Primordial Nucleosynthesis as a test of variable rest masses 5-dimensional cosmology

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Abstract

The deviation of primordial Helium production due to a variation on the difference between the rest masses of the nucleons is presented. It is found an upper bound $\delta(M_n - M_p) \lesssim 0.129$ MeV, between the present and nucleosynthesis epochs. This bound is used to analyze Wesson's theory of gravitation; as a result, it is ruled out by observation.

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The so-called Hot Big Bang model provides a consistent description of the evolution of the Universe. This model depends on a set of universal parameters known as *fundamental constants*. However, a great number of theories of gravitation have appeared in which some of these *constants* do vary with time. In particular, scalar-tensor theories of gravity [1–3] make predictions which are in complete agreement with present experimental data while predicting a variation of Newton's gravitational constant over cosmological time scales. The physical embodiment of these theories allows a natural generalization of General Relativity and thus provides a convenient set of representations for the observational limits on possible deviations of Einstein's theory, making them a profitable arena for cosmology.

The hypothesis that the gravitational interaction has changed over the history of the Universe (i.e. the gravitational parameter G or the rest masses of elementary particles depend on the time t) can be also analyzed in the framework of a 5-dimensional cosmology, proposed by Wesson [4]. This theory is founded either on dimensional analysis [5,6] as well as on reinterpretation of the five dimensional vacuum equations [7,8]. A consequence of this relation is that the rest mass of a given body varies from point to point in space-time, in agreement with the ideas of Mach [9,10]. This is a definite and testable prediction, especially when time intervals of cosmological order are considered [4,11].

In this letter, we find an upper bound to the variation of the masses of the nucleons over cosmological time intervals, from a comparison of the observed primordial abundance of ^{4}He with the theoretical variation induced by a changing mass. Subsequently, this upper bound is compared with the prediction of the 5–dimensional Wesson's theory of gravitation.

If we consider that a variation of the rest masses of the particles had occurred between the epoch of primordial nucleosynthesis and ours, we can compute the deviation in the ^{4}He production from the Hot Big Bang model prediction due to this fact. The method we are going to apply is a generalization of the calculation made by Casas, Bellido and Quirós [12] to fix nucleosynthesis bounds on the variation of the gravitational constant in Jordan-Brans-Dicke theory of gravity. The same method was recently used to limit other scalar-tensor theories with more general couplings functions [13]. We know that – in the Hot Big Bang model – the primordial ^{4}He production is given by

$$Y_{\rm p} = \lambda \left(\frac{2x}{1+x} \right)_{t_{\rm f}} \tag{1}$$

where $\lambda = \exp(-(t_{\rm nuc} - t_{\rm f})/\tau)$ stands for the fraction of neutrons which decayed into protons between $t_{\rm f}$ and $t_{\rm nuc}$, with $t_{\rm f}$ ($t_{\rm nuc}$) the time of freeze out of the weak interactions (nucleosynthesis) [14], τ the neutron mean lifetime, and $x = \exp(-(M_n - M_p)/kT)$ the neutron to proton ratio. If we make a variation on the rest masses of the particles, i.e. if we consider that a difference between the rest masses in the present and nucleosythesis epochs does exist, it is straightforward to compute an expression for the deviation in the ^{4}He primordial production, that reads

$$\delta Y_{\rm p} = Y_{\rm p} \ln \left(\frac{2\lambda}{Y_{\rm p}} - 1 \right) \left[-1 + \frac{Y_{\rm p}}{2\lambda} \right] \frac{\delta(\Delta Q)}{\Delta Q} \tag{2}$$

where we have defined $\Delta Q = M_n - M_p$.

