

Field-theoretical approach to non-local interactions: 1d electrons and fermionic impurities

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Abstract

We apply a recently proposed path-integral approach to non-local bosonization to a Thirring-like system modeling non-relativistic massless particles interacting with localized fermionic impurities. We consider forward scattering processes described by symmetric potentials including interactions between charge, current, spin and spin-current densities. In the general (spin-flipping) problem we obtain an effective action for the collective modes of the model at $T = 0$, containing WZW-type terms. When spin-flipping processes are disregarded the structure of the action is considerably simplified, allowing us to derive exact expressions for the dispersion relations of collective modes and two point fermionic correlation functions as functionals of the potentials. Finally, as an example, we compute the momentum distribution for the case in which electrons and impurities are coupled through spin and spin-current densities only. The formulae we get suggest that our formalism could be useful in order to seek for a mechanism able to restore Fermi liquid behavior.

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1 Introduction

In the last years there has been a renewed interest in the study of low-dimensional field theories. This is due, in part, to crucial advances in the field of nanofabrication which have allowed to build ultranarrow semiconductor structures [1] in which the motion of the electrons is confined to one dimension [2]. One of the main tools for the theoretical understanding of the one-dimensional (1d) electron system is the Tomonaga-Luttinger (TL) model [3],[4], which can be considered as the paradigm of Luttinger Liquid (LL) behavior [5],[2]. This model describes a non-relativistic gas of massless particles (the electrons) with linear free dispersion relation and two-body, forward-scattering interactions.

In a recent work [6], we have presented a Thirring-like model with fermionic currents coupled by general (symmetric) bilocal potentials. We were able to obtain a completely bosonized effective action for this non-local field theory, and we showed that it contains the TL model as a special case.

The main purpose of the present article is to extend the path-integral approach to non-local bosonization proposed in [6], to the case in which an interaction between the electrons and a finite density of fermionic impurities is included in the action. This generalization of the non-local bosonization procedure provides a new way to examine the low-energy physics of the TL model in the presence of localized impurities, that could allow to make contact with recent very interesting studies [7],[8] on the response of a LL to localized perturbations.

We describe the impurities following the work of Andrei [9], who introduced a new fermionic field with vanishing kinetic energy to represent a finite density of impurities, arbitrarily (not randomly) situated. This treatment has been previously employed, for example, in the path-integral bosonization of the Kondo problem [10].

We introduce a non-local diagonal potential matrix binding impurities and electrons through their corresponding fermionic currents. This procedure allows to treat a wide range of possible interactions, depending on the precise functional form of the potential matrices. The complete coupling term includes interactions between charge, current, spin and spin-current densities. Our functional approach enables us to obtain an effective action governing the dynamics of the collective modes, providing then a practical framework to face a non-perturbative analysis of bosonic degrees of freedom

in the presence of impurities. We start from the partition function

$$Z = \int D\bar{\Psi} D\Psi D\bar{d} Dd e^{-S}, \quad (1.1)$$

where the action S can be splitted as

$$S = S_0 + S_{int}, \quad (1.2)$$

with

$$S_0 = \int d^2x [\bar{\Psi} i\not{\partial}\Psi + d^\dagger i\partial_t d] \quad (1.3)$$

and

$$S_{int} = - \int d^2x d^2y [J_\mu^a(x) V_{(\mu)}^{ab}(x, y) J_\mu^b(y) + J_\mu^a(x) U_{(\mu)}^{ab}(x, y) S_\mu^b(y)], \quad (1.4)$$

where the electron field Ψ is written as

$$\Psi = \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix},$$

with Ψ_1 (Ψ_2) in the fundamental representation of $U(N)$, describing right (left) movers, whereas the impurity field d is given by

$$d = \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}.$$

Note the absence of a spatial derivative in the free piece of the impurity action, meaning that the corresponding kinetic energy is zero. Concerning the electronic kinetic energy, we have set the Fermi velocity equal to 1.

The interaction pieces of the action have been written in terms of $U(N)$ currents J_μ^a and S_μ^a , defined as

$$\begin{aligned} J_\mu^a &= \bar{\Psi} \gamma_\mu \lambda^a \Psi, \\ S_\mu^a &= \bar{d} \gamma_\mu \lambda^a d, \quad a = 0, 1, \dots, N^2 - 1, \end{aligned} \quad (1.5)$$

with $\lambda^0 = I/2$, $\lambda^j = t^j$, t^j being the $SU(N)$ generators normalized according to $tr(t^i t^j) = \delta^{ij}/2$. $V_{(\mu)}^{ab}(x, y)$ and $U_{(\mu)}^{ab}(x, y)$ are $N^2 \times N^2$ matrices whose elements are symmetric bilocal arbitrary potentials describing the electron-electron (e-e) and the electron-impurity (e-i) interactions, respectively. Note that no sum over repeated indices is implied when a subindex (μ) is involved.

In the next Section we shall show how to extend the above mentioned approach to non-local bosonization [6], originally developed for the impurity-free Thirring model, to the case in which e-i interactions are considered. We will express the partition function (1.1) in terms of two fermionic determinants, one associated to conduction electrons and the other one to the fermionic impurities.

In Section 3 we solve these determinants for the non-Abelian ($U(2)$) case. This model describes the general forward-scattering problem of spin 1/2 electrons coupled to impurities, with both e-e and e-i spin-changing interactions. We obtain an effective bosonic action including Wess-Zumino-Witten-type terms. Although the structure of this action is quite involved, it exhibits spin-charge decoupling explicitly.

In Section 4 we specialize the results of previous sections to the case in which the maximal Abelian subgroup of $U(2)$ is considered. This model represents a many-body system of spin 1/2 particles when spin-flipping processes are disregarded but non-trivial interactions with fermionic impurities are taken into account. In Subsection 4.1 we obtain the completely bosonized action describing the collective modes of the system. From this action we get the dispersion relations of these oscillations as functions of e-e and e-i interaction potentials. Two-point fermionic correlations and electronic momentum distributions are computed in 4.2 and 4.3, respectively. In this last Subsection we illustrate a possible application of our work by discussing a "potential tuning" mechanism that leads to a restoration of the Fermi edge. Although we were able to show this effect only for a very special choice of the couplings, which evidently weakens its experimental relevancy, we think it deserves attention as a first step towards a reconciliation between the standard TL model and the FL phenomenology.

In Section 5 we summarize our results and conclusions.

We also include an Appendix that gathers the main results on fermionic and bosonic Jacobians associated to the changes of variables used throughout the paper.

