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Quasiparticle operators with non-Abelian braiding statistics

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Abstract

We study the gauge invariant fermions in the fermion coset representation of $SU(N)_k$ Wess-Zumino-Witten models which create, by construction, the physical excitations (quasiparticles) of the theory. We show that they provide an explicit holomorphic factorization of $SU(N)_k$ Wess-Zumino-Witten primaries and satisfy non-Abelian braiding relations.

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i) Introduction

The notion of generalized statistics has attracted a great deal of attention in the last 15 years (see *e.g.* [1, 2] and references therein), both in $1+1$ and in $2+1$ dimensional theories.

The most natural generalization of the traditional classification of particles into bosons and fermions corresponds to the possibility of having an arbitrary phase when two particles are interchanged. This is allowed in $2+1$ dimensions since the statistics is characterized by the Braid group, B_n, (instead of S_n as it is in higher dimensions), which admits more general representations.

A further generalization is to consider the case of higher dimensional (non-Abelian) representations of B_n which would correspond to non-Abelian statistics. In fact, the concept of non-Abelian statistics was introduced in the context of the Fractional Quantum Hall Effect (FQHE) in [3] as the statistics of the quasiparticles of the so-called Pfaffian state, (see also [4], where more general examples were treated). The approach in these papers is based on the construction of trial ground state wave functions of FQHE systems using the Conformal Blocks (CB) of a Conformal Field Theory, and the statistics appears as a consequence of the braiding properties of these CB's, which are known [5, 6]. In particular, in [4] the mentioned CB's correspond to those of the $SU(N)_k$ Wess-Zumino-Witten (WZW) theories [7].

In the present Letter we address this issue from a different point of view: we aim to construct a concrete realization of quasiparticle operators satisfying non-Abelian braiding relations.

To this end we first show that the holomorphic and anti-holomorphic factors of the WZW primary fields in a $SU(N)_k$ WZW theory can be constructed using its fermionic coset description. In fact, the natural observables in the fermionic description, which are the gauge invariant fermions (GIF's) [8, 9, 10], correspond to the holomorphic factors of the WZW primaries.

Finally, by evaluating the Operator Product Algebra (OPA) of the GIF's, we show that they satisfy non-Abelian braiding relations [11, 6], thus giving the desired explicit operator realization of quasiparticle operators with non-Abelian braiding.

The present approach could be useful in the context of the FQHE for systems whose quasiparticles obey non-Abelian statistics.

As for the holomorphic factorization that we present, it corresponds to a very interesting issue from a more formal point of view, and has been the subject of recent investigations [12].

The present approach could also be useful in connection with the spinon construction developed in [13] and with the construction of quasiparticle representations of the characters for Conformal Field Theories [14].

ii) $SU(N)_k$ Wess-Zumino-Witten (WZW) theory as a fermionic coset

To make the paper self-consistent and set our conventions we will first recall the fermionic coset representation of the $SU(N)_k$ WZW theory [15]. The action is given by

$$S = \frac{1}{\sqrt{2}\pi} \int d^2x \bar{\psi}^{i\alpha} \left((i\partial\!\!\!/ + a\!\!\!/)\delta_{ij}\delta_{\alpha\beta} + \delta_{ij}\, B\!\!\!/_{\alpha\beta} \right) \psi^{j\beta}, \tag{1}$$

where the fermions $\psi^{i\alpha}$ $(i = 1, \cdots, N,\ \alpha = 1, \cdots, k)$ are in the fundamental representation of $U(Nk)$ and the $U(1)$ gauge field a_μ and the $SU(k)$ gauge field B_μ act as Lagrange multipliers implementing the constraints

$$j_\mu|phys>= 0, \qquad J^a_\mu|phys>= 0 \quad (a = 1, \cdots, k^2 - 1)\ , \tag{2}$$

for the $U(1)$ and the $SU(k)$ currents respectively. This corresponds to the identification:

$$SU(N)_k \equiv \frac{U(Nk)}{SU(k)_N \times U(1)}, \tag{3}$$

which is understood as an equivalence between the correlation functions of corresponding fields in the two theories.

