Supersymmetric Non-Abelian Born-Infeld Theory

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Abstract

Using the natural curvature invariants as building blocks in a superfield construction, we show that the use of a symmetric trace is mandatory if one is to reproduce the square root structure of the non-Abelian Dirac-Born-Infeld Lagrangian in the bosonic sector. We also discuss the BPS relations in connection with our supersymmetry construction.

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Dirac-Born-Infeld (DBI) type actions arise in the study of low-energy dynamics of D-branes [1]-[5] (see [6]-[7] for a complete list of references). In the case of superstring theory, one has to deal with a supersymmetric extension of DBI actions and, when a number of D-branes coincide, there is a symmetry enhancement [8] and the Abelian DBI action should be generalized to the non-Abelian case.

Several possibilities for extending the Abelian Born-Infeld action to the case of a non-Abelian gauge symmetry have been discussed in the literature [9]-[17]. Basically, they differ in the way the group trace operation is defined. In the string context, a symmetric trace operation as that advocated by Tseytlin [12] seems to be the appropriate one. Among its advantages, one can mention:

(i) It eliminates unwelcome odd powers of the curvature, this implying that the field strength F (although possibly large) should be slow varying since $F^3 \sim [D, D]F^2$. With this kind of Abelian approximation (it implies commuting $F's$) one can make contact with the tree level open string effective action.

(ii) It is the only one leading to an action which is linearized by BPS conditions and to equations of motion which coincide with those arising by imposing the vanishing of the β-function for background fields in the open superstring theory [13]-[15].

It should be mentioned, however, that there are some unsolved problems concerning the use of a symmetric trace for the non-Abelian Born-Infeld action. In particular, some discrepancies between the results arising from a symmetrized non-Abelian Born Infeld theory and the spectrum to be expected in brane theories are pointed out in ref. [16].

As noticed in [15], the fact that the symmetric trace is singled out as that leading to BPS relations should be connected with the possibility of supersymmetrizing the Born-Infeld theory. In this respect, we construct in this work the supersymmetric version of the non-Abelian Dirac-Born-Infeld action and discuss the trace issue and the Bogomol'nyi structure of the resulting bosonic sector.

Our analysis, close to that developed in [18], extends to the non-Abelian case the results presented for the Abelian Supersymmetric Born-Infeld theory in ref. [19].

As it is well-known, the basic object for constructing supersymmetric gauge theories is the (non-Abelian) gauge vector superfield V which we shall write (in $d=4$ Minkowski space) in the form

$$V(x,\theta,\bar\theta) = C + i\theta\chi - i\bar\theta\bar\chi + \frac{i}{2}\theta\theta(M+iN) - \frac{i}{2}\bar\theta\bar\theta(M-iN) - \\ \theta\sigma^\mu\bar\theta A_\mu + i\theta\theta\bar\theta(\bar\lambda + \frac{i}{2}\bar{\not\partial}\chi) - i\bar\theta\bar\theta\theta(\lambda + \frac{i}{2}\not\partial\bar\chi) + \frac{1}{2}\theta\theta\bar\theta\bar\theta(D + \frac{1}{2}\Box C) \quad (1)$$

In the case of SUSY Yang-Mills theory, (generalized) gauge invariance allows to work in the Wess-Zumino gauge, for which C, χ, M and N are all set to zero, thus remaining a multiplet with the gauge field A_μ, the Majorana fermion field λ and the auxiliary real field D, all taking values in the Lie algebra of the gauge group which we take for definiteness as $SU(N)$,

$$A_\mu = A_\mu^a t^a \qquad \lambda = \lambda^a t^a \qquad D = D^a t^a \quad (2)$$

with t^a the (hermitian) $SU(N)$ generators,

$$[t^a, t^b] = i f^{abc} t^c \quad (3)$$

$$\mathrm{tr}\, t^a t^b = \mathcal{N}\delta^{ab} \quad (4)$$

It is convenient to define a chiral variable y^μ in the form

$$y^\mu = x^\mu + i\theta\sigma^\mu\bar\theta \quad (5)$$

so that the usual derivatives D and $\bar D$ can be defined as

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + 2i\left(\sigma^\mu\bar\theta\right)_\alpha \frac{\partial}{\partial y^\mu}, \qquad \bar D_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} \quad (6)$$

when acting on functions of $(y,\theta,\bar\theta)$ and

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha}, \qquad \bar D_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} - 2i\left(\theta\sigma^\mu\right)_{\dot\alpha} \frac{\partial}{\partial y^{\dagger\mu}} \quad (7)$$

on functions of $(y^\dagger,\theta,\bar\theta)$.

