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Dynamics of the Edgeworth-Kuiper Belt beyond 50 AU

Spread of a primordial thin disk

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Abstract. In this paper we report numerical simulations of the dynamical evolution of the region $a > 50$ AU. We found that some dynamical effects such as high-order secular resonances with the rate of precession of Neptune's node of the form $k\dot{\Omega} - \dot{\Omega}_{\rm Nep}$ with $k = 4, 5, ...$ or combined mean motion resonances with Uranus and Neptune of the form $kn_{\rm N} + jn_{\rm U} + mn \sim 0$ may increase the area of a very thin primordial disk in this region by a factor of up to 2 after 4.5 Gy of evolution.

Key words. Kuiper Belt – comets: general – solar system: general

1. Introduction

Since the discovery of the first Edgeworth-Kuiper Belt object (EKBO) in 1992 (Jewitt & Luu 1993) the orbital distribution in the trans-Neptunian region has gradually been explored. Some ∼500 objects have been discovered at present. The observations indicate the presence of $\approx 10^5$ objects with diameter greater than 100 km orbiting the Sun between 30 AU and 50 AU with a combined mass of approximately 0.1 $M_\oplus$, in good agreement with the dynamical estimates (Gladman et al. 1998; Jewitt et al. 1998; Chiang & Brown 1999; Allen et al. 2000; Trujillo et al. 2001; Gladman et al. 2001; Trujillo & Brown 2001).

The evolutionary studies indicate that the primordial trans-Neptunian population was more massive in early times (Stern & Colwell 1998a, 1998b; Kenyon & Luu 1999). The growth of the observed population of ∼100 km size EKBO's, Pluto and Charon-like objects is possible in a disk one to two orders of magnitude more massive; it should contain 1 $M\oplus$ to 30 $M\oplus$ of material. Therefore, more than 90% of the primordial mass inside the region $a < 50$ AU must be lost.

Current models of the formation of the outer planets suggest that just after the end of the accretion phase, Neptune's eccentricity was probably higher than the one observed at present ($e_{\rm N} \sim 0.009$) (Fernández & Ip 1984, 1986; Thommes et al. 1999). A possible mechanism to accomplish this is the one proposed by Ward & Hahn (1998), who have shown that if the Edgeworth-Kuiper disk extends beyond 50 AU, a couple of Earth masses in the region between 48 AU and 75 AU would be consistent with the present value of Neptune's orbital eccentricity. Neptune's apsidal precession triggers density waves in the disk, which would carry away Neptune's orbital angular momentum, damping its orbital eccentricity. If the surface density of the primordial EKB was consistent with models of the minimum mass solar nebula (i.e. $N(r) \propto r^{-q}$ with $q = 3/2$ or $q = 2$, depending on the distribution of orbital eccentricities and inclinations of the planetesimal disk), and assuming that the primordial EKB extended continuously beyond Neptune, 2 $M_\oplus$ of mass in the region $a > 50$ AU is consistent with a primordial mass in the inner EKB of ∼2.3–2.6 $M_\oplus$, i.e. within acceptable values in accretion models (Stern & Colwell 1998a, 1998b; Kenyon & Luu 1999).

However, a puzzling feature of the presently observed distribution of EKBOs is that no Classical EKBO has been discovered with a semi major axis beyond 50 AU (Gladman et al. 1998; Jewitt et al. 1998; Chiang & Brown 1999; Allen et al. 2000; Trujillo et al. 2001; Gladman et al. 2001; Trujillo & Brown 2001), although an eventual detection would be within the reach of these surveys. Objects with the correspondent sky-plane velocities and apparent magnitudes should have been detected, and, indeed, distant objects at equivalent heliocentric distances have already been observed in the Scatter disk.

There are several possible explanations for the lack of detection of far Classical EKBOs (Jewitt et al. 1998): (1) The size distribution of objects in the Belt becomes steper beyond 50 AU. In this way, most of the mass in this region resides in the smallest, undetectable objects; (2) The accretion time scales in this region are too long to form large, detectable objects; (3) The objects beyond 50 AU may be darker, and therefore remain undetected; (4) The orbital eccentricities could be lower in the outer belt, making detection difficult;

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(5) The radial distribution of matter in the ecliptic plane is steper than the assumed $p = -3/2$; (6) There is an abrupt drop in the surface density of objects at $R = 50$ AU (Brunini & Melita 2002) and (7) A dense, primordial disk beyond 50 AU is dynamically cold, thus appearing as a very thin disk. The Invariant Plane of the solar system would be the most likely plane where this primordial cold disk could be located. As most deep surveys were performed around the Ecliptic plane, and the Invariant Plane being inclined up to few degrees from the Ecliptic, distant EKBOs could have evaded detection.