The fundamental field g of the bosonic $SU(N)_k$ WZW theory is represented in terms of fermions by the bosonization formula [1] [15]

$$g^{ij} = \psi_L^{i\dagger}\psi_R^j \tag{4}$$

where the $SU(k)$ indices are summed out. Their conformal dimensions are given by $(h_g, \bar{h}_g) = (\frac{N^2-1}{2N(k+N)}, \frac{N^2-1}{2N(k+N)})$.

[1]Our conventions are $\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$ and γ_i are the Pauli matrices, $\gamma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and $\gamma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$.

Other integrable representations are constructed as appropriately symmetrized products of these fundamental fields. In the particular $N=2$ case, higher spin integrable representations are constructed as

$$\begin{aligned} g^{(j)\ i_1,\ldots,i_{2j}}_{j_1,\ldots,j_{2j}} &= \mathcal{S}\left(:g^{i_1}_{j_1}\ldots g^{i_{2j}}_{j_{2j}}:\right) \\ &= \mathcal{S}\left(:\psi^{i_1\dagger}_L\psi^{j_1}_R\ldots\psi^{i_{2j}\dagger}_L\psi^{j_{2j}}_R:\right), \end{aligned} \tag{5}$$

where $\mathcal{S}$ stands for symmetrization over the left and right indices separately and j takes the values $j=0,1/2,1,....,k/2$. This restriction in the spin of the representation has its origin in the selection rules imposed by the affine (Kac-Moody) symmetry [16, 17]. It is interesting to note that in the fermion description of $SU(N)_k$ the presence of a second index α in the fermion fields $\psi^{i\alpha}$, running from 1 to k, allows for the construction of symmetrized products of at most k bilinears. In this way we obtain only the allowed integrable representations, other representations being forbidden by the Pauli principle.

It will be useful for later purposes to review the decoupled picture. The partition function is given by

$$\mathcal{Z}=\int\mathcal{D}\psi^\dagger\mathcal{D}\psi\mathcal{D}a_\mu\mathcal{D}B_\mu\exp(-S), \tag{6}$$

where S is given in eq. (1).

The decoupling transformations are[2]

$$\begin{array}{ll} \psi_L=hV^{-1}\chi_L & \psi_R=\bar{h}U^{-1}\chi_R, \\ a=i\bar{h}\partial\bar{h}^{-1} & \bar{a}=ih\bar{\partial}h^{-1}, \\ B=iU^{-1}\partial U & \bar{B}=iV^{-1}\bar{\partial}V. \end{array} \tag{7}$$

Taking into account the gauge fixing procedure and the Jacobians associated to the change of variables above [18] one arrives at the desired decoupled form for the partition function:

$$\mathcal{Z}=\mathcal{Z}_{ff}\mathcal{Z}_{fb}\mathcal{Z}_{WZW}\mathcal{Z}_{gh}, \tag{8}$$

where

$$\mathcal{Z}_{ff} = \int\mathcal{D}\bar{\chi}\mathcal{D}\chi\exp\left(-\frac{1}{\pi}\int(\chi^{i\alpha\dagger}_L\bar{\partial}\chi^{i\alpha}_L+\chi^{i\alpha\dagger}_R\partial\chi^{i\alpha}_R)d^2x\right),$$

[2] Further conventions are: $z=x_1+ix_2$, $\bar{z}=x_1-ix_2$, $\partial\equiv\frac{\partial}{\partial z}$, $\bar{\partial}\equiv\frac{\partial}{\partial\bar{z}}$, $a=(a_1-ia_2)/2$, $\bar{a}=(a_1+ia_2)/2$, $B=(B_1-iB_2)/2$, $\bar{B}=(B_1+iB_2)/2$ and $h=\exp(\phi+i\eta)$, $\bar{h}=\exp(-\phi+i\eta)$.

