Lambda$  is the cosmological constant (one easily sees that with our conventions  $\Lambda < 0$  corresponds, in the absence of matter, to anti-de Sitter space).

The equations of motion that follow from (2.3) are

$$E_{\mu\nu} + \Lambda G_{\mu\nu} = \alpha_0 (T_{\mu\nu}^{YM} + T_{\mu\nu}^H)$$

$$\begin{aligned}
g_{ij} \tilde{D}_\rho D^\rho H^j &= \partial_i V(H) \\
\frac{1}{e^2} D_\rho F_a^{\mu\rho} &= g_{ij} D^\mu H^i R(X_a)^j{}_k H^k
\end{aligned} \tag{2.4}$$

where the matter energy-momentum tensor is

$$\begin{aligned}
T_{\mu\nu}^{YM} &= \frac{1}{e^2} (-F_{\mu\rho}^a F_{a\nu}{}^\rho + \frac{1}{2} G_{\mu\nu} F_{\rho\sigma}^a F_a^{\rho\sigma}) \\
T_{\mu\nu}^H &= g_{ij} D_\mu H^i D_\nu H^j + G_{\mu\nu} L_H
\end{aligned} \tag{2.5}$$

and  $\tilde{D}_\mu^i{}_j \equiv D_\mu \delta_j^i + \Gamma[g]_{jk}^i D_\mu H^k$  contains the Christoffel symbol of  $g_{ij}$ .

Let us take now for definiteness  $G = SU(2)$ ,  $g_{ij} = \delta_{ij}$  for the Higgs fields in the adjoint representation “3”, and a basis where  $f^c{}_{ab} = \epsilon_{abc}$  with  $\langle X_a, X_b \rangle \equiv \frac{1}{2} \text{tr}_3 X_a X_b = -\delta_{ab}$ . The most general static spherically symmetric form for the metric in 3 spatial dimensions together with the t’Hooft-Polyakov-Julia-Zee ansatz for the gauge and Higgs fields in the usual vector notation reads

$$\begin{aligned}
G &= -\mu(x) A(x)^2 dt^2 + \mu(x)^{-1} dr^2 + r^2 d^2\Omega_2 \\
\vec{A} &= dt e h_0 J(x) \check{e}_r - d\theta (1 - K(x)) \check{e}_\varphi + d\varphi (1 - K(x)) \sin\theta \check{e}_\theta \\
\vec{H} &= h_0 H(x) \check{e}_r
\end{aligned} \tag{2.6}$$

where we identify the constant  $h_0$  (assumed non zero) with the minimum of the potential

$$V(H) = \frac{\lambda}{4} (\vec{H}^2 - h_0^2)^2 \tag{2.7}$$

and we introduce the dimensionless coordinate  $x \equiv e h_0 r$ .

Using this ansatz, the equations of motion take the form

$$\begin{aligned}
(x \mu(x))' &= 1 + 3 \gamma_0 x^2 - \alpha_0 h_0^2 \left( \mu(x) V_1 + V_2 + \frac{x^2}{2} \frac{J'(x)^2}{A(x)^2} + \frac{J(x)^2 K(x)^2}{\mu(x) A(x)^2} \right) \\
x A'(x) &= \alpha_0 h_0^2 \left( V_1 + \frac{J(x)^2 K(x)^2}{\mu(x)^2 A(x)^2} \right) A(x)
\end{aligned} \tag{2.8}$$

$$\begin{aligned}
(\mu(x) A(x) K'(x))' &= A(x) K(x) \left( \frac{K(x)^2 - 1}{x^2} + H(x)^2 - \frac{J(x)^2}{\mu(x) A(x)^2} \right) \\
(x^2 \mu(x) A(x) H'(x))' &= A(x) H(x) \left( 2 K(x)^2 + \frac{\lambda}{e^2} x^2 (H(x)^2 - 1) \right) \\
\mu(x) \left( \frac{x^2 J'(x)}{A(x)} \right)' &= \frac{2 J(x) K(x)^2}{A(x)}
\end{aligned} \tag{2.9}$$

where for convenience we have defined the dimensionless parameter

$$\gamma_0 \equiv -\frac{\Lambda}{3e^2 h_0^2} \tag{2.10}$$

and

$$\begin{aligned}
V_1 &= K'(x)^2 + \frac{x^2}{2} H'(x)^2 \\
V_2 &= \frac{(K(x)^2 - 1)^2}{2 x^2} + \frac{\lambda}{4e^2} x^2 (H(x)^2 - 1)^2
\end{aligned} \tag{2.11}$$

