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ASSESING HP FILTER PERFORMANCE FOR ARGENTINA AND U.S. MACRO AGGREGATES

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Hodrick-Prescott filter has been the favourite empirical technique among researchers studying "cycles". Software facilities and the optimality criterion, from which the filter can be derived, can explain its wide use. However, different shortcomings and drawbacks have been pointed out in the literature, as alteration of variability and persistence and detecting spurious cycles and correlations. This paper discusses these criticisms from an empirical point of view trying to clarify what the filter can and cannot do. In particular, a less mechanical use for descriptive analysis is proposed: testing how the estimated cyclical component behaves and using autocorrelation adjusted standard errors to evaluate cross correlations to differentiate the "genuine" from "spurious" case. Simulation results to test these bivariate correlations when there is a "genuine" relationship are presented. Some examples of descriptive analysis for macro aggregates (real activity, trade flows and money) of Argentina and USA are reported to show that not always the filter is appropriate. Simple tools are used to appreciate how the filtered series result and to evaluate cross correlations.

JEL classification codes: C4, E3
Key words: HP filter, cycles, spurious cycles, genuine cross correlation

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I. Introduction

Almost twenty years after its first presentation in the literature, Hodrick-Prescott (HP)[1] filter is still the favourite empirical technique among researchers who attempt to separate cyclical behaviour from the long run path of economic series. Applied to both "true" and "artificial" data, filtered series have been studied mainly to discover "stylised facts" in business cycles by observing and comparing univariate and cross moments: variability, autocorrelation, bivariate correlation, etc.

In spite of its wide use, not "mechanical" HP filtering has been exceptional given nowadays software facilities and invoking as justification the optimality criterion from which the filter can be derived. At the same time, a large literature has pointed out several "problems" of applying the "popular" filter, as alteration of variability and persistence and detecting spurious cycles and correlations, among the most important ones. Different papers have analysed shortcomings and drawbacks of the filter. A good summary of them is offered by Ravn and Uhlig (1997): the filter might generate most of the cycles, the filter is only "optimal" (minimum –square– error) in special cases and may produce extreme second order properties of detrended data. They, however, suggest that "none of these shortcomings and undesirable properties are particularly compelling: the HP filter has withstood the test of the time and the fire of discussion remarkably well" (op. cit., p 1).

As this type of filter (and the decomposition it assumes) has a long history, the controversy about "filtering" is neither new. Hodrick and Prescott dated the filter in 1923 and similar approaches even in the last century. At the same time the decomposition of economic series is mainly based of 1919 work of Persons, based on the idea of different causal forces of cyclical and trend components[2].

The purpose of this paper is to discuss the filter from an empirical point of view trying to clarify what it can and cannot do and suggest some guidelines

[1] Hodrick and Prescott (1981), reprinted in Hodrick and Prescott (1997).

[2] See Singleton (1988) for a discussion of this work an its criticism supported by the famous debate between Burns and Mitchell and Koopmans.

for evaluation. Next section describes the filter. Section III analyses the problem of the decomposition of series. Section IV considers the question of spurious correlation; presents simulation results to evaluate bivariate correlations of filtered series and shows some examples of descriptive analysis for macro aggregates (real activity, trade flows and money) of Argentina and United States. Section V discusses the filter in econometric models. Section VI concludes.

II. The HP Filter

The conceptual framework presented by Hodrick and Prescott can be summarised as follows,

$$y_t = g_t + c_t \qquad (1)$$

a given series y_t is the sum of growth component g_t and cyclical component c_t.

The growth component is determined from solving the next problem,

$$Min_{\{g_t/t=1...T\}} \left(\sum_{t=1}^{T} c_t^2 + \sum_{t=1}^{T}[(g_t - g_{t-1}) - (g_{t-1} - g_{t-2})]^2\right) \qquad (2)$$

where the cyclical components are deviations from the long run path (expected to be near zero on average over long time period) and smoothness of the growth component is measured by the sum of squares of its second difference:

$$\Delta^2 g_t = (1-L)^2 g_t = (g_t - g_{t-1}) - (g_{t-1} - g_{t-2})$$

where L denotes the lag operator, $Lx_t = x_{t-1}$.

The parameter λ is a positive number which penalises variability in the growth component: the larger its value, the smoother g_t. In the limit as λ approaches infinity, the first difference $\Delta g_t = (g_t - g_{t-1})$ tends to a constant and the solution of the problem to a least square fit of a linear trend. In this original framework a prior value of the smoothing parameter is obtained by assuming a probability model in which:

$$c_t \sim IN(0,\ {}^{2}_{c}) \tag{3a}$$

$$\Delta^2 g_t =\ {}_{t} \sim IN(0,\ {}^{2}_{g}) \tag{3b}$$

The expected value of g_t given observations is the solution of the problem in equation (2) when $\lambda^{1/2} = \sigma_c / \sigma_g$.

