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Bogomol'nyi Bounds and Killing Spinors in $d = 3$ Supergravity

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Abstract

We discuss the connection between the construction of Bogomol'nyi bounds and equations in three dimensional gravitational theories and the existence of an underlying $N = 2$ local supersymmetric structure. We show that, appart from matter self duality equations, a first order equation for the gravitational field (whose consistency condition gives the Einstein equation) can be written as a consequence of the local supersymmetry. Its solvability makes possible the evasion of the no-go scenario for the construction of Killing spinors in asymptotically conical spacetimes. In particular we show that the existence of non-trivial supercovariantly constant spinors is guaranteed whenever field configurations saturate the topological bound.

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The connection between Bogomol'nyi (topological) bounds [1] for the energy or the action of classical solutions in bosonic field theories and the $N = 2$ supersymmetry algebra of the corresponding supersymmetric extension is by now well understood. The topological charge of the purely bosonic theory coincides with the central charge of the supersymmetry algebra in the $N = 2$ extended model [2, 3, 4]. The Bogomol'nyi bound then arises from positivity of the squared supersymmetry charge algebra. Moreover, supersymmetry of physical states leads to Bogomol'nyi equations saturating the bound (first order self-duality equations for the bosonic theory). Half of the supersymmetries are broken on field configurations that solve the Bogomol'nyi equations. The necessary conditions for the $N = 2$ supersymmetric extension, coincide with those ensuring the existence of Bogomol'nyi bounds. Let us finally stress that the classical aproximation to the mass spectrum given by these bounds is expected to be exact at the quantum level since supersymmetry ensures that there are no quantum corrections.

The same scenario should hold when one includes gravity, thus considering the case of *local* supersymmetry. Indeed, several models with self-gravitating solitons have been studied following the approach described above [5, 6, 7] and Bogomol'nyi bounds were found by carefully analysing the $N = 2$ supergravity algebra. As in the global case, half of the supersymmetries are broken when the bound is saturated. The presence of remnant unbroken supersymmetries is equivalent to the existence of Killing spinors [8]. It is important to stress at this point that the local algebra posses certain problems which, as we shall see, can be unambiguosly solved following the Hamiltonian formulation proposed by Deser and Teitelboim [9, 10, 11]).

In three dimensional spacetimes the connection between Bogomol'nyi bounds and local supersymmetry exhibits certain subtleties. To see this, let us consider a spinor η (such that $\gamma^0\eta = \pm\eta$) which is parallel transported around a closed curve Γ at infinity. Then, η acquires a phase given by the circulation of the spinorial connection ω^a_μ

$$\eta(x)|_{2\pi} = \mathcal{P}\exp\left(-\frac{i}{2}\oint_\Gamma \omega^a_\mu\gamma^a dx^\mu\right)\eta(x)|_0. \tag{1}$$

Here $\mathcal{P}$ denotes path-ordered integration. Now, static massive solutions to Einstein equations correspond, in three dimensional spacetime, to asymptotically conical geometries so that the circulation of the spinorial connection is proportional to the deficit angle δ. The resulting non-trivial holonomy gives raise to an ill-defined η. But precisely these spinors are necessary for defining supercharges that generate unbroken supersymmetries [12]. The impossibility of finding such Killing spinors would then difficult an extension of the global approach [3, 4] to locally supersymmetric models in 3 dimensions.

The present work intends to clarify these points giving general arguments that show the conditions under which covariantly constant Killing spinors are well defined in a variety of three dimensional supergravity models. As an example of our approach we discuss at the end of our work the $d = 3$ CP^n model coupled to $N = 2$.