## The boundary conditions

Ansatz (2.6) will lead to well behaved solutions for the matter fields if, at  $x = 0$ , one imposes

- $H(x)/x$  and  $J(x)/x$  are regular;
- $1 - K(x)$  and  $K'(x)$  go to zero.

On the other hand we want the system to go asymptotically to anti-de Sitter space which corresponds to the solution of the Einstein equations with  $\Lambda < 0$  in absence of matter (see next Section); for this to happen we must impose that the matter energy-momentum tensor vanishes at spatial infinity. From eq.(2.5) one can see that the appropriate conditions for  $x \rightarrow \infty$  are

$$\begin{aligned} K(x) &\rightarrow O(x^{-\alpha_1}) \\ H(x) &\rightarrow H_\infty + O(x^{-1-\alpha_2}) \\ J(x) &\rightarrow J_\infty + O(x^{-\alpha_3}) \end{aligned} \tag{2.12}$$

with  $\alpha_i > 0$ ,  $i = 1, 2, 3$ .

## 3 A dyonic AdS black hole in four dimensions

Before discussing regular solutions to equations of motion (2.9) satisfying the boundary conditions specified above, let us discuss a singular solution which has very interesting properties. Indeed, one can easily see that the restriction

$$\begin{aligned} K(x) &= 0 \\ H(x) &= 1 \end{aligned} \tag{3.1}$$

leads to a singular solution that exhibits a Dirac string starting at  $x = 0$  and so the Abelian Dirac monopole character of this solution, related to a Wu-Yang-like dyon [21] which, for asymptotically flat space was considered long ago [5]-[6]. The equations for the metric are trivially solved by (3.1)

$$\begin{aligned} A(x) &= 1 \\ \mu(x) &= 1 + \gamma_0 x^2 - \frac{a}{x} + \frac{\alpha_0 h_0^2 (1 + b^2)}{2 x^2} \end{aligned} \tag{3.2}$$

leading to a metric of the Reissner-Nordström type (with cosmological constant). The constant  $a$  in (3.2) is related to the mass of this AdS charged black hole.

Concerning the electric potential

$$J(x) = -\frac{b}{x} + J_\infty \tag{3.3}$$

With this, the electric ( $Q_e$ ) and magnetic ( $Q_m$ ) charges of the solution are respectively

$$Q_e \equiv -\frac{1}{4\pi} \int_{S^2} *F^r = b \quad (3.4)$$

$$Q_m \equiv -\frac{1}{4\pi} \int_{S^2} F^r = 1 \quad (3.5)$$

We then see that the parameter  $b$  determines the electric charge while  $J_\infty$  sets the scale for  $J$ . In fact, the metric function  $\mu(x)$  (eq. (3.2)) can be rewritten in the form

$$\mu(x) = 1 + \gamma_0 x^2 - \frac{a}{x} + \frac{\alpha_0 h_0^2 (Q_m^2 + Q_e^2)}{2 x^2} \quad (3.6)$$

making explicit the role of the (unit) magnetic and electric charges, the same as in the  $\Lambda = 0$  case [12]. At the radii for which the metric function vanishes, there will be in general event horizons, as it happens in asymptotically flat space. Now, the condition  $\mu(x) = 0$  leads here to a quartic algebraic equation for  $x$ ,

$$\gamma_0 x^4 + x^2 - ax + Z^2 = 0 \quad (3.7)$$

where we have defined

$$Z^2 = \sigma_0 (Q_m^2 + Q_e^2), \quad \sigma_0 = \frac{\alpha_0 h_0^2}{2} \quad (3.8)$$

The explicit expression for the roots of this equation is not very illuminating. One can see that for  $\gamma_0 \ll a^{-2}$  (or  $|\Lambda| \ll (1/a^2)e^2 h_0^2$ ) the horizons have, qualitatively, the same behavior as in the asymptotically flat case: there is a critical value  $Z_{crit}^2$  for  $Z^2$  such that there are two horizons for  $Z^2 < Z_{crit}^2$  and two complex conjugate roots for  $Z^2 > Z_{crit}^2$ . The value of  $Z_{crit}^2$  and the two horizons can be determined as a power expansion in  $a^2 \gamma_0$  [22]. Let us end this section by noting that, although the gauge field is singular at the origin, the solution can be considered regular as a black hole, so that the singularity could be hidden behind a horizon.