Thus the authors suggest for quarterly data: $\lambda^{1/2} = [(5/(1/8)]$ and $\lambda = 1600$. However, they recognise the restriction imposed by these assumptions. Sensitivity analysis of results to such "λ" is explored which confirms it is a reasonable value for the case studied.

Three aspects merits to be remarked in this formulation: a) given equation (1), no irregular component is assumed in the decomposition of the series, which is therefore subsumed as part of the cyclical component; b) the minimisation problem, equation (2), and as consequence of a), supposes c_t as residual of the growth estimation (growth and cycle both unobservable) and c) the value of λ is not determined, in principle, by optimisation but it is matter of choice of empirical investigators, in general only based on "prior beliefs", although its adequacy for a particular data set can be tested. These issues are later discussed.

A useful insight of the HP filter can be derived from its representation on time domain as presented in King and Rebelo (1993) who consider the case of "infinite sample" ignoring "applied" questions of endpoints treatment (see Hodrick and Prescott, 1981,1997). In this case growth component can be expressed as,

$$g_t = \sum_{t=-\infty}^{\infty} w_j y_{t-j} = G(L) y_t \tag{4}$$

that is, g_t is a two side weighted moving average of the original series y_t and therefore,

$$c_t = [1 - G(L)]y_t = C(L)y_t \tag{5}$$

the cyclical component is also a moving average of the series. *G(L)* and *C(L)* are "linear filters".

Since the information set of this optimisation problem is the whole sample, the first order condition , from (2) given (1),

$$0 = -2(y_t - g_t) + 2 \quad [(g_t - g_{t-1}) - (g_{t-1} - g_{t-2})] - \tag{6}$$

$$-\,4 \quad [(g_{t+1} - g_t) - (g_t - g_{t-1})] + 2 \quad [(g_{t+2} - g_{t+1}) - (g_{t+1} - g_t)]$$

which can be written as

$$F(L) = g_t = y_t$$

F(L) is the lag polynomial,

$$\begin{aligned} F(L) &= \quad L^{-2} - 4 \quad L^{-1} + (6 \quad +1) - 4 \quad L + \quad L^2 \\ &= [\quad (1-L)^2 (1-L^{-1})^2 + 1] \\ &= [\quad \Delta^{4*} + 1] \end{aligned} \tag{7}$$

where * indicates "centred" or "a forward second difference of the backward second difference",

$$\begin{aligned} \Delta^{4*} &= L^{-2} - 4L^{-1} + 6 - 4L + L^2 \\ &= [(1-L^2)(1-L^{-1})^2] \end{aligned}$$

Thus,

$$F(L)^{-1} = G(L)$$

and

$$C(L) = [F(L) - 1]\, F(L)^{-1} \tag{8}$$
$$= \Delta^{4*}/[\ \Delta^{4*} + 1]$$

Hence, King and Rebelo indicate that this cyclical filter "is capable of rendering stationary any integrated process up to fourth order, since there are four differences in the numerator".[3]

Notwithstanding the above derivation of HP filter (minimising a cost function which penalises both departure of actual series from growth and changes in the rate of growth), there is another –less formal– interpretation of the filter: the long run component, the "trend", is what an analyst would draw by hand through the plot of the data (see, Kydland and Prescott, 1990)[4].

III. Problems of the Decomposition of Series with HP Filter

Suppose that an investigator is ready to apply HP filter in order to separate growth from cycle of economic series taking advantage from "easy use" software facilities. What can be learnt for empirical work? First of all, "a more critical and less mechanical" use of the filter is required.

Since modelling of unobservable components "g_t" and "c_t" from "y_t" is the issue, the additive (or log additive) decomposition, equation (1) should be assumed as the univariate representation and therefore, Persons' views on different driven forces of components should be shared. This also implies that the seasonal component -if present- has been somehow removed (whose effects could merit additional discussions) and the irregular has been absorbed by "c_t".

Moreover, since both components are determined from a given "y_t", the

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[3] They also found that HP filter is optimal -in the sense of minimising the mean square error- for a limited class of ARIMA models, which "are unlikely to be even approximately true in practice" (op. cit. p. 230).

[4] It is also virtually identical to a "natural cubic spline" for a given λ (see Doornik and Hendry, 1996).

separation supposes that the fraction "1/[Δ^{4*}+1]y_t" corresponds to "g_t" and " Δ^{4*}/[Δ^{4*}+1]y_t" corresponds to "c_t" (from equations (7) and (8)). The "weights" are the same for all series except for λ, which reflects the trade off between minima series departures from long run and minima departures of last growth rate (equation (2)). Are these terms those that matter for the cost function? Does it make sense to peg the rate of growth or the level of the long run component to their past values (as in the case of "exponential smoothing", see King and Rebelo, 1993)? Are such terms the only ones or cross terms should also be included in the relevant function to minimise? Although these questions are difficult to answer a priori, a good practice would indicate to check if what is obtained by "filtering" is what is expected to be.

