Monopoles in non-Abelian Dirac-Born-Infeld theory

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We investigate monopole solutions for the Born-Infeld Higgs system. We analyze numerically these solutions and compare them with the standard 't Hooft–Polyakov monopoles. We also discuss the existence of a critical value of β (the Born-Infeld ''absolute field parameter'') below which no regular solution exists. [S0556-2821(99)05912-3]

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I. INTRODUCTION

Classical solutions for the Dirac-Born-Infeld (DBI) theories are useful in the understanding of brane dynamics [1−3]. In this respect, bions and soliton-like solutions have recently attracted much attention [2−4]; in particular, vortex and monopole solutions have been investigated [2−7].

Concerning monopoles, it was shown in [6] that it is possible to construct a DBI action coupled to a Higgs scalar in such a way that the usual Bogomol'nyi-Prasad-Sommerfield (BPS) monopole solution to the Yang-Mills-Higgs theory also solves the resulting (first-order) equations of motion. To this end, one has to endow the Higgs field with dynamics also described by a square-root Born-Infeld like Lagrangian and also consider Prassad-Sommerfield $\lambda \rightarrow 0$ limit for the symmetry breaking potential. The solution being that of a BPS monopole, one does not capture any features associated with Born-Infeld dynamics and, in particular, the resulting solution is insensitive to the value of β, the ''absolute field'' parameter in Born-Infeld models. In contrast, a critical value β_c was discovered in a previous investigation of vortex solutions in Abelian DBI theories [7], such that no soliton solution exists for $\beta \lesssim \beta_c$, thus showing how DBI dynamics determines the nature of soliton solutions.

It should be stressed that our main interest is to investigate the existence of topologically non-trivial regular monopole solutions in Dirac-Born-Infeld theories. This requires the gauge group to be non-Abelian and faces us with the choice of an appropriate trace operation to define a DBI scalar action. Moreover, a Higgs scalar has to be introduced in order to ensure symmetry breaking and the consequent non-triviallity of the homotopy group related to magnetic charge.

In this work we shall discuss monopole solutions in $SO(3)$ DBI gauge theories coupled to a Higgs triplet which enters through the usual kinetic energy term, $L_{Higgs} \sim \mathrm{tr}(D_\mu \phi D^\mu \phi)$. Concerning the way in which the non-Abelian DBI scalar Lagrangian is defined, there exist different possibilities, among which we consider taking (i) the usual trace over internal indices of the square root DBI Lagrangian defined through its power series expansion and (ii) the ''symmetric trace'' advocated by Tseytlin [8] as a way to make contact with the low energy effective action derived from superstring theories.

Concerning the scalar action, bion and catenoid solutions for DBI actions with scalars arising from the ''transverse cordinates'' of the DBI action taken as a brane action have been discussed [2]. Here, instead, we consider the addition to the gauge field DBI action of the ''usual'' Higgs field Lagrangian seeking for regular monopole solutions in contrast with bion-like singular solutions needing sources. This choice of Lagrangian implies that the solutions we find do not have a direct application to string theory. However, understanding the peculiarities of DBI monopole solutions for a Lagrangian as the one we choose may help to understand solutions arising when separated branes are considered in such a way that the gauge symmetry is spontaneously broken (see [2], Sec. VII B).

The paper is organized as follows: we present in Sec. II the $SO(3)$ DBI-Higgs action, discuss the spherically symmetric *Ansatz* and derive the radial equations of motion both for the usual and the symmetric trace. In Sec. III we describe our numerical solutions and discuss their main properties. We give in Sec. IV analytical arguments giving support to the existence of critical values for β below which the monopole solution ceases to exist. Finally we present in Sec. V a summary of our results and the conclusions.

II. LAGRANGIAN AND THE MONOPOLE *ANSATZ*

The 't Hooft–Polyakov monopole solution [9,10] to the equations of motion of the Yang-Mills-Higgs Lagrangian owes its existence and main properties to the non-Abelian character of an *Ansatz* for the gauge and scalar fields, mixing space-time and internal indices in such a way that ensures topological non-triviality and regularity of the resulting solution. In order to look for analogous solutions in the DBI

theory, one should necessarily start from a non-Abelian version of the Born-Infeld theory and also decide how the Higgs field will be coupled to the gauge field.