## 4 The system in AdS space

### Statement of the problem

In the  $\alpha_0 h_0^2 \rightarrow 0$  limit the gravitational equations decouple from the matter ones leading for the metric to the solution

$$\begin{aligned} A(x) &= 1 \\ \mu(x) &= 1 + \gamma_0 x^2 - \frac{a}{x} \end{aligned} \quad (4.1)$$

which is nothing but the vacuum solution of the Einstein equations with a cosmological constant (assumed negative), and corresponds to a neutral Schwarzschild black hole in AdS

space. Concerning the integration constant  $a$ , it is related to the mass of the black hole and will be put to zero in what follows. This metric, in turn, acts as a (AdS) background with radius  $r_0$ ,

$$r_0 = \sqrt{-3/\Lambda} \quad (4.2)$$

for the Yang-Mills-Higgs system.

For simplicity we start studying eqs (2.9) in the BPS limit which corresponds to  $\lambda/e^2 = 0$  with  $h_0$  fixed.

$$\begin{aligned} (\mu(x)K'(x))' &= K(x) \left( \frac{K(x)^2 - 1}{x^2} + H(x)^2 - \frac{J(x)^2}{\mu(x)} \right) \\ (x^2\mu(x)H'(x))' &= 2 H(x) K(x)^2 \\ \mu(x) (x^2J'(x))' &= 2 J(x) K(x)^2 \end{aligned} \quad (4.3)$$

The total amount of matter  $M$  can be associated with the generator of translations in time,  $\partial_t$  (which appears in the AdS algebra). It takes the form (see for example [15])

$$M = \int_{\Sigma_t} d^3x \sqrt{g^{(3)}} T_{00} \quad (4.4)$$

where  $g^{(3)}$  is the determinant of the induced metric on surfaces  $\Sigma_t$  of constant time  $t$  with normal vector  $e_0 = \mu(x)^{-\frac{1}{2}}\partial_t$  and  $T_{00} \equiv e_0^\mu e_0^\nu T_{\mu\nu} = T_{tt}/\mu(x)$  is the local energy density as seen by an observer moving on the flux lines of  $\partial_t$ . For the spherically symmetric configuration we are considering, it takes the form

$$M = \frac{4\pi h_0}{e} \int_0^\infty dx \frac{x^2}{(1 + \gamma_0 x^2)^{\frac{3}{2}}} \frac{T_{tt}}{e^2 h_0^4} \quad (4.5)$$

We quote for completeness the explicit expressions for  $T_{tt} = T_{tt}^{(YM)} + T_{tt}^{(H)}$

$$\begin{aligned} \frac{T_{tt}^{(YM)}}{e^2 h_0^4} &= \frac{\mu(x)}{2} J'(x)^2 + \frac{J(x)^2 K(x)^2}{x^2} + \frac{\mu(x)^2 K'(x)^2}{x^2} + \frac{\mu(x)}{2 x^4} (K(x)^2 - 1)^2 \\ \frac{T_{tt}^{(H)}}{e^2 h_0^4} &= \frac{\mu(x)^2}{2} H'(x)^2 + \frac{\mu(x)}{x^2} H(x)^2 K(x)^2 \end{aligned} \quad (4.6)$$

It is not difficult to see from these expressions that the boundary conditions imposed through eqs.(2.12) are precisely those required for finiteness of  $M$ .