$$
\begin{aligned}
\mathcal{Z}_{fb} &= \int \mathcal{D}\phi \exp(\frac{Nk}{2\pi}\int \phi\Delta\phi d^2x), \\
\mathcal{Z}_{WZW} &= \int \mathcal{D}\tilde{g} \exp\left((2k+N)\Gamma[\tilde{g}]\right). \qquad (9)
\end{aligned}
$$

$-(2k+N)\Gamma[\tilde{g}]$ is the level $-(2k+N)$ WZW action [7] for the gauge invariant combination $\tilde{g} = VU^{-1}$, and Z_{gh} corresponds to the Fadeev-Popov ghosts partition function, whose explicit form will not be needed.

In particular, the central charge is easily evaluated as the sum of four independent contributions coming from the different sectors, $c_{ff} = Nk$, $c_{fb} = 1$, $c_{WZW} = (2k+N)(k^2-1)/(k+N)$ and $c_{gh} = -2k^2$, thus giving

$$
c = \frac{k(N^2-1)}{k+N}, \qquad (10)
$$

which corresponds to the central charge of the $SU(N)_k$ WZW action.

iii) Gauge invariant fermions and holomorphic factorization

The coset theory, defined by the Lagrangian (1), is manifestly invariant under gauge transformations $U(1) \times SU(k)$. Consequently, the original fermion fields ψ are not "physical" operators, as they do not commute with the associated BRST charges. It is then natural to find a new set of fermionic variables invariant under the BRST symmetry that, by construction, create the physical excitations of the theory.

For the fermionic coset theory there is a natural candidate for this gauge invariant fermion field. The gauge degrees of freedom a_μ and B_μ that enter in the Lagrangian (1) are of topological nature since they do not have kinetic terms. In fact, their equations of motion are just the flat connection conditions $F^a_{\mu\nu}[B] = F_{\mu\nu}[a] = 0$. Then, we can attach to the fermion an infinite flux line that, due to the zero-curvature condition, is only dependent on the end point, guaranteeing its locality. This infinite Wilson line absorbs the gauge variation of the fermion field and then the compound object has the desired properties. We will show later that, in addition, this GIF's are also chiral and can be identified with the vertex operators of the coset model in the sense defined in [11].

Let us start defining the gauge invariant fermion fields [8]:

$$\begin{aligned}\hat{\psi}^{i\alpha}(x) &= e^{i\int_x^\infty dz^\mu a_\mu} P\left(e^{i\int_x^\infty dz^\mu B_\mu}\right)^{\alpha\beta} \psi^{i\beta}(x)\\ \hat{\psi}^{i\alpha\dagger}(x) &= \psi^{j\beta\dagger}(x) P\left(e^{i\int_x^\infty dz^\mu B_\mu}\right)^{\beta\alpha\dagger} e^{-i\int_x^\infty dz^\mu a_\mu}\end{aligned} \tag{11}$$

As we stated above, the Schwinger line integrals in (11) do not depend on the choice of the path due to the zero curvature condition satisfied by the gauge connections a_μ and B_μ.

In order to analyze the properties of the GIF's it is useful to work with decoupled variables (7), where things are more easily tractable.

In terms of these variables, the gauge invariant fermions are given by

$$\hat{\psi}_L^{i\alpha}(x) = e^{i\int_x^\infty dz^\mu a_\mu} P\left(e^{i\int_x^\infty dz^\mu B_\mu}\right)^{\alpha\beta} h(V^{-1})^{\beta\gamma}\chi_L^{i\gamma} \tag{12}$$

$$\hat{\psi}_R^{i\alpha}(x) = e^{i\int_x^\infty dz^\mu a_\mu} P\left(e^{i\int_x^\infty dz^\mu B_\mu}\right)^{\alpha\beta} \bar{h}(U^{-1})^{\beta\gamma}\chi_R^{i\gamma} \tag{13}$$

Using the equations of motion for the decoupled fields one can prove that the fields $\hat{\psi}_L^i$ ($\hat{\psi}_R^i$) are holomorphic (anti-holomorphic). In order to show it we analyze in detail one of them, say $\hat{\psi}_L^i$, (the same analysis can be carried out for the other components in a similar way).

