Effect of the 3K background radiation on ultrahigh energy cosmic rays

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Abstract

In this work we re-examine the opacity of the cosmic background radiation to the propagation of extremely high energy cosmic rays. We use the continuous energy loss approximation to provide spectral modification factors for several hypothesized cosmic ray sources. Earlier problems with this approximation are resolved including the effects of resonances other than the $\Delta$.

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I. INTRODUCTION

With the discovery of the microwave background radiation (MBR) [1], it was realized that its effect on the propagation of ultrahigh energy cosmic rays (UHECR’s) must be considered. The first treatments in 1966 by Greisen, Zatsepin and Kuz’min [2,3] indicated a sharp cutoff for cosmic rays with energies above $5 \times 10^{19}$ eV, due to the process $\gamma + p \rightarrow \Delta \rightarrow p/n\pi$. Physically, the thermal photons are seen highly blue-shifted by the cosmic rays in their rest frames so that this reaction becomes possible.

Recently, several extensive air showers have been clearly observed which imply the arrival of cosmic rays of energies above $10^{20}$ eV. In particular, the Akeno Giant Air Shower Array (AGASA) experiment recorded an event with energy $1.7 - 2.6 \times 10^{20}$ eV [4,5], while the Fly’s Eye experiment reported the highest energy cosmic ray (CR) event ever detected on Earth, with an energy $2.3 - 4.1 \times 10^{20}$ eV [6,7]. Since the GZK cutoff provides an important constraint on the proximity of UHECR sources, the origin of such energetic particles has became one of the most pressing questions of cosmic ray astrophysics.

The study of the modification of the spectrum of cosmic rays due to their interactions with the MBR en route to us here on Earth, is essential if one is to obtain a deeper understanding of the origin of the highest energy cosmic rays. Various treatments of the propagation of nucleons in the intergalactic medium exist, including purely analytical ones [8–13], as well as purely numerical ones [14–17]. In this paper, we re-analyze the effects of meson photoproduction, the dominant energy loss mechanism, using the continuous energy loss approximation which is expected to be reasonable for the problem at hand. This allows us to discuss, in a general way, the modification of the spectrum of CR’s from nearby sources due to interactions with the cosmic background radiation.
II. PROPAGATION OF COSMIC RAYS IN THE MICROWAVE BACKGROUND RADIATION

The propagation of cosmic rays through the intergalactic medium is described by a kinetic transport equation which takes into account three mechanisms for energy losses: redshift, pair production, and pion photoproduction. Nevertheless, for nearby sources corresponding to propagation times \( \tau \approx 3 \times 10^8 \) years, or distances of less than 100 Mpc, redshift effects can be neglected.

If one assumes that the highest energy cosmic rays are indeed nucleons, the fractional energy loss due to interactions with the cosmic background radiation at temperature \( T \) and redshift \( z = 0 \), is determined by the integral of the nucleon loss energy per collision times the probability per unit time for a nucleon collision moving through an isotropic gas of photons \[18\]. This integral can be explicitly written as follows,

\[
-\frac{1}{E} \frac{dE}{dt} = \frac{c}{2T^2} \int_0^{w_m} dw_r \, K(w_r) \sigma(w_r) \, w_r \int_{w_r/(2\Gamma)}^{w_m} dw \, \frac{n(w)}{w^2}
\]

where \( w_r \) is the photon energy in the rest frame of the nucleon, and \( K(w_r) \) is the average fraction of the energy lost by the photon to the nucleon in the laboratory frame (i.e. the frame in which the microwave background radiation is at \( \approx 3 \) K). \( n(w) \) stands for the number density of photons with energy between \( w \) and \( dw \), following a Planckian distribution at temperature \( T \) \[15\], \( \sigma(w_r) \) is the total cross section of interaction, \( \Gamma \) is the usual Lorentz factor of the nucleon, and \( w_m \) is the maximum energy of the photon in the photon gas.

Thus, the fractional energy loss is given by

\[
-\frac{1}{E} \frac{dE}{dt} = -\frac{ckT}{2\pi^2T^2(ch)^3} \int_{w_0}^{\infty} dw_r \, \sigma(w_r) \, K(w_r) \, w_r \ln(1 - e^{-w_r/(2\Gamma kT)})
\]

where \( k \) and \( h \) are Boltzmann’s and Planck’s constants respectively, and \( w_0 \) is the threshold energy for the reaction in the rest frame of the nucleon.

The characteristic time for the energy loss due to pair production at \( E > 10^{19} \) eV is \( t \approx 5 \times 10^9 \) yr \[20\] and therefore it does not affect the spectrum of nucleons arriving from
nearby sources. Consequently, for nucleons with \( E > 3 \times 10^{19}\text{eV} \) (taking into account the interaction with the tail of the Planck distribution), meson photoproduction is the dominant mechanism for energy loss. Notice that we do not distinguish between neutrons and protons; in addition, the inelasticity due to the neutron \( \beta \) decay is negligible.