The definition of the DBI theory for a non-Abelian gauge group is not unique and several alternatives have been discussed in the literature [8,11–16]. The simplest extension amounts to defining the gauge field Lagrangian in the form

$$L_{DBI}^{tr} = \beta^2 \text{tr} \left(1 - \sqrt{1 + \frac{1}{2\beta^2} F_{\mu\nu} F^{\mu\nu} - \frac{1}{8\beta^4} (F_{\mu\nu} \widetilde{F}^{\mu\nu})^2} \right). \tag{1}$$

Here $F_{\mu\nu}$ is the field strength taking values in the Lie algebra of the gauge group [which we take for simplicity as $SO(3)$],

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + e[A_\mu, A_\nu], \tag{2}$$

$$A_\mu = A_\mu^a t^a = \vec{A}_\mu \cdot \vec{t}, \quad \vec{t} = \frac{\vec{\sigma}}{\sqrt{2}}, \tag{3}$$

$$\widetilde{F}_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\alpha\beta} F^{\alpha\beta} \tag{4}$$

and ''tr'' in Eq. (1) represents the usual trace on $SO(3)$ indices, with generators normalized so that

$$\text{tr}(t^a t^b) = \delta^{ab}. \tag{5}$$

However, unlike the Abelian case, expression (1) cannot be written in the familiar form $L \approx \sqrt{-\det(I+F)}$.

A second possibility is precisely to define the Born-Infeld Lagrangian in terms of a determinant and then making contact with the tree level open string effective action for branes [8],

$$L_{DBI}^{Str} = \beta^2 \text{Str} \left[1 - \sqrt{-\det\left(g_{\mu\nu} + \frac{1}{\beta} F_{\mu\nu}\right)} \right]. \tag{6}$$

In this case the determinant is computed over the space-time indices and the trace over the internal indices, denoted by ''Str,'' is the symmetric trace operation defined as

$$\text{Str}(t_1, t_2, \ldots, t_N) \equiv \frac{1}{N!} \sum_\pi \text{tr}(t_{\pi(1)} t_{\pi(2)} \cdots t_{\pi(N)}) \tag{7}$$

with the sum extending over all permutations π of the product of N given t's.

Of course, $g_{\mu\nu}$ in Eq. (6) is the 3+1 usual Minkowski space-time metric, $g_{\mu\nu} = \text{diag}(1, -1, -1, -1)$, and not the pullback of the $(d+1)$-dimensional Minkowski metric to the $(p+1)$ dimensional world volume of the p-brane. It should be mentioned that odd powers of the field strength F are absent from the expansion of L_{Str}, this implying that F (although possibly large) should be slow varying since $F^3 \sim [D,D]F^2$. In this sense using Str amounts to some kind of Abelian approximation. It should be noted that some unsolved problems related to the use of a symmetric trace have been signaled. They refer to discrepancies between the re-

sults that arise from a symmetrized non-Abelian Born-Infeld theory and the expected spectrum of brane theories [12].

By expanding the determinant in Eq. (6) we can write the DBI Lagrangian as

$$L_{DBI}^{Str} = \beta^2 \text{Str} \left(1 - \sqrt{1 + \frac{1}{2\beta^2} F_{\mu\nu} F^{\mu\nu} - \frac{1}{8\beta^4} (F_{\mu\nu} \widetilde{F}^{\mu\nu})^2} \right) \tag{8}$$

which only differs from the Lagrangian (1) in the prescription of the trace operation. Notice however that in spite of its apparent similarity both forms are operationally extremely different. In the Lagrangian (1) the square root can be expanded in power series, eventually ressumed, and then the trace can be taken. However, in the Lagrangian (8) the re-summation can only be done *after* taken the symmetric trace since the Str operation has only sense acting on the series expansion.