To this end we rewrite eq. (12) as

$$\hat{\psi}_L^{i\alpha}(z) = e^{\varphi(z)}\mathcal{Q}^{\alpha\beta}(z)\chi_L^{i\beta}, \tag{14}$$

where we defined (see footnote 2 for notation)

$$\varphi(z) = \phi + i\int_x^\infty dz_\mu \epsilon_{\mu\nu}\partial_\nu\phi, \tag{15}$$

(note that the field η cancels out as it should do, since it corresponds to the gauge degree of freedom), and

$$\mathcal{Q}^{\alpha\beta}(z) = P\left(e^{i\int_x^\infty dz^\mu B_\mu}\right)^{\alpha\gamma}(V^{-1})^{\gamma\beta}. \tag{16}$$

The field $\varphi(z)$ is the holomorphic component of the free boson ϕ. The condition $\bar{\partial}\varphi = 0$ directly follows from the ϕ equation of motion.

The holomorphic character of the field $\mathcal{Q}^{\alpha\beta}(z)$ follows from the equation of motion of B_μ, (zero curvature condition). Calling $\mathcal{U}(x) = P\left(e^{i\int_x^\infty dz^\mu B_\mu}\right)$ one finds

$$i\partial_\mu \mathcal{U}(x) - \mathcal{U}(x)B_\mu(x) = 0. \tag{17}$$

Then,

$$i\partial_\mu\left(\mathcal{U}V^{-1}\right) = \mathcal{U}\left(B_\mu - iV^{-1}\partial_\mu V\right) \tag{18}$$

shows that

$$\bar{\partial}\mathcal{Q} = 0, \tag{19}$$

while the z derivative can be written as

$$\partial\mathcal{Q} = \mathcal{Q}\tilde{g}\partial\tilde{g}^{-1} \tag{20}$$

where $\tilde{g}$ is the field in the (negative level) WZW sector of the theory (see eq. (9)).

Putting all these things together and using the equation of motion of the free fermion $\bar{\partial}\chi_L = 0$, we conclude that

$$\bar{\partial}\hat{\psi}_L = 0, \qquad i.e.\;\; \hat{\psi}_L = \hat{\psi}_L(z). \tag{21}$$

Similarly the other fields can be written as

$$\hat{\psi}_R(\bar{z}) = e^{-\bar{\varphi}(\bar{z})}\bar{\mathcal{Q}}(\bar{z})\chi_R(\bar{z}), \tag{22}$$

where

$$\bar{\varphi}(\bar{z}) = \phi - i\int_x^\infty dz_\mu \epsilon_{\mu\nu}\partial_\nu\phi \tag{23}$$

$$\bar{\mathcal{Q}}(\bar{z}) = P\left(e^{i\int_x^\infty dz^\mu B_\mu}\right)U^{-1}. \tag{24}$$

One can easily show that $\bar{\varphi}$ and $\bar{\mathcal{Q}}$ are antiholomorphic following the same steps as in (15-19).