## Attempts towards a perturbative solution

In flat Minkowski space eqs.(4.3) become BPS equations with the well-honoured Prasad-Sommerfield solution saturating the Bogomol'nyi bound [4]

$$\begin{aligned} K_0(x) &= \frac{x}{\sinh x} \\ H_0(x) &= \cosh \gamma f(x) \end{aligned}$$

$$J_0(x) = \sinh \gamma f(x) \quad (4.7)$$

where the constant  $\gamma$  defines the boundary condition at infinity and

$$f(x) \equiv \coth x - \frac{1}{x} \quad (4.8)$$

For asymptotically flat space-times one can prove that self-gravitating monopoles saturating the Bogomol'nyi bound do not exist [15]. In contrast, solutions to the second order equations of motion can be found numerically, starting from the flat space solution (4.7) and taking  $\alpha_0$  sufficiently small. This regular monopole (or dyon) solution ceases to exist for some critical value  $\alpha_0 = \alpha_0^c$  [10]-[14].

One should expect that something similar happens in AdS space, at least for small enough  $\gamma_0$  (we are already in the  $\alpha_0 h_0^2 \rightarrow 0$  limit): the existence of a small cosmological constant (with respect to the Higgs mass) should alter a little the Prasad-Sommerfeld solution leading to a dyonic regular solution of the second order equations of motion in AdS space. To analyse this possibility one can try a perturbative solution around the Prasad-Sommerfeld solution in the form

$$\begin{aligned} K(x) &= K_0(x) + \sum_{m \geq 1} \gamma_0^m K_m(x) \\ H(x) &= H_0(x) + \sum_{m \geq 1} \gamma_0^m H_m(x) \\ J(x) &= J_0(x) + \sum_{m \geq 1} \gamma_0^m J_m(x) \end{aligned} \quad (4.9)$$

Inserting these expansions in equations (4.3) and comparing  $\gamma_0$  powers, one gets a recursive set of inhomogeneous linear second order differential equations which can be written as

$$\begin{pmatrix} \frac{K_m(x)}{xf(x)} \\ \frac{H_m(x)}{f(x)} \\ \frac{J_m(x)}{f(x)} \end{pmatrix}'' + 2 (\ln(xf(x)))' \begin{pmatrix} \frac{K_m(x)}{xf(x)} \\ \frac{H_m(x)}{f(x)} \\ \frac{J_m(x)}{f(x)} \end{pmatrix}' - V(x) \begin{pmatrix} \frac{K_m(x)}{xf(x)} \\ \frac{H_m(x)}{f(x)} \\ \frac{J_m(x)}{f(x)} \end{pmatrix} = \frac{1}{xf(x)} \vec{q}_m(x) \quad (4.10)$$

for all  $m \geq 1$ . Here  $V(x)$  is a  $m$ -independent matrix defined as

$$\begin{aligned} V(x) &= (2 f(x) \coth x - 1) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ &+ \frac{2 f(x)}{\sinh x} \begin{pmatrix} 0 & \cosh \gamma & -\sinh \gamma \\ 2 \cosh \gamma & 0 & 0 \\ 2 \sinh \gamma & 0 & 0 \end{pmatrix} \end{aligned} \quad (4.11)$$

Concerning the inhomogeneous term in the r.h.s. of (4.10), vectors  $\vec{q}_m$  can be determined at order  $m$  from the solution at lower orders from the closed expressions

$$q_m^1 = \sum_{p=0}^{m-1} (-x^2)^{m-1-p} (K_p(x) - 2x K_p'(x))$$

$$\begin{aligned}
& + \sum_{k,l,s=0}^{m-1} (-x^2)^{m-k-l-s} K_k(x) \left( \frac{K_l(x)K_s(x)}{x^2} + H_l(x)H_s(x) \right) \\
& - (m+1-k-l-s)J_l(x)J_s(x) \\
q_m^2 & = 2 \sum_{p=0}^{m-1} (-x^2)^{m-p} H_p'(x) + \frac{2}{x} \sum_{k,l,s=0}^{m-1} (-x^2)^{m-k-l-s} K_k(x)K_l(x)H_s(x) \\
q_m^3 & = \frac{2}{x} \sum_{k,l,s=0}^{m-1} (-x^2)^{m-k-l-s} K_k(x)K_l(x)J_s(x) \tag{4.12}
\end{aligned}$$