Apart from this alternatives related to the way the trace operation is defined, one has to decide how the Higgs field dynamics is introduced. In previous analysis, DBI monopoles were constructed by demanding that the *usual* Yang-Mills-Higgs BPS relations also hold in the DBI case [6]. This amounts to define a Higgs field Lagrangian in a Born-Infeld-like way (i.e., also under a square root) in such a way that the model has a supersymmetric extension [4,14–17]. The BPS relations being the same as in the Yang-Mills-Higgs case, the resulting DBI monopole solutions are identical to the well-honored Prassad-Sommerfield exact solutions and have no specific features resulting from the DBI dynamics. Instead, we shall consider here the usual $SO(3)$ Higgs field Lagrangian and a symmetry breaking potential not necessarily in the BPS limit. We then propose the following Lagrangian for the Higgs field:

$$\mathcal{L}_{Higgs} = \frac{1}{2} D^\mu \vec{\phi} \cdot D_\mu \vec{\phi} - V[\phi] \tag{9}$$

with the scalar triplet written in the form

$$\phi = \phi^a t^a = \vec{\phi} \cdot \vec{t}, \tag{10}$$

the symmetry breaking potential given by

$$V[\phi] = \frac{\lambda}{4} (\vec{\phi} \cdot \vec{\phi})^2 - \frac{\mu^2}{2} \vec{\phi} \cdot \vec{\phi} \tag{11}$$

and the covariant derivative defined as

$$D_\mu \vec{\phi} = \partial_\mu \vec{\phi} + e \vec{A}_\mu \wedge \vec{\phi}. \tag{12}$$

A. Equations of motion for $L_{DBI-Higgs}^{tr}$

When the trace operation ''tr'' is used, the DBI-Higgs Lagrangian reads

$L^{tr}_{DBI\text{-}Higgs}$

$$= \beta^2 \text{tr}\left(1 - \sqrt{1 + \frac{1}{2\beta^2}F_{\mu\nu}F^{\mu\nu} - \frac{1}{8\beta^4}(F_{\mu\nu}\tilde{F}^{\mu\nu})^2} \right)$$

$$+ \frac{1}{2}D^\mu\vec{\phi}\cdot D_\mu\vec{\phi} - V[\phi]. \tag{13}$$

From here on we shall consider purely magnetic configurations for which $F_{\mu\nu}\tilde{F}^{\mu\nu}=0$. Then the equations of motion take the form

$$D^\mu\left(\frac{\vec{F}_{\mu\nu}}{\sqrt{1 + \frac{1}{4\beta^2}\vec{F}^{\mu\nu}\cdot\vec{F}_{\mu\nu}}} \right) = e\vec{\phi}\times D_\nu\vec{\phi}, \tag{14}$$

$$D^\mu D_\mu\vec{\phi} = \mu^2\vec{\phi} - \lambda\phi^2\vec{\phi}. \tag{15}$$

We shall consider the usual spherically symmetric 't Hooft–Polyakov *Ansätze* [9,10]

$$\vec{A}_i(\vec{r}) = \frac{K(r)-1}{e}\vec{\Omega}\wedge\partial_i\vec{\Omega}, \tag{16}$$

$$\vec{A}_0(\vec{r}) = 0, \tag{17}$$

$$\vec{\phi}(\vec{r}) = \frac{H(r)}{er}\vec{\Omega}, \tag{18}$$

$$\vec{\Omega} = \vec{\Omega}(\theta,\varphi) = \frac{1}{r}\vec{r}, \tag{19}$$

with the appropriate boundary conditions for K and H,

$$\lim_{r\to\infty}K(r) = 0, \quad \lim_{r\to\infty}\frac{1}{r}H(r) = \frac{\mu e}{\sqrt{\lambda}} \tag{20}$$

together with the conditions at the origin:

$$K(0) = 1, \quad H(0) = 0. \tag{21}$$

Inserting *Ansätze* (16)–(18) into the equations of motion (14),(15) one gets

$$r^2K'' - r^2\frac{R'}{R}K' = K(RH^2 + K^2 - 1),$$

$$r^2H'' = 2HK^2 - \mu^2r^2H\left(1 - \frac{\lambda}{e^2\mu^2r^2}H^2 \right) \tag{22}$$

where

$$R = \sqrt{1 + \frac{1}{\beta^2e^2r^4}\left(r^2K'^2 + \frac{1}{2}(K^2-1)^2 \right)}. \tag{23}$$