The previous heuristic definition of the gauge invariant fermions can be also motivated by the following natural argument. One can observe that although in the decoupling change of variables eq. (7) the fields U and V appear, only the gauge invariant combination $\tilde{g} = VU^{-1}$ is relevant. Besides, the above semiclassical analysis shows that we can also factorize the field $\tilde{g}$ into its holomorphic and antiholomorphic components as follows

$$\tilde{g} = \mathcal{Q}^{-1}(z)\bar{\mathcal{Q}}(\bar{z}). \tag{25}$$

Assuming that this factorization into holomorphic and antiholomorphic parts is valid at the quantum level (cf. [3, 12]) it is natural to construct the GIF's with $\mathcal{Q}$ and $\bar{\mathcal{Q}}$ considered as the holomorphic-antiholomorphic factors in (25) instead of the (functionals of) the factors U and V that appear in the decoupling change of variables. Eq. (11) can then be considered as the classical counterpart of the present definition (14,22,25).

It is worthwhile to notice that these new fields create physical excitations since they commute with the $SU(k)$ BRST charges, Q_{BRST} and $\bar{Q}_{BRST}$. The expressions for these charges in the decoupled picture is [19]

$$
\begin{aligned}
Q_{BRST} &= \oint dz : [c^a(z) \left(\chi_L^\dagger T^a \chi_L - \frac{2k+N}{2} \partial \mathcal{Q}^{-1} \mathcal{Q} \right) - 1/2 f^{abc} c^a b^b c^c] :, \\
\bar{Q}_{BRST} &= \oint d\bar{z} : [\bar{c}^a(\bar{z}) \left(\chi_R^\dagger T^a \chi_R - \frac{2k+N}{2} \bar{\partial} \bar{\mathcal{Q}}^{-1} \bar{\mathcal{Q}} \right) - 1/2 f^{abc} \bar{c}^a \bar{b}^b \bar{c}^c] :
\end{aligned} \quad (26)
$$

where $c, \bar{c}, b, \bar{b}$ are ghost fields.

To show that the gauge invariant fermions defined above commute with these charges it is sufficient to show that the Operator Product Expansion (OPE) with the integrands in eq. (26) is regular. Using the expressions (14,22) for the gauge invariant fermions, and the transformation properties of $\mathcal{Q}$ and $\bar{\mathcal{Q}}$ under left and right $SU(k)$ chiral rotations, one can show that this is indeed true.

Let us now show how the $SU(N)_k$ WZW fields can be built up from the gauge invariant fermions. This can be done by simply taking the product

$$
\sum_{\alpha=1}^{k} (\hat{\psi}_L^\dagger)^{i\alpha} \hat{\psi}_R^{j\alpha} = \sum_{\beta,\gamma=1}^{k} (\chi_L^\dagger)^{i\beta} e^{-2\phi} \left(\mathcal{Q}^{-1} \bar{\mathcal{Q}} \right)^{\beta\gamma} \chi_R^{j\gamma} \quad (27)
$$

and using the explicit expressions for $\mathcal{Q}$ as given by eq. (16). We finally obtain

$$
\sum_{\alpha=1}^{k} (\hat{\psi}_L^\dagger)^{i\alpha} \hat{\psi}_R^{j\alpha} = \sum_{\alpha=1}^{k} (\psi_L^\dagger)^{i\alpha} \psi_R^{j\alpha} = g^{ij} \quad (28)
$$

where the last equality follows from eq. (4).

This simple calculation shows that the gauge invariant fermions, which create physical states in the holomorphic or in the antiholomorphic sectors, can be used to construct the $SU(N)_k$ WZW primary fields g^{ij}. This construction exhibits the holomorphic factorization of the fields g^{ij} [3, 12]. The same conclusion applies to the WZW primaries corresponding to other integrable representations, since they all can be constructed from the field in the fundamental representation, g^{ij}, by taking suitable symmetrized normal ordered products.

We will now evaluate the OPE between the energy momentum tensor and the GIF's in order to prove their primary character and to obtain their conformal dimensions. The energy momentum tensor can be written as the sum of three independent contributions, a free fermion part, a free boson part and a WZW part. Then, the conformal dimensions of the GIF's are given by the sum of the free fermion contribution, the vertex operator of the free boson contribution and that of the composite operators $\mathcal{Q}$.