As previously seen the terms in the cost function is weighted by λ, which is the only parameter under "control". Unless the researcher performed a maximum likelihood approach to estimate simultaneously λ, its value should be "guessed"[5]. The default value has been set at 1600 for quarterly data accordingly to the basic probability model of HP summarised in section II, which depends on the assumption about the ratio of variance between cyclical and growth rate white noises (see also King and Rebelo, 1993, p. 224). In the Hodrick-Prescott's paper λ takes values from 400 to ∞ (perfect smoothing) for the sensitivity analysis of the filtered data. However, the range for this periodicity might be considerably wider if a different representation is assumed, as in Nelson and Plosser (1982), which closest value is about 1 (see also Canova, 1994).

Other frequencies are still more controversial. From the default value of 1600 for quarterly data, linear or quadratic adjustments have been used in applied works (say λ = 400 or 100 for annual data). Recently, a power adjustment of 4 (λ = 6.25 for annual data) has been proposed since the transfer function is in this way invariant to the sampling frequency (Ravn and Uhlig, 1997).

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[5] Harvey and Jaeger found usually too low values of λ when applying maximun likelihood.

Given such a range of values, empirical work should evaluate whether or not estimated growth and cycle reject their conjectured behaviour. Minima criteria are: all long run components (low frequencies) should be part of "g_t" whereas other components of shorter periodicity (higher frequencies) should be left to "c_t". However, it is expected to be not too "noisy" (not too much weight on the highest frequencies). Although Hodrick and Prescott present unit root tests of the cyclical components, it is not common to see such tests, spectra or just correlograms. In other cases, neither a visual inspection is offered to evaluate "how well" the investigator "draw by hand" the trend, using a specific λ.

The probability model from which Hodrick and Prescott derived a prior for λ has some interpretation problem as it has been used as a "paradigm" in this literature even though these authors recognised the limitations of assuming such a representation (op. cit., p. 4). As shown in section II, equations (6) and (7),

$$(\lambda\Delta^{4*} + 1)\, g_t = y_t \tag{9}$$

$$\lambda\Delta^{4*} g_t = y_t - g_t = c_t$$

$$\lambda\, [1 - L^{-1}]^2\, [1 - L]^2\, g_t = c_t$$

and given the data generating process (DGP) assumed for the long run component, (equation (3b))

$$\lambda\, [1 - L^{-1}]^2\, \varepsilon_t = c_t \tag{10}$$

$$\lambda\, [\, \varepsilon_t - 2\, \varepsilon_{t+1} + \varepsilon_{t+2}\,] = c_t$$

Therefore c_t is not white noise as assumed in the DGP (equation (3a)) but λ times a non-invertible MA(2) whose roots are, therefore, outside the range of those showing cyclical behaviour. Note that the information in t+1 and t+2 is known since the optimisation is over the whole sample t = -1... T (equation (2)).

Another view of the same question is obtained when deriving growth (equations (6) and (7)) from the DGP assumed (equations (3a) and (3b)),

$$(\lambda\Delta^{4*} + 1)\, g_t = y_t = (\varepsilon_t / \Delta^2) + c_t \qquad (11)$$

$$\Delta^2 g_t = \varepsilon_t + (1 - L^2)\, c_t - \lambda\, (1 - L^{-1})^2 (1 - L)^4 g_t$$

then $\Delta^2 g_t$ cannot be white noise as supposed in the DGP.

Similar considerations apply to the structural representation whenever it is equivalent to $g_t \approx$ ARIMA (0, 2, 1) and $c_t \approx$ ARMA (2, 1), subject to restrictions (the AR part corresponding to complex roots (Harvey and Jaeger, 1993, p. 234) and a difference stationary (as that analysed by Cogley and Nason, 1995) or second difference stationary representation of y_t. In the first case, equation (11) can be generalised and $\Delta^2 g_t$ does not result as MA(1). For the latter, assuming

$$\Delta^2 y_t = \eta_t \qquad \eta_t \sim \text{IID}\,(0, \sigma^2_\eta)$$

and

$$c_t = \{ \lambda\Delta^{4*} / [\lambda\Delta^{4*} + 1] \}\, y_t = \lambda\Delta^{4*} \eta_t / [\lambda\Delta^{4*} + 1]\, \Delta^2$$

or

$$c_t = \lambda\, (1 - L^{-1})^2\, \eta_t + \lambda\, (1 - L^{-1})^2\, (1 - L)^2 c_t \qquad (12)$$

and therefore, c_t does not appear as a “typical” cycle within the class of ARMA models.

Nelson and Plosser (1982) suggested this kind of problem when expressed "HP strategy implicitly imposes a components model on the data without investigating what restrictions are implied (a difficult task in their model) and whether those restrictions are consistent with the data", p.158.

Therefore, researchers on the HP filter should have in mind a DGP which differs from those that can be expressed in terms of the family of the ARIMA class, since much of the debate can be put in terms of the conjectured DGP. Then, the task is to look for tools to test that the results obtained do not reject the conjectures. Evaluating the behaviour of estimated components -as above discussed- would be one part of the question. The other is the possibility of spurious cross correlation between spurious cycles.

