

Boson Stars in General Scalar-Tensor Gravitation: Equilibrium Configurations

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Abstract

We study equilibrium configurations of boson stars in the framework of general scalar-tensor theories of gravitation. We analyse several possible couplings, with acceptable weak field limit and, when known, nucleosynthesis bounds, in order to work in the cosmologically more realistic cases of this kind of theories. We found that for general scalar-tensor gravitation, the range of masses boson stars might have is comparable with the general relativistic case. We also analyse the possible formation of boson stars along different eras of cosmic evolution, allowing for the effective gravitational constant far out from the star to deviate from its current value. In these cases, we found that the boson stars masses are sensitive to this kind of variations, within a typical few percent. We also study cases in which the coupling is implicitly defined, through the dependence on the radial coordinate, allowing it to have significant variations in the radius of the structure.

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I. INTRODUCTION

Boson stars, stellar structures first proposed by Ruffini and Bonazzola [1], are gravitationally bound macroscopic quantum states made up of scalar bosons. They differ from neutron stars, their fermionic counterparts, in that their pressure support derives from the uncertainty relation rather than Pauli's exclusion principle. Although the seminal work of reference [1] was published in 1969, it was followed up only in the past decade. In these recent years, cosmology has been refurnished with the introduction of several ideas concerning the critical role scalar field may have in the evolution of the universe. This revived the possibility of constructing stellar objects made up of these scalars instead of conventional fermions.

Considering bosons as described by a non-interacting, massive, complex scalar field, Ruffini and Bonazzola solved the equations of motion given by the Einstein field equations and the Klein-Gordon equation. The general setting they used is the same as the one explained below. They found that the masses of such boson stars were of order $M \simeq M_{pl}^2/m$, where M_{pl} is the Planck mass and m is the boson mass. This model served to open the possibility that boson stars might indeed exist in nature, although their masses were small enough as to discard them as a viable solution to the dark matter problem. The order of magnitude of these boson stars coincides with simple computations. For a quantum state confined into a region of radius R , and with units given by $\hbar = c = 1$, the boson momentum is $p = 1/R$. If the star is moderately relativistic $p \simeq m$, then $R \simeq 1/m$. If we equate R with the Schwarzschild radius $2M_{pl}^{-2}M$ (recall that $G = M_{pl}^{-2}$) we get $M \simeq M_{pl}^2/m$. Later work made by Colpi, Shapiro and Wasserman [2] introduced a self-interaction term for the scalar field. With this addendum, stellar equilibrium configurations had masses of order M_{pl}^3/m^2 , which are of the same order than the Chandrasekhar mass ($\simeq 1$ solar masses). The stability of these objects have also been analysed with similar results to those of the neutronic case [3]. Taking Ruffini and Bonazzola and Colpi et. al. works as starting points, several extensions have been proposed. Jetzer and van der Bij [4] considered the inclusion of a U(1) gauge

charge and Jetzer [5] studied then its stability properties. Non-minimally couplings for the scalars have also been analysed in [6]. These and other related models are reviewed in [7]. In addition, provided many of this objects may would have a primordial origin, while being formed in a gas of boson and fermions, it is expected that boson-fermions stars might exist. This was studied by Henriques et. al. [8] without interaction and, recently, by de Sousa et. al. with current-current type interaction [9]. Finally, the possible understanding of galactic halo properties by means of boson stars models have also been proposed [10].

Boson stars solutions have been, however, scarcely analysed in the framework of alternative theories of gravitation. We are particularly interested in scalar-tensor gravity, in which the effective gravitational constant is a field variable [11]. Historically, most interest has been given to Brans-Dicke (BD) gravity, in which the coupling function $\omega(\phi)$, free function these theories have, is constant. To ensure that the weak field limit of this theory agrees with current observations, ω has to be big enough [12]. But in general, when ω varies, we need that $\omega \rightarrow \infty$ and $\omega^{-3}d\omega/d\phi \rightarrow 0$ when $t \rightarrow \infty$, to allow the weak field limit of scalar-tensor gravitation accord well with general relativity (GR) tested predictions. Afterwards, soon was realized that these general scalar-tensor theories would admit significant deviations from GR in the past [11] and that they could be a useful tool in the understanding of early universe models. The interest on them was recently rekindled by inflationary scenarios [13] and fundamental theories that seek to incorporate gravity with other forces of nature [14]. In general, almost all studies made on scalar-tensor gravitation focus in the cosmological models they lead. This is in order to put several constraints upon the coupling function. Observational bounds, mainly coming from weak field tests [12] and nucleosynthesis [15–18] are more restrictive if exact analytical solutions are known for the cosmological equations. A few years ago, Barrow [19], Barrow and Mimoso [20] and Mimoso and Wands [21] derived algebraic numerical methods that allow Friedmann-Robertson-Walker (FRW) solutions to be found in models with matter content in the form of a barotropic fluid for any kind of coupling $\omega(\phi)$. Some of these methods were recently extended to incorporate non-minimally coupled theories, even in the cases in which the functions involved in the lagrangian do