It will be convenient to define new dimensionless variables and parameters,

$$\rho = \frac{e\mu r}{\sqrt{\lambda}},$$

$$\lambda_0 = \lambda/e^2,$$

$$\beta_0 = \frac{\beta\lambda}{e\mu^2}, \tag{24}$$

so that one finally has

$$\rho^2K'' = K(RH^2 + K^2 - 1) + \rho^2\frac{R'}{R}K', \tag{25}$$

$$\rho^2H'' = 2HK^2 - \lambda_0H(\rho^2 - H^2), \tag{26}$$

$$R = \sqrt{1 + \frac{1}{\beta_0^2\rho^4}\left(\rho^2K'^2 + \frac{1}{2}(K^2-1)^2 \right)}. \tag{27}$$

With this *Ansatz*, we can write the energy for the monopole solution in the form

$$E = \frac{4\pi\mu}{\sqrt{\lambda}e}\int d\rho\rho^2\left\{ 2\beta_0^2(R-1) + \frac{1}{2\rho^2} \right.$$

$$\left. \times\left[\left(H' - \frac{H}{\rho}\right)^2 + \frac{2}{\rho^2}H^2K^2 \right] + \frac{\lambda_0}{4}\left(\frac{H^2}{\rho^2} - 1\right)^2 \right\}. \tag{28}$$

This expression reduces to the 't Hooft–Polyakov monopole mass formula in the $\beta\to\infty$ limit, as expected.

The electromagnetic $U(1)$ field strength $\mathcal{F}_{\mu\nu}$ is defined as usual [9] in the form

$$\mathcal{F}_{\mu\nu} = \frac{1}{|\phi|}\vec{\phi}\cdot\vec{F}_{\mu\nu} - \frac{1}{e|\phi|^3}\vec{\phi}\cdot(D_\mu\vec{\phi}\wedge D_\mu\vec{\phi}). \tag{29}$$

Now, since we are considering DBI dynamics, we have to distinguish between the magnetic induction $\vec{B}$ and the magnetic intensity $\vec{H}$,

$$B^i = \frac{1}{2}\varepsilon^{ijk}\mathcal{F}_{jk}, \quad H^i = \frac{1}{2}\varepsilon^{ijk}\frac{\mathcal{F}_{jk}}{R}. \tag{30}$$

Using *Ansätze* (16)–(18) one easily finds that

$$B^i = \frac{x^i}{er^3}, \tag{31}$$

so that the magnetic flux at infinity,

$$\Phi \equiv \int_{S^2_\infty}dS_iB^i = \frac{4\pi}{e}, \tag{32}$$

corresponds to that of a unit magnetic monopole located at the origin. The magnetic flux Φ can alternatively be defined in terms of $\vec{H}$, this leading to the same answer (32).

B. Equations of motion for $L_{DBI\text{-}Higgs}^{Str}$

When the symmetric trace operation is used, the DBI-Higgs Lagrangian is defined as

$L_{DBI\text{-}Higgs}^{Str}$

$$= \beta^2 \text{Str}\left(1 - \sqrt{1 + \frac{1}{2\beta^2} F_{\mu\nu} F^{\mu\nu} - \frac{1}{8\beta^4}(F_{\mu\nu}\tilde{F}^{\mu\nu})^2} \right)$$

$$+ \frac{1}{2} D^\mu \vec{\phi} D_\mu \vec{\phi} - V[\phi]. \tag{33}$$

Again we will only consider purely magnetic configurations; so $F_{\mu\nu}\tilde{F}^{\mu\nu}=0$. Because of the use of the symmetric trace, deriving the equations of motion in this case becomes rather involved. Indeed, one has first to expand the square root in L^{Str} in powers of $1/\beta^2$ and at each order N, consider the $N!$ terms which are included in Str. For example, up to order $1/\beta^2$ one has for the purely DBI Lagrangian

$$\mathcal{L}_{DBI}^{Str} = -\frac{1}{4}\vec{F}_{\mu\nu}\cdot\vec{F}^{\mu\nu} + \frac{1}{96\beta^2}[(\vec{F}_{\mu\nu}\cdot\vec{F}^{\mu\nu})^2 + 2(\vec{F}_{\mu\nu}\cdot\vec{F}_{\rho\sigma})$$

$$\times (\vec{F}^{\mu\nu}\cdot\vec{F}^{\rho\sigma})] + O\left(\frac{F^6}{\beta^4}\right). \tag{34}$$