IV. Simulation Results: Testing "Genuine" Cross Correlations

Cogley and Nason (1995) alerted against the possibility of obtaining "spurious cycles" when filtering "difference stationary data" (like a random-walk representation).

Harvey and Jaeger (1993) considered the spurious cyclical behaviour from applying HP filter as "a classic example" of the Yule-Slutsky effect (op. cit., p. 234). Slutsky in 1937 (see Sargent (1979) for an exposition) showed how a cyclical behaviour can be obtained starting with a white noise, taking a two period moving sum n-times and then first differences m-time. Given equations (4) and (5) such results cannot be excluded a priori when applying HP filters. Furthermore, they extended this analysis and showed the possibility of "spurious sample cross correlation" between spurious cycles. Thus, these authors put an additional warning to the "uncritical use of mechanical detrending" (op. cit., p. 231)[6].

In their work a simulation exercise is made assuming independent random

[6] These authors also interpreted HP filter in terms of "structural time series models" (Harvey, 1989) which would correspond to a special (restricted) case of them.

walks and first differences random walks as DGP. They found that spurious correlation between spurious cycles may not be negligible. Harvey and Jaeger evaluated the cross correlation of these independent processes using asymptotic standard errors (SE) (Brockwell and David, 1987, p. 400) and recommend reporting SE in addition to point estimates of cross correlations (p. 245).

While their simulation concentrates on rejecting the null ($\rho_{xy} = 0$) when it is true by construction, the other side of the test should be performed: not rejecting the null when it is false, but the DGP should be different of random walk or difference random walk, otherwise it makes no sense the HP filtering. Next, the results of a simulation exercise is presented using U.S GNP series, for which "the HP filter is tailor-made for extracting the business cycle component" (Harvey and Jaeger, 1993, p. 236).

Sample cross correlations could be evaluated taking into account the following asymptotic distribution

$$r_{xy}(h) \sim AN\left(0, T^{-1}\left(1 + 2\sum_{j=1}^{\infty} \rho_x(j)\ \rho_y(j)\right)\right)$$

where $r_{xy}(h)$ is the sample cross correlation at lag h between two series with sample T and $\rho_x(j)$, $\rho_y(j)$ are the autocorrelation of stationary processes x_t and y_t at lag j. In this way the probability of finding large spurious correlation between independent spurious cycles could be taken into account.

In order to consider how the adjusted SE could perform in the case of evaluating a "genuine" correlation between two series with "typical" cyclical behaviour (for which the HP filter would be most appropriate) four series were generated assuming that the cyclical component of the US GNP[7] (now X_t) contributes 80%, 50%, 20% and 10% to the variance of the artificial series. Thus, normal random numbers were added as errors to obtain y_{80t}, y_{50t}, y_{20t} and y_{10t}. Appendix 1 shows cross plots and autocorrelations for each series

[7] The series is taken from the National Bureau of Economic Research.

for the sample 59(1) - 98(3). Table 1 reports simulated sample cross correlations and the autocorrelation adjusted SE times the limit of the 95% confidence interval for $\rho_{xy}(0) = 0$ where ρ_{xy} denotes population cross correlation coefficient between two independent stationary series. The empirical (large sample) approximations to the SE are made considering the sample autocorrelation $r_x(j)$ and $r_y(j)$ with j = 1 to J, J = T/4[8].

Table 1. Simulated "Sample" Cross Correlations and Autocorrelation Adjusted SE

	$r_{xy}(0)$	$(1.96\ T^{-1/2})(1 + 2 \sum_{j=1}^{j=J} r_x(j)\ r_y(j))^{1/2}$
y_{80t}	0.896	0.28
y_{50t}	0.699	0.24
y_{20t}	0.436	0.22
y_{10t}	0.309	0.16

The first column reports sample cross correlations between x_t (the "c_t" of the US GNP) and y_t (artificial series generated for explained variances of 80, 50, 20 and 10%); the second column shows autocorrelation adjusted SE times the limit (absolute value) of 95% confidence interval; J = 40 and $(1.96\ T^{-1/2}) = 0.16$

In each case, even for smallest, cross correlation can be empirically detected as significant (not inside the 95% confidence interval for $\rho_{xy} = 0$). As the exercise suggests, using these "autocorrelation adjusted" SE could help to

[8] Note that it is usual to make similar approximations to evaluate univariate autocorrelations (see Nelson,1973).

evaluate cross correlations between cyclical components[9] having protection from the "spurious correlation problem". Note that the same SE can be used for different h.

The evaluation previously suggested for descriptive-explorative analysis is illustrated with macro-aggregates of Argentina and U.S.A.

A. Argentina

For Argentina a subset of series[10] (GDP, consumption, investment, trade flows and M1) used by Kydland and Zarazaga (1997) are analysed. Firstly, the univariate behaviour of the cyclical components "c_t" is studied by observing autocorrelations and performing usual unit-root tests. Then, cross correlations are evaluated using the SE which allow statistically differentiation from the "spurious correlation" case[11].