For the free fermions χ_L and χ_R the dimensions are $(1/2, 0)$ and $(0, 1/2)$ and for the vertex operators $e^{\pm\varphi}$ and $e^{\pm\bar{\varphi}}$, they are $(\frac{-1}{2Nk}, 0)$ and $(0, \frac{-1}{2Nk})$ respectively.

As for the conformal dimension of the composites $\mathcal{Q}(z)$ and $\bar{\mathcal{Q}}(\bar{z})$, they are determined simply by the conformal transformation properties of the WZW field $\tilde{g}$. Since this field is a Virasoro primary of the WZW Conformal Field Theory and the left and right Virasoro algebras commute with each other, the holomorphic and antiholomorphic operators $\mathcal{Q}(z)$ and $\bar{\mathcal{Q}}(\bar{z})$ are primaries of the left and right Virasoro algebras separately.

In fact, $\mathcal{Q}$ transforms in the fundamental representation of the affine Lie algebra $SU(k)_{-N-2k}$, *i.e.,* its OPE with the affine current is given by

$$\tilde{J}^a(z)\mathcal{Q}(w) = \frac{t^a\mathcal{Q}(w)}{z-w} + \mathcal{N}[\tilde{J}^a\mathcal{Q}](w) + r.t., \tag{29}$$

where $\mathcal{N}$ denotes normal ordering and $r.t.$ stands for regular terms.

Using this equation and the Sugawara representation of the energy momentum tensor [20], we obtain:

$$\tilde{T}(z)\mathcal{Q}(w) = -\frac{k^2-1}{2k(k+N)}\frac{\mathcal{Q}(w)}{(z-w)^2} + \frac{2}{z-w}\partial_w\mathcal{Q}(w) + r.t., \tag{30}$$

from which one can read off the dimension of $\mathcal{Q}$, $h_{\mathcal{Q}} = -\frac{k^2-1}{2k(k+N)}$.

Adding to $h_{\mathcal{Q}}$ the contributions from the free fermions and that of the vertex operator of the free boson, we get for the dimension of the GIF operator the expression

$$h_{\hat{\psi}} = -\frac{k^2-1}{2k(k+N)} + \frac{1}{2} - \frac{1}{2Nk} = \frac{N^2-1}{2N(k+N)}. \tag{31}$$

This shows that the GIF's have the conformal dimensions corresponding to the holomorphic or antiholomorphic factors of the $SU(N)_k$ WZW primaries (4).

According to [11] this means that the GIF's are the vertex operators of the WZW theory. Usually the so-called Chiral Vertex Operators are introduced in this context [11, 5] and can be constructed as appropriate projections of the GIF's. They formally correspond to (considering for simplicity the case of $N = 2$)

$$\Phi\begin{pmatrix} & j & \\ i & & k \end{pmatrix} = \Pi_i \mathcal{S}\left(\hat{\psi}\hat{\psi}..\hat{\psi}\right)\Pi_k \ , \tag{32}$$

where Π_i stands for the projector on the integrable representation of spin i and $\mathcal{S}$ is the Young symmetrizer that projects the product of $2j$ GIF's onto the representation of spin j.

iv) Braiding relations among GIF's

We now evaluate the OPA satisfied by the GIF's, which can be easily calculated using the explicit expressions in eqs. (14,22).

Let us consider first the OPE of two fields $\hat{\psi}_L^{i\alpha}$:

$$\hat{\psi}_L^{i\alpha}(z)\hat{\psi}_L^{j\beta}(w) = e^{\varphi(z)}\mathcal{Q}^{\alpha\alpha'}(z)\chi_L^{i\alpha'}(z)e^{\varphi(w)}\mathcal{Q}^{\beta\beta'}(w)\chi_L^{j\beta'}(w). \tag{33}$$