2 Partition function in terms of fermionic determinants

Our first goal is to express the functional integral (1.1) in terms of fermionic determinants. To this end we define new currents

$$K_\mu^a(x) = \int d^2y V_{(\mu)}^{ab}(x, y) J_\mu^b(y), \quad (2.1)$$

and

$$P_\mu^a(x) = \int d^2y U_{(\mu)}^{ab}(x, y) J_\mu^b(y). \quad (2.2)$$

The usual procedure in order to match the quartic interactions between fermions consists in introducing auxiliary fields. In the present case the action contains two terms quadratic in currents, so one needs two fields $A_\mu^a(x)$ and $B_\mu^a(x)$, which are incorporated in the functional integrand in the form

$$\begin{aligned} Z &= \int D\bar{\Psi} D\Psi D\bar{d} Dd e^{-S_0} \\ &\int DA_\mu^a \delta[A_\mu^a - K_\mu^a] \exp\left[\int d^2x J_\mu^a A_\mu^a\right] \\ &\int DB_\mu^a \delta[B_\mu^a - P_\mu^a] \exp\left[\int d^2x S_\mu^a B_\mu^a\right]. \end{aligned} \quad (2.3)$$

In order to fix the number of impurities to be n , we introduce a Lagrange multiplier a , enforcing the constraint $d^\dagger d = n$. It becomes then natural to redefine the free piece of the action as

$$S'_0 = S_0 + \int d^2x a(d^\dagger d - n). \quad (2.4)$$

On the other hand, we represent the δ functionals as integrals of exponentials over two new fields $C_\mu^a(x)$ and $D_\mu^a(x)$, thus obtaining

$$\begin{aligned} Z &= \int D\bar{\Psi} D\Psi D\bar{d} Dd Da DA_\mu^a DB_\mu^a e^{-S'_0(\Psi, d)} \\ &\exp\left[\int d^2x J_\mu^a A_\mu^a + \int d^2x S_\mu^a B_\mu^a\right] \int DC_\mu^a \exp\left[-\int d^2x (A_\mu^a - K_\mu^a) C_\mu^a\right] \\ &\int DD_\mu^a \exp\left[-\int d^2x (B_\mu^a - P_\mu^a) D_\mu^a\right]. \end{aligned} \quad (2.5)$$

At this point one sees that the fermionic piece of the action (the free part and the terms involving the currents J, S, K and P) can be cast in a local form by defining the "potential transformed" fields

$$\bar{C}_\mu^a(x) = \int d^2y V_{(\mu)}^{ab}(x, y) C_\mu^b(y), \quad (2.6)$$

$$\bar{D}_\mu^a(x) = \int d^2y U_{(\mu)}^{ab}(x, y) D_\mu^b(y), \quad (2.7)$$

We then get

$$\begin{aligned} Z = & \int D\bar{\Psi} D\Psi D\bar{d} Dd Da DA DB D\bar{C} D\bar{D} \\ & \exp\left\{-\int d^2x [\bar{\Psi}(i\rlap{\not{D}} + (\rlap{\not{A}} + \bar{C} + \bar{D}))\Psi + \bar{d}(i\gamma_0\partial_t + \gamma_0 a + \rlap{\not{B}})d + \right. \\ & \left. + C_\mu^a(x)A_\mu^a(x) + B_\mu^a(x)D_\mu^a(x) - a.n]\right\}. \end{aligned} \quad (2.8)$$

This equation, in turn, suggests the following change of variables

$$\begin{aligned} A_\mu^a + \bar{C}_\mu^a + \bar{D}_\mu^a &= \tilde{A}_\mu^a, \\ A_\mu^a - \bar{C}_\mu^a + \bar{D}_\mu^a &= \tilde{C}_\mu^a, \\ A_\mu^a + \bar{C}_\mu^a - \bar{D}_\mu^a &= \tilde{D}_\mu^a, \end{aligned} \quad (2.9)$$

giving

$$\begin{aligned} Z = & \int D\tilde{A} DB D\tilde{C} D\tilde{D} \det(i\rlap{\not{D}} + \tilde{A}) \det(i\gamma_0\partial_t + i\gamma_0 a + \rlap{\not{B}}) \\ & \int Da \exp \int d^2x a.n \exp \int d^2x d^2y \left\{ -\frac{a_{(\mu)}^{ab}(x, y)}{2} [\tilde{A}_\mu^a(x) - \tilde{D}_\mu^a(x)] B_\mu^b(y) \right\} \\ & \exp \int d^2x d^2y \left\{ -\frac{b_{(\mu)}^{ab}(x, y)}{4} [\tilde{A}_\mu^a(x) - \tilde{C}_\mu^a(x)] [\tilde{D}_\mu^b(y) + \tilde{C}_\mu^b(y)] \right\}, \end{aligned} \quad (2.10)$$

where we have defined the inverse potentials a and b through the identities

$$\int V_{(\mu)}^{ac}(x, y) b_{(\mu)}^{cd}(x, z) d^2x = \delta^{ad} \delta^{(2)}(y - z), \quad (2.11)$$

$$\int U_{(\mu)}^{ac}(x, y) a_{(\mu)}^{cd}(x, z) d^2x = \delta^{ad} \delta^{(2)}(y - z), \quad (2.12)$$

In the above expression for Z one sees that, by virtue of the change of variables (2.9), the fields \tilde{C} and \tilde{D} play no direct role in the fermionic determinants. They are actually artefacts of our method, whereas the fields A and

B describe the physically relevant bosonic degrees of freedom. Therefore, the next step is to perform the integrals in \tilde{C} and \tilde{D} . This can be easily done, as usual, by conveniently shifting the fields. The field \tilde{C} describes negative metric states, a situation already encountered in the impurity-free problem [6]. The important difference is that here the ghost-field is coupled with both A and B . One has then to first perform the corresponding integration in order to extract the dependence on the relevant fields, and only when this is done, to absorb the decoupled ghost partition function in the overall normalization constant. This treatment of the negative metric states is in agreement with the prescription of Klaiber [11] which, in the operator framework, precludes the use of an indefinite-metric Hilbert space. Taking all these considerations into account, and setting from now on $\tilde{A} = A$, one finally gets

$$Z = \int DA DBDa e^{-S'[A,B]} \det(i\cancel{\partial} + \cancel{A}) \det(i\gamma_0 \partial_t + a\gamma_0 + \cancel{B}). \quad (2.13)$$

with

$$\begin{aligned} S'[A, B] = & \int d^2x a.n + \int d^2x d^2y [B_\mu^a(x) a_{(\mu)}^{ab}(x, y) A_\mu^b(y) + \\ & - B_\mu^a(x) \int d^2u d^2v a_{(\mu)}^{ac}(x, u) (b_{(\mu)}^{-1})^{cd}(u, v) a_{(\mu)}^{db}(v, y) B_\mu^b(y)] \end{aligned} \quad (2.14)$$

This is our first non-trivial result. We have been able to express Z in terms of two fermionic determinants, one dependent on the original electronic fields coupled to A , and the other one defined through the impurity degrees of freedom interacting with a and B . Note that although these fermionic determinants involve independent bosonic fields, the bosonic action provides an interaction term coupling them. The important point is that now, the non-local character of the interactions is entirely located in this bosonic action, whose functional form depends on the original couplings. Interestingly enough, however, one can go further quite a long way without specifying the potentials. This will be shown in the next Sections, where we shall deal explicitly with the determinants.