(Sums above are restricted according to  $k+l+s \leq m$ ). One can easily find the explicit form in the  $m=1$  case,

$$\vec{q}_1 = \begin{pmatrix} \frac{x^3}{\sinh x} \left( 1 - 2f(x)^2 + f(x)^2 \sinh^2 \gamma \right) \\ -2 \cosh \gamma \left( 1 - 2 \frac{x^2}{\sinh^2 x} + \frac{x^3}{\sinh^3 x} \cosh x \right) \\ -2 \sinh \gamma \frac{x^3 f(x)}{\sinh^2 x} \end{pmatrix} \tag{4.13}$$

At each order  $m$ , eqs.(4.10) corresponds to a coupled linear system of three differential equations with three unknowns, obeying boundary conditions defined by eqs.(2.12). Now, this system can be reduced by diagonalizing the off diagonal part of  $V(x)$ ; indeed, defining  $(h_m^1, h_m^\pm)$  by

$$\begin{aligned}
K_m(x) & = \frac{1}{\sqrt{2}} x f(x) h_m^+(x) \\
H_m(x) & = f(x) (\sinh \gamma h_m^1(x) + \cosh \gamma h_m^-(x)) \\
J_m(x) & = f(x) (\cosh \gamma h_m^1(x) + \sinh \gamma h_m^-(x)) \tag{4.14}
\end{aligned}$$

for all  $m \geq 1$ , eqs.(4.10) become

$$\left( x^2 f(x)^2 h_m^1'(x) \right)' = x f(x) (-\sinh \gamma q_m^2(x) + \cosh \gamma q_m^3(x)) \tag{4.15}$$

$$\begin{aligned}
h_m^{+''}(x) + 2 (\ln(xf(x)))' h_m^{+'}(x) & = (2 f(x) \coth x - 1) h_m^+(x) + 2 \sqrt{2} \frac{f(x)}{\sinh x} h_m^-(x) \\
& + \frac{\sqrt{2}}{xf(x)} q_m^1(x) \tag{4.16}
\end{aligned}$$

$$\begin{aligned}
h_m^{-''}(x) + 2 (\ln(xf(x)))' h_m^{-}'(x) & = 2 \sqrt{2} \frac{f(x)}{\sinh x} h_m^+(x) \\
& + \frac{1}{xf(x)} (\cosh \gamma q_m^2(x) - \sinh \gamma q_m^3(x)) \tag{4.17}
\end{aligned}$$

Identifying  $H_\infty \equiv \cosh \gamma$  and  $J_\infty \equiv \sinh \gamma$ , boundary conditions can be simply stated in terms of these functions: solutions must be regular at  $x=0$  and vanish for  $x \rightarrow \infty$ .

Eq. (4.15) can be easily integrated, this giving  $h_m^1$ ; one is then left with the reduced system (4.16)-(4.17), with two unknowns  $h_m^\pm$ . Although we were not able to solve this system in closed form, the first order  $m=1$  solution of eq.(4.15),  $h_1^1$ , can be easily integrated with the result

$$h_1^1(x) = \frac{x}{f(x)} \tag{4.18}$$



which is well behaved in  $x = 0$  but grows linearly with  $x$  at infinity! This could be taken as a signal that a solution of the type proposed in (4.9) does not exist, indicating that the theory with cosmological constant strictly zero has a completely different behavior from that with  $\Lambda \neq 0$ , regardless how small  $\Lambda$  could be. Another indication in the same direction comes from the comparison of the AdS boundary conditions (2.12) with those corresponding to the flat space case,

$$\begin{aligned} K(x) &\rightarrow O(x^{-\alpha_1}) \\ H(x) &\rightarrow H_\infty + O(x^{-\alpha_2}) \\ J(x) &\rightarrow J_\infty + O(x^{-\alpha_3}) \end{aligned} \quad x \gg 1 \quad (4.19)$$

Comparing these flat space conditions with those corresponding to AdS space (conditions (2.12)) we see that the latter require the Higgs field to reach its vacuum expectation value faster than in flat space. It is seemingly that the unbounded behaviour of  $\mu(x) - 1$  for  $\gamma_0 \neq 0$  (eq.(4.1)) is at the root of this situation. It is worth to note at this point that away from the BPS limit ( $\lambda/g_0^2 \neq 0$ ) the vanishing of the Higgs potential at infinity requires exactly the same boundary both in AdS and in flat space.