The OPE of the two $U(1)$ bosonic vertex operators appearing in (33) is simply given by

$$e^{\varphi(z)}e^{\varphi(w)} = \frac{1}{(z-w)^{\frac{1}{N-k}}}e^{2\varphi(w)} + \ldots \ , \tag{34}$$

and that of the free fermions, which is conveniently separated into symmetric and antisymmetric combinations, by

$$\begin{aligned}\chi_L^{i\alpha}(z)\chi_L^{j\beta}(w) = 1/2(:\chi_L^{i\alpha}(w)\chi_L^{j\beta}(w): - :\chi_L^{j\alpha}(w)\chi_L^{i\beta}(w):) + \\ 1/2(:\chi_L^{i\alpha}(w)\chi_L^{j\beta}(w): + :\chi_L^{j\alpha}(w)\chi_L^{i\beta}(w):) + \ldots \ .\end{aligned} \tag{35}$$

Finally for the OPE of the WZW vertex operators we have

$$\mathcal{Q}^{\alpha\alpha'}(z)\mathcal{Q}^{\beta\beta'}(w) = \sum_l \mathcal{C}^{(l)}_{\mathcal{Q}\mathcal{Q}} z^{h_l - 2h_{\mathcal{Q}}} \left[\Phi_l^{\alpha\beta\alpha'\beta'}(w) + ...\right] \tag{36}$$

where the Φ_l's are the primary fields associated with the integrable representations of the affine chiral algebra [20], and the dots stand for their descendants. In our case, we have

$$\mathcal{Q}^{\alpha\alpha'}(z)\mathcal{Q}^{\beta\beta'}(w) = \frac{\mathcal{C}^A_{\mathcal{Q}\mathcal{Q}}}{(z-w)^{-h_A+2h_{\mathcal{Q}}}}\Phi_A^{\alpha\beta\alpha'\beta'}(w) + \frac{\mathcal{C}^S_{\mathcal{Q}\mathcal{Q}}}{(z-w)^{-h_S+2h_{\mathcal{Q}}}}\Phi_S^{\alpha\beta\alpha'\beta'}(w) + ... \tag{37}$$

where Φ_A and Φ_S are the antisymmetric and symmetric channels of the product in the right hand side of (36) with dimensions $h_A = \frac{(2-k)(k+1)}{k(k+N)}$, $h_S = \frac{(k+2)(1-k)}{k(k+N)}$ respectively.

Combining eqs. (33,34,37) we obtain

$$\hat{\psi}_L^{i\alpha}(z)\hat{\psi}_L^{j\beta}(w) = (z-w)^{-h_{\mathcal{S}}}\mathcal{S}^{ij}_{\alpha\beta}(w) + (z-w)^{-h_{\mathcal{A}}}\mathcal{A}^{ij}_{\alpha\beta}(w) + ... \;, \tag{38}$$

where $\mathcal{S}^{ij}_{\alpha\beta}$ ($\mathcal{A}^{ij}_{\alpha\beta}$) is symmetric (antisymmetric) in the indices α, β and antisymmetric (symmetric) in the indices i, j. Their conformal weights $h_{\mathcal{S}}$ and $h_{\mathcal{A}}$ are given respectively by:

$$h_{\mathcal{S}} = \frac{1+N}{N(N+k)}, \qquad \text{and} \qquad h_{\mathcal{A}} = \frac{1-N}{N(N+k)} \;. \tag{39}$$

As already stressed, the Chiral Vertex Operators $\Phi\begin{pmatrix} & i & \\ j & & k\end{pmatrix}$ introduced in [11, 5] correspond to suitable projections of the GIF's over the integrable representations of the $SU(N)_k$ affine algebra (see eq. (32)). With these operators one can verify explicitly the $N=2$ braiding relations [11, 5]

$$\Phi\begin{pmatrix} & k_1 & \\ j_1 & & p\end{pmatrix}(z_1)\Phi\begin{pmatrix} & k_2 & \\ p & & j_2\end{pmatrix}(z_2) = \sum_{p'} B_{pp'}\begin{bmatrix} k_1 & k_2 \\ j_1 & j_2\end{bmatrix}\Phi\begin{pmatrix} & k_2 & \\ j_1 & & p'\end{pmatrix}(z_2)\Phi\begin{pmatrix} & k_1 & \\ p' & & j_2\end{pmatrix}(z_1) \;. \tag{40}$$