3 The spin-flipping (non-abelian) case

In the context of 2d QFT's, the bosonization procedure [12], originally developed in the operator language [13], has been also fruitfully implemented in

the path-integral framework [14]. In this Section we show how to use this last method, based on decoupling changes in the functional measures, in order to get an effective bosonic action for the general forward-scattering problem defined in the previous sections. The technique depends non-trivially on the nature of the groups involved. Here we consider the case in which fermion fields are taken in the fundamental representation of U(2) and the vector fields take values in the Lie algebra of U(2). Although our procedure can be easily extended to the U(N) case, bearing in mind that we want to make contact with the spin 1/2 TL model, we set $N = 2$ from the beginning.

Let us start by considering the first determinant in (2.13), associated with the conduction electrons. We decompose the field A_μ as

$$A_\mu = \Lambda_\mu + \mathcal{A}_\mu, \quad (3.1)$$

with

$$\Lambda_\mu = \Lambda_\mu^0 \lambda^0, \quad (3.2)$$

and

$$\mathcal{A}_\mu = \mathcal{A}_\mu^i \lambda^i, \quad (3.3)$$

where $\lambda^0 = I/2$ and λ^i ($i = 1, 2, 3$) are the SU(2) generators. The U(1) part of the vector field can be decoupled from the fermionic fields by performing the following change in the path-integral variables:

$$\begin{aligned} \Psi(x) &= e^{i\eta^0(x) + \gamma_5 \Phi^0(x)} \chi \\ \bar{\Psi}(x) &= \bar{\chi} e^{-i\eta^0(x) + \gamma_5 \Phi^0(x)} \\ \Lambda_\mu^0(x) &= -\epsilon_{\mu\nu} \partial_\nu \Phi^0 + \partial_\mu \eta^0. \end{aligned} \quad (3.4)$$

where Φ^0 and η^0 are scalars. The Jacobian associated to the change in the bosonic variables is trivial, whereas that related to the fermionic variables gives

$$J_{U(1)}^F = \exp -\frac{1}{2\pi} \int d^2x (\partial_\mu \Phi^0)^2. \quad (3.5)$$

We then have

$$\int DA_\mu \det(i\cancel{\partial} + \mathcal{A}) = \int D\Phi^0 D\eta^0 D\mathcal{A} e^{-\frac{1}{2\pi} \int d^2x (\partial_\mu \Phi^0)^2} \det(i\cancel{\partial} + \mathcal{A}) \quad (3.6)$$

As explained in the Appendix, the SU(2) components of the vector field can be also decoupled by using a non-Abelian extension of equations (3.4) in the

form

$$\begin{aligned}
\Psi_1 &= h^{-1}\chi_1, \\
\bar{\Psi}_1 &= \bar{\chi}_1 h, \\
\Psi_2 &= g^{-1}\chi_2, \\
\bar{\Psi}_2 &= \bar{\chi}_2 g, \\
\mathcal{A}_+ &= h^{-1}\partial_+ h \\
\mathcal{A}_- &= g^{-1}\partial_- g
\end{aligned} \tag{3.7}$$

where g and h are elements of $SU(2)$ and $\partial_{\pm} = \partial_0 \pm i\partial_1$. In contrast to the Abelian change, now both the bosonic and fermionic Jacobians are non-trivial. Gathering their contributions one obtains:

$$\begin{aligned}
\int D\mathcal{A} \det(i\cancel{\partial} + \mathcal{A}) &= \det(i\cancel{\partial}) \int Dh Dg \exp\{-5(W[hg^{-1}] + \\
&+ \alpha \operatorname{tr} \int d^2x h^{-1}\partial_+ h g^{-1}\partial_- g)\}
\end{aligned} \tag{3.8}$$

where W is the Wess-Zumino-Witten action defined in the Appendix and α is a regularization parameter (for a careful study of regularization ambiguities see [15]).

The evaluation of the determinant corresponding to the impurity field is more subtle, and it can be performed through different methods. (See for example the Wilson loop approach and the geometric quantization described in [10]). In the present context we prefer to apply a technique similar to the analytic continuation sketched in [10]. As a first step, by writing the scalar field a in terms of a new bosonic field Φ as

$$a = 2i\partial_t \Phi, \tag{3.9}$$

it is easy to decouple it from the remaining determinant

$$\det(i\gamma_0\partial_t + \gamma_0 a + \cancel{\mathcal{B}}) = \exp\left\{\frac{1}{\pi} \int d^2x (\Phi\partial_t^2\Phi)\right\} \det(i\gamma_0\partial_t + \cancel{\mathcal{B}}) \tag{3.10}$$

while the change in the path-integral measure reads

$$Da = \det(2i\partial_t) D\Phi. \tag{3.11}$$

Note that the integral in the bosonic field Φ can be performed and the result absorbed in an overall normalization constant \mathcal{N} .

As in the previous case we write the field as the sum of two components, one abelian vector field Δ_μ and three vector fields \mathcal{B}_μ^i . In order to decouple the field B one can write the determinant in a factorized form

$$\begin{aligned} \det(i\gamma_0\partial_t + \mathcal{B}) &= (\det i\cancel{\phi})^{-1} \det \begin{pmatrix} 0 & i\partial_+ \\ i\partial_t + B_- & 0 \end{pmatrix} \\ &\quad \det \begin{pmatrix} 0 & i\partial_t + B_+ \\ i\partial_- & 0 \end{pmatrix}. \end{aligned} \quad (3.12)$$

Now each one of these determinants can be easily decoupled from the Δ_\pm ($\Delta_\pm = \Delta_0 \pm i\Delta_1$) fields. In the first determinant it is the change of variables

$$\Delta_+^0 = 2i\partial_t\zeta^0 \quad (3.13)$$

which allows us to separate the U(1) field, whereas in the second one we choose

$$\Delta_-^0 = -2i\partial_t\theta^0. \quad (3.14)$$

The corresponding Jacobians read

$$\exp^{-\frac{1}{2\pi} \int d^2x \zeta^0 \partial_- \partial_t \zeta^0}, \quad (3.15)$$

and

$$\exp^{-\frac{1}{2\pi} \int d^2x \theta^0 \partial_t \partial_+ \theta^0} \quad (3.16)$$

respectively.

The resulting determinants depend on \mathcal{B} , and can be treated in a similar way as the Λ field to obtain

$$\int D\mathcal{B}_+ \det \begin{pmatrix} 0 & i\partial_t + \mathcal{B}_+ \\ \partial_- & 0 \end{pmatrix} = \det \begin{pmatrix} 0 & i\partial_t \\ i\partial_- & 0 \end{pmatrix} \int DU e^{-5\dot{W}[U]}, \quad (3.17)$$

with $\dot{W}[U]$ a WZW-type action given by

$$\dot{W}[U] = \frac{1}{8\pi} \text{tr} \int d^2x \partial_t U \partial_- U^{-1} + \Gamma[U], \quad (3.18)$$