In this paper, we will center our attention on points (4) and (7), analyzing if the distant EK disk is dynamically cold as expected. In the following section, we shall describe the numerical experiments we have carried out to test the global dynamical behavior of the region of $a > 50$ AU, focusing our attention on the problem of the evolution of the orbital inclinations and eccentricities. In Sect. 3 we will discuss the results. The last section is devoted to conclusions.

2. Initial conditions and numerical method

We use our numerical code EVORB that consists of a second order symplectic integrator based on Wisdom & Holman (1991) algorithm, with a Bulirsch-Stoer routine that computes every close encounter between a test body and a planet within 3 Hill radii. The accuracy of the integration of the massive objects (planets) was checked by the evolution of its total energy, which kept nearly constant in all our numerical integrations (it showed oscillations of at most one part in 10^8). The precision of the integration for the case of particles encountering a planet was evaluated by computing the evolution of the Jacoby constant in the frame of the circular, restricted, three-body problem. After several hundreds of encounters the particles can undergo relative changes in the Jacoby constant at most of the order of 10^{-5} to 10^{-6} with a time step $P/50$, where P is the revolution period of the planet, and this holds even for orbits with very small perihelion distances. Maximum changes of an order of magnitude greater can occur but only if $e > 0.96$, a situation that never occurs in the simulations reported in this paper. The integrator was also tested by computing the orbital evolution of objects already studied by other authors and also reproducing the circumstances of the next 2 or 3 encounters with Earth of some potential hazardous asteroids (PHA), as predicted in the JPL NEO web site (`neo.jpl.nasa.gov/neo/pha.html`). Further details of the performance of the integrator can be found in Fernández et al. (2002).

3. Results

We have performed several numerical experiments. In the first one, the initial conditions were generated as follows: 250 massless particles were uniformly distributed at equidistant intervals in semimajor axis between 50 AU and 75 AU. The particles were placed on circular orbits on the invariant plane of the solar system (IP), and the rest of the angular elements were fixed ($\Omega = w = M = 0°$). The giant planets were included with orbital elements and masses taken from the Ephemerides of