The setting of the problem and its solution

Let us start by noting that the problem described above concerning the proper definition of Killing spinors is absent in the three dimensional models that have been already used as laboratories to investigate the relation above mentioned [6, 7, 13]. In particular, this is the case of the Abelian Higgs model coupled to $N = 2$ supergravity for which the existence of global unbroken supercharges, for certain solitons states, has been recently shown [6, 7]. Also in 3 dimensions, a new class of (p, q)-extended Chern-Simons Poincaré supergravities has been constructed and the existence of Killing spinors in the $(2, 0)$ theory has been proven [13]. In all these examples, unbroken supersymmetries strikingly survive the coupling to supergravity, despite the problems posed by the conical geometry.

In the case of the $2 + 1$ supersymmetric Abelian Higgs model coupled to gravity, the reasons behind this behavior were clarified already in [6] but one could ask whether it obeys to peculiarities of a particular three dimensional model or if it can be understood on general grounds and for other models. To explore this last possibility we consider a $2 + 1$ dimensional field theory describing gravity coupled to gauged matter with the property that a topological charge can be defined. In fact, one can construct in general a topologically conserved current J_μ for theories containing a gauge potential A^λ:

$$J_\mu = \epsilon_{\mu\nu\lambda}\partial^\nu A^\lambda \qquad (2)$$

The corresponding topological charge takes the form

$$T = \int_\Sigma J^\mu d\Sigma_\mu = \oint_{\partial\Sigma} A_\mu dx^\mu \qquad (3)$$

where the integrals are performed over a space-like surface Σ and its contour $\partial\Sigma$. Of course, the base manifold and the gauge group should be such as to ensure non-triviality of T. The class of theories admitting such contruction include the model analysed in [6, 7], the CP^n model discussed below and others.

Since we consider the coupling of the gauge-matter system system to $N = 2$ supergravity, the gauge potential belongs to a vector multiplet, so that one has a supercovariant derivative defined as [14]

$$\hat{\mathcal{D}}_\mu(\omega, \mathcal{A}) \equiv \mathcal{D}_\mu(\omega) + i\frac{\kappa^2}{4}\mathcal{A}_\mu \tag{4}$$

with $\mathcal{A}_\mu = \sum \mathcal{A}^i_\mu$, being $\mathcal{A}^i_\mu$ the spin-1 component of each vector multiplet coupled to the Einstein supermultiplet. We are denoting by $\mathcal{D}_\mu(\omega)$ the (ordinary) supergravity covariant derivative containing the spinorial connection. By looking to the action of the covariant derivative (4) on the gravitino field one can conclude that, in a sense, the gravitino becomes *charged* [1]. As we shall see, the presence of the gauge connection in the supercovariant derivative (4) is at the root of the existence of supercovariantly constant spinors. Indeed, the Bohm-Aharonov holonomy of $\mathcal{A}_\mu$ will cancel out that of the spinorial connection thus eliminating the effects of conical structure at infinity. This is in fact what happens in the Abelian Higgs model coupled to $N = 2$ supergravity [6, 7] and in Chern-Simons formulation of $(2, 0)$ Poincare supergravity [13]. We argue here that this cancellation occurs whenever a topological object satisfiying a topological bound, is present. Note that the A_μ field needs not to be a dynamical gauge field: one just needs a connection entering in the supercovariant derivative in such a way as to cancel out the contribution of the spinorial connection . In particular, we shall see that this is what happens for the CP^n model, where A_μ is an auxiliary field with no dynamics.

To begin with our construction, we shall look for the supercharge algebra of the above defined gauge-matter theory coupled to $N = 2$ supergravity. From this, we shall see how Bogomol'nyi bounds and Bogomol'nyi equations can be established. Finally we will explain how the problem posed by a correct definition of a covariantly constant spinor (1) can be circumvented.

[1]For example, in the model of Ref.[13], it corresponds to the so-called automorphism charge appearing in $(2, 0)$ Poincaré supergravity.