(please see the Appendix for the definition of the functional $\Gamma[U]$). The bosonic field U is related to \mathcal{B}_+ through

$$\mathcal{B}_+ = U^{-1} \partial_+ U. \quad (3.19)$$

In the same way, defining \mathcal{B}_- as

$$\mathcal{B}_- = Q^{-1}\partial_-Q, \quad (3.20)$$

we can decouple the corresponding determinant in the form

$$\int D\mathcal{B}_- \det \begin{pmatrix} 0 & i\partial_+ \\ \partial_t + \mathcal{B}_- & 0 \end{pmatrix} = \det \begin{pmatrix} 0 & i\partial_+ \\ i\partial_t & 0 \end{pmatrix} \int DQ e^{-5\tilde{W}[Q]}, \quad (3.21)$$

where $\tilde{W}[Q]$ is another WZW-type action:

$$\tilde{W}[Q] = \frac{1}{8\pi} \text{tr} \int d^2x \partial_-Q \partial_t Q^{-1} + \Gamma[Q]. \quad (3.22)$$

Putting all these results together one finally has

$$\begin{aligned} Z &= \mathcal{N} \det(i\not{\partial}) \det(2i\partial_t) \det(\gamma_0 i\partial_t) \int D\Phi^0 D\eta^0 D\zeta^0 D\theta^0 e^{-S^0[\Phi^0, \eta^0, \zeta^0, \theta^0]} \\ &\quad \int Dh Dg DU DQ e^{-S_{eff}[h, g, U, Q]}, \end{aligned} \quad (3.23)$$

with

$$\begin{aligned} S^0[\Phi^0, \eta^0, \zeta^0, \theta^0] &= \frac{1}{2\pi} \int d^2x [(\partial_\mu \Phi^0)^2 - \zeta^0 \partial_t \partial_- \zeta^0 - \theta^0 \partial_t \partial_+ \theta^0] \\ &\quad + \int d^2x d^2y \left(\partial_1 \Phi^0 a_{(0)}^{00} i \partial_0 (\zeta^0 - \theta^0) - \partial_0 \Phi^0 a_{(1)}^{00} \partial_0 (\zeta^0 + \theta^0) + \right. \\ &\quad + \partial_0 \eta^0 i a_{(0)}^{00} \partial_0 (\zeta^0 - \theta^0) - \partial_1 \eta^0 a_{(1)}^{00} \partial_0 (\zeta^0 + \theta^0) + \partial_0 \zeta^0 c_-^{00} \partial_0 \zeta^0 + \\ &\quad \left. + \partial_0 \theta^0 c_-^{00} \partial_0 \theta^0 - 2\partial_0 \zeta^0 c_+^{00} \partial_0 \theta^0 \right) \end{aligned} \quad (3.24)$$

and

$$\begin{aligned} S_{eff}[h, g, U, Q] &= 5\{W[hg^{-1}] + \alpha \text{tr} \int d^2x h^{-1} \partial_+ h g^{-1} \partial_- g + \\ &\quad + \dot{W}[U] + \tilde{W}[Q]\} + \\ &\quad + \frac{1}{4} \int d^2x d^2y \left\{ (g^{-1} \partial_+ g)^a(x) (a_0 - a_1)^{ab}(x, y) (U^{-1} \partial_+ U)^b(y) + \right. \\ &\quad + (g^{-1} \partial_+ g)^a(x) (a_0 + a_1)^{ab}(x, y) (Q^{-1} \partial_- Q)^b(y) + \\ &\quad + (h^{-1} \partial_- h)^a(x) (a_0 + a_1)^{ab}(x, y) (U^{-1} \partial_+ U)^b(y) + \\ &\quad + (h^{-1} \partial_- h)^a(x) (a_0 - a_1)^{ab}(x, y) (Q^{-1} \partial_- Q)^b(y) \left. \right\} - \\ &\quad + \frac{1}{4} \int d^2x d^2y \left\{ (U^{-1} \partial_+ U)^a(x) (c_0 - c_1)^{ab}(x, y) (U^{-1} \partial_+ U)^b(y) + \right. \\ &\quad - 2(U^{-1} \partial_+ U)^a(x) (c_0 + c_1)^{ab}(x, y) (Q^{-1} \partial_- Q)^b(y) + \\ &\quad \left. - (Q^{-1} \partial_- Q)^a(x) (c_0 - c_1)^{ab}(x, y) (Q^{-1} \partial_- Q)^b(y) \right\}, \end{aligned} \quad (3.25)$$

where we have defined the following combinations of potential matrix elements:

$$(c_0 \pm c_1)^{ab}(x, y) = \int d^2u d^2v \quad \{ \quad a_0^{ac}(x, u)(b_0^{-1})^{cd}(u, v)a_0^{db}(v, y) \\ \pm \quad a_1^{ac}(x, u)(b_1^{-1})^{cd}(u, v)a_1^{db}(v, y) \}. \quad (3.26)$$

Equations (3.23),(3.24) and (3.25) constitute our second non-trivial result. We have obtained a completely bosonized effective action for the collective excitations of a TL model including both e-e and e-i spin-flipping interactions. Concerning the partition function of the model, it has been written as a product of a free determinant and two purely bosonic functionals: one in terms of fields Φ^0, η^0, ζ^0 and θ^0 , and the other, which includes the effects of non-locality, in terms of h, g, U and Q . The other two determinants appearing in (3.23) do not depend on the bosonic degrees of freedom and can then be absorbed in a normalization constant. Nevertheless, they must be taken into account when studying other aspects of the model as, for example, its conformal properties, or when implementing a finite-temperature analysis.

We can see from (3.23) that nonlocality arises only in the bosonic action for g, h, U and Q , and it includes a variety of interactions between the bosonic degrees of freedom, related through the arbitrary potentials.

Let us stress that, despite the involved form of the action, in our framework, the occurrence of spin and charge separation becomes evident for this general forward-scattering problem. Indeed, it is straightforward to show, for instance by direct comparison with the impurity-free case [6], that the fields Φ^0 and η^0 are related to the charge-density fluctuations of the system (please recall that they were introduced to parametrize Λ , the U(1) component of A , just the one associated to charge conservation). Thus, the factorized form of the partition function (3.23) is just a direct consequence of the decoupling between charge and spin-density modes. In other words, there is an explicit separation between J_μ^0 and J_μ^i fluctuations. The relationship linking these currents to charge and spin densities for branch i ($i = 1, 2$) particles can be seen from their definitions

$$\begin{aligned} \rho_i &= \Psi_{i\uparrow}^\dagger \Psi_{i\uparrow} + \Psi_{i\downarrow}^\dagger \Psi_{i\downarrow}, \\ \sigma_i &= \Psi_{i\uparrow}^\dagger \Psi_{i\uparrow} - \Psi_{i\downarrow}^\dagger \Psi_{i\downarrow}. \end{aligned} \quad (3.27)$$

By explicitly writing the currents of the conduction electrons J_μ^a ($a = 0, 1, 2, 3$) defined in (1.5) we can identify $J_0^0 = \rho_1 + \rho_2$ and $J_1^0 = i(\rho_2 - \rho_1)$ as those

connected with charge densities, and $J_0^1 = \sigma_1 + \sigma_2$ and $J_1^1 = i(\sigma_2 - \sigma_1)$ with spin densities. The remaining currents, J_μ^2 and J_μ^3 , mix spin-down and up fermionic components; they are associated to spin conservation in processes with spin flips.