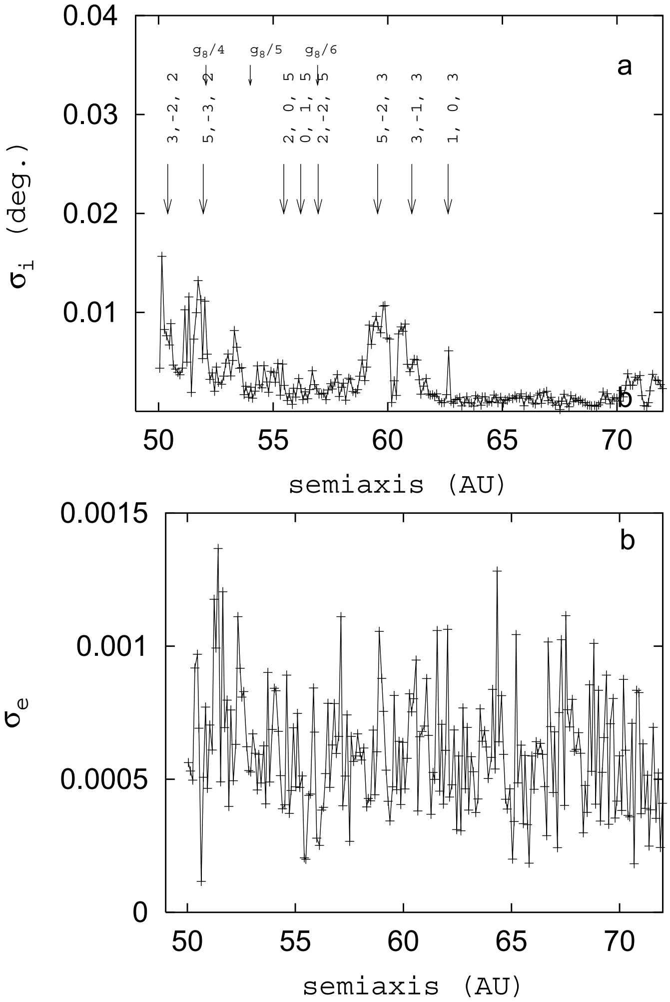

Fig. 1. a) Dispersion of the maximum value of the orbital inclinations for the sample with initial $i = 0°$. **b)** The same quantity but for the orbital eccentricities. See the text for details of how these quantities were computed.

Minor Planets for 2001. The integration was performed with a step-size of 0.1 y.

We have performed a survey of the gross stability of orbits in the explored region. This was made by integrating the orbits during a relative short time interval of $T = 5 \times 10^8$ y. This interval was divided in sub-intervals of $\delta t = 10^8$ y, computing, for each subinterval, the maximum value of the orbital inclination, max(i), and of the orbital eccentricity, max(e), for each particle. Then, we computed the dispersions σ_i and σ_e of the five values of each quantity, in the usual way. The results are shown in Figs. 1a,b.

Depending on the sequence of events that given rise to our planetary system, a collisionally relaxed population such as the one we are studying would have small free eccentricity and inclination, rather than small osculating eccentricities and inclinations relative to the IP. Nevertheless, we have also computed numerical free eccentricities and inclinations for the sample particles, obtaining for the whole sample $e_{\rm free} < 0.01$ and $i_{\rm free} < 0.3°$.

The first conclusion that can be drawn is that the region between 50 and ~63 AU is not dynamically inert as previously

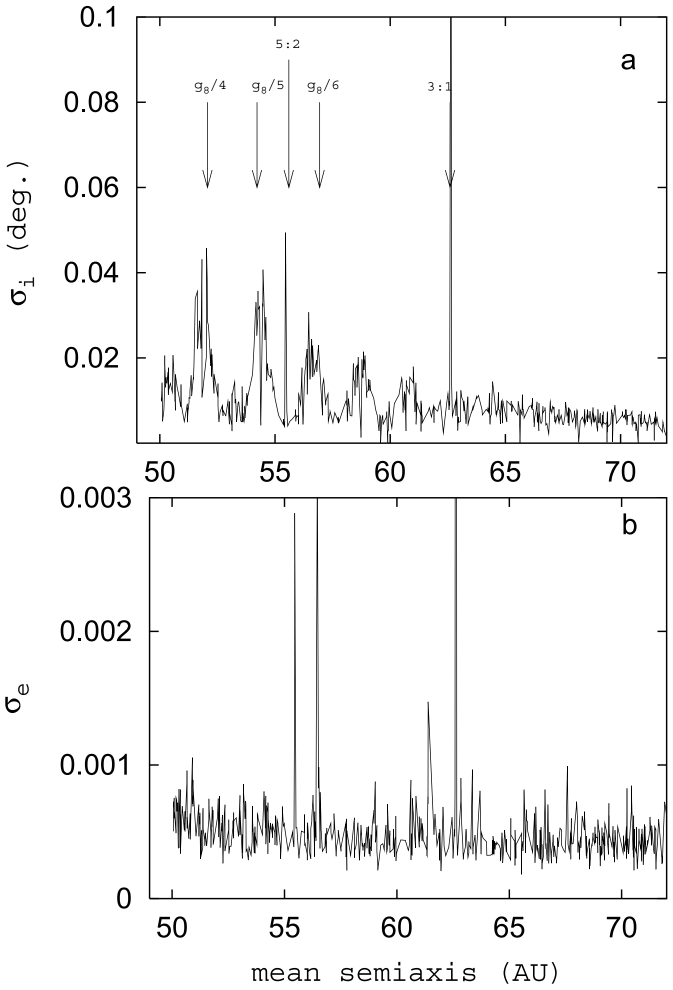

Fig. 2. The same as Fig. 1 but for sample II.