From this we can conclude that the GIF's can be considered as quasiparticle operators with non-Abelian braiding in the sense of [3, 4]. Correlators of these operators can be computed following the same steps as in [20].

v) Conclusions

We have shown in this paper that the fermionic coset representation of the $SU(N)_k$ WZW theory allows for the explicit construction of the vertex operators [11], fundamental physical fields that can be used to build up the WZW primaries. These gauge invariant fermion fields are holomorphic and transform as the holomorphic part of the primaries under the action of the full chiral algebra.

Moreover, they create physical excitations and their modes can then be used to construct the physical Hilbert space, which will consist of states of left and right moving quasiparticles. Due to the braiding relations satisfied by the GIF's, they can be interpreted as quasiparticle operators with non-Abelian statistics according to [3, 4].

The connection between the Fock space obtained through this procedure could be compared with the spinon Fock space constructed in [13], to understand the relation of the quasiparticles created with the modes of the GIF's and the non-Abelian spinons.

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References

[1] *Fractional Statistics and Anyon Superconductivity*, ed. F. Wilczek, World Scientific, 1989.

[2] J. M. Leinaas, preprint hep-th/9611167.

[3] G. Moore and N. Read, Nucl. Phys. **B 360** (1991) 362.

[4] B. Block and X. G. Wen, Nucl. Phys. **B 374** (1992) 615; X. G. Wen, Y-S. Wu and Y.Hatsugai, Nucl. Phys. **B 422** (1994) 476.

[5] G. Moore and N. Seiberg, Phys. Lett. **212B** (1988) 451; Commun. Math. Phys.**123** (1989) 177.

[6] L. Alvarez-Gaume, C. Gomez and G. Sierra, Phys. Lett. **220B** (1998) 142.

[7] E. Witten, Commun. Math. Phys. **92** (1984) 455.

[8] D. C. Cabra and K. D. Rothe, Phys. Rev. **D51** (1995) R2509; Ann. Phys. **251** (1996) 337.

[9] D. C. Cabra and E. F. Moreno, Nucl. Phys. **B475** (1996) 522.

[10] D. C. Cabra and G. L. Rossini, Mod. Phys. Lett. **A12** (1997) 265.

[11] A. Tsuchiya and Y. Kanie, Lett. Math. Phys **13** (1987) 303.

[12] M. B. Halpern and N. A. Obers, Int. J. Mod. Phys. **A12** (1997) 4317.

[13] P. Bouwknegt, A. W. W. Ludwig and K. Schoutens, Phys. Lett. **359B** (1995) 304.

[14] R. Kedem, T. R. Klassen, B. M. McCoy and E. Melzer, Phys. Lett. **304B** (1993) 263; **307B** (1993) 68.

[15] S. G. Naculich and H. J. Schnitzer, Nucl. Phys **B347** (1990) 687.

[16] D. Gepner and E. Witten, Nucl. Phys. **B278** (1986) 493.

[17] G. Felder, K. Gawedzki and A. Kupiainen, Commun. Math. Phys. **117** (1988) 127.

[18] A. Polyakov and P. Wiegmann, Phys. Lett. **141B** (1984) 223; R. E. Gamboa Saraví F. A. Schaposnik and J. E. Solomin, Nucl.Phys. **B185** (1981) 239.

[19] D. Karabali and H. Schnitzer, Nucl. Phys. **B329**, (1990) 649.

[20] V. G. Knizhnik and A. B. Zamolodchikov, Nucl. Phys. **B247** (1984) 83.