4 Bosonization without spin-flips (the maximal abelian subgroup)

When one considers the maximal abelian subgroup of $U(N)$, the interactions between currents acquire a simple form. Although our procedure can be easily used for arbitrary N , we shall perform the analysis as in the previous Section, for the special case $N = 2$. In this case the model describes a many-body system of spin- $\frac{1}{2}$ fermions when spin-flipping processes are not allowed (A different functional approach to this problem -in the impurity-free case- is given in [16]). Now, the potential matrices are diagonal whose elements can be written in terms of the g -functions defined by Sólyom [17] as

$$\begin{aligned}
V_{(0)}^{00} &= \frac{1}{4}(g_{4\parallel} + g_{4\perp} + g_{2\parallel} + g_{2\perp}), \\
V_{(0)}^{11} &= \frac{1}{4}(g_{4\parallel} - g_{4\perp} + g_{2\parallel} - g_{2\perp}), \\
V_{(1)}^{00} &= \frac{1}{4}(-g_{4\parallel} - g_{4\perp} + g_{2\parallel} + g_{2\perp}), \\
V_{(1)}^{11} &= \frac{1}{4}(-g_{4\parallel} + g_{4\perp} + g_{2\parallel} - g_{2\perp}).
\end{aligned} \tag{4.1}$$

It is straightforward to verify that the e-e interaction term in (1.4) contains the whole set of diagrams associated to forward scattering processes without spin-flips. Let us recall that the coupling constants for incident fermions with parallel spins are denoted by the subscript \parallel and that for fermions with opposite spins by the subscript \perp . In the g_2 processes the two branches (left and right moving particles) are coupled, while in the g_4 processes all four participating fermions belong to the same branch. The Tomonaga-Luttinger model, with charge-density fluctuations only, corresponds to $V_{(0)}^{11} = V_{(1)}^{11} = 0$. In a completely analogous way we introduce the potentials that couple

electron and impurity currents in the form

$$\begin{aligned}
U_{(0)}^{00} &= \frac{1}{4}(h_{4\parallel} + h_{4\perp} + h_{2\parallel} + h_{2\perp}), \\
U_{(0)}^{11} &= \frac{1}{4}(h_{4\parallel} - h_{4\perp} + h_{2\parallel} - h_{2\perp}), \\
U_{(1)}^{00} &= \frac{1}{4}(-h_{4\parallel} - h_{4\perp} + h_{2\parallel} + h_{2\perp}), \\
U_{(1)}^{11} &= \frac{1}{4}(-h_{4\parallel} + h_{4\perp} + h_{2\parallel} - h_{2\perp}).
\end{aligned}$$

This description includes both charge and spin density interactions, as well as spin-current interactions. A Kondo-like interaction, i.e. the coupling between spin densities only, corresponds to the case $U_{(0)}^{00} = U_{(1)}^{00} = 0$.

4.1 Bosonized action and dispersion relations

In order to carry out the bosonization of the model we shall start from the partition function (2.13) with the action given by (2.14). The fermionic determinant involving the field A_μ can be readily computed by employing the same path-integral technique depicted in Section 3. In the present case the decoupling of the vectorial field is achieved by the following transformation

$$\begin{aligned}
\Psi &= e^{(\gamma_5 \Phi + i\eta)} \chi, \\
\bar{\Psi} &= \bar{\chi} e^{(\gamma_5 \Phi - i\eta)}, \\
A_\mu &= -\epsilon_{\mu\nu} \partial_\nu \Phi + \partial_\mu \eta,
\end{aligned} \tag{4.2}$$

with $\Phi = \Phi^i \lambda^i$, $\eta = \eta^i \lambda^i$, $i = 0, 1$.

Due to the essentially non-Abelian character of this change of variables, the bosonic Jacobian is field-independent, and the fermionic determinant is decoupled in the usual way:

$$\det(i\cancel{\partial} + \cancel{A}) = (\det i\cancel{\partial}) \exp \frac{1}{2\pi} \int d^2x \Phi \square \Phi. \tag{4.3}$$

In order to treat the determinant associated with the impurity, once again it is convenient to first decouple the scalar field a and then to write the determinant in a factorized form, as in (3.12), and consider each factor separately. It is then easy to show that the changes $B_+ = 2i\partial_t \alpha$ in one of the

determinants, and $B_- = -2i\partial_t\beta$ in the other one, with α and β scalars, together with the corresponding chiral rotations in the fermion variables, allow to extract free fermion contributions. The fermionic Jacobians associated to these transformations are

$$\exp\left[-\frac{1}{2\pi}\int d^2x \alpha\partial_+\partial_t\alpha\right]. \quad (4.4)$$

and

$$\exp\left[-\frac{1}{2\pi}\int d^2x \beta\partial_-\partial_t\beta\right]. \quad (4.5)$$

All these changes determine a vacuum to vacuum functional depending on eight bosonic fields (Φ^i, η^i, α^i and β^i), multiplied by the free determinants coming from the Jacobians associated with the changes in the integration measures. In the present context these factors can be incorporated in the normalization constant. Of course, one must be more careful when considering a finite-temperature study, since in this case the free determinants would depend on temperature.

The action corresponding to the variables α and β turns out to be quadratic and therefore the functional integrations over these fields can be readily performed. One is then left with a partition function in terms of the bosonic fields Φ^i and η^i which, by comparison with the impurity-free case, one naturally identifies with the collective modes of the system [6]. The result is

$$Z = \int D\Phi^i D\eta^i \exp - \left\{ S_{eff}^{00} + S_{eff}^{11} \right\}, \quad (4.6)$$

where the actions, in Fourier space, are given by

$$\begin{aligned}
S_{eff}^{ii} &= \frac{1}{(2\pi)^2} \int d^2p [\hat{\Phi}^i(p) A^{ii}(p) \hat{\Phi}^i(-p) + \hat{\eta}^i(p) B^{ii}(p) \hat{\eta}^i(-p) + \\
&+ \hat{\Phi}^i(p) \frac{C^{ii}(p)}{2} \hat{\eta}^i(-p) + \hat{\eta}^i(p) \frac{C^{ii}(p)}{2} \hat{\Phi}^i(-p)], \tag{4.7}
\end{aligned}$$

where

$$\begin{aligned}
A(p) &= \frac{1}{\Delta(p)} \left\{ \frac{p^2}{\pi} \Delta - a_0 a_1 \frac{p_1^2}{\pi} + \frac{1}{2\pi} (a_0^2 p_1^2 - a_1^2 p_0^2) - 2a_0^2 a_1^2 \left(\frac{p_1^2}{b_1} + \frac{p_0^2}{b_0} \right) \right\}, \\
B(p) &= \frac{1}{\Delta(p)} \left\{ \frac{p_1^2}{\pi} a_0 a_1 + \frac{1}{2\pi} (a_0^2 p_0^2 - a_1^2 p_1^2) - 2a_0^2 a_1^2 \left(\frac{p_1^2}{b_0} + \frac{p_0^2}{b_1} \right) \right\}, \\
C(p) &= \frac{1}{\Delta(p)} \left\{ \frac{a_0 a_1}{\pi} \left(\frac{p_1^3}{p_0} - p_0 p_1 \right) + \frac{p_0 p_1}{\pi} (a_0^2 + a_1^2) + 4p_0 p_1 a_0^2 a_1^2 \left(\frac{1}{b_0} - \frac{1}{b_1} \right) \right\}, \\
\Delta(p) &= \frac{p_1^2}{4\pi^2 p_0^2} + 4 \left(\frac{1}{4\pi} - \frac{a_1^2}{b_1} \right) \left(\frac{1}{4\pi} + \frac{a_0^2}{b_0} \right). \tag{4.8}
\end{aligned}$$

For the sake of clarity we have omitted *ii* superindices in the above expressions, which are written in terms of the Fourier transforms of the inverse potentials defined in (2.11) and (2.12). (Note that $b_\mu(p) = V_{(\mu)}^{-1}(p)$ and $a_\mu(p) = U_{(\mu)}^{-1}(p)$).