We must also take into account that a variation in the rest masses of the particles will affect the neutron lifetime. The latter fact was not considered above since (2) represents only the explicit derivative with respect to ΔQ. A calculation of how a variation of the neutron lifetime affects the prediction on primordial ^{4}He has been already done in [12,15]. It is given by,

$$\delta Y_{\rm p} = 0.185 \frac{\delta \tau}{\tau} \tag{3}$$

Noting that the dependence of τ upon the masses is $\tau \propto G_{\rm F}^{-2} \, \Delta Q^{-5} \propto M_W^4 \, \Delta Q^{-5}$, where $G_{\rm F}$ is the Fermi constant and M_W is the mass of the bosonic mediator of weak interactions, it is easy to obtain,

$$\frac{\delta \tau}{\tau} = -\frac{\delta \Delta Q}{\Delta Q} \tag{4}$$

Thus, any variable rest mass theory will predict a primordial Helium abundance given by,

$$Y_{\mathrm{p,var-rest-mass}} = Y_{\mathrm{p,std}} + \delta Y_{\mathrm{p}} \tag{5}$$

where $Y_{\mathrm{p,std}}$ is the value predicted by the standard big–bang nucleosynthesis theory.

We can find an upper bound for δY_{p} summing up the two deviations referred above, *i.e.* equations (2) and (3), and comparing with the observed value Y_{obs},

$$|\delta Y_{\mathrm{p}}| \leq |Y_{\mathrm{obs}} - Y_{\mathrm{std}}| + \epsilon \leq \sigma \tag{6}$$

where ϵ is an estimate of the observational error and σ includes also estimates of theoretical errors. From [15,16] we estimate $\sigma \leq 0.01$. But, it may also be possible that, due to small changes in nucleon masses, small changes in nuclear cross sections do occur. Since the functional dependence of cross sections with masses is generally unknown, we shall take into account these changes by arbitrarily doubling the theoretical error. Thus, we obtain

$$\left| \frac{\delta(\Delta Q)}{\Delta Q_0} \right| \leq 10\% \tag{7}$$

with $\Delta Q_0 \simeq 1.294$ MeV [16], or equivalently

$$\delta(\Delta Q) \leq 0.129 \text{ MeV} \tag{8}$$

At this stage, we must work out the cosmological solution for the radiation dominated era. We shall consider the warped product $M^4 \times R$, where M^4 is the ordinary 4-dimensional spacetime manifold and R correspond to the extra dimension. This leads to the line element

$$ds^2 = -dt^2 + \frac{A^2(t)}{(1 + k(x_1^2 + x_2^2 + x_3^2)/4)^2}(\, dx_1^2 + dx_2^2 + dx_3^2\,) + e^{\zeta(t)} dx_5^2 \tag{9}$$

where $A^2(t)$ is the expansion scale factor and $e^{\zeta(t)}$ must be associated with the mass scale factor. This could be simplified considering a flat spatial section in M^4 (i.e. $k = 0$). This assumption is justified when one compares the order of magnitude of the different terms in the Einstein equation evaluated in the radiation era [19].

The suitable generalization of Einstein equation for the theory can be written as[1]

[1]In what follows, latin indices $A, B, \ldots$, run from 0 to 4, greek indices from 0 to 3 and latin indices $i, j, \ldots$, from 1 to 3.

$$G_{AB} = 8\,\pi\,G\;T_{AB} \tag{10a}$$

$$T_{AB} = \mathrm{diag}(\rho,\,-p,\,-p,\,-p,\,0) \tag{10b}$$

with ρ (p) the density (pressure) of the radiation fluid, and G is Newton's gravitational constant. The equation of state is $p = 1/3\rho$. The boundary condition that ought to be imposed is a smoothly matching at $t = t_{\rm eq}$ (equivalence time) with the dust-filled solution obtained in a previous paper [20]. It is important to stress that in our previous work we required that both ζ and $d\zeta/dt$ vanish at $t =$ today. With these conditions, the masses of the fundamental particles can be set to their present experimental value [16], and mass variations are negligible in short time scales (see eq.(14)), which is consistent with the bound $|\dot{m}/m|_{\rm today} \lesssim 10^{-12}$ yr^{-1} [21]. However, our results are to a large extent independent of the epoch in which the initial conditions for ζ and $d\zeta/dt$ were imposed. [2]