## Re-statement of the problem and asymptotic expansions

A hint about the possible non regular behaviour of the solution in terms of the parameter  $\gamma_0$  comes from the following analysis.

If and only if  $\gamma_0$  is different from zero and positive (e.g. in the case of AdS space) one can introduce the following functions

$$\begin{aligned} k(y) &\equiv \sqrt{\mu(x)} K(x)|_{x=\frac{y}{\sqrt{\gamma_0}}} \\ h(y) &\equiv x \sqrt{\mu(x)} H(x)|_{x=\frac{y}{\sqrt{\gamma_0}}} \\ j(y) &\equiv x J(x)|_{x=\frac{y}{\sqrt{\gamma_0}}} \end{aligned} \quad (4.20)$$

in term of which the equations of motion take the form

$$\begin{aligned} y^2 (1 + y^2)^2 k''(y) &= k(y) \left( k(y)^2 + h(y)^2 - j(y)^2 - 1 \right) \\ y^2 (1 + y^2)^2 h''(y) &= 2 h(y) \left( k(y)^2 + y^2 \left( y^2 + \frac{3}{2} \right) \right) \\ y^2 (1 + y^2)^2 j''(y) &= 2 j(y) k(y)^2 \end{aligned} \quad (4.21)$$

The dependence on  $\gamma_0$  has completely disappeared from (4.21) through the non analytical change (4.20)

Note the monopole mass can be written as

$$M = \frac{4 \pi h_0}{e} f(0, \gamma_0) = \frac{4 \pi}{e^2 r_0} f_0 \quad (4.22)$$

where, extending the usual flat space notation [1] we have introduced the dimensionless function  $f = f(\lambda/e^2, \gamma_0)$  and which in the Prasad-Sommerfield limit can be written as

$f(0, \gamma_0) = \sqrt{\gamma_0} f_0$  with  $f_0$  a numerical constant which can be in principle calculated and is of course finite if the appropriate boundary conditions hold. Thus, having AdS space a natural scale  $r_0$ , the system trades (in the Prasad-Sommerfield limit)  $h_0$  for the AdS radius  $r_0 = \sqrt{-3/\Lambda}$  which now sets the scale for the mass.

Returning to eqs.(4.21), one should note that in the  $y \ll 1$  region, which in the original variable corresponds to  $x \ll 1/\sqrt{\gamma_0}$ , one could hope to find the BPS solution since  $\gamma_0$  is effectively very small and the domain is approximately flat; in fact it is easy to check that the system reduces in this region to the BPS equations. Therefore, the problem can be settled as follows: a finite mass monopole (or dyon) in AdS space should interpolate between the BPS solution near  $y = 0$ ,

$$\begin{aligned} k(y) &= K_0(y) \\ h(y) &= y H_0(y) \\ j(y) &= y J_0(y) \end{aligned} \tag{4.23}$$

and a solution that asymptotically behaves as

$$\begin{aligned} k(y) &\rightarrow O(y^{1-\alpha_1}) \\ h(y) &\rightarrow H_\infty y^2 + O(y^{1-\alpha_2}) \\ j(y) &\rightarrow J_\infty y + O(y^{1-\alpha_3}) \end{aligned} \tag{4.24}$$

In the intermediate ( $y \sim 1$ ) region (4.21) corresponds to a constant coefficient system, this ensuring the existence of a solution in the neighborhood, according to standard theorems on non-linear differential equations systems.

## The pure magnetic monopole.

Let us still simplify the problem by considering neutral ( $j = 0$ ) magnetic monopoles. We try at large distances a power series solution of the form

$$\begin{aligned} k(y) &= \sum_{n=1}^{\infty} k_n y^{-n}, \quad y \gg 1 \\ h(y) &= H_\infty y^2 + \sum_{n=0}^{\infty} h_n y^{-n}, \quad y \gg 1 \end{aligned} \tag{4.25}$$

Plugging these expansions into eqs.(4.21) one gets a compatible set of recursion relations. It is worth to note that this procedure does not work in flat space: in order system to close, it should necessary to add also positive powers, because of the exponential decay behaviour of the solution (4.7). This signals a very different qualitative behaviour of the solution in AdS with respect to flat space, its origin being of course the quadratically divergent behaviour of  $\mu(x)$  in AdS space, which produces a power six (instead of two) in the r.h.s. of (4.21).