Appendix 2 reports Dickey-Fuller autocorrelations and statistics (Tables 2.1 and 2.2). For all the series these statistics (or their augmented versions when necessary) reject the null of a unit root (at traditional levels) for the cyclical component except in the case of investment. For the last series, different λ were tried (from 400 to 6400) but the null cannot be rejected in any case. Visual inspection of the respective autocorrelations confirms that estimated cycles look like "stationary series" being also far from a "noisy" behaviour.

Table 2.3 and 2.4 in the Appendix 2 show the relation between the cyclical component of GDP and those of the other macro-aggregates. They are very similar to the obtained by Kydland and Zarazaga, both volatility and correlations for all the series. In particular cross correlations look quite high.

[9] They can be useful only if the cyclical components result as "stationary".

[10] The series are taken from the Statistical Appendix of the Economic Ministry and the Central Bank of Argentina.

[11] For simplicity only "time domain" tools are presented.

However, when “genuine” correlation (see Table 2.4) are evaluated using the adjusted SE, the case of M1 cannot be detected as significant (the 95% confidence for a zero cross-correlation includes the computed value). Then, in the case of money, correlation cannot be empirically distinguished from the spurious case. The rest of the evaluated series remains showing significant correlations[12].

B. U.S.A.

Appendix 3 shows the sample autocorrelations (Table 3.1) and the unit-root tests statistics (Table 3.2) for the macro-aggregates of U.S.A. All the cyclical component obtained by HP filtering ($\lambda = 1600$) can be considered as “stationary” from inspecting sample autocorrelations and Dickey-Fuller statistics (augmented if required), that reject the null in all cases.

Table 3.3 reports cross correlations and the autocorrelation adjusted SE for these series (each one with the GNP). For two aggregates, the cross correlation cannot be undoubtedly distinguished from the spurious case: exports (at least for the 95% interval) and, in particular, M1.

In both examples, Argentina and U.S.A., the cross correlations between output and M1 are not significant different from zero. Thus, the use of autocorrelation adjusted SE could be more critical to differentiate the “genuine” from the “spurious” when evaluating cross correlations that involve money aggregates.

V. Reasons not to Use Filtered Series when Estimating Econometric Relationships

Although almost nobody could disagree about using seasonally adjusted data to better understand economic series behaviour as part of a descriptive

[12] Given the reported Dickey-Fuller statistic, the case of investment cannot be evaluated.

analysis, their use is more debatable for econometric modelling in a multivariate framework. Ericsson, Hendry and Tran (1994) summarises polar positions: Wallis (1974) considers the implications of estimation with seasonally adjusted data when the DGP relationship involves unadjusted data whereas Sims analysed the converse situation: estimation with unadjusted data when the DGP relationship involves the non-seasonal components. In each case the model is mis-specified (the dynamics alters) and estimates are generally inconsistent[13]. The same considerations are relevant for filtered data: separate cycle and growth may be useful for descriptive analysis but their use for econometric relationships depends on the conjectured DGP. A similar reasoning can be made about the alteration of variability and persistence: filtered and raw data have different sample moments but which is the appropriate depends on the beliefs about the DGP (as in King and Rebelo, 1993).

Whenever economic agents were supposed not to separate components a "Wallis effect" (see Hendry and Mizon, 1979) may alter econometric relationships as follows when used "filtered series",

$$\alpha(L)\, y_t = \beta(L)\, x_t + u_t \qquad u_t \sim IID(0, \sigma^2_u)$$

$$y^a_t = \delta(L)\, y_t$$

$$x^a_t = \gamma(L)\, x_t$$

where $\alpha(L)$ and $\beta(L)$ are polynomials in L and $\delta(L)$ and $\gamma(L)$ are linear filter, such as G(L) or C(L) (see equations (4) and (5). Then

$$\alpha(L)\, y^a_t = \beta(L)\, x^a_t + \beta(L)\,[\,\delta(L) - \gamma(L)\,]\; x_t + \delta(L)\, u_t$$

[13] Ericsson, Hendry and Tran compared both type of models. They found no differences for cointegrating relationships but alteration of dynamics and exogeneity status.

Using filtered series in dynamic econometric models implies -for different filters- an "omitted variable problem" (from the second term) and an "autocorrelation problem" (from the third term), both as part of the error term. This shows that a necessary condition to obtain consistent estimators is to adjust the series using the same λ (here δ (L) = γ (L)). Such requirement may be critical if different parameters are appropriate for each series, according to their univariate behaviour.

However, even for the same λ, autocorrelation would still be present[14] and, therefore, inconsistent and inefficient estimates would arise in dynamic equations like one of a VAR system.

This explains the results in Singleton (1988) that pre-filtering has important effects on the dynamic interrelation among series (assuming a VAR representation), in particular he found inconsistent estimates of parameters[15].

Therefore, all depends on the "beliefs" about the DGP: whether or not "unadjusted" data enter the model. But, this would be testable since the presence of autocorrelation (different from first order) when using filtered data would be an indication of the presence of a "Wallis effect".