This is one of our main results. We have obtained a completely bosonized action for the collective modes corresponding to a system of electrons which interact not only between themselves, but also with fermionic localized impurities at $T = 0$. This effective action describes the dynamics of charge density (Φ^0 and η^0) and spin density (Φ^1 and η^1) fields. As we can see, these modes remain decoupled as in the impurity free case. Their dispersion relations can be obtained from the poles of the corresponding propagators. Alternatively, one can write the effective Lagrangian as

$$L_{eff}^{ii} = \frac{1}{2\pi} \begin{pmatrix} \hat{\Phi}^i & \hat{\eta}^i \end{pmatrix} \begin{pmatrix} A^{ii} & C^{ii}/2 \\ C^{ii}/2 & B^{ii} \end{pmatrix} \begin{pmatrix} \hat{\Phi}^i \\ \hat{\eta}^i \end{pmatrix}, \tag{4.9}$$

with A, B and C as defined above, and solve the equation

$$C^2(p) - 4A(p)B(p) = 0. \tag{4.10}$$

Going to real frequencies: $p_0 = i\omega, p_1 = q$, this equation has the following pair of relevant solutions:

$$\omega_\rho^2(q) = q^2 \frac{1 + \frac{2}{\pi}V_{(0)}^{00} + \frac{1}{2\pi^2}\{(U_{(0)}^{00})^2 - 2U_{(0)}^{00}U_{(1)}^{00}\}}{1 + \frac{2}{\pi}V_{(1)}^{00} - \frac{1}{2\pi^2}(U_{(1)}^{00})^2}, \quad (4.11)$$

$$\omega_\sigma^2(q) = q^2 \frac{1 + \frac{2}{\pi}V_{(0)}^{11} + \frac{1}{2\pi^2}\{(U_{(0)}^{11})^2 - 2U_{(0)}^{11}U_{(1)}^{11}\}}{1 + \frac{2}{\pi}V_{(1)}^{11} - \frac{1}{2\pi^2}(U_{(1)}^{11})^2}, \quad (4.12)$$

The first equation gives the dispersion relation associated to charge-density fluctuations $(\hat{\Phi}^0, \hat{\eta}^0)$, whereas the second one corresponds to spin-density modes $(\hat{\Phi}^1, \hat{\eta}^1)$. To understand this identification one can write the fermionic currents in terms of the charge and spin densities as we did in (3.27).

As a confirmation of the validity of our approach, we note that the above dispersion relations, involving both e-e and e-i interaction potentials, coincide with the well-known result for the spectrum of charge and spin excitations in the TL model without impurities, obtained by choosing $V_{(1)} = U_{(0)} = U_{(1)} = 0$ and $V_{(0)} = v(q)$, in the above formulae (see [6]).

4.2 Two point fermionic correlations

Let us now consider the fermionic 2-point function

$$\langle \Psi(x)\bar{\Psi}(y) \rangle = \begin{pmatrix} o & G_1(x, y) \\ G_2(x, y) & 0 \end{pmatrix} \quad (4.13)$$

where

$$G_{1(2)}(x, y) = \begin{pmatrix} G_{1(2)\uparrow}(x, y) & 0 \\ 0 & G_{1(2)\downarrow}(x, y) \end{pmatrix} \quad (4.14)$$

The subindex 1(2) means that we consider electrons belonging to the branch 1(2), and \uparrow (\downarrow) indicates that the field operator carries a spin up (down) quantum number. Let us recall that in the present case we have disregarded those processes with spin-flip. This is why the fermionic Green function do not have non-zero components with mixed spin indices.

To be specific we consider $G_{1\uparrow}$ (similar expressions are obtained for $G_{2\uparrow}$, $G_{1\downarrow}$ and $G_{2\downarrow}$). When the decoupling chiral change is performed, the components

of the Green functions are factorized into fermionic and bosonic contributions in the form

$$\begin{aligned}
G_{1\uparrow}(x, y) &= \langle \Psi_{1\uparrow}(x) \Psi_{1\uparrow}^\dagger(y) \rangle \\
&= G_{1\uparrow}^{(0)}(x, y) \langle e^{\{[\Phi^0(y) - \Phi^0(x)] + i[\eta^0(y) - \eta^0(x)]\}} \rangle_{00} \times \\
&\times \langle e^{\{[\Phi^1(y) - \Phi^1(x)] + i[\eta^1(y) - \eta^1(x)]\}} \rangle_{11},
\end{aligned} \tag{4.15}$$

where $G_{1\uparrow}^{(0)}(x, y)$ is the free propagator, which involves the Fermi momentum p_F , and is given by

$$G_{1\uparrow}^{(0)}(x, y) = \frac{e^{ip_F z_1}}{2\pi |z|^2} (z_0 + iz_1). \tag{4.16}$$

The symbol $\langle \rangle_{ii}$ means v.e.v. with respect to the action (4.7, 4.8). Exactly as we did in the impurity-free case ([6]), the bosonic factors in (4.15) can be evaluated by appropriately shifting the fields. Indeed, working in momentum-space, and defining the non-local operator

$$D(p; x, y) = e^{-ip \cdot x} - e^{-ip \cdot y}, \tag{4.17}$$

the functional integrations can be performed, yielding

$$\begin{aligned}
\langle \Psi_{1\uparrow}(x) \Psi_{1\uparrow}^\dagger(y) \rangle &= G_{1\uparrow}^{(0)}(x, y) \exp\left\{-\int \frac{d^2 p}{(2\pi)^2} D^2 \frac{A^{00} - B^{00} + iC^{00}}{4A^{00}B^{00} - (C^{00})^2}\right\} \\
&\exp\left\{-\int \frac{d^2 p}{(2\pi)^2} D^2 \frac{A^{11} - B^{11} + iC^{11}}{4A^{11}B^{11} - (C^{11})^2}\right\},
\end{aligned} \tag{4.18}$$

with A^{ii} , B^{ii} and C^{ii} given by (4.8). In order to continue the calculation one needs, of course, to specify the couplings and perform the integrals. This means that our formula could be used to test the effect of different e-e and e-i potentials on the behavior of the fermionic propagator.

4.3 Momentum distribution

As a final illustration of our procedure, in this Subsection we shall show how to compute the electronic momentum distribution, for a quite peculiar choice

of e-e and e-i potential matrix elements.