The complex supercharge associated to supersymmetry transformations can be written as the circulation of the gravitino gauge field [7]

$$\mathcal{Q}[\epsilon] = -\frac{2}{\kappa}\oint_{\partial\Sigma} \overline{\epsilon}\psi_\mu dx^\mu. \tag{5}$$

This expression can be obtained from dimensional reduction of the four dimensional one obtained in Ref.[10]. Alternatively, it can be constructed by applying the Hamiltonian approach described in [10] to the model under consideration. This last approach needs a carefull treatment of improper supersymmetry transformations [11] which difficult the computation of the supercharge algebra if one naively evaluates Poisson brackets. We will discuss this issue bellow and for the moment we simply compute the supercharge algebra by acting on the integrand of (5) with a supersymmetry transformation

$$\{\bar{\mathcal{Q}}[\epsilon], \mathcal{Q}[\epsilon]\}| \equiv i\delta_\epsilon \mathcal{Q}[\epsilon]| = \frac{2i}{\kappa}\oint_{\partial\Sigma} \overline{\epsilon}\delta_\epsilon\psi_\mu| dx^\mu. \tag{6}$$

For a given functional $\mathcal{F}$ depending both on bosonic and fermionic fields, $\mathcal{F}|$ means $\mathcal{F}|_{\{\Psi\}=0}$. Being the transformation of the gravitino field given by:

$$\delta_\epsilon\psi_\mu = \frac{2}{\kappa}\hat{\mathcal{D}}_\mu(\omega, \mathcal{A})\epsilon \tag{7}$$

we obtain:

$$\{\bar{\mathcal{Q}}[\epsilon], \mathcal{Q}[\epsilon]\}| = \frac{4i}{\kappa^2}\oint_{\partial\Sigma} \overline{\epsilon}\hat{\mathcal{D}}_\mu(\omega, \mathcal{A})\epsilon dx^\mu. \tag{8}$$

It is interesting to note that the above expression is an identity between the supercharge algebra evaluated in the purely bosonic sector and the circulation of the so-called generalized Nester form

$$\Omega = \overline{\epsilon}\hat{\mathcal{D}}_\mu(\omega, \mathcal{A})\epsilon dx^\mu. \tag{9}$$

Using the explicit expression for the covariant derivative system given in (4), eq.(8) can be written as

$$\{\bar{\mathcal{Q}}[\epsilon], \mathcal{Q}[\epsilon]\}| = \frac{4i}{\kappa^2}\oint_{\partial\Sigma} \overline{\epsilon}\mathcal{D}_\mu(\omega)\epsilon dx^\mu - \oint_{\partial\Sigma} \overline{\epsilon}\epsilon\mathcal{A}_\mu dx^\mu. \tag{10}$$

In order to compute these integrals, we consider the contour of integration at large but finite radius R (to avoid infrared problems), with appropiate asymptotic conditions on the fields. We choose by convenience $\gamma^0\epsilon = \epsilon$ and an asymptotic behaviour

$$\epsilon \to \Theta(R)\epsilon_\infty \tag{11}$$

where $\Theta(R)$ will be determined using the so-called Witten condition [15].

Then, for static configurations one can write eq.(10) in the following form

$$\{\bar{\mathcal{Q}}[\epsilon], \mathcal{Q}[\epsilon]\}| = (M - \oint_{\partial\Sigma} \mathcal{A}_\mu dx^\mu)\overline{\epsilon}_\infty \epsilon_\infty \Theta^2(R). \tag{12}$$

where we have defined M (which, as we shall see corresponds to the ADM mass) as

$$M = \frac{2}{\kappa^2} \oint_{\partial\Sigma} \omega_\mu^0 \gamma_0 dx^\mu. \tag{13}$$

We now use Stokes' theorem, to rewrite the r.h.s. of (8) as

$$\{\bar{\mathcal{Q}}[\epsilon], \mathcal{Q}[\epsilon]\}| = i \int_\Sigma \epsilon^{\mu\nu\beta} \hat{\mathcal{D}}_\beta (\overline{\epsilon} \hat{\mathcal{D}}_\mu \epsilon) d\Sigma_\nu. \tag{14}$$