Generalized gauge transformation will be written in the form

$$\exp(2i\Lambda) = \exp(2i\Lambda^a t^a) \quad (8)$$

where $\Lambda(y,\theta)$ is a chiral left-handed superfield and $\Lambda^\dagger(y^\dagger,\bar\theta)$ its right-handed Hermitian conjugate,

$$\bar D_{\dot\alpha}\Lambda = D_\alpha\Lambda^\dagger = 0 \tag{9}$$

Explicitly,

$$\Lambda(y,\theta) = \frac{1}{2}(A - iB) + \theta\chi + \theta\theta\frac{1}{2}(F + iG) \tag{10}$$

Here A, B, F, G are real scalar fields and χ is a Majorana spinor. Under such a transformation, superfield V transforms as

$$\exp(2V) \to \exp(-2i\Lambda^\dagger)\exp(2V)\exp(2i\Lambda) \tag{11}$$

From V, the non-Abelian chiral superfield W_α can be constructed,

$$W_\alpha\,(y,\theta) = \frac{1}{8}\bar D_{\dot\alpha}\bar D^{\dot\alpha}\exp(-2V)D_\alpha\exp(2V) \tag{12}$$

In contrast with (11), under a gauge transformation W_α transforms covariantly,

$$W_\alpha \to \exp(-2i\Lambda)W_\alpha\exp(2i\Lambda) \tag{13}$$

Concerning the hermitian conjugate, $\bar W_\alpha$, it transforms as

$$\bar W_{\dot\alpha} \to \exp(-2i\Lambda^\dagger)\bar W_{\dot\alpha}\exp(2i\Lambda^\dagger) \tag{14}$$

Written in components, W_α reads

$$W_\alpha\,(y,\theta) = i\lambda_\alpha - \theta_\alpha D - \frac{i}{2}\left(\theta\sigma^\mu\bar\sigma^\nu\right)_\alpha F_{\mu\nu} - \theta\theta\left(\not{\nabla}\bar\lambda\right)_\alpha \tag{15}$$

with

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu] \tag{16}$$

and

$$\left(\not{\nabla}\bar\lambda\right)_\alpha = (\sigma^\mu)_{\alpha\dot\alpha}\left(\partial_\mu\bar\lambda^{\dot\alpha} + i[A_\mu, \bar\lambda^{\dot\alpha}]\right) \tag{17}$$

As it is well-known, the SUSY extension of $N = 1$ Yang-Mills theory can be constructed from W by considering W^2 and its Hermitian conjugate $\bar W^2$. Indeed, W^2 reads

$$\begin{aligned} W^\alpha W_\alpha \;=\; & -\lambda\lambda - i(\theta\lambda D + D\theta\lambda) + \frac{1}{2}(\theta\sigma^\mu\bar\sigma^\nu\lambda F_{\mu\nu} + F_{\mu\nu}\theta\sigma^\mu\bar\sigma^\nu\lambda) + \\ & \theta\theta\left(-i\lambda\;\not{\nabla}\bar\lambda - i(\not{\nabla}\bar\lambda)\lambda + D^2 - \frac{1}{2}(F_{\mu\nu}F^{\mu\nu} + i\tilde F_{\mu\nu}F^{\mu\nu})\right) \end{aligned} \tag{18}$$

with

$$\tilde{F}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}F^{\alpha\beta} \tag{19}$$

Or, writing explicitly the $SU(N)$ generators,

$$W^\alpha W_\alpha = \{t^a, t^b\}\Big(-\frac{1}{2}\lambda^a\lambda^b - i\theta\lambda^a D^b + \frac{1}{2}\theta\sigma^\mu\bar{\sigma}^\nu\lambda^a F^b_{\mu\nu} - \\ i\theta\theta\lambda^a\sigma^\mu(\delta^{bc}\partial_\mu + f^{bcd}A^d_\mu)\bar{\lambda}^c + \frac{1}{2}\theta\theta\left(D^a D^b - \frac{1}{2}(F^a_{\mu\nu}F^{b\,\mu\nu} + i\tilde{F}^a_{\mu\nu}F^{b\,\mu\nu})\right)\Big) \tag{20}$$