There is another question related to "exogeneity" when using a filtered series as explanatory variable. Conditioning on (sequential data) "x_t" is here modified since "x_t^a", that is filtered "x_t", supposes an information set which includes future information within sample but not known at each t.

It is worth noting that, even when suitable evaluated cross correlations (taking into account adjusted SE) could be part of a "explorative" analysis, there is no guarantee of obtaining unbiased estimates of such linear relationships if more variables contribute to explain them. The bivariate correlations are also more likely to be unstable as Bardsen, Fisher and Nymoen (1995) showed for activity -inflation and real wages- unemployment using the U.K and Norwegian data.

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[14] Unless the assumption about the original disturbance is not correct and the same filter for y_t makes this term white noise.

[15] He proposed to study secular and cyclical frequencies simultaneously.

To sum up, regressions that use "HP filtered series" require not only the same λ when adjusting all the series but also a careful study of the residual autocorrelation in dynamic models to avoid inconsistent estimators. This testing could be useful to evaluate whether or not evidence rejects the conjectured model involving unadjusted variables.

Finally, an alternative approach to pre-filtering series for multivariate dynamic econometric modelling is to leave the data "inform" about different filters. Seasonality, long-run, and cyclical behaviour can be jointly modelled following a "general to particular" approach (see Hendry, 1995). "Linear filters" -as G(L) or C(L) of equation (4) y (5)- can be embedded in "linear (dynamic) models" without "constraining" the lag weights (the "w_j"). However, these "data-based" filters would use only "past" information ($j > 0$) in the "conditioning" set.

VI. Conclusions

Different shortcomings and drawbacks of the Hodrick-Prescott filter have been pointed out in the literature which at the same time do not appear to have had great effects on its wide use in empirical research. This paper discusses the filter trying to see what can be learnt for applied work. The following recommendations can be derived. First, researchers should be aware of the decomposition of the series that the filter assumes (different from the ARIMA type). Second, a less mechanical use is proposed by testing how the estimated cyclical component behaves (at least applying univariate tests of unit roots, inspecting correlograms, etc.). Then, autocorrelation adjusted standard errors are suggested to evaluate cross correlations in order to differentiate the "genuine" from the "spurious" case.

Examples of descriptive analysis for macro–aggregates of Argentina and USA show that not always the filter, mechanically applied, is appropriate. Simple tools could be informative about how the filtered series result and to evaluate "significant" cross correlations.

Although the role of the filter as part of a descriptive analysis cannot be denied (as it cannot be the use of seasonally adjusted series), econometric dynamic modelling of filtered series is more problematic if the data generating process involves unfiltered series.

Appendix 1. Simulation Results

$Y_{it} = X_t + u_{it}$

where: i = 80, 50, 20, 10, $u_{it} \sim IN(0, \sigma^2_{ui})$ and $X_t = c$ US GNP