Then, from

$$[\hat{\mathcal{D}}_\mu(\omega, \mathcal{A}), \hat{\mathcal{D}}_\nu(\omega, \mathcal{A})] = \frac{1}{2} R_{\mu\nu}{}^{ab}\Sigma_{ab} + \frac{i\kappa^2}{2}\partial_{[\mu}\mathcal{A}_{\nu]}, \tag{15}$$

Einstein equations and the supersymmetry transformations of the fermionic fields, we have

$$i\epsilon^{\mu\nu\beta}\hat{\mathcal{D}}_\beta(\overline{\epsilon}\hat{\mathcal{D}}_\mu\epsilon) = i\epsilon^{\mu\nu\beta}\overline{\hat{\mathcal{D}}_\beta\epsilon}\hat{\mathcal{D}}_\mu\epsilon + \frac{\kappa^2}{2}\sum_{\{\Psi\}} \delta_\epsilon \bar{\Psi}\gamma^\nu \delta_\epsilon \Psi \tag{16}$$

We now specialize our spacelike integration surface Σ so that $d\Sigma_\mu = (d\Sigma_t, \vec{0})$. Then, we just need to compute the time component of eq.(16) which, after some Dirac algebra, reads

$$\begin{aligned} i\epsilon^{t\nu\beta}\hat{\mathcal{D}}_\beta(\overline{\epsilon}\hat{\mathcal{D}}_\mu\epsilon) &= \left(\gamma^i\hat{\mathcal{D}}_i\epsilon\right)^\dagger \left(\gamma^j\hat{\mathcal{D}}_j\epsilon\right) - g^{ij}\left(\hat{\mathcal{D}}_i\epsilon\right)^\dagger \left(\hat{\mathcal{D}}_j\epsilon\right) \\ &+ \frac{\kappa^2}{2}\sum_{\{\Psi\}} \delta_\epsilon\bar{\Psi}^\dagger \delta_\epsilon \Psi \end{aligned} \tag{17}$$

At this point, we impose the generalized Witten condition [15] on the spinorial parameter ϵ

$$\gamma^i \hat{\mathcal{D}}_i(\omega, \mathcal{A})\epsilon = 0, \tag{18}$$

so that the asymptotic behaviour of ϵ can be determined. That is, the function $\Theta(R)$ can be computed. On the other hand, being the r.h.s of eq.(17) a sum of bilinear terms once (18) is applied, positivity of the l.h.s. in eq.(14) is obtained:

$$\{\bar{\mathcal{Q}}[\epsilon], \mathcal{Q}[\epsilon]\}| \geq 0, \tag{19}$$

the equality being saturated if and only if

$$\delta_\epsilon \Psi = 0 \quad , \quad \Psi \in \{\Psi\} \tag{20}$$

$$\hat{\mathcal{D}}_i(\omega, \mathcal{A})\epsilon = 0 \tag{21}$$

Since the argument holds for any integration surface Σ, this last condition becomes in fact

$$\hat{\mathcal{D}}_\mu(\omega, \mathcal{A})\epsilon = 0 \tag{22}$$

We recognize in (22) the condition that ϵ must satisfy in order to be a Killing spinor. Moreover, one can see that eqs.(20)-(22) are the Bogomol'nyi equations of our theory and that eq.(19) allows the obtention of a Bogomol'nyi bound. To see this we shall establish at this point a link between our approach and that of Deser and Teitelboim [9] based on Dirac formalism for constrained systems. Let us start by recalling that, it was proven in refs.[9]-[10] that supergravity charges obey a supersymmetry algebra at space-like infinity which is nothing but the usual (global) flat-space algebra. In our notation,

$$\{\bar{\mathcal{Q}}[\epsilon], \mathcal{Q}[\epsilon]\}| = \bar{\epsilon}(\infty)\gamma^\mu \epsilon(\infty) P_\mu + i\bar{\epsilon}(\infty) Z \epsilon(\infty) \tag{23}$$