where

$$\{t^a, t^b\} = t^a t^b + t^b t^a \tag{21}$$

From eq.(18) and an analogous one for $\bar{W}_{\dot{\alpha}}\bar{W}^{\dot{\alpha}}$ one sees that the supersymmetric Yang-Mills Lagrangian can be written in the form

$$L_{SYM} = \frac{1}{4e^2\mathcal{N}}\mathrm{tr}\!\int\left(d^2\theta W^2 + d^2\bar{\theta}\bar{W}^2\right) \tag{22}$$

with an on-shell purely bosonic part giving

$$L_{SYM}|_{bos} = -\frac{1}{4e^2}F^a_{\mu\nu}F^{a\mu\nu} \tag{23}$$

We are ready now to extend the treatment in [18]-[19] and find a general gauge invariant non-Abelian $N=1$ supersymmetric Lagrangian of the DBI type. This Lagrangian will be basically constructed in terms of W, $\bar{W}$ and $\exp(\pm 2V)$. It is important to note that at this stage the trace operation on internal ($SU(N)$) indices to be used in order to define a scalar Lagrangian could differ, in principle, from the ordinary trace "tr" defined in (4) and used in eq.(22).

In order to get a DBI like Lagrangian (written as space-time determinant) in the bosonic sector of the theory, one should include terms which cannot be expressed in terms of $F_{\mu\nu}F^{\mu\nu}$ and $F_{\mu\nu}\tilde{F}^{\mu\nu}$ like for example those containing $F^4 \equiv F_\mu^{\ \alpha}F_\alpha^{\ \beta}F_\beta^{\ \gamma}F_\gamma^{\ \mu}$. Indeed, ignoring for the moment ambiguities arising in the definition of a non-Abelian space-time determinant, one has, concerning even powers,

$$-\det\left(g_{\mu\nu}I + F_{\mu\nu}\right)|_{even\ powers} = I + \frac{1}{2}F^2 + \frac{1}{4}(\frac{1}{2}(F^2)^2 - F^4) \tag{24}$$

In the Abelian case, the F^4 term in the r.h.s. of eq.(24) can be written in terms of F^2 and $F\tilde{F}$ but this is not the case in the non-Abelian case. Moreover, odd powers of F which were absent in the former are present in the latter case.

Let us start at this point our search for a supersymmetric extension of the non-Abelian DBI model. To begin with, in order to get higher (even) powers of $F_{\mu\nu}F^{\mu\nu}$ and $\tilde{F}_{\mu\nu}F^{\mu\nu}$ which necessarily arise in a DBI-like Lagrangian, we shall have to include higher powers of W and $\bar{W}$ combined in such a form as to respect gauge-invariance. In the Abelian case, this was achieved by combining $W^2\bar{W}^2$ with adequate powers of D^2W^2 and $\bar{D}^2\bar{W}^2$ [18]-[19]. In the present non-Abelian case, in view of transformation laws (11),(13),(14), the situation is a little more involved . Consider then the possible gauge-invariant superfields that can give rise to quartic terms. There are two natural candidates,

$$Q_1 = \int d^2\theta d^2\bar{\theta} W^\alpha W_\alpha \exp(-2V)\bar{W}_{\dot{\beta}}\bar{W}^{\dot{\beta}} \exp(2V) \tag{25}$$

$$Q_2 = \int d^2\theta d^2\bar{\theta} W^\alpha \exp(-2V)\bar{W}^{\dot{\beta}} \exp(2V) W_\alpha \exp(-2V)\bar{W}_{\dot{\beta}} \exp(2V) \tag{26}$$

with purely bosonic components

$$\mathrm{tr}\ Q_1|_{bos} = \frac{1}{4}\left(\mathrm{tr}(F^2)^2 + \mathrm{tr}(F\tilde{F})^2\right) \tag{27}$$

$$\mathrm{tr}\ Q_2|_{bos} = \frac{1}{4}\left(\mathrm{tr}F_{\mu\nu}F_{\rho\sigma}F^{\mu\nu}F^{\rho\sigma} + \mathrm{tr}F_{\mu\nu}F_{\rho\sigma}\tilde{F}^{\mu\nu}\tilde{F}^{\rho\sigma}\right) \tag{28}$$