not possess analytical inverse [22–24]. These extensions showed the possibility of classifying the cosmological behaviour of scalar-tensor theories in equivalence sets, where the field itself is a class variable.

In an astrophysical setting, if a scalar-tensor theory describes gravitation, the value of the effective gravitational constant far out from the star must not necessarily be the Newton constant, but the value given by the evolution of a cosmological model at the time of formation of the stellar object. As we shall see below, this may change the boundary conditions of the problem. Within this gravitational framework, boson stars were analysed only in the simplest case. Gunderson and Jensen [25] addressed the possible existence of such objects in Brans-Dicke gravity, with and without self-interaction. They adopted a fixed boundary condition for the field equal to 1 –dimensionless Newton constant– and found that in general, for almost all ω , the Brans-Dicke model of boson stars gives a maximum mass smaller than the general relativistic model in a few percent [25]. A similar work addressed the existence of boson stars in a gravitational theory with dilaton [26] and its results coincide with the previous case.

The aim of this work is then, to present a comprehensive study on the possible existence of boson stars solutions in *general scalar-tensor theories*. In the case of their existence, we want to analyse the values they give for masses of typical objects and other dynamical variables of interest, like the typical behavior of the scalar. We also want to see if modifications in the boundary condition for the Brans-Dicke scalar –due to cosmological evolution– produce any noticeable deviation in the masses of equilibrium configurations. We also implicitly define the coupling in order to allow substantive variations of it in the radius of the structure. We then study equilibrium configurations in these schemes.

The rest of the paper is organized as follows. In Sec. II we introduce the formalism for boson stars construction together with the numerical recipes used. In Sec. III we make choices of coupling functions and in Sec. IV the results for them are presented. The last section deals with our conclusions.

II. FORMALISM

A. Gravitational theory and boson system

We first derive the equations that corresponds to the general scalar-tensor theory. The action for this kind of generalized BD theories is

$$S = \frac{1}{16\pi} \int \sqrt{-g} dx^4 \left[\phi R - \frac{\omega(\phi)}{\phi} g^{\mu\nu} \phi_{,\mu} \phi_{,\nu} + 16\pi \mathbf{L}_m \right], \quad (1)$$

where $g = \text{Det } g_{\mu\nu}$, R is the scalar curvature, ω is the coupling function and L_m represents the matter content of the system. We take this L_m to be the lagrangian density of a complex, massive, self-interacting scalar field. This lagrangian reads as:

$$L_m = -\frac{1}{2} g^{\mu\nu} \psi_{,\mu}^* \psi_{,\nu} - \frac{1}{2} m^2 |\psi|^2 - \frac{1}{4} \lambda |\psi|^4. \quad (2)$$

Varying the action with respect to the dynamical variables $g^{\mu\nu}$ and ϕ we obtain the field equations:

$$R_{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = \frac{8\pi}{\phi} T_{\mu\nu} + \frac{\omega(\phi)}{\phi} \left(\phi_{,\mu} \phi_{,\nu} - \frac{1}{2} g_{\mu\nu} \phi^{,\alpha} \phi_{,\alpha} \right) + \frac{1}{\phi} (\phi_{,\mu;\nu} - g_{\mu\nu} \square \phi) \quad (3)$$

$$\square \phi = \frac{1}{2\omega + 3} \left[8\pi T - \frac{d\omega}{d\phi} \phi^{,\alpha} \phi_{,\alpha} \right], \quad (4)$$

where we have introduced $T_{\mu\nu}$ as the energy-momentum tensor for matter fields and T as its trace. This energy-momentum tensor is given by

$$T_{\mu\nu} = \frac{1}{2} (\psi_{,\mu}^* \psi_{,\nu} + \psi_{,\mu} \psi_{,\nu}^*) - \frac{1}{2} g_{\mu\nu} \left(g^{\alpha\beta} \psi_{,\alpha}^* \psi_{,\beta} + m^2 |\psi|^2 + \frac{1}{2} \lambda |\psi|^4 \right). \quad (5)$$