where Z is the central charge. Now, it is well-known that the central charge in $N = 2$ supersymmetric theories, is related to the topological charge of the field configuration T [2]-[4]

$$Z = iT. \tag{24}$$

Then, for static configurations, eq.(23) can be written as

$$\{\bar{\mathcal{Q}}[\epsilon], \mathcal{Q}[\epsilon]\}| = (M - T)\bar{\epsilon}_\infty \epsilon_\infty \Theta^2(R). \tag{25}$$

which is nothing but our eq.(12) originally obtained by computing the Poisson bracket through the formula (6). Using these results and eq.(19) one easily writes a Bogomol'nyi bound for the mass,

$$M \geq T. \tag{26}$$

In view of eqs.(12), (23)-(24) we are lead to identify the topological charge T of the configuration with the circulation of the vector $\mathcal{A}_\mu$:

$$T \equiv \oint_{\partial\Sigma} \mathcal{A}_\mu dx^\mu. \tag{27}$$

Note that eq.(27) does not imply $\mathcal{A}_\mu = A_\mu$. Indeed, they can differ in terms with vanishing circulation[2].

Taking into account (26), we can identify eqs.(20) as the Bogomol'nyi equations of the matter fields. Indeed, given a particular model, conditions (20) lead to a first order differential equation whose solutions satisfy the second order Euler-Lagrange equations. Moreover, eq.(22) is to be interpreted as the Bogomol'nyi equation of the gravitational field in the sense that its consistency equation coincides with Einstein equation for the gravitational field coupled to matter.

As pointed out in the introduction, one cannot naively assume the existence of non-trivial solutions to the Killing spinor equation in view of the problems posed by conical geometry in three dimensional spacetime. To examine in detail this issue let us consider a spinor η parallel transported around a closed curve surrounding the matter sources which, after a gauge transformation, reads:

$$\eta(x)|_{2\pi} = \mathcal{P}\exp\left(-\frac{i}{2}\oint_\Gamma \omega_\mu^0\gamma_0 dx^\mu + i\frac{\kappa^2}{4}\oint_\Gamma \mathcal{A}_\mu dx^\mu\right)\eta(x)|_0. \tag{28}$$

Then, as a consequence of the fact that the gravitino has acquired a charge, a Bohm-Aharonov phase appears, in addition to the usual phase arising from the spinorial connection.

Now, if one assumes, for static configurations, $\gamma^0\eta = \eta$, and then uses eq. (13) together with eq.(27), it is easy to see that whenever the Bogomol'nyi bound is saturated, the holonomies in (28) cancel each other. That is, unbroken supersymmetries can be defined over Bogomol'nyi saturated states. It is worthwhile to remind that, as noted above, the existence of Killing spinors implies the existence of non-trivial solutions to Einstein equations.

[2]For example, in the Abelian Higgs model coupled to $N = 2$ supergravity , $\mathcal{A}_\mu$ has a contribution proportional to the Higgs current whose circulation at infinity vanishes for finite energy configurations [6, 7].

Concerning eqs.(20), we recognize the usual Bogomol'nyi equations for bosonic matter fields. Let us stress at this point that solutions of eqs.(20) break half of the supersymmetries; indeed, writing the spinor parameter as

$$\epsilon \equiv \begin{pmatrix} \epsilon_+ \\ \epsilon_- \end{pmatrix}$$

one can easily see that

$$\delta_{\epsilon_+}\Psi = 0 \Longrightarrow \delta_{\epsilon_-}\Psi \neq 0 \tag{29}$$

for non-trivial configurations. Note that, as pointed out in [16] for the Abelian Higgs model [6, 7], the possibility of defining global conserved supercharges for solitonic states that saturate a Bogomol'nyi bound, does not imply Bose-Fermi degeneracy.