One can see now that a particular combination of Q_1 and Q_2 generates the quartic terms one expects in the expansion of a square root DBI Lagrangian, provided this last is defined using a symmetric trace. Indeed, one has

$$\frac{1}{24}\mathrm{tr}\,(2Q_1 + Q_2)\Big|_{bos} = \mathrm{Str}\left(1 - \sqrt{1 + \frac{1}{2}(F_{\mu\nu}, F^{\mu\nu}) - \frac{1}{16}(F_{\mu\nu}, \tilde{F}^{\mu\nu})^2}\right)\Bigg|^{4th\ ord.} \tag{29}$$

where

$$\mathrm{Str}\,(t_1, t_2, \ldots, t_N) = \frac{1}{N!}\sum_\pi \mathrm{tr}\left(t_{\pi(1)}t_{\pi(2)}\ldots t_{\pi(N)}\right)\ , \tag{30}$$

Another feature in favour of using the symmetric trace is that it gives the natural way of resolving ambiguities arising in the definition of the DBI Lagrangian as a determinant. Indeed, one has [2],[15],

$$\mathrm{Str}\left(1-\sqrt{1+\frac{1}{2}(F_{\mu\nu},F^{\mu\nu})-\frac{1}{16}(F_{\mu\nu},\tilde{F}^{\mu\nu})^2}\right)=\mathrm{Str}\left(1-\sqrt{\det(g_{\mu\nu}+F_{\mu\nu})}\right) \tag{31}$$

with the r.h.s. univoquely defined through the Str prescription.

Eq.(29) is one of the main steps in our derivation: it shows that in order to reproduce the quartic term in the expansion of a DBI-type square root, one has to choose a particular combinacion of the normal trace. But this combination of normal traces corresponds precisely to the symmetric trace, originally proposed by Tseytlin [2] for the DBI theory in order to make contact with the low energy effective theory arising from superstring theory. It is worthwhile to notice that the r.h.s. of (29) vanishes for $F = \pm i\tilde{F}$. This will guarantee, at least at the quartic order we are discussing up to now, that the supersymmetric Lagrangian will reduce to SUSY Yang-Mills when the Bogomol'nyi bound (in the Euclidean version) is saturated, as it should be [16]-[17], [20].

The analysis above was made for the purely bosonic sector. It is then natural to extend it by considering the complete superfield combination $2Q_1 + Q_2$ with a trace that again accomodates in the form of a symmetric trace

$$\begin{aligned}&\frac{1}{3}(2\mathrm{tr}Q_1+\mathrm{tr}Q_2)=\\&\qquad\mathrm{Str}\left(W^\alpha,W_\alpha,\exp(-2V)\bar{W}_{\dot\beta}\exp(2V),\ \exp(-2V)\bar{W}^{\dot\beta}\exp(2V)\right)\end{aligned} \tag{32}$$

Now, in order to construct higher powers of F^2 and $F\tilde{F}$, necessary to obtain the DBI Lagrangian, we define, extending the treatment in [19], superfields X and Y,

$$\begin{aligned}X &= \frac{1}{16}\left(\bar{D}^2\left(\exp(-2V)\bar{W}_{\dot\alpha}\bar{W}^{\dot\alpha}\exp(2V)\right)+\right.\\&\quad\left.\exp(-2V)D^2\left(\exp(2V)W^\alpha W_\alpha\exp(-2V)\right)\exp(2V)\right)\end{aligned} \tag{33}$$

$$\begin{aligned}Y &= \frac{i}{32}\left(\bar{D}^2\left(\exp(-2V)\bar{W}_{\dot\alpha}\bar{W}^{\dot\alpha}\exp(2V)\right)-\right.\\&\quad\left.\exp(-2V)\ D^2\left(\exp(2V)W^\alpha W_\alpha\exp(-2V)\right)\exp(2V)\right)\end{aligned} \tag{34}$$

Both fields transform like W_α under generalized gauge transformations

$$X \to \exp(-2i\Lambda) X \exp(2i\Lambda) \ , \quad Y \to \exp(-2i\Lambda) Y \exp(2i\Lambda) \tag{35}$$

and their $\theta = 0$ component give, as in the Abelian case, the two basic invariants

$$X|_{\theta=0} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \ , \quad Y|_{\theta=0} = \frac{1}{8} \tilde{F}_{\mu\nu} F^{\mu\nu} \tag{36}$$