Let us consider the momentum distribution of electrons belonging to branch 1 and with spin-up. This distribution is given by

$$N_{1\uparrow}(q) = C(\Lambda) \int_{-\infty}^{+\infty} dz_1 e^{-iqz_1} \lim_{z_0 \rightarrow 0} G_{1\uparrow}(z_0, z_1). \quad (4.19)$$

We shall set

$$V_{(1)}^{00} = V_{(1)}^{11} = 0, \quad V_{(0)}^{00} = \frac{\pi}{2}r, \quad V_{(0)}^{11} = \frac{\pi}{2}s,$$

which corresponds to an e-e interaction including only charge-density fluctuations (the usual TL model). Concerning the interaction between electrons and impurities, we shall take into account only spin-density and spin-current interactions,

$$U_{(0)}^{00} = U_{(1)}^{00} = 0, \quad (U_{(0)}^{11})^2 = (U_{(1)}^{11})^2 = 2\pi^2 t.$$

Note that for repulsive electron-electron interactions one has $r > 0$ and $s > 0$, whereas $t > 0$ for both ferromagnetic and antiferromagnetic couplings.

Taking the limit $z_0 \rightarrow 0$ in (4.15) and replacing eqs. (4.15) and (4.16) in (4.19) one gets

$$N_{1\uparrow}(q) = C(\Lambda) \int dz_1 \frac{e^{-i(q-p_F)z_1}}{z_1} e^{-\int dp_1 \frac{1-\cos p_1 z_1}{p_1} \Gamma(p_1)}, \quad (4.20)$$

where $C(\Lambda)$ is a normalization constant depending on an ultraviolet cutoff Λ , and $\Gamma(p_1)$ depends on p_1 through the potentials in the form

$$\begin{aligned} \Gamma(r, s, t) &= \frac{(|1+r|^{1/2}-1)^2}{|1+r|^{1/2}} + \frac{(|1+s-t|^{1/2}-|1-t|^{1/2})^2}{|1+s-t|^{1/2}|1-t|^{1/2}} - \\ &- \frac{2t(t-t_c)}{|1+s-t|^{3/2}|1-t|^{1/2}}. \end{aligned} \quad (4.21)$$

If we define

$$f(s, t) = \frac{(|1+s-t|^{1/2}-|1-t|^{1/2})^2}{|1+s-t|^{1/2}|1-t|^{1/2}}, \quad (4.22)$$

$$h(s, t) = t \frac{2t(t-1) + s(s-1)}{|1+s-t|^{3/2}|1-t|^{1/2}}, \quad (4.23)$$

we can write eq. (4.21) in the form

$$\Gamma(r, s, t) = f(r, 0) + f(s, t) - h(s, t). \quad (4.24)$$

Once again, in order to go further and make the integration in (4.20), one has to specify the functional form of r , s and t . At this point one observes that we are in a position of discussing, through this simple example, an interesting aspect of the general model under consideration. Indeed, we can try to determine under which conditions it is possible to have a restoration of the Fermi edge. To this end, and as a first approximation, we shall consider contact interactions (r , s and t constants) and search for those relations between potentials giving $\Gamma(r, s, t) = 0$. In this last case one obtains the well-known normal Fermi-liquid (FL) behavior

$$N_{1\uparrow}(q) \approx \Theta(q - p_F). \quad (4.25)$$

At this point some remarks are in order. In the impurity free case ($t = 0$), $\Gamma(r, s, t)$ cannot vanish for any value of r and s other than $r = s = 0$, which corresponds to the non-interacting Fermi gas. This result is consistent with the well-known LL behavior of the TL model.

In order to have collective modes with real frequencies ($\omega^2 > 0$), one finds two regions where the FL edge could be restored: $t > 1 + s$ and $t < 1$.

In eq.(4.22) one can observe that $f(r, 0) > 0$, thus setting $\Gamma = 0$ yields the condition

$$F(s, t) = h(s, t) - f(s, t) > 0. \quad (4.26)$$

A simple numerical analysis of $F(s, t)$ shows that the above inequality is not fulfilled for $0 < t < 1$. The electron-impurity coupling is not strong enough in this region as to eliminate the LL behavior. On the contrary, for $t > 1 + s$ equation (4.22) can be always satisfied. Moreover, in this region, we obtain a surface in which the condition $\Gamma = 0$ provides the following analytical solution for r in terms of $F(s, t)$

$$r = F^2/2 + 2F + (1 + F/2)(F^2 + 4F)^{1/2}. \quad (4.27)$$

The above discussion can be summarized by identifying the following three regions in the space of couplings:

Region I, given by $0 \leq t < 1$, in which one necessarily has LL behavior. Region II, with $1 \leq t < s + 1$, in which the frequency of the spin density excitations becomes imaginary; and region III, given by $t > s + 1$, where the FL behavior is admitted. In this region Eq(4.23) defines a surface in the space of potentials on which FL behavior takes place. One particular

solution belonging to this surface is obtained by choosing $s = 0$ in (4.27), which yields

$$r(t) = \frac{2t(3t - 2) + 2(2t - 1)\sqrt{3t^2 - 2t}}{(t - 1)^2}.$$

corresponding to the case in which the dispersion relation of the spin density excitations is given by $\omega^2 = q^2$. For t large, r approaches a minimum value $r_{min} = 6 + 4\sqrt{3}$, a feature that is shared with each curve $s = \text{constant}$ on the "FL surface".

In summary, we have shown how the e-i couplings can be tuned in order to have a restoration of the Fermi edge in a TL model of electrons interacting with fermionic impurities. Unfortunately, we could analytically work out this mechanism only for a very peculiar choice of the e-i couplings, which evidently weakens its experimental relevancy. Besides, a more realistic study should include at least the backward-scattering processes. However, we think this discussion deserves attention as a first step towards a possible reconciliation between the standard TL model and the FL phenomenology.

5 Summary and conclusions

In this paper we have shown how to extend a path-integral approach to non-local bosonization [6], to the case in which, besides the fermionic auto-interactions, an additional coupling with another set of fermion fields is included in the initial action. Taking into account the results of [6], we identify the self-interacting fields as 1d electrons, and following the work of Andrei [9], we interpret the new fields as a finite density of (not randomly) localized (with zero kinetic energy) fermionic impurities. Concerning the scattering processes involving both, electrons with themselves and electrons with impurities, we restrict our analysis to the forward-scattering case.

As a necessary condition for the application of our technique, in Section 2 we were able to write the partition function of the system in terms of two fermionic determinants, one associated to electrons and the other one to impurities, and both connected through bosonic degrees of freedom.

In Section 3 we used the well-established functional methods for the evaluation of fermionic determinants [14] in order to decouple fermions from bosons, getting thus a bosonized action describing the dynamics of the collective modes, for the general problem, including spin-flipping arbitrary (sym-

metric in coordinates) potentials. As it happens in the impurity-free case, being the problem a non-Abelian one (the group is $U(2)$), the analysis of the physical spectrum is not straightforward. Of course, in the present case the structure of the action is even more complicated since one not only has the usual WZW functional arising from the electronic bosonization, but also some other functionals coming from the interaction with the impurities. Some of these terms had been previously found in a path-integral treatment of the Kondo problem [10] which followed the same Andrei's scheme to represent the impurity. Despite these drawbacks, our method has the merit of explicitly displaying the expected decoupling between charge-density and spin degrees of freedom. On the other hand, our effective action could be used as a starting point for perturbative or semiclassical analysis.