Already at this order, this DBI Lagrangian differs from the one arising when one expands L_{DBI}^{tr},

$$\mathcal{L}_{DBI}^{tr} = -\frac{1}{4}\vec{F}_{\mu\nu}\cdot\vec{F}^{\mu\nu} + \frac{1}{32\beta^2}(\vec{F}_{\mu\nu}\cdot\vec{F}^{\mu\nu})^2 + O\left(\frac{F^6}{\beta^4}\right). \tag{35}$$

The $1/\beta^4$ term in the expansion of L_{DBI}^{Str} involves 120 terms containing the sixth power of the field strength, this making the search of a solution using a numerical approach too complicated. We shall here consider the problem to the $1/\beta^2$ order given in Eq. (34) and analyze how the solution differs from the one obtained using the more simple ''tr'' operation.

The equations of motion for the gauge field resulting from Eq. (34) read

$$D_\mu \left(\vec{F}^{\mu\nu} - \frac{1}{12\beta^2}[(\vec{F}_{\rho\sigma}\cdot\vec{F}^{\rho\sigma})\vec{F}^{\mu\nu} + 2(\vec{F}_{\rho\sigma}\cdot\vec{F}^{\mu\nu})\vec{F}^{\rho\sigma}] \right)$$

$$= e\,\vec{\phi}\times\vec{D}^\nu\phi \tag{36}$$

while those associated to the Higgs field remain unchanged.

After using the spherically symmetric *Ansätze* (16)–(18), Eq. (36) becomes

$$K''(\rho) = -[3K(\rho) - 6\rho^4\beta_0^2 K(\rho) + 6\rho^4\beta_0^2 H(\rho)^2 K(\rho) - 17K(\rho)^3 + 6\rho^4\beta_0^2 K(\rho)^3 + 45K(\rho)^5 - 71K(\rho)^7 + 70K(\rho)^9$$

$$- 42K(\rho)^{11} + 14K(\rho)^{13} - 2K(\rho)^{15} - 4\rho K'(\rho) + 8\rho K(\rho)^2 K'(\rho) - 4\rho K(\rho)^4 K'(\rho) + 2\rho^2 K(\rho)^3 K'(\rho)^2 - 8\rho K'(\rho)^3$$

$$- 4\rho^3 K'(\rho)^3 + 32\rho K(\rho)^2 K'(\rho)^3 - 48\rho K(\rho)^4 K'(\rho)^3 + 32\rho K(\rho)^6 K'(\rho)^3 - 8\rho K(\rho)^8 K'(\rho)^3 - 12\rho^2 K(\rho)K'(\rho)^4$$

$$+ 36\rho^2 K(\rho)^3 K'(\rho)^4 - 36\rho^2 K(\rho)^5 K'(\rho)^4 + 12\rho^2 K(\rho)^7 K'(\rho)^4 - 8\rho^3 K'(\rho)^7 - 2\rho^2 K(\rho)K'(\rho)^2]\frac{1}{S}, \tag{37}$$

$$S = \rho^2[1 - 6\rho^4\beta^2 - 2K(\rho)^2 + K(\rho)^4 + 6K'(\rho)^2 + 6\rho^2 K'(\rho)^2 - 24K(\rho)^2 K'(\rho)^2 + 36K(\rho)^4 K'(\rho)^2 - 24K(\rho)^6 K'(\rho)^2$$

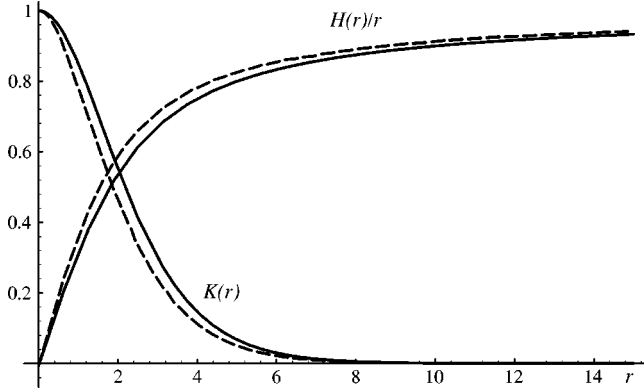$$+ 6K(\rho)^8 K'(\rho)^2 + 28\rho^2 K'(\rho)^6] \tag{38}$$