Coming back to the expansions, one gets for for  $k_n$  the recursive relations

$$(2 - H_\infty^2) k_1 = 0$$

$$\begin{aligned}
(6 - H_\infty^2) k_2 &= 0 \\
(12 - H_\infty^2) k_3 &= (2 h_0 H_\infty - 4) k_1 \\
(20 - H_\infty^2) k_4 &= 2 H_\infty h_1 k_1 + (2 h_0 H_\infty - 12) k_2 \\
(30 - H_\infty^2) k_5 &= (2 h_2 H_\infty + h_0^2 - 3) k_1 \\
&\quad + 2 H_\infty h_1 k_2 + (2 h_0 H_\infty - 24) k_3 \\
&\dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \\
(n(n+1) - H_\infty^2) k_n &= f_1^{(n)} k_1 + \dots + f_{n-2}^{(n)} k_{n-2}
\end{aligned} \tag{4.26}$$

where the coefficients  $f_i^{(n)}$  are determined by the coefficients  $k_k$  with  $k < n-2$  and also by coefficients  $h_n$  (Although inspection of (4.26) shows no dependence on  $k_k$  for  $n = 1, \dots, 5$ , the situation changes for  $n > 5$ ).

It is convenient at this point to redefine  $h_1 = H_\infty A$ . Then, the solution for  $h(y)$  takes the form

$$h(y) = H_\infty \left( y^2 + \frac{1}{2} + \frac{A}{y} - \frac{1}{8 y^2} - \frac{A}{10 y^3} + \left( \frac{1}{48} + \frac{k_1^2}{9} \right) \frac{1}{y^4} - \frac{11}{280} \frac{A}{y^5} + O\left(\frac{1}{y^6}\right) \right) \tag{4.27}$$

with  $A$  a free parameter. The analysis of the system (4.26) shows up a remarkable feature. One easily verifies that, unless

$$H_\infty^2 = m(m+1) \quad , \quad m = 1, 2, \dots \tag{4.28}$$

one has  $k(y) \equiv 0$ .

Now, when  $k(y) = 0$  the gauge field corresponds to a Dirac monopole configuration while  $h(y)$  is just the solution of the linear equation

$$h''(y) = \frac{3 + 2 y^2}{(1 + y^2)^2} h(y) \tag{4.29}$$

given by

$$h = y \sqrt{1 + y^2} \tag{4.30}$$

which corresponds to  $H(y) = 1$ . This shows that this solution coincides with that discussed in Section 3. (There is a second solution for  $h$  which has to be discarded since it is not regular at the origin).

More interesting are the solutions for which  $H_\infty$  is quantized according to (4.28), which implies that the Higgs field takes at infinity the value (see eq.(2.6))

$$\left| \vec{H}(\infty) \right|_\Lambda^2 = m(m+1) \left( \frac{1}{e r_0} \right)^2 \tag{4.31}$$

to be compared with the flat space answer

$$\left| \vec{H}(\infty) \right|_{\Lambda=0}^2 = h_0^2 \tag{4.32}$$

Now, in the Prasad-Sommerfield flat space case, being the Higgs potential absent,  $h_0$  is introduced both in order to set a scale and as a vestige of the vanishing potential. This is achieved precisely through the asymptotic condition (4.32) which is not imposed by the potential (as in the  $\lambda \neq 0$  case) but nevertheless results to be a consistent asymptotic condition for monopoles without electric charge. With a similar purpose we introduced  $h_0$  in curved space as a scale, finding that the consistent asymptotic value of the Higgs scalar is constrained by the AdS geometry to obey (4.31) instead of (4.32). Moreover the natural scale for the Higgs v.e.v. is set by the cosmological constant. The fact that the asymptotic conditions change in AdS space is already encountered for the simple case of a free massive field where one finds that the scalar cannot approach a constant at infinity [23]. In the present case, where the scalar is coupled to a gauge field, the Higgs scalar has to behave at infinity according to (4.31) and even the lowest possible non-trivial value for  $m$ ,  $m = 1$ , gives in anti-de Sitter space a squared Higgs field v.e.v. which is twice the flat space value measure in the natural units of each problem. In some sense this behavior resembles, for the purely magnetic solution in AdS space, to the consistent asymptotic condition for flat space Prasad-Sommerfield dyons which is not  $h_0$  but  $h_0 \cosh \gamma$  [4].