thought. There are some large and narrow spikes associated with the 5:2 and the 3:1 mean motion resonances with Neptune, and some other resonances with Uranus and Neptune, of the form $kn_{\rm N} + ln_{\rm U} + mn \sim 0$ (denoted with the label k, l, m in Fig. 1a). These resonances, being of high order, are very narrow. Other interesting features are also evident. Knezévic et al. (1991) have shown that no first order secular resonance with the major planets is present outside $a > 42$ AU. We have computed the proper frequencies of the nodes (or perihelion) in the explored region, according to the first order secular theory of Brouwer & van Woerkom (1950). It is given by

$$\dot{\Omega}(a) = \frac{n}{4} \sum_{i=1}^{N_p} \frac{m_i}{M_\odot} \alpha_i b_{3/2}^{(1)}(\alpha_i) \qquad (1)$$

where the sum is extended to all planets, m_i and a_i being the mass and semi major axis of them. n is the mean motion at the distance a, $\alpha_i = a_i/a$ and $b_{3/2}^{(1)}(\alpha_i)$ is the Laplace coefficient. As our orbits are of very low eccentricity and inclination, this approximation is accurate enough to give the node rotation rate $\dot{\Omega}$. The nodes precess at a rate that varies from $\dot{\Omega} \sim 0.206'' \text{ y}^{-1}$ at 50 AU to $\dot{\Omega} \sim 0.036'' \text{ y}^{-1}$ at 75 AU. The rate of precession of Neptune's node is $g_8 \sim 0.677'' \text{ y}^{-1}$. We have identified each one of the features observed in Figs. 1 and 2, and not associated with any mean motion resonance, to secular resonances of the form $k\dot{\Omega} - g_8$, with $k = 4, 5, 6, ...$

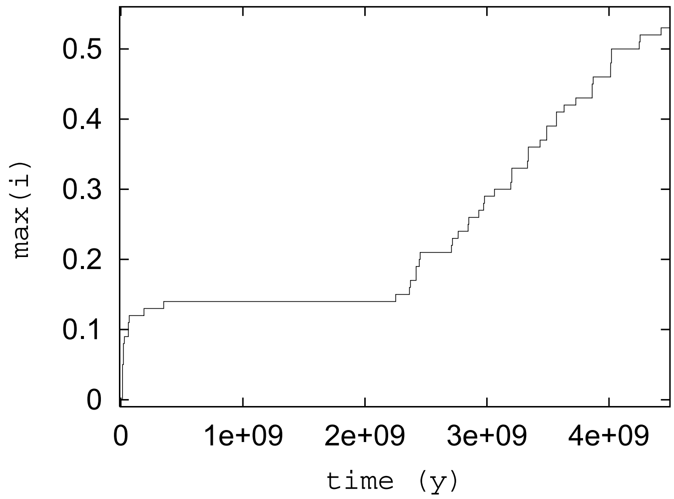

Fig. 3. Evolution of the maximum of the amplitude of the orbital inclination, for an object inside the $4\dot{\Omega} - g_{18}$ secular resonance, and with initial $i \leq 1°$ from the Invariant Plane of the solar system.

At these locations, the secular behavior of the orbital inclination cannot be properly represented by the secular linear theory.

Another sample of 300 particles (labelled "sample II") was generated as follows: the semi major axes were, as before, uniformly distributed between 50 AU $\leq a \leq$ 75 AU. The eccentricities were generated at random in the range $e \leq 0.05$, and the orbital inclinations in the range $i \leq 2°$ with respect to the IP. The other angular elements were generated at random in the interval $(0°, 360°)$ In total, we generated 300 particles with these initial conditions. To compute the same "stability maps" as before we used the technique designed by Morbidelli & Nesvorný (1999), computing "mean" semi major axes, by averaging the semi major axis of each particle over the whole interval. This is a sort of "proper" semi major axis. In this way, we have avoided some noise in the figures due to the fact that the resonances are wider at high eccentricities. Nevertheless, the resonances continue to be very narrow. In fact, only 5 objects out of a total of 3000 are inside the 3:1 resonance (a = 62.7 AU), whereas only 2 are inside the 5:2 one (a = 55.6 AU).

Due to the combined action of the identified dynamical effects, some spread in the amplitude of the oscillations of the orbital inclinations should be expected after 4.5 Gy of evolution. In fact, for $\sigma_i \geq 0.01°$ some growth in the amplitude of the order of $\geq 0.1°$ would be possible (in doing this estimation, we are considering that the sub-intervals used for the computation of σ_i were of 10^8 y, so $\delta\max(i) \sim \sigma_i \sqrt{4.5 \times 10^9/10^8}$).