FIG. 1. Plot of the functions $K(r)$ and $H(r)/r$ (in dimensionless variables) for the monopole solution with $\lambda = 0$. The solid line corresponds to the solution with $\beta = 10$ and the dashed line corresponds to the the solution with $\beta = 0.5$.
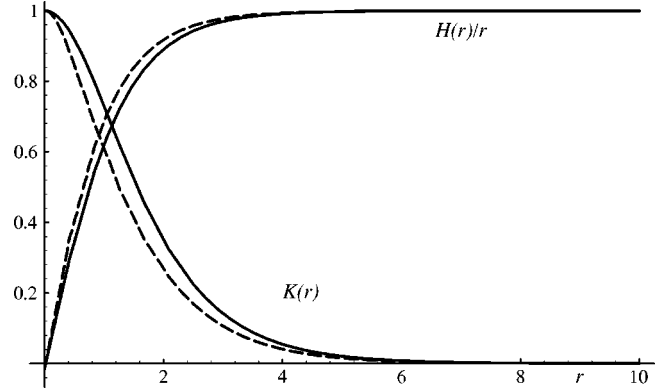

FIG. 2. Plot of the functions $K(r)$ and $H(r)/r$ (in dimensionless variables) for the monopole solution with $\lambda = 0.5$. The solid line corresponds to the solution with $\beta = 10$ and the dashed line corresponds to the the solution with $\beta = 0.8$.

while the equation for $H(\rho)$ is still given by Eq. (26). Finally, the energy associated with the monopole is given by

$$E = \frac{4\pi\mu}{\sqrt{\lambda}e} \int d\rho \left\{ \frac{1}{\rho^2} \left(\rho^2 K'^2 + \frac{1}{2}(K^2-1)^2 \right) - \frac{1}{6\rho^6 \beta_0^2} \right.$$
$$\times \left[\left(\rho^2 K'^2 + \frac{1}{2}(K^2-1)^2 \right)^2 + \left(\rho^2 K'^4 + \frac{1}{2}(K^2-1)^4 \right)^2 \right]$$
$$\left. + \frac{1}{2} \left[\left(H' - \frac{H}{\rho} \right)^2 - \frac{2H^2 K^2}{\rho^2} \right] - \frac{\lambda_0}{4} \left(\frac{H^2}{\rho^2} - 1 \right)^2 \right\} + O(\beta_0^4).$$

(39)

III. NUMERICAL RESULTS

To obtain a detailed profile of the monopole solution, we solved numerically the differential equations (25)–(27) for the case of the trace operation "tr" and Eqs. (37), (38), and (26) for the symmetric trace "Str". We employed a relaxation method for boundary value problems [18]. Such a method determines the solution by starting with an initial guess and improving it iteratively. The natural initial guess was the exact Prassad-Sommerfield solution [19] (which corresponds to $\lambda_0 = 0$ and $\beta \to \infty$).

A. Usual trace

For $\beta \gtrsim 10$, the solutions to Eqs. (25)–(27) do not differ appreciably from the 't Hooft–Polyakov monopole solution (see for example [20] for a plot of the 't Hooft–Polyakov solution). As β decreases, the solution changes slowly: the monopole radius decreases and the (radial) magnetic field $\vec{H}$ concentrates at the origin. Some of the solutions profile are depicted in Figs. 1 and 2.

For $\beta \sim 1$ new features become apparent from our numerical analysis. In particular, we found that:

(1) For $\lambda_0 = 0$ the behavior of the Higgs field at large distances depends on β,

$$H(r) \to \phi_0 r + c(\beta/\mu^2) \quad \text{for} \quad r \to \infty \quad (40)$$

[here ϕ_0 is the Higgs field vacuum expectation value (VEV) in the Prassad-Sommerfield limit]. Of course, for $\beta \to \infty$, $c(\beta/\mu^2) \to 1$ and one has the usual asymptotic behavior of the Higgs field for the 't Hooft–Polyakov monopole. For finite β, however, it is interesting to note that the $1/r$ falloff, which in the Yang-Mills-Higgs case is related to the massless dilaton associated with the scale invariance of the BPS regime, has now a β dependent coefficient.

(2) There is a critical value of β, which we shall denote as β_c, such that for $\beta \lesssim \beta_c$ there is no (numerical) solution to the equations of motion (25)–(27). Some values for β_c are $\beta_c = 0.41$ for $\lambda_0 = 0$, $\beta_c = 0.62$ for $\lambda_0 = 0.5$.