Before illustrating our ideas with the analysis of a simple model, we shall descibe as promissed, how the Deser-Teitelboim (DT) Hamiltonian approach [9]-[10] eliminates ambiguities afflicting the supercharge algebra, so that all the expressions we have used involving supercharge generators are well-defined. Within the DT formalism, one starts from the non-improved (naive) Hamiltonian H_0 and then one adds Lagrange multipliers for the total energy-momentum, angular momentum and supercharge. Then, one fixes the gauge by means of coordinate conditions that imply a preferred local time so that space-like surfaces, over whose contour the generators are defined, become meaningful. One should note that gauge conditions make the asymptotic form of the Lagrange multipliers determine the multipliers everywhere.

Since, as noted above, the expression (5) for the supercharge coincides with that arising in pure supergravity, the ambiguity problem aflicting our supercharge algebra is just the same as that treated by Deser and Teitelboim [9]. That is, if one writes the charge, within the Hamiltonian approach in terms of the generator of gauge transformations,

$$Q_{H_0} = \int d^2x \bar{\epsilon}\epsilon_{ij}\hat{\nabla}_j\psi_i \tag{30}$$

Poisson brackets cannot be calculated for ϵ not vanishing at infinity: there are surface terms preventing the naive evaluation of Poisson brackets. Within the DT approach, this problem is solved by adding to H_0 a boundary terms of the form [10, 9]

$$H_s = \overline{\psi}_0(\infty)(\frac{2}{\kappa}\oint_{\partial\Sigma}\psi_\mu dx^\mu) = -\mathcal{Q}[\psi_0(\infty)] \tag{31}$$

Analogous terms have to be added in connection with energy-momentum and angular momentum but we will include them since they play no role in our arguments. Then, the improved Hamiltonian takes the form:

$$H_{imp} = H_0 + H_s \tag{32}$$

Concerning supercharges, this ammounts to define the supercharge as

$$\mathcal{Q}_H = Q_{H_0} - \frac{2}{\kappa}\oint \overline{\epsilon}\psi_i dx^i \tag{33}$$

In this way $\mathcal{Q}_H$ becomes second class and the supersymmetry algebra can be obtained by computing Dirac brackets. Since the boundary term in H_s is the same as that considered in [9] for pure supergravity and $\mathcal{Q}$ coincides with the one defined there, one naturally arrives to Deser-Teitelboim results forthe models we are discussing. It becomes clear within this approach that the inequality in eq.(19) is referred to the ADM mass.

The CP^n example

We now illustrate the ideas described in the precedent section by analysing the gravitating CP^n model. The $N = 2$ supersymmetric action in three dimensional spacetime can be obtained by dimensional reduction of the $N = 1$ supersymmetric CP^n model in $d = 4$ spacetime [17]. We will quote here just the bosonic part of the $N = 2$ action

$$S = \int d^3x \left(-\frac{1}{4\kappa^2}eR + \frac{e}{2}(D_\mu\mathbf{z})^*(D^\mu\mathbf{z})\right) \tag{34}$$

where $\mathbf{z} = (z^1 \dots z^n)$ are n complex scalar fields, e is the determinant of the dreibein, R is the scalar curvature and $D_\mu = \partial_\mu - iA_\mu$, with A_μ an auxilliary field. The following condition on the scalar field holds

$$|\mathbf{z}|^2 = \sum_a |z^a|^2 = 1 \tag{35}$$

together with its corresponding supersymmetric partner. The set of supersymmetry transformations for the fermionic partners of those fields appearing in (34) are

$$\delta\psi_\mu| \equiv \frac{2}{\kappa}\hat{\mathcal{D}}_\mu(\omega, A)|\epsilon = \frac{2}{\kappa}\left(\mathcal{D}_\mu(\omega)| + \frac{\kappa^2}{4}\mathbf{z} \overset{\leftrightarrow}{D}_\mu \mathbf{z}^*\right)\epsilon \tag{36}$$

and

$$\delta\chi| = \frac{1}{2} \not{D}\mathbf{z}\epsilon \tag{37}$$

where χ is the supersymmetric partner of $\mathbf{z}$ and ϵ is a complex (Dirac) spinor, reflecting the existence of an extended supersymmetry. Eq.(36) reflects nothing but the extension of the supercovariant derivative whenever a vector multiplet is coupled to the Einstein multiplet as discussed after eq.(4) with

$$\mathcal{A}_\mu \equiv -i\mathbf{z} \overset{\leftrightarrow}{D}_\mu \mathbf{z}^* \tag{38}$$