$^{2}_{ui} = 0.25 \;\; ^{2}_{x}$ *for* $R^2 = 0.80$

$^{2}_{ui} = \;\; ^{2}_{x}$ *for* $R^2 = 0.50$

$^{2}_{ui} = 4 \;\; ^{2}_{x}$ *for* $R^2 = 0.20$

$^{2}_{ui} = 9 \;\; ^{2}_{x}$ *for* $R^2 = 0.10$

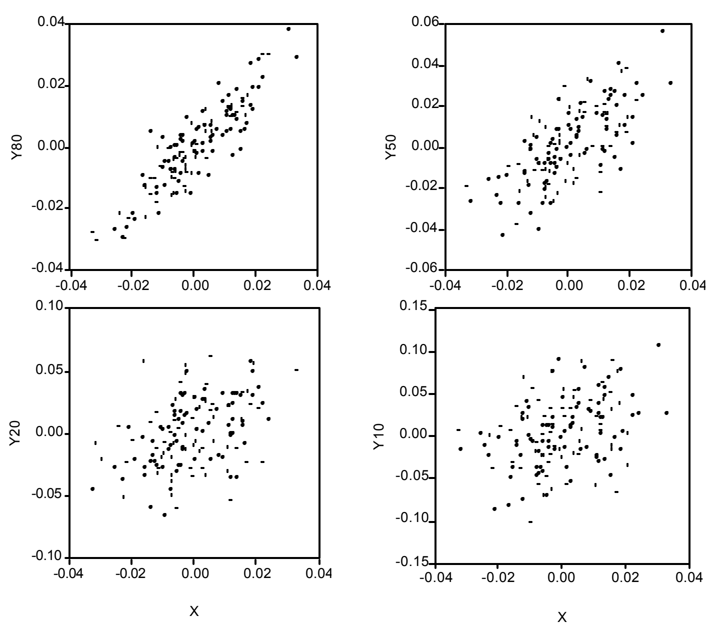

Table 1.1. Autocorrelations

T = 159 J = 40

Lag	X	Y10	Y20	Y50	Y80
1	0.778	0.007	0.154	0.319	0.610
2	0.512	-0.154	0.114	0.110	0.380
3	0.256	0.072	0.057	0.153	0.188
4	0.035	0.052	0.028	0.018	0.020
5	-0.163	-0.010	0.056	-0.048	-0.132
6	-0.260	-0.031	0.019	-0.091	-0.231
7	-0.299	-0.020	-0.147	-0.125	-0.248
8	-0.300	0.134	-0.264	-0.036	-0.256
9	-0.240	-0.024	-0.111	-0.147	-0.202
10	-0.180	-0.211	-0.147	-0.267	-0.106
11	-0.166	-0.012	-0.068	-0.126	-0.055
12	-0.210	0.015	-0.192	-0.093	-0.128
13	-0.209	-0.031	-0.043	-0.119	-0.109
14	-0.181	-0.159	-0.012	-0.174	-0.066
15	-0.137	0.066	0.087	-0.008	-0.058
16	-0.050	0.037	0.123	0.065	-0.108
17	-0.002	-0.010	0.156	0.028	-0.052
18	0.044	-0.100	-0.018	0.000	-0.043
19	0.054	0.017	0.023	0.054	-0.038
20	0.091	0.081	0.144	0.077	-0.029
21	0.067	-0.076	0.059	-0.033	0.046
22	0.030	-0.046	-0.067	-0.007	0.070
23	-0.005	-0.005	0.008	0.003	0.035
24	0.009	0.137	-0.085	0.084	0.097
25	0.006	0.082	0.048	0.090	0.059
26	-0.019	0.079	0.114	0.107	0.014
27	-0.013	-0.059	-0.036	0.007	0.039
28	-0.010	0.008	-0.062	0.019	-0.009
29	-0.023	-0.087	0.029	-0.044	-0.018
30	-0.038	-0.106	0.135	-0.083	0.005
31	0.001	0.014	0.040	-0.004	-0.004
32	-0.020	0.066	-0.017	0.002	0.018
33	-0.062	-0.084	-0.029	-0.100	-0.028
34	-0.052	0.058	0.036	-0.013	-0.016

Table 1.1. (Continue) Autocorrelations

T = 159 J = 40

Lag	X	Y10	Y20	Y50	Y80
35	-0.050	-0.087	-0.023	-0.142	-0.026
36	-0.053	-0.172	0.049	-0.156	0.000
37	-0.055	0.066	-0.142	0.012	-0.001
38	-0.023	-0.137	-0.128	-0.119	0.011
39	-0.008	0.018	-0.107	-0.036	-0.026
40	0.023	-0.010	0.040	-0.025	-0.026

Appendix 2: Argentine Macro-Aggregates: Cyclical Components

Table 2.1. Autocorrelations

J = 17

Lag	GDP	Total Consumption	Exports	Imports	M1	Investment
1	0.783	0.804	0.413	0.876	0.910	0.821
2	0.562	0.555	0.154	0.669	0.790	0.680
3	0.384	0.37o	0.163	0.447	0.613	0.485
4	0.136	0.145	-0.039	0.213	0.424	0.247
5	-0.098	-0.038	0.046	0.009	0.199	0.025
6	-0.155	-0.096	0.073	-0.151	0.012	-0.131
7	-0.263	-0.223	-0.240	-0.281	-0.192	-0.284
8	-0.411	-0.361	-0.285	-0.382	-0.351	-0.418
9	-0.403	-0.375	-0.185	-0.409	-0.490	-0.451
10	-0.356	-0.359	-0.203	-0.422	-0.562	-0.482
11	-0.341	-0.369	-0.232	-0.452	-0.624	-0.499
12	-0.305	-0.358	-0.262	-0.461	-0.617	-0.486
13	-0.217	-0.317	-0.270	-0.433	-0.593	-0.427
14	-0.210	-0.294	-0.187	-0.379	-0.508	-0.396
15	-0.165	-0.231	-0.088	-0.328	-0.412	-0.322
16	-0.106	-0.174	-0.127	-0.268	-0.275	-0.239
17	-0.068	-0.131	-0.061	-0.215	-0.165	-0.140

Table 2.2. Unit – Root Tests

Serie	ADF(j)
GDP	ADF(1) = 2.998*
Total Consumption	ADF(1) = 3.229*
Exports	ADF(0) = -5.164**
Imports	ADF(1) = -3.116*
M1	ADF(3) = -3.224*
Investment	ADF(1) = -2.34

All cases include the constant and j indicates the lags of the Augmented Dickey-Fuller test.

* indicates significance at 5 per cent.

** indicates significance at 1 per cent.

Relations between Macro-Aggregates and GDP

Table 2.3. Volatilities and Correlations

Series	Absolute Volatility	Relative Volatility	Contemporaneous Correlation
GDP	0.044		
Total Consumption	0.052	1.182	0.962 (Procyclical)
Exports	0.075	1.705	-0.602 (Countercyclical)
Imports	0.182	4.136	0.804 (Procyclical)
M1	0.646	14.681	-0.391 (Countercyclical)
Investment	0.129	2.932	0.936 (Procyclical)

Absolute volatility corresponds to the standard deviation of the series; relative volatility represents the ratio between the absolute volatility of the variable of reference and the absolute volatility of GDP and contemporaneous correlation measures the direction and closeness of the linear relationship between the variable of reference and the GDP.