This peculiar characteristic of the solutions appears to be a consequence of the high nonlinearity of the equations and not a fictitious artifact of the numerical method. Moreover, the energy of the solutions seems to be singular at $\beta = \beta_c$ (see Fig. 3).
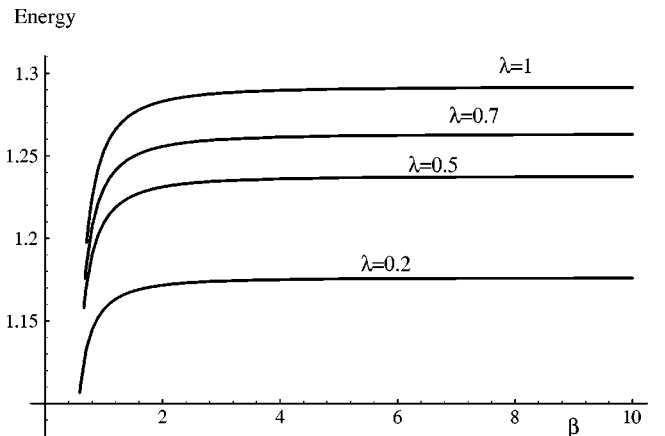

FIG. 3. Energy of the monopole configuration as a function of β for different values of λ.

B. Symmetric trace

For this case we solved Eqs. (37), (38) and (26) with the same numerical approach. Since the equations are valid to order $1/\beta^2$, our analysis cannot be reliable for small β. We see that for $\beta \gtrsim 4$ the solutions do not differ notably from those arising when the trace operation ''tr'' is considered. The profile of the solutions are indistinguishable from the solid-line curves of Figs. 1 and 2. In view of our approximation, we could not analyze the region where one expects the existence of β_c in this case.

The use of the Str prescription makes the equations of motion so involved as to prevent an exact treatment similar to that developed for the normal trace. In particular, even when the fields take the simplified form of the 't Hooft–Polyakov *Ansatz*, the series expansion of the square root of Eq. (33) cannot be resummed as in the normal trace case and the Born-Infeld Lagrangian only makes sense as an infinite series. The approximate treatment shows, however, that the main properties of the monopole solution are common to both prescriptions although it does not guarantee the existence of a critical value β_c.

IV. ANALYSIS OF β_c

As noted in the Introduction, the existence of a critical value of the absolute field, below which the solution to the equations of motion of the DBI-Higgs system ceases to exist, was already noticed for vortices in the Abelian case [7] and should be considered as a distinctive feature of soliton solutions in DBI theories.

In order to better understand the origin of β_c let us introduce the following scaling argument. For a monopole-like solution, there is a characteristic radius R_W that can be associated with the monopole core; outside this core, the gauge field approaches its asymptotic value. For Yang-Mills-Higgs monopoles (or Nielsen-Olesen vortices), this radius should be necessarily related to μ, the sole parameter carrying dimensions $R_W \sim 1/\mu$. The size R_W is fixed so as to minimize the sum of the energy stored in the magnetic field outside the core and the energy due to the scalar field gradient inside the core. The resulting value R_W is in this case $R_W = 1/M_W$, with M_W the mass of the gauge boson, $M_W = (e/\sqrt{\lambda})\mu$. A second length playing a role in the monopole configuration is related to the size of the region outside of which the Higgs scalar takes practically its vacuum expectation value. We shall call the radius of this region R_H. For the Yang-Mills-Higgs system one has $R_H = 1/\mu$. From R_W and R_H we can define a dimensionless parameter v measuring the relative intensity of the two coupling constants in the theory,

$$v \equiv \frac{R_H}{R_W} = \frac{e}{\sqrt{\lambda}}. \qquad (41)$$

(In the vortex case $1/v$ coincides with the Ginzburg-Landau parameter separating the two types of superconductivity.) For $v \sim 1$ one has a well-defined monopole configuration.