The radiation dominated era solution is given by

$$A^2(t) = 2\,\beta\,t \tag{11a}$$

$$\zeta(t) = 2\,\ln\left[-\frac{1}{2}\left[\frac{t_{\rm eq}}{\sqrt{t_{{}_0}}} - \sqrt{t_{{}_0}}\right]t^{-1/2} + \sqrt{\frac{t_{\rm eq}}{t_{{}_0}}}\right] \tag{11b}$$

$$\rho(t) = \frac{1}{16\,\pi\,G}\left\{\frac{3}{2\,t^2} + \frac{3}{2\,t}\;\frac{\frac{1}{4}\,[\frac{t_{\rm eq}}{\sqrt{t_0}} - \sqrt{t_0}\,]\;t^{-3/2}}{-\frac{1}{2}\,[\frac{t_{\rm eq}}{\sqrt{t_0}} - \sqrt{t_0}\,]\;t^{-1/2} + \sqrt{\frac{t_{\rm eq}}{t_0}}}\right\} \tag{11c}$$

(with β a constant)[3].

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[2]This can be seen by comparing the prediction of this theory for $\dot{m}/m = \dot{\zeta}/2$ at $t =$ today in the case where the initial conditions for ζ and its derivative were imposed for instance at the Earth formation epoch, with the current experimental limits on $\dot{m}/m$ [21].

[3]This solution was previously obtained by Mann and Vincent [17] imposing different boundary conditions. It was also obtained by Grøn [18]; from his work it is clear that the rate of change of the fifth metric coefficient depends on initial conditions.

The way in which the mass is introduced in this formalism has an inherent ambiguity. The proposal of Ma [6]

$$m(t) = \frac{c^2}{G} \int_{x_5}^{x_5 + l} \sqrt{g_{55}} \, dx^5 \tag{12}$$

or, in the case of an x_5-independent metric,

$$m(\tau) = \frac{c^2}{G} \sqrt{g_{55}} \ \Delta l_0 \tag{13}$$

(Δl_0 is the -finite- "length" of the body in the x_5 direction [6]), does not specify the tensorial character of the mass, which is implicitly introduced into the theory. In other words, there is no conclusive reason to mantain the covariant form of g_{55} under the square root, and so $m(t)$ can be also scale as

$$m(\tau) = \frac{c^2}{G} \sqrt{g^{55}} \ \Delta \tilde{l}_0 \tag{14}$$

The bottom line in this idea is that the dimensional analysis used to define the relationship between the mass and the extra dimension $x^5 = c^2 \, m \, / \, G$ does not exclude the covariant formulation $x_5 = c^2 \, m \, / \, G$. We shall adopt hereafter the most favorable definition for the theory.

Hence, we can estimate the variation of the mass from eq. (11b). Since $t_0 \gg t_{\rm eq} \gg t_{\rm nuc}$ (t_0 is the present age of the universe), we find

$$\zeta_{\rm nuc} \approx 2 \ln \frac{1}{2} \sqrt{\frac{t_0}{t_{\rm nuc}}} \approx 2 \ln 10^8 \tag{15}$$

Defining the quotient $\Delta m / m$ as

$$\frac{\Delta m}{m_0} = \frac{m(t_{\rm nuc}) - m(t_0)}{m(t_0)} = e^{-\zeta_{\rm nuc}/2} - 1 \ \simeq \ -1 \tag{16}$$

we are able to see that the theory predicts

$$\left| \frac{\delta \Delta Q}{\Delta Q_0} \right| \simeq 100\% \tag{17}$$

which is in disagreement with the previous bound (7). Using a covariant formulation for the mass scale factor in (16) one would get an even worse disagreement.

Thereupon, the assumed relation connecting the $5^{\rm th}$ dimension and the particles rest masses is false, at least in the case we have explored. More general (i.e. x_5 dependant) metrics deserve more thorough analysis. We hope to report on this issues in a forthcoming work.

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