Table 2.4. Sample Cross Correlations and Adjusted SE

	$r_{xy}(0)$	$(1.96\ T^{-1/2})\ (1 + 2 \sum_{j=1}^{j=J} r_x\ (j)\ r_y\ (j))^{1/2}$
Total Consumption	0.962	0.541
Exports	-0.602	0.429
Imports	0.804	0.584
M1	-0.391	0.619

The first column reports sample cross correlations between x_t (the "c_t" of the GDP) and y_t (macro-aggregates), the second column shows autocorrelation adjusted SE times the limit (absolute value) of 95% confidence interval; J = 17.

Appendix 3: USA Macro-Aggregates: Cyclical Components

Table 3.1. Autocorrelations

J = 41

GNP	Imports	Exports	Consumption	Investment	M1
0.77	0.70	0.79	0.87	0.88	0.94
0.50	0.41	0.67	0.71	0.69	0.82
0.24	0.16	0.49	0.52	0.46	0.67
0.02	-0.05	0.29	0.31	0.23	0.49
-0.17	-0.16	0.09	0.10	0.03	0.30
-0.27	-0.21	-0.06	-0.06	-0.15	0.11
-0.31	-0.22	-0.22	-0.22	-0.28	-0.09
-0.29	-0.24	-0.34	-0.33	-0.37	-0.27
-0.23	-0.22	-0.41	-0.37	-0.41	-0.42
-0.17	-0.20	-0.47	-0.39	-0.44	-0.53
-0.15	-0.12	-0.45	-0.40	-0.46	-0.61
-0.19	-0.17	-0.45	-0.40	-0.47	-0.66

Table 3.1. (Continue) Autocorrelations

J = 41

GNP	Imports	Exports	Consumption	Investment	M1
-0.19	-0.15	-0.42	-0.37	-0.44	-0.67
-0.16	-0.11	-0.33	-0.35	-0.38	-0.65
-0.12	-0.12	-0.27	-0.29	-0.28	-0.60
-0.04	-0.08	-0.20	-0.23	-0.19	-0.53
-0.01	-0.11	-0.18	-0.20	-0.12	-0.44
0.03	-0.08	-0.10	-0.15	-0.06	-0.35
0.04	-0.06	-0.05	-0.11	0.01	-0.24
0.07	-0.01	0.00	-0.07	0.07	-0.11
0.05	0.01	0.07	-0.02	0.12	0.00
0.02	0.01	0.11	0.04	0.14	0.10
0.00	0.02	0.15	0.06	0.14	0.19
0.01	0.08	0.18	0.09	0.14	0.25
0.01	0.13	0.20	0.12	0.15	0.30
-0.01	0.16	0.16	0.15	0.15	0.34
0.00	0.17	0.17	0.16	0.15	0.37
-0.01	0.12	0.15	0.15	0.14	0.37
-0.02	0.08	0.13	0.11	0.15	0.36
-0.04	0.07	0.11	0.05	0.15	0.33
-0.01	0.06	0.07	0.02	0.16	0.29
-0.03	-0.03	0.04	-0.03	0.15	0.25
-0.07	-0.14	0.01	-0.07	0.12	0.20
-0.07	-0.18	-0.02	-0.09	0.07	0.15
-0.06	-0.16	-0.01	-0.10	0.03	0.10
-0.05	-0.14	0.01	-0.10	-0.02	0.07
-0.04	-0.07	0.01	-0.07	-0.06	0.04
-0.01	0.00	0.02	-0.02	-0.08	0.00
0.01	0.07	0.05	0.02	-0.12	-0.04
0.04	0.11	0.03	0.06	-0.13	-0.08
0.07	0.13	0.03	0.10	-0.14	-0.12

Table 3.2. Unit – Root Tests

Serie	ADF(j)
GNP	ADF(0) = -5.510**
Total Consumption	ADF(1) = -3.828**
Exports	ADF(0) = -4.414**
Imports	ADF(1) = -5.677**
M1	ADF(1) = -4.079**
Investment	ADF(1) = -4.845**

All cases include the constant and j indicates the lags of the Augmented Dickey-Fuller test.
*indicates significance at 5 per cent.
** indicates significance at 1 per cent.

Table 3.3. Sample Cross Correlations and Adjusted SE

	$r_{xy}(0)$	$(1.96\ T^{-1/2})\ (1 + 2 \sum_{j=1}^{j=J} r_x(j)\ r_y(j))^{1/2}$
Total Consumption	0.590	0.327
Exports	0.320	0.322
Imports	0.576	0.285
M1	0.362	0.444
Investment	0.756	0.331

The first column reports sample cross correlations between x_t (the "c_t" of the GDP) and y_t ("c_t" of macro-aggregates), the second column shows autocorrelation adjusted SE times the limit (absolute value) of 95% confidence interval; J = 41.

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