It is immediate to compute the supercharge $\mathcal{Q}[\epsilon]$ from the conserved supercurrent

$$\mathcal{J}^\mu = -\frac{1}{2}\epsilon^{\mu\nu\rho}\overline{\Psi}_\rho\hat{\mathcal{D}}_\nu(\omega, A)\epsilon - \frac{1}{2\sqrt{2}}\bar{\chi}^a\gamma^\mu \not{D}\phi^a\epsilon \tag{39}$$

which, after Euler-Lagrange equations, can be written as the surface integral given in (5). One can then compute the supercharge algebra from the circulation of a generalized Nester form and follow the steps previously described to get, after static conditions are imposed, the Bogomol'nyi bound $M \geq |T|$ with T given by:

$$T = -i\oint_{\partial\Gamma} \mathbf{z} \overset{\leftrightarrow}{D}_\mu \mathbf{z}^* dx^\mu. \tag{40}$$

Note that in the present model, $\mathcal{A}_\mu$ coincides with the vector potential of the topological current.

Following the general prescriptions described above, it is straigthforward to find the self-duality equation for the scalar fields, after choosing a spinorial parameter ϵ_+:

$$\delta_{\epsilon_+}\chi| = 0 \Longrightarrow D_i\mathbf{z} = i\epsilon_{ij}D_j\mathbf{z} \tag{41}$$

as well as the Bogomol'nyi equation for the gravitational field

$$\delta_{\epsilon_+}\psi_\mu| = 0 \Longrightarrow \left(\mathcal{D}_\mu(\omega)| + \frac{\kappa^2}{4}\mathbf{z} \overset{\leftrightarrow}{D}_\mu \mathbf{z}^*\right)\epsilon_+ = 0 \tag{42}$$

whose integrability condition

$$\left[\mathcal{D}_\mu(\omega)| + \frac{\kappa^2}{4}\mathbf{z} \overleftrightarrow{D}_\mu \mathbf{z}^*, \mathcal{D}_\nu(\omega)| + \frac{\kappa^2}{4}\mathbf{z} \overleftrightarrow{D}_\nu \mathbf{z}^*\right]\epsilon_+ = 0 \tag{43}$$

reproduces the Einstein equation once eq.(41) is imposed. Had we considered a spinorial parameter ϵ_-, we would have end with self-gravitating antisolitonic configurations. Let us end our discussion on the CP^n model stressing that eq.(42) admits non-trivial solutions. It is clear, since the holonomy produced by the conical nature of spacetime at infinity is explicitely cancelled by the holonomy of $\mathcal{A}_\mu$ (see eq.(38)) over Bogomol'nyi saturated configurations.

We conclude this work summarizing its main results. We have proven on general grounds that Bogomol'nyi bounds and equations in three dimensional gravitational theories, can be viewed as the fingerprint of an underlying $N = 2$ supersymmetric structure. We have shown that saturation of the bound is accompanied by the existence of non-trivial supercovariantly constant spinors even for asymptotically conical spacetime geometries, due to a cancellation of holonomies. Unbroken supercharges can then be defined for Bogomol'nyi saturated states, this fact leading to the vanishing of the cosmological constant without having Bose-Fermi degeneracy [12, 16]. The presence of Killing spinors implies the existence of non-trivial solutions to Einstein equations. Finally, we have presented as an example the CP^n model which clearly illustrates our arguments.

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