Inspired in the result (32) obtained in order to reproduce the adequate quartic power in F and $\tilde{F}$, we propose the following supersymmetric non-Abelian Lagrangian as a candidate to reproduce the DBI dynamics in its bosonic sector,

$$\begin{aligned} L_S = L_{SYM} + \Bigg(& \sum_{n,m} C_{nm} \int d^4\theta \mathrm{Str} \, (W^\alpha, W_\alpha, \\ & \exp(-2V) \bar{W}_{\dot\beta} \exp(2V), \exp(-2V) \bar{W}^{\dot\beta} \exp(2V), X^n, Y^m \Big) + \mathrm{h.c.} \Bigg) \end{aligned} \tag{37}$$

The arbitrary coefficients C_{nm} remain to be determined. One should retain at this point that expression (37) gives a general Lagrangian corresponding to the supersymmetric extension of a bosonic gauge invariant Lagrangian depending on the field strength F through the algebraic invariants $F_{\mu\nu}F^{\mu\nu}$ and $\tilde{F}_{\mu\nu}F^{\mu\nu}$, in certain combinations constrained by supersymmetry. The Abelian version of (37) was engeneered in [18]-[19] so that the Dirac-Born-Infeld Lagrangian could be reproduced by an appropriate choice of coefficients C_{nm}. The same happens in the non-Abelian case: a particular choice of C_{nm} corresponds to the non-Abelian Born-Infeld theory,

$$\begin{aligned} C_{0\,0} &= \frac{1}{8} \\ C_{n-2m\,2m} &= (-1)^m \sum_{j=0}^{m} \binom{n+2-j}{j} q_{n+1-j} \\ C_{n\,2m+1} &= 0 \ , \end{aligned} \tag{38}$$

$$\begin{aligned} q_0 &= -\frac{1}{2} \\ q_n &= \frac{(-1)^{n+1}}{4n} \frac{(2n-1)!}{(n+1)!(n-1)!} \quad \text{for} \ \ n \geq 1 \end{aligned} \tag{39}$$

With this choice one has for the purely bosonic part of Lagrangian (37),

$$L_S|_{bos} = \mathrm{Str}\left(1 - \sqrt{-\det(g_{\mu\nu} + F_{\mu\nu})}\right) \tag{40}$$

This is the non-Abelian Dirac-Born-Infeld Lagrangian in the form originally proposed in ref.[2].

As in the Abelian case, there are other choices of coefficients C_{mn} which also give consistent causal supersymmetric gauge theories with non-polynomial gauge-field dynamics. In particular, the alternative proposal for a $SO(N)$ DBI action recently presented in [21] should correspond to one of such choices.

We have then been able to construct a $N = 1$ supersymmetric Lagrangian (eq.(37)) within the superfields formalism, with a bosonic part expressed in terms of the square root of $\det(g_{\mu\nu} + F_{\mu\nu})$. We have employed the natural curvature invariants as building blocks in the superfield construction arriving to a Lagrangian which, in its bosonic sector, depends only on the invariants $F_{\mu\nu}F^{\mu\nu}$ and $F_{\mu\nu}\tilde{F}^{\mu\nu}$ and can be expressed in terms of the symmetric trace of a determinant. Odd powers of the field strength F were excluded in our treatment due to the fact that it is not possible to construct a superfield functional of W ($\bar{W}$) and DW ($\bar{D}\bar{W}$) containing F^3 terms in its higher θ component.

As mentioned above, the trace structure of the non-Abelian Born-Infeld theory was fixed in [15] by demanding the action to be linearized by BPS-like configurations (instantons, monopoles, vortices). In the present work we have seen that the symmetric trace naturally arises in the superfield formalism in the route to the construction of the square root Dirac-Born-Infeld Lagrangian. This confluence of results is nothing but the manifestation of the well-known connection between supersymmetry and BPS relations. Then, in order to complete our work, we shall now describe the BPS aspects in the model.