Now, when DBI-Higgs monopoles are considered, there is, apart from μ, a second dimensionful parameter, β, $[\beta]$

$= [\mu]^2$. Then R_W and R_H could in principle depend both on μ and β and the configuration minimizing the energy will result from the matching of both parameters determining the size of the monopole. It may happen that in some region of the (β, μ) domain such a matching becomes not possible. The outcome will be the non-existence of solutions in a range of values of β with size related to β_c. In view of the complexity of the non-linear coupled system (25)–(27), let us analyze this possibility by using an approximate monopole configuration sharing the main features of the true solution:

$$K_{app}(r) = \left(1 - \frac{r}{R}\right)\theta(R - r), \qquad (42)$$

$$H_{app}(r) = r\left(1 - \frac{r_0^2}{r^2}\right)^2 \theta(r - r_0). \qquad (43)$$

Here R and r_0 are parameters controlling the shape of the gauge field and scalar field configurations and they have to be determined by minimizing the energy E of the configuration. One can relate R and r_0 with R_W and R_H by searching in Eqs. (42),(43) for the values of r for which the gauge and scalar field configurations differ in $1/e$ from its asymptotic. This gives $R_W \sim (1/2)R$, $R_H \sim 3r_0$ and in this sense one can think that $R = R(\beta, \mu)$ and $r_0 = r_0(\beta, \mu)$. Let us finally note that using Eq. (41) one can write v in terms of r_0 and R,

$$v \sim 6\frac{r_0}{R} \equiv 6x. \qquad (44)$$

We have seen that for large β (say $\beta \gtrsim 5$), the DBI-Higgs theory just gives the same answer as the Yang-Mills-Higgs model so that one should always find in this region values for R and r_0 minimizing E. In particular, in the $\beta \to \infty$ case we found using our approximate configurations that one has, for $\lambda_0 = 0.5$, $x = 0.128$. This giving for v the result approximate result $v_{app} \sim 6x \sim 0.8$ to be compared with the ''exact'' result for Yang-Mills-Higgs theory, $v = 1/\sqrt{\lambda_0} = \sqrt{2}$.

Now, for small β the situation radically changes. Indeed, using Eqs. (42) and (43) one finds for the energy, to second order in β (apart of an irrelevant additive constant),

$$E = -R[-0.33 + 0.03\beta + 3.04x + 3.33x^2 - 8.53x^3 + 2x^4$$
$$-0.13x^6 + 0.01x^8 + 8x^2 \log(x)] + 2.1R^3\lambda x^3 + O(\beta^3)$$
$$= 2.1R^3\lambda x^3 - R(f(x) + 0.03\beta) + O(\beta^3). \qquad (45)$$

It is not difficult to show that for small β (and any λ) the above expression does not have a minimun for any R and x.

We conjecture that this phenomenon, occurring for an approximate configuration, also takes place for the actual monopole solution: below a critical β value, there is no possible matching between the monopole core and the size of the region where the Higgs scalar is different from its vacuum value, in such a way the energy is minimized.

V. SUMMARY AND CONCLUSIONS

We have discussed in this work monopole solutions for an $SO(3)$ Dirac-Born-Infeld gauge theory coupled to a Higgs scalar. We considered two alternative Lagrangians for the theory, differing in the way the trace over group indices is taken. Concerning the Higgs field, we have chosen the usual kinetic energy term and symmetry breaking potential.

As in the case of the Yang-Mills-Higgs system, a spherically symmetric *Ansatz* leads to a system of coupled nonlinear radial equations that have to be solved numerically. We have seen that the magnetic field corresponds, as in the 't Hooft–Polyakov case, to that of a monopole with unit charge. When the absolute field β parameter is large ($\gtrsim 5$) the profile of the monopole solution is practically the same as the corresponding to the Yang-Mills-Higgs model. As β decreases, the monopole radius becomes smaller and the magnetic field concentrates more and more near the origin.

A remarkable effect takes place for small β: there exists a critical value β_c such that for $\beta \lesssim \beta_c$ the solution ceases to exist. The actual value of β_c depends on the choice of the other free parameters. We presented an scaling argument that supports this result: using an approximate solution that depends only on the dimensions of the configuration we showed that for small values of β it is impossible to adjust the size parameters to minimize the energy.

The monopole solution we have presented has many remarkable features that make worth a thorough investigation. In particular, the analysis of dyon solutions, which implies the inclusion of the $(F\widetilde{F})^2$ term in the DBI action, should reveal new features related to the existence of the dimension-full parameter β. We hope to discuss this problem in a future work.

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