The covariant derivative of this tensor is null. That may be proved either from the field equations, recalling the Bianchi identities, or by intuitive arguments since the minimally coupling between the field ϕ and the matter fields. This implies:

$$\psi_{,\mu}^{;\mu} - m^2 \psi - \lambda |\psi|^2 \psi^* = 0. \quad (6)$$

We now introduce the background metric. That is the corresponding to a spherically symmetric system, because of the symmetry we impose upon the star. Then:

$$ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2d\Omega^2. \quad (7)$$

We also demand a spherically symmetric form for the field which describe the boson, *i.e.*, we adopt:

$$\psi(r, t) = \chi(r) \exp[-i\varpi t]. \quad (8)$$

Semiclassically, we are able to think about χ expanded in creation and annihilation operators and $T_{\mu\nu}$ as an expectation value in a given configuration with a large number of bosons [1]. Using the metric (7) and the equation defining the form of the boson field (8) together with the energy-momentum tensor (5) in the field equations (3,4) we get the equations of structure of the star. Before we explicitly write them we are going to introduce a rescaled radial coordinate by:

$$x = mr. \quad (9)$$

From now on, a prime will denote a differentiation with respect to the variable x . We also define dimensionless quantities by

$$\Omega = \frac{\varpi}{m}, \quad \Phi = \frac{\phi}{M_{pl}^2}, \quad \sigma = \sqrt{4\pi}\chi(r), \quad \text{and} \quad \Lambda = \frac{\lambda}{4\pi} \left(\frac{M_{pl}}{m} \right)^2, \quad (10)$$

where M_{pl} is the Planck mass. In order to consider the total amount of mass of the star within a radius x we change the function A in the metric to its Schwarzschild form:

$$A(x) = \left(1 - \frac{2M(x)}{x} \right)^{-1}. \quad (11)$$

Then, the total mass will be given by $M(\infty)$ and will corresponds to

$$M_{star} = \frac{M(\infty)}{m} M_{pl}^2, \quad (12)$$

for a given value of m . With all these definitions, the equations of structure reduce to the following set:

$$\sigma'' + \sigma' \left(\frac{B'}{2B} - \frac{A'}{2A} + \frac{2}{x} \right) + A \left[\left(\frac{\Omega^2}{B} - 1 \right) \sigma - \Lambda \sigma^3 \right] = 0 \quad (13)$$

$$\Phi'' + \Phi' \left(\frac{B'}{2B} - \frac{A'}{2A} + \frac{2}{x} \right) - \frac{2A}{2\omega + 3} \left[\left(\frac{\Omega^2}{B} - 2 \right) \sigma^2 - \frac{\sigma'^2}{A} - \Lambda \sigma^4 \right] + \frac{1}{2\omega + 3} \frac{d\omega}{d\Phi} \Phi'^2 = 0 \quad (14)$$

$$\begin{aligned} \frac{B'}{xB} - \frac{A'}{x^2} \left(1 - \frac{1}{A} \right) &= \frac{A}{\Phi} \left[\left(\frac{\Omega^2}{B} - 1 \right) \sigma^2 + \frac{\sigma'^2}{A} - \frac{\Lambda}{2} \sigma^4 \right] + \frac{\omega}{2} \left(\frac{\Phi'}{\Phi} \right)^2 - \frac{A}{\Phi} \frac{2}{2\omega + 3} \times \\ &\left[\left(\frac{\Omega^2}{B} - 2 \right) \sigma^2 - \frac{\sigma'^2}{A} - \Lambda \sigma^4 \right] + \left(\frac{\Phi''}{\Phi} - \frac{1}{2} \frac{\Phi'}{\Phi} \frac{A'}{A} \right) + \frac{1}{2\omega + 3} \frac{d\omega}{d\Phi} \frac{\Phi'^2}{\Phi} \end{aligned} \quad (15)$$