For definiteness we shall concentrate on instanton configurations in $d = 4$ dimensional space-time. In the Wess-Zumino gauge, the $N = 1$ supersymmetry vector multiplet is (A_μ, λ, D), with λ a Majorana fermion. Now, in order to look for BPS relations, we should consider a $N = 2$ supersymmetric model which includes, appart from these fields, those belonging to a chiral scalar multiplet. Indeed, in analogy with what was done to obtain the $N = 1$ general supersymmetric Lagrangian (37), one can construct a general $N = 2$

SUSY Lagrangian by adding to the vector multiplet a chiral multiplet as in the $N = 2$ SUSY Yang-Mills Lagrangian case. We shall not detail this construction here but just consider the relevant $N = 2$ SUSY transformation laws in order to derive BPS relations.

A complete $N = 2$ vector multiplet can be accomodated in terms of the fields described above in the form $(A_\mu, \lambda, \phi, D, F)$ with λ now a Dirac fermion, $\lambda = (\lambda_1, \lambda_2)$, ϕ a complex scalar, $\phi = M + iN$, D and F auxiliary real fields. The gaugino supersymmetric transformation law reads (we call $\xi = (\xi_1, \xi_2)$ the $N = 2$ transformation parameter)

$$\delta\lambda_i = (\Gamma^{\mu\nu} F_{\mu\nu} + \gamma_5 D)\xi_i + i\varepsilon_{ij}(F + \gamma^\mu \nabla_\mu (M + \gamma_5 N))\xi_j \tag{41}$$

where

$$\Gamma^{\mu\nu} = \frac{i}{4}[\gamma^\mu, \gamma^\nu] \tag{42}$$

Instanton configurations correspond to $D = F = \phi = 0$ so that (41) simplifies to

$$\delta\lambda = \Gamma^{\mu\nu} F_{\mu\nu} \xi \tag{43}$$

or

$$\delta\lambda = \frac{1}{2}\Gamma^{\mu\nu}(F_{\mu\nu} + i\gamma_5 \tilde{F}_{\mu\nu})\xi \tag{44}$$

In order to look for BPS relations one imposes as usual $\delta\lambda = 0$ thus obtaining

$$\begin{aligned} (F_{\mu\nu} + i\tilde{F}_{\mu\nu})\xi_1 &= 0 \\ (F_{\mu\nu} - i\tilde{F}_{\mu\nu})\xi_2 &= 0 \end{aligned} \tag{45}$$

In Euclidean space, eqs.(45) become

$$\begin{aligned} (F_{\mu\nu} + \tilde{F}_{\mu\nu})\xi_1 &= 0 \\ (F_{\mu\nu} - \tilde{F}_{\mu\nu})\xi_2 &= 0 \end{aligned} \tag{46}$$

with ξ_1 and ξ_2 two Euclidean Weyl fermion independent parameters. As usual, these conditions lead to instanton or anti-instanton self-dual equations

$$F_{\mu\nu} = \pm\tilde{F}_{\mu\nu} \tag{47}$$

each one of its solutions breaking half of the supersymmetries.

The fact that Yang-Mills self-dual equations arise also when the dynamics of the gauge field is governed by a non-Abelian Born-Infeld Lagrangian was already observed in [13]-[17]. In the context of supersymmetry, this can be understood following [20] where it is shown how the supersymmetry transformation law for the gaugino (and for the Higgsino in the case of the example discussed in [20]), together with the (algebraic) equation of motion for the auxiliary fields, make the BPS relations remain unchanged irrespectively of the specific choice for the gauge field Lagrangian. Moreover, one can see that the $N = 2$ SUSY charges for a general non-polynomial theory, obtained via the Noether construction, coincide, *on shell*, with those arising in Maxwell or Yang-Mills theories.

In summary, using the superfield formalism, we have derived a supersymmetric non-Abelian Dirac-Born-Infeld Lagrangian which shows the expected BPS structure, namely that of the (normal) Yang-Mills theory. In our construction, we have seen that the natural superfield functionals from which supersymmetric non-Abelian gauge theories are usually built, combine in the adequate, square root DBI form in such a way that the symmetric trace is singled out as the one to use in defining a scalar superfield Lagrangian. It should be stressed that the fact that the purely bosonic Lagrangian depends on the basic invariants F^2 and $F\tilde{F}$ and not on odd powers of F is not the result of the choice of a symmetric trace but the consequence of using W and DW as building blocks for the supersymmetric Lagrangian. Finally, let us mention that not only the supersymmetric DBI Lagrangian but a whole family of non-polynomial Lagrangians are then included in our main result, eq.(37) and all of them are linearised by BPS configurations which coincide with those of the normal Yang-Mills theory.

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