In order to determine the effect of meson photoproduction on the spectrum of cosmic rays, we first examine the kinematics of photon-nucleon interactions. Assuming that reactions mediated by baryon resonances have spherically symmetric decay angular distributions \([13,16]\), the average energy loss of the nucleon after \( n \) resonant collisions is given by

\[
K(m_{R_0}) = 1 - \frac{1}{2^n} \prod_{i=1}^{n} \left( 1 + \frac{m^2_{R_i} - m^2_M}{m^2_{R_{i-1}}} \right)
\]

where \( m_{R_i} \) denotes the mass of the resonant system of the chain, \( m_M \) the mass of the associated meson, \( m_{R_0} = \sqrt{s} \) is the total energy of the reaction in the centre of mass, and \( m_{R_n} \) the mass of the nucleon. It is well established from experiments \([21]\) that, at very high energies (\( \sqrt{s} \) above ~ 3 GeV), the incident nucleons lose one-half their energy via pion photoproduction independent of the number of pions produced.

In the region dominated by baryon resonances, the cross section is described by a sum of Breit-Wigner distributions constructed from the experimental data in the Table of Particle Properties \([22]\) considering the main resonances produced in \( N\gamma \) collisions with \( \pi N, \pi\pi N, \) and \( \pi K N \) final states. For the cross section at high energies we used the fits to the high-energy cross section \( \sigma_{\text{total}}(p\gamma) \) made by the CERN, DESY HERA, and COMPAS Groups \([22]\). In this energy range, the \( \sigma_{\text{total}}(n\gamma) \) is to a good approximation identical to \( \sigma_{\text{total}}(p\gamma) \).

The numerical integration of Eq. (4) is performed taking into account the aforementioned resonance decays and the production of multipion final states at higher centre of mass energies (\( \sqrt{s} \sim 3 \) GeV). A \( \chi^2 \) fit of the numerical results of equation (4) with an exponential behaviour, \( A \exp\{-B/E\} \), proposed by Berezinsky and Grigor’eva \([10]\) for the region of resonances, gives: \( A = (3.66 \pm 0.08) \times 10^{-8} \text{yr}^{-1} \), \( B = (2.87 \pm 0.03) \times 10^{20} \text{eV} \) with a \( \chi^2/\text{dof} = 3.9/10 \). At high energies the fractional energy loss was fitted with a constant,
$C = (2.42 \pm 0.03) \times 10^{-8}$ yr$^{-1}$. These results differ from those obtained in [10], due to a refined expression for the total cross section. From the values determined for the fractional energy loss it is straightforward to compute the energy degradation of UHECR’s in terms of their flight distance. The results shown in Fig. 1, are consistent with those previously obtained by Cronin [23], in which just single pion production is considered. Our results therefore support the assumption that the energy loss is dominated by single pion photoproduction.

III. MODIFICATION OF THE COSMIC RAY SPECTRUM

Let us now turn to the modification that the MBR produces in the UHECR spectrum. The evolution of the spectrum is governed by the balance equation

$$\frac{\partial N}{\partial t} = \frac{\partial (b(E)N)}{\partial E} + D \nabla^2 N + Q \quad (4)$$

that takes into account the conservation of the total number of nucleons in the spectrum. In the first term on the right, $b(E)$ is the mean rate at which particles lose energy. The second term, the diffusion in the MBR, is found to be extremely small due to the low density of relic photons and the fact that the average cosmic magnetic field is less than $10^{-9}$ G [24] and is neglected in the following. The third term corresponds to the particle injection rate into the intergalactic medium from some hypothetical source. Since the origin of cosmic rays is still unknown, we consider three possible models: 1) the universal hypothesis, which assumes that cosmic rays come from no well-defined source, but rather are produced uniformly throughout space, 2) single point sources of cosmic rays, and 3) sources of finite size approximating clusters of galaxies. In all the cases it has been assumed that nucleons propagate in a straight line through the intergalactic medium due to the reasons mentioned above that allow us to neglect the diffusion term.