$$\begin{aligned} \frac{2BM'}{x^2} &= \frac{B}{\Phi} \left[\left(\frac{\Omega^2}{B} + 1 \right) \sigma^2 + \frac{\sigma'^2}{A} + \frac{\Lambda}{2} \sigma^4 \right] + \frac{\omega B}{2A} \left(\frac{\Phi'}{\Phi} \right)^2 + \frac{B}{\Phi} \frac{2}{2\omega + 3} \times \\ &\left[\left(\frac{\Omega^2}{B} - 2 \right) \sigma^2 - \frac{\sigma'^2}{A} - \Lambda \sigma^4 \right] - \frac{B}{A(2\omega + 3)} \frac{d\omega}{d\Phi} \frac{\Phi'^2}{\Phi} - \frac{1}{2} \frac{\Phi'}{\Phi} \frac{B'}{A}. \end{aligned} \quad (16)$$

It is important to note that these equations reduce to the known BD ones of reference [25] when ω is taken as a constant and to those of GR of reference [2] when $\Phi \rightarrow \Phi_0$, constant.

B. Numerical procedure and boundary conditions

We shall carry out a numerical integration from the centre of the star outwards towards radial infinity. The boundary condition for the system are the following. Concerning σ , we require a finite mass, which implies $\sigma(\infty) = 0$ and non-singularity at the origin, *i.e.* $\sigma(0)$ a finite constant and $\sigma'(0) = 0$. We shall look for zero node solutions because, as remarked in [2], it is reasonable to suppose them as the lowest energy bound states. We shall demand asymptotic flatness, which means $B(\infty) = 1$ and $A(\infty) = 1$, this last condition, ensured by the equations themselves. Non-singularity at the origin also requires $M(0) = 0$. Finally, $\Phi(\infty)$ must take the value of Φ in an appropriate cosmological model at the time of stellar formation. If not otherwise specified, it will be considered as 1. Note that this boundary condition differs from the others in being less restrictive, in fact, preliminarily, it impose only an *acceptable boundary value prescription*. We shall manage this boundary condition by stopping the integration at the asymptotic region, where $\Phi = \Phi(\infty)$, ($\pm 10^{-6}$), and the derivative $\Phi'(\infty)$ tends to zero. All these boundary conditions generate an eigenvalue problem for Ω . In order to obtain accurate results, this eigenvalue has to be specified with

at least seven significant figures in a typical case, this is within the capability of a double precision numerical method and was also the case in general relativistic cases [8]. Note that due to the form of the equations, which are linear in B , we can integrate the system without impose the boundary condition on B from the beginning. Instead, we ultimately rescale B and Ω in order to satisfy flatness requirements.

The numerical method we shall use is a fourth order Runge-Kutta and is based in the recipes of [27]. Some of the subroutines were modified in order to test the possibility of satisfying the boundary conditions at each step of the integration (see [1] for details on this) and some others were built in order to search for the eigenvalue and satisfy the specific boundary conditions. The program was tested in the limiting cases of equations (13-16), *i.e.* GR and BD, and agreement was found with reported results.

III. THE COUPLING FUNCTION

As there is no *a priori* prescription about the form or the value of ω , we are interested in ascertaining the general behaviour displayed by a wide range of scalar-tensor stars. The first Group of couplings we are going to analyse have the property of tending to infinity as $\phi \rightarrow \phi_0$, where ϕ_0 may be taken as the present value $\phi(t)$ or, equivalently, as the inverse of the Newton constant. We shall take the forms that Barrow and Parsons recently analysed in a cosmological setting [28]. They are:

- Theory 1. $2\omega + 3 = 2B_1|1 - \phi/\phi_0|^{-\alpha}$, with $\alpha > 0$ and $B_1 > 0$ constants.
- Theory 2. $2\omega + 3 = 2B_2 \ln |\phi/\phi_0|^{-2\delta}$, with $\delta > 0$ and $B_2 > 0$ constants.
- Theory 3. $2\omega + 3 = 2B_3|1 - (\phi/\phi_0)^\beta|^{-1}$, with $\beta > 0$ and $B_3 > 0$ constants.