A first analysis of eqs.(4.26) indicates that a non trivial monopole solution should exist in asymptotically anti-de Sitter space. Indeed,  $k(y)$  can be written at large distances, in the form

$$k(y) = \frac{k_m}{y^m} \left( 1 - \frac{a_m}{y^2} + O(1/y^3) \right) \quad (4.33)$$

where  $a_m$  and the higher order terms can be straightforwardly computed from (4.26):  $a_1 = 1/5, a_2 = 3/7, a_3 = 2/3, \dots$ . We see that  $k(y)$  exhibits an  $y^{-m}$  behavior. The corresponding coefficient as well as  $A$  are free parameters which should be in principle determined by matching this behavior with the solution for small  $y$ .

Concerning the case in which an electric field is included, direct inspection of the third equation in (4.21) reveals that an ansatz of the form

$$j(y) = J_\infty y - b + \frac{j_5}{y^5} + \dots \quad (4.34)$$

together with the ansatz (4.25) for  $k(y)$  and  $h(y)$  should work as well. (The high  $1/y$  power is dictated by the necessity of cancelling the  $\mu(x)$  factor)

## 5 Discussion

We have discussed in this work monopole and dyon configurations for Yang-Mills-Higgs theory coupled to gravity when a cosmological constant is included so that space-time is, asymptotically, anti-de Sitter space. Making the usual spherically symmetric ansatz to separate the equations of motion, we have investigated both singular and regular configurations carrying magnetic and electric charge.

Concerning singular solutions, we have constructed a Wu-Yang like dyon solution with a metric of the Reissner-Nordström type (with cosmological constant). The event horizons

can be determined as a power expansion in  $a^2\gamma_0$ , with  $a$  related to the mass of the black hole and  $\gamma_0$  proportional to the cosmological constant. Although the gauge field is singular at the origin, the solution can be considered regular as a black hole, with the singularity hidden behind the horizon.

In order to find regular solutions to the coupled equations of motion, we have investigated different regimes. With vanishing gravitational constant, Einstein equations decouple from matter and the solution for the metric corresponds to a neutral Schwarzschild black hole in AdS. This metric acts as a background for the Yang-Mills-Higgs system, this changing radically the properties of the solution with respect to the asymptotically flat space case.

First, we have tried to find solutions to the second order equations of motion, close to the BPS ones, which have been shown to exist in the small Newton constant regime for asymptotically flat space. Now, when a cosmological constant is included (no matter how small this constant is) no solution close to the BPS configuration can be found. As we showed, it is the change in the asymptotic behavior of the Higgs field due to a non zero cosmological constant that prevents such a solution.

We have then studied the problem of matching the conditions that the solutions have to satisfy in asymptotically AdS space with those required at the origin, in order to have finite mass. Working for simplicity in the Prasad-Sommerfield limit, we have found a remarkable result: the v.e.v.  $|\vec{H}(\infty)|$  of the Higgs scalar should obey  $|\vec{H}(\infty)|^2 = m(m+1)(er_0)^{-2}$  with  $m = 1, 2, \dots$  (note that this result is obtained when an asymptotic power series behaviour is assumed). When this condition is fulfilled, our analysis shows that a monopole solution can be constructed with a finite mass whose scale is set by the AdS radius. We leave the numerical analysis of such monopole and dyon solution for a forthcoming work.

## Acknowledgments

This work is partially supported by CONICET, CICBA and ANPCYT (PICT 97/2285) (Argentina). A.R.L. would like to thank ISAS (Trieste) where part of this work was done as well as helpful discussions with L. Bonora and A. Bressan.

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