There exists evidence that the source spectrum of cosmic rays has a power-law dependence $Q(E) = \kappa E^{-\gamma}$ (see for instance [14]). With the hypothesis that cosmic rays are produced from sources located uniformly in space, with this power law energy dependence (which implies a steady state process), the solution of Eq. (4) is found to be
For the case of a single point source, the solution of equation (4) reads [11],

\[ N(E, t) = \frac{1}{b(E)} \int_{E}^{\infty} Q(E_g, t') dE_g \]  

with

\[ t' = t - \int_{E}^{E_g} \frac{d\tilde{E}}{b(\tilde{E})} \]  

and \( E_g \) the energy of the nucleon when emitted by the source. The injection spectrum of a single source located at \( t_0 \) from the observer can be written as

\[ Q(E, t) = \kappa E^{-\gamma} \delta(t - t_0) \]

and for simplicity, we consider the distance as measured from the source, that means \( t_0 = 0 \).

At very high energies, i.e. where \( b(E) = C E \) (where \( C \) is the constant defined above in the discussion of fractional energy loss) the total number of particles at a given distance from the source is given by

\[ N(E, t) \approx \frac{\kappa b(E)}{E_g} \int_{E}^{\infty} \frac{E_g^{-\gamma}}{\delta(t - 1) C \ln \frac{E_g}{E}} dE_g \]

or equivalently

\[ N(E, t) \approx \kappa E^{-\gamma} e^{-\gamma} \]

This means that the spectrum is uniformly damped by a factor depending on the proximity of the source.

At low energies, in the region dominated by baryon resonances, the parametrization of \( b(E) \) does not allow a complete analytical solution. However using the change of variables

\[ \tilde{t} = \int_{E}^{E_g} \frac{d\tilde{E}}{b(E)} \]  

with \( E_g = \xi(E, \tilde{t}) \) and \( d\tilde{t} = dE_g/b(E_g) \), we easily obtain,

\[ N(E, t) = \frac{\kappa}{b(E)} \int_{0}^{\infty} \xi(E, \tilde{t})^{-\gamma} \delta(\tilde{t} - t) b[\xi(E, \tilde{t})] d\tilde{t} \]

and then, the compact form,
\[ N(E, t) = \frac{\kappa}{b(E)} E^{-\gamma} b(E) \] (12)

In our case, where we have assumed an exponential behaviour of the fractional loss energy, \( E_g \) is fixed by the constraint: \( A t - Ei (B/E) + Ei (B/E_g) = 0 \), \( Ei \) being the exponential integral [2], and \( B \) the constant defined above in the parametrization of Berezinsky and Grigor’eva.

Studies are underway of the case of a nearby source (i.e. within about 100 Mpc) for which the fractional energy loss \( \Delta E/E \) is small. Writing \( E_g = E + \Delta E \), and neglecting higher orders in \( \Delta E \), it is possible to obtain an analytical solution and the most relevant characteristics of the modified spectrum. Therefore, an expansion of \( b(E + \Delta E) \) in powers of \( \Delta E \) in Eq. (7) allows one to obtain, an expression for \( \Delta E \),

\[ \Delta E \approx \left\{ \exp \left[ \frac{t b(E) (E + B)}{E^2} \right] - 1 \right\} \frac{E^2}{(E + B)} \] (13)

To describe the modification of the spectrum, it is convenient to introduce the factor \( \eta \), the ratio between the modified spectrum and the unmodified one, that results in

\[ \eta = \left( \frac{E + \Delta E}{E} \right)^{-\gamma} \frac{b(E + \Delta E)}{b(E)} \] (14)

Equations (13) and (14) describe the spectral modification factor up to energies of \( \approx 95 \) EeV (\( \approx 85 \) EeV) with a precision of 4% (9%) for a source situated at 50 Mpc (100 Mpc).

In Fig. 2 we plot the modification factors for the case of nearby sources with power law injection (\( \gamma = 2.5 \)) to compare with the corresponding results of ref. [10]. The most significant features of \( \eta \) are the bump and the cutoff. It has been noted in the literature that the continuous energy loss approximation tends to overestimate the size of the bump [12,14]. Figure 2 shows how the bump is less pronounced in our treatment. As the bump is a consequence of the sharp (exponential) dependence of the free path of the nucleon on energy, we attribute these differences to the replacement of a cross section approximated by the values at threshold energy [10], by a more detailed expression taking into account the main baryon resonances that turn out to be important.
Another alternative for the particle injection is clusters of galaxies. These are usually modelled by a set of point sources with spatially uniform distribution, although in reality the distribution of galaxies inside the clusters is somewhat non-uniform. In our treatment we shall assume that the concentration of potential sources at the center of the cluster is higher than that near the periphery, and we adopt a spatial gaussian distribution. With this hypothesis, the particle injection rate into the intergalactic medium is given by

\[ Q(E, t) = \kappa \int_{-\infty}^{\infty} \frac{E^{-\gamma}}{\sqrt{2 \pi \sigma}} \delta(t - T) \exp \left\{ \frac{-(T - t_0)^2}{2 \sigma^2} \right\} dT \] (15)