The behavior of these theories in a FRW metric was analitically studied in [28] and weak field limit constraints upon the parameters were provided there also. Note that, while using these couplings in our equations, the functions dependences shift from cosmic time to radial

coordinate. Cosmological solutions for this group allows ϕ approach to ϕ_0 from below, *i.e.* $\phi \in (0, \infty)$ or from above, $\phi \in (\infty, 0)$. This implies that the boundary condition in Φ may be equal to, less than or bigger than 1. Theories 1.–3. approach BD when $\Phi \rightarrow 0$ and to next theory (Theory 4.) when $\Phi \rightarrow \infty$. Note also that the weak field constraints are, in fact, independent of the form of the cosmological solutions provided $\Phi \rightarrow \Phi_0$, when t is big enough. It is this latter requirement which introduce further restrictions upon the the parameter space, specially in the exponents, which vary as a function of the cosmic era [28].

The second Group of coupling functions will be represented by:

- Theory 4. $2\omega + 3 = \omega_0 \phi^n$, with $n > 0$ and ω_0 constants.

It also has analytical solutions [20] and even we know nucleosynthesis bounds for it [18]. This group differs from the first in that, although growing with time, they only reach GR when $\phi \rightarrow \infty$, that is, when $t \rightarrow \infty$, ϕ do not tends to ϕ_0 . To normalize we may set $\phi(t = \text{today}) = 1$.

Finally, the third Group we shall analyse consists of *local implicitly defined functions* of the form:

- Theory 5. $\omega = \omega(x) = \omega(x(\phi))$

The aim in doing so is explicitly see how can one manage the behavior of ω within the radius of the star. Note that these are implicit definitions, being ultimately necessary to invert $\phi(x)$ to get the correct dependence of the coupling function. If $\phi(x)$ is a monotonous function, then, the existence of this inverse is analitically ensured. It is worth recalling that the limit $x \rightarrow \infty$ if of crucial importance. Far out from the star we would want to recover a scalar-tensor theory with a cosmological well-behaved evolution.

IV. RESULTS

A. Group I and II Couplings

Although formally and conceptually different, when considering a cosmological setting, the theories described as Group I and Group II resulted to be similar concerning boson stars solutions. In addition, most of the similarities arise inside Group I couplings, where simulations based on these theories suggest that anyone can be mapped into the other for convenient choices of each particular set of their free parameters. Concerning Theory 4. some differences have to be remarked, and so we do below, but its general behaviour do not differ very much from Group I couplings. Taking this into account, we shall present in deep detail only the case of Theory 1. and we shall make some comments on others special situations.

We shall first consider boson stars based on Theory 1. gravitation. We take $B_1 = 5$ and $\alpha = 2$ and look for models in equilibrium for different values of the central density and strength of the self-interaction. What we found is sketched in Fig. 1. Recall that the value $\alpha = 2$ is one extremum of the interval which admits convergency to GR in a cosmological evolution with a perfect fluid model as matter source ($\gamma \in (0, 4/3); p = (\gamma - 1)\rho$) [28]. We can note from there that the general form of the graph is preserved when compared with both, GR and BD cases. The boson stars masses increase from the BD case with $\omega = 6$ presented by Gunderson and Jensen. In fact, results for values of α greater than 1 are extremely similar to those of general relativity and the BD scalar is almost numerically constant along the boson star structure. This is something that could be expected in the case solutions may exist, because of the rapidly approaching scheme to GR that Theory 1. develops when α is big enough. Things change when considering values of α smaller than 1. Table I presents computations for models with $B_1 = 5$ and $\alpha = 0.5$. Recall that this value of α is the smaller value that preserves the weak field limit in a cosmological evolution and one of the extremum which guarantees convergency to GR in the case of the radiation era [28]. Note that the equilibrium configuration in each case always choice a bigger value of Φ at the center of the star, which implies less gravitationally bounded objects than those