In Section 4 we have considered a simpler but still interesting version of the model which consists in restricting the generators to those expanding the maximal Abelian subgroup of $U(2)$. From the physical point of view this means that both electrons and impurities do not change their spins in the scattering process. In this case, the action governing the dynamics of the collective modes is quadratic and we could then obtain some analytical results. In particular we got exact expressions for the dispersion relations of charge-density (plasmons) and spin-density modes as functionals of e-e and e-i interaction potentials. We have also computed the two-point electronic correlation function and the corresponding momentum distribution. We want to emphasize that our results are valid for arbitrary bilocal e-e and e-i potentials, i.e. in our approach one does not need to specify the couplings in order to get closed formulae for the bosonized action and Green functions. This is interesting because it opens the possibility of employing some of these results as a testing bench to check the validity of different potentials. Finally, as a short digression, in order to illustrate another possible application of our work, we have explored the behavior of the momentum distribution for a particular choice of the potentials that corresponds to a TL model of electrons coupled to impurities through spin-densities and spin-current-densities only. In this example we verified the existence of a region in the space of couplings in which the momentum distribution recovers the normal free-fermion form (the so called Fermi liquid behavior). However, concerning this result we have to be very cautious, since we have disregarded both backward-scattering and spin-flipping processes, and the resulting region of parameter space in which FL behavior takes place corresponds to very

large values of the couplings. Nevertheless we think that this example could be useful as a first step towards the construction of a modified TL model for which LL and FL behaviors could manifest as different phases within the same space of potentials.

This work could be followed in many directions, although we think that the main challenge will be to incorporate backward and umklapp processes within this framework. We hope to report on these issues in the close future.

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Appendix

The decoupling method that allows to separate the bosonic part from the fermionic one in a Dirac determinant [14] is quite well-known in the context of QFT's. Basically, the method consists in performing a change in the fermionic and bosonic path-integral variables, in such a way that the partition function becomes factorized into a bosonic factor times a fermionic one. For a field A in the Lie algebra of a group G , parametrized in terms of fields h and g belonging to G as

$$\begin{aligned} A_+ &= h^{-1}\partial_+h, \\ A_- &= g^{-1}\partial_-g, \end{aligned} \tag{A.1}$$

where $A_\pm = A_0 \pm iA_1$, $\partial_\pm = \partial_0 \pm i\partial_1$, the change of variables (A.1) together with the fermionic one

$$\begin{aligned} \Psi_1 &= h^{-1}\chi_1, \\ \bar{\Psi}_1 &= \bar{\chi}_1h, \\ \Psi_2 &= g^{-1}\chi_2, \\ \bar{\Psi}_2 &= \bar{\chi}_2g, \end{aligned} \tag{A.2}$$

give

$$\begin{aligned} \det(i\cancel{\partial} + \cancel{A}) &= \det i\cancel{\partial} \exp\{- (W[h] + W[g] \\ &+ (\frac{1}{4\pi} + \alpha) \text{tr} \int d^2x h^{-1} \partial_+ h g^{-1} \partial_- g)\}. \end{aligned} \quad (\text{A.3})$$

In this bosonic action $W[h]$ is a Wess-Zumino-Witten action term given by

$$W[h] = \frac{1}{8\pi} \text{tr} \int d^2x \partial_\mu h^{-1} \partial^\mu h + \Gamma[h], \quad (\text{A.4})$$

with

$$\Gamma[h] = \frac{1}{12\pi} \text{tr} \int_B d^3y \epsilon_{ijk} h^{-1} \partial^i h h^{-1} \partial^j h h^{-1} \partial^k h, \quad (\text{A.5})$$

and α is an arbitrary constant related to the regularization ambiguities which appear when computing the fermionic Jacobian [15]. Concerning the Jacobians associated with the changes in the bosonic measure [18], they read

$$\begin{aligned} DA_+ &= Dh \exp -2CW[h], \\ DA_- &= Dg \exp -2CW[g]. \end{aligned} \quad (\text{A.6})$$

where C is the quadratic Casimir of the group under consideration, ($f^{acd} f^{bcd} = \delta^{ab} C$). We can use this results, with $C = N$ for $G = \text{SU}(N)$, to write the final form of the partition function as

$$\begin{aligned} \int DAdet(i\cancel{\partial} + \cancel{A}) &= \det i\cancel{\partial} \int Dg Dh \exp\{-(1 + 2N) (W[h] + W[g] \\ &+ (\frac{1}{4\pi} + \alpha) \text{tr} \int d^2x (h^{-1} \partial_+ h g^{-1} \partial_- g)\}. \end{aligned} \quad (\text{A.7})$$

Both determinants appearing in (3.12) associated with the \mathcal{B} field can be decoupled by the use of analytic continuations and the method described above.

Let us first consider

$$\det \begin{pmatrix} 0 & i\partial_+ \\ i\partial_t + \mathcal{B}_- & 0 \end{pmatrix} \quad (\text{A.8})$$

By means of the analytic continuation $\partial_- \equiv \partial_t$, and the identification $\mathcal{B}_+ = 0$ and $\mathcal{B}_- = iQ^{-1} \partial_t Q$ one obtains

$$\int D\mathcal{B}_- \det \begin{pmatrix} 0 & i\partial_+ \\ i\partial_t + \mathcal{B}_- & 0 \end{pmatrix} = \det \begin{pmatrix} 0 & i\partial_+ \\ i\partial_t & 0 \end{pmatrix} \int DQ e^{-(1+2N)\tilde{W}[Q]}, \quad (\text{A.9})$$

where $\tilde{W}[Q]$ is a WZW-type action which differs from (A.4), because of the analytic continuation, in the first term:

$$\tilde{W}[Q] = \frac{1}{8\pi} \text{tr} \int d^2x \partial_- Q \partial_t Q^{-1} + \Gamma[Q]. \quad (\text{A.10})$$

In the same vein, using the analytic continuation $\partial_+ \equiv \partial_t$, and the identification $\mathcal{B}_- = 0$ and $\mathcal{B}_+ = iU^{-1}\partial_t U$ one obtains

$$\int D\mathcal{B}_+ \det \begin{pmatrix} 0 & i\partial_t + \mathcal{B}_+ \\ i\partial_- & 0 \end{pmatrix} = \det \begin{pmatrix} 0 & i\partial_t \\ i\partial_- & 0 \end{pmatrix} \int DU e^{-(1+2N)\dot{W}[U]}, \quad (\text{A.11})$$

where $\dot{W}[U]$ also differs from the usual WZW action in the surface term

$$\dot{W}[U] = \frac{1}{8\pi} \text{tr} \int d^2x \partial_t U \partial_- U^{-1} + \Gamma[U]. \quad (\text{A.12})$$

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