A delta function expansion around \( t_0 \), with derivatives denoted by lower case Roman superscripts,

\[ \delta(t - T) = \delta(t - t_0) + \delta^{(i)}(t - t_0)(T - t_0) + \frac{1}{2!} \delta^{(ii)}(t - t_0)(T - t_0)^2 + \ldots \] (16)

leads to a convenient form for the injection spectrum, which is given by,

\[ Q(E, t) = \kappa E^{-\gamma} \left[ \delta(t' - t_0) + \frac{\sigma^2}{2!} \delta^{(ii)}(t' - t_0) + \frac{\sigma^4}{4!} \delta^{(iv)}(t' - t_0) + \ldots \right] \] (17)

From Eqs. (8) and (9), it is straightforward to compute an expression for the modification factor,

\[ \eta = \frac{E^{-\gamma} b(E_g)}{E^{-\gamma} b(E)} \left\{ 1 + \frac{\sigma^2 A^2 e^{-2B/E_g}}{2!} F_1(E_g) + \frac{\sigma^4 A^4 e^{-4B/E_g}}{4!} F_2(E_g) + O(6) \right\} \] (18)

where

\[ F_1(E_g) = 2B^2 E_g^{-2} + (2 - 3\gamma) B E_g^{-1} + (1 - \gamma)^2 \]

and

\[ F_2(E_g) = 24B^4 E_g^{-4} + (4 - 50\gamma) B^3 E_g^{-3} + (35\gamma^2 - 25\gamma + 8) B^2 E_g^{-2} + (-10\gamma^3 + 20\gamma^2 - 15\gamma + 4) B E_g^{-1} + (1 - \gamma)^4 \]

The modification factor for the case of extended sources described by Gaussian distributions of widths 2 and 6 Mpc at a distance of 18.3 Mpc is shown in Fig. 3. These may be
taken as very crude models of the Virgo cluster [26], assuming that there is no other significant energy loss mechanism for cosmic rays traversing parts of the cluster beyond those due to interactions with the cosmic background radiation.

A power law injection ($\gamma = 2.5$) was used again, as in the pointlike case. The curves can be understood qualitatively. Both peaks are in about the same place, occurring around the onset of pion photoproduction. The narrower curve, corresponding to the broader source distribution, reflects the losses suffered by the more remote part of the distribution in traversing a greater distance to us. We note that the modification factors for nearby extended sources can appear similar to those for more distant narrower sources.

**IV. CONCLUSIONS**

We have presented a recalculation of the effects of the cosmic background radiation on the propagation of cosmic rays using the continuous energy loss approximation first used by Berezinsky [18] and our best knowledge of $\gamma$-nucleon cross sections. This approximation is important as it is the only way to allow semi-analytical treatments in a wide range of situations.

While earlier calculations within this approximation suffered from problems described in the main text, these do not appear with our improved treatment of cross sections, and the results are in reasonable agreement with previous calculations using Monte Carlo methods without the continuous energy loss approximation.

The eventual observation of the GZK cutoff is of fundamental interest in cosmic ray physics, providing constraints on the distance to sources of ultrahigh energy cosmic rays. The future Pierre Auger Project [27] should provide a decisive test of the ideas discussed in this paper.
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REFERENCES


[19] Concerning the density of photons, data from the Far-InfraRed Absolute Spectrometer
(FIRAS) of the Cosmic Background Explorer (COBE) shows that the spectrum of MBR is that of a blackbody of temperature $T = 2.73 \pm 0.06$ K, with no deviation from a Planckian spectrum greater than 0.25 % of the peak brightness and a limit on the Compton $y$-parameter of $y < 0.0004$ (95 % CL). See, R. A. Shafer et al., *Bull. Am. Phys. Soc* **36**, 1398 (1991); H. P. Gush, M. Halpern and E. H. Wishnow, *Phys. Rev. Lett.* **65**, 537 (1990).


Figure 1. Energy attenuation length of nucleons in the intergalactic medium. Notice that, independently of the initial energy of the nucleon, the mean energy values approach to 100 EeV after a distance of $\approx 100$ Mpc.

Figure 2. Modification factor of single-source energy spectra for different values of propagation distance and power law index $\gamma = 2.5$.

Figure 3. Modification factor of extended-source energy spectrum for a propagation distance $\approx 18.3$ Mpc and power law index $\gamma = 2.5$. Solid line stands for a gaussian distribution of width $\sigma = 2$ Mpc, while dashed line a width of $\sigma = 6$ Mpc.
FIG. 1.

The figure shows a graph with the x-axis labeled as $E$ [eV] and the y-axis labeled as $D$ [Mpc]. The graph plots the relationship between distance ($D$) and energy ($E$) on a logarithmic scale.
FIG. 2.