of GR. This was also the case of BD models. The values of the masses are smaller than GR ones but still greater than the ones which do not behave as required for a cosmological setting, as for instance the BD case with $\omega = 6$. So, Theory 1. has viable solutions for boson stars structures where masses are compatible with simplest cases. Concerning the behaviour of σ as a function of x , it has the same convexity properties commented for GR and BD models. Fig. 2. shows its behaviour for typical values of the parameter space. The same does Fig. 3. for the behaviour of Φ . The dependences of the masses of the equilibrium structures upon the parameters of the gravitational theory was tested in further detail. It was found that for values of α greater than 1, changes in B_1 do not produce noticeable changes in the mass. The opposite happens for smaller values of α . Table II represents these trends in a more quantitative form for $\Lambda = 100$ and $\sigma(0) = 0.100$. Finally, we address the possible variation of $M(\infty)$ with a deviation in the boundary condition for Φ , the effective gravitational constant far out from the star. This is aimed in getting a first insight of possible boson stars formation along different eras of cosmic evolution. As Theory 1. admits cosmological solutions with values of Φ greater or smaller than 1 we consider both cases as possible boundary conditions. Table III shows $M(\infty)$ and the corresponding $\Phi(0)$ value for each choice of the boundary condition in three particular models. It is interesting to note that, in the first place, *masses are sensitive to variations in the boundary condition of the scalar within a few percent as typical order of magnitude*, and second, *the behavior of models varies with Λ* . If Λ is big enough (greater than 10) a growing mass appears with a growing boundary condition. Otherwise, the models show a peak in the masses within the range explored for $\Phi(\infty)$.

Concerning Theory 4. it has to be noted that the parameter space is not mainly constrained by weak field test [20], which do not limit the values of n –provided $n > 0$ –, but it is by nucleosynthesis processes [18]. This bounds, which resulted in lower limits for n , are provided once the cosmological parameters Ω_0 and the Hubble constant H_0 are given. A common characteristic of these Group is that masses of equilibrium configurations are smaller than the cases previously studied and much more smaller when compared with GR.

A typical example is the $\Lambda = 100$ and $\sigma(0) = 0.100$ model. For $\omega_0 = 2$ and $n = 3$, $M(\infty) = 1.870$. Note, however, that this value of n produce acceptable nucleosynthesis consequences only in a range of $\Omega_0 h^2 < 0.25$. Another thing to note is that Theory 4. resulted in the ones which more dependence on the parameters show for small values of n and ω_0 . There, variations may reach a typical 10% in mass.

B. Group III Couplings

The third and last group of couplings we shall analyse consists in implicitly defined functions of the form of Theory 5. As an example we choose several forms of the couplings, results for them can be seen in Table IV, for the model given by $\Lambda = 0$ and $\sigma(0) = 0.325$. These functions are enough to get a feeling of which the idea is. Locally there is no prescription upon ω , while far out from the star we would want to recover a scalar-tensor theory cosmologically well-behaved. Scalar-tensor theories which deviates more from GR are those which can be compared with BD theories of small ω . The choice of the different functions is focused to encompass GR at the asymptotic region while admitting severe deviation inside the structure. For all cases, when $x \rightarrow \infty$, $\omega \rightarrow \infty$, making these theories cosmologically acceptable. If not otherwise specified, all cases present monotonous Φ -functions. The correct dependence of the coupling, $\omega(\Phi)$, may be lately obtained from the inverse of the function $x(\Phi)$. It is worth recalling that the dependence of ω with Φ changes whenever the model change. For instance, in passing through different values of Λ , the functional form of $\Phi(x)$ change, and the same does its inverse. This implies that even without changing $\omega = \omega(x)$ we are changing $\omega = \omega(\Phi)$. For the cases studied in Table IV we found that the order of magnitude of boson stars masses remains the same, although some cases with very small masses may arise –also in the cases of $\Lambda \neq 0$. These, in general, mild variations in the boson star properties must be explained in terms of the complex structure of the differential system. The terms proportional to the derivatives of the couplings are also proportional to the derivative of Φ , which in turn must be obtained from the solution of the system.

V. CONCLUSIONS

In the last few years, the possibility of constructing complete cosmologies, by encompassing exact analytical solutions of general scalar-tensor gravitation, have raised an enormous interest on these kind of theories, which has to be added to the developed by the applicability of them to inflationary scenarios. Once the cosmological setting is fixed, we have to analyse possible astrophysical consequences of having, for instance, a different value for the gravitational constant or a different rate of expansion. As an example, we should mention a recent work about primordial formation and evaporation of black holes [29].

In this work, we have analysed the possible existence of boson stars solutions within the framework of general scalar-tensor theories. It then extends a previous paper by Gunderson and Jensen [25] where solutions to the Brans-Dicke equations were considered. We have shown the theoretical construction of such systems in general cases of alternative gravity, all them contrasted with the gravity tests known up to date. Different kind of couplings with exact cosmological solutions and others that allow a significant variation within the radius of the star were considered. We found that the order of magnitude of the general relativistic boson star masses do not vary when these more realistic cases of scalar-tensor gravity are the basis of the gravitational theory. In general, and because general forms of couplings can be expanded in the form of a series of Group I and/or Group II couplings, we may state that boson stars might exist for any of these gravitational settings. We also found an interesting situation concerning the evolution of boson stars masses as a function of the time of formation of the stellar object. It appreciable vary, within a typical few percent, when cosmological time scales are considered. If this fact may provide a useful basis for searching new observational consequences and/or bounds upon the coupling function is currently under study. Finally it remains to be considered the question of stability, for which we do not know results, even in the Brans-Dicke case. We hope to report on it in a forecoming work.

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TABLES

TABLE I. Boson stars masses for Theory 1. with $B_1 = 5$ and $\alpha = 0.5$.

Λ	$\sigma(0)$	$B(0)$	$\Phi(0)$	$M(\infty)$
0	0.325	0.4231	1.0007	0.627
10	0.225	0.4163	1.0010	0.919
100	0.100	0.3853	1.0011	2.248
200	0.070	0.4256	1.0009	3.128

TABLE II. Dependences of boson stars masses upon the parameter space for Theory 1. with $\Lambda = 100$, $\sigma(0) = 0.100$.

B_1	α	$\Phi(0)$	$M(\infty)$
2	0.5	1.0053	2.245
5	0.5	1.0010	2.248
8	0.5	1.0004	2.249
2	1.0	1.0000	2.250
8	1.5	1.0000	2.250
8	2.0	1.0000	2.250

TABLE III. Boson stars masses as a function of the boundary condition for Φ . First set shows the model with $\Lambda = 100$ and $\sigma(0) = 0.100$, the second set shows: $\Lambda = 10$ and $\sigma(0) = 0.225$, while the third, $\Lambda = 0$ and $\sigma(0) = 0.325$. These models are for Theory 1. with $B_1 = 5$ and $\alpha = 0.5$.

$\Phi(\infty)$	$\Phi(0)$	$M(\infty)$	$\Phi(0)$	$M(\infty)$	$\Phi(0)$	$M(\infty)$
0.90	0.9127	2.096	0.9128	0.875	0.9111	0.610
0.95	0.9593	2.164	0.9593	0.893	0.9581	0.616
1.00	1.0009	2.253	1.0010	0.920	1.0007	0.627
1.05	1.0615	2.263	1.0617	0.916	1.0600	0.618
1.10	1.1167	2.295	1.1170	0.921	1.1145	0.614

TABLE IV. Masses of boson stars for implicitly defined scalar-tensor theories. Results for the model $\Lambda = 0$ and $\sigma(0) = 0.325$ are shown. A small star point that the Φ -function is not monotonous, typically in the innermost region. Boundary condition on the BD scalar was setted equal to 1 although deviations provided by an asymptotic derivative of Φ of order 10^{-4} were accepted.

Theory 5.: $\omega = \omega(x)$	$M(\infty)$	$\Phi(0)$
0.1 x	0.538	1.1011
10 x	0.624	1.0075
$\log(x)^*$	0.577	1.0754
$\exp(0.01x)$	0.539	1.0760

FIGURES

FIG. 1. Boson stars masses of Theory 1. for $B_1 = 5$ and $\alpha = 2$ and different values of Λ and $\sigma(0)$. There are 34 models for each value of Λ . Numerical values in this graph are very similar to the ones derived for General Relativity boson stars masses.

FIG. 2. Behaviour of σ as a function of the radial coordinate for two typical models of scalar-tensor boson stars; Theory 1. with $B_1 = 5$ and $\alpha = 0.5$.

FIG. 3. Behaviour of Φ as a function of the radial coordinate for two typical models of scalar-tensor boson stars; Theory 1. with $B_1 = 5$ and $\alpha = 0.5$.

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