To check this prediction, we have selected from this last sample those objects with $\sigma_i \geq 0.01°$ (a total of 39 objects), and the numerical integration was followed up to t = 4.5 Gy. In Fig. 3 the evolution of the maximum of the amplitude of the orbital inclination is shown for one object inside the $4\dot{\Omega} - g_8$ secular resonance, and with initial $i \leq 1°$ from the IP. We can observe that the inclination increases almost linearly with time. We suspect that the maximum is not reached during the

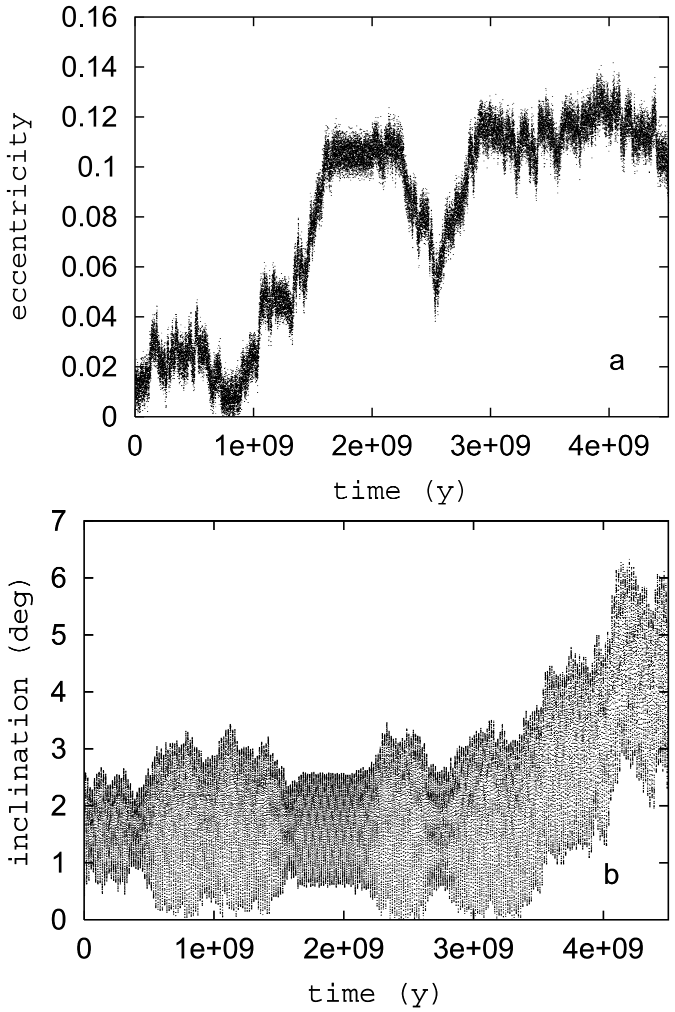

Fig. 4. a) Evolution of the orbital eccentricity of a particle at the 3:1 mean motion resonance with Neptune (a = 62.6 AU). **b)** The evolution of the orbital inclination of the same particle.

integration. The average in the increase of the amplitude of the orbital inclination for the whole sample is of 0.62°. Extracting two objects at the 3:1 mean motion resonance, the same quantity drops to 0.34°.

The behavior of an object inside the 3:1 mean motion resonance is shown in Figs. 4a,b.

It is possible to observe that this resonance pumps the eccentricity up to values near $e = 0.15$ and the inclinations up to $i = 6°$. These values, even being large, are not able to bring objects to Neptune-crossing orbits. Therefore, even if the warming of the distant belt is considerable, we cannot expect any scatter disk member from this region, although the perihelion of these class of objects could reach ~50 AU. Figures 5 (a to c) shows the behavior of a particle at the $4\dot{\Omega} - g_8$ secular resonance. Figure 5c shows that the particle is always near the 4:1 secular resonance with the node frequency of Neptune. It is evident that the orbital inclination suffers a slow "diffusion", as already predicted by the stability indicators (e.g. see Fig. 1a). This kind of secular resonances are not as strong as the first order ones. Nevertheless, they introduce a slow "spread" of the distant belt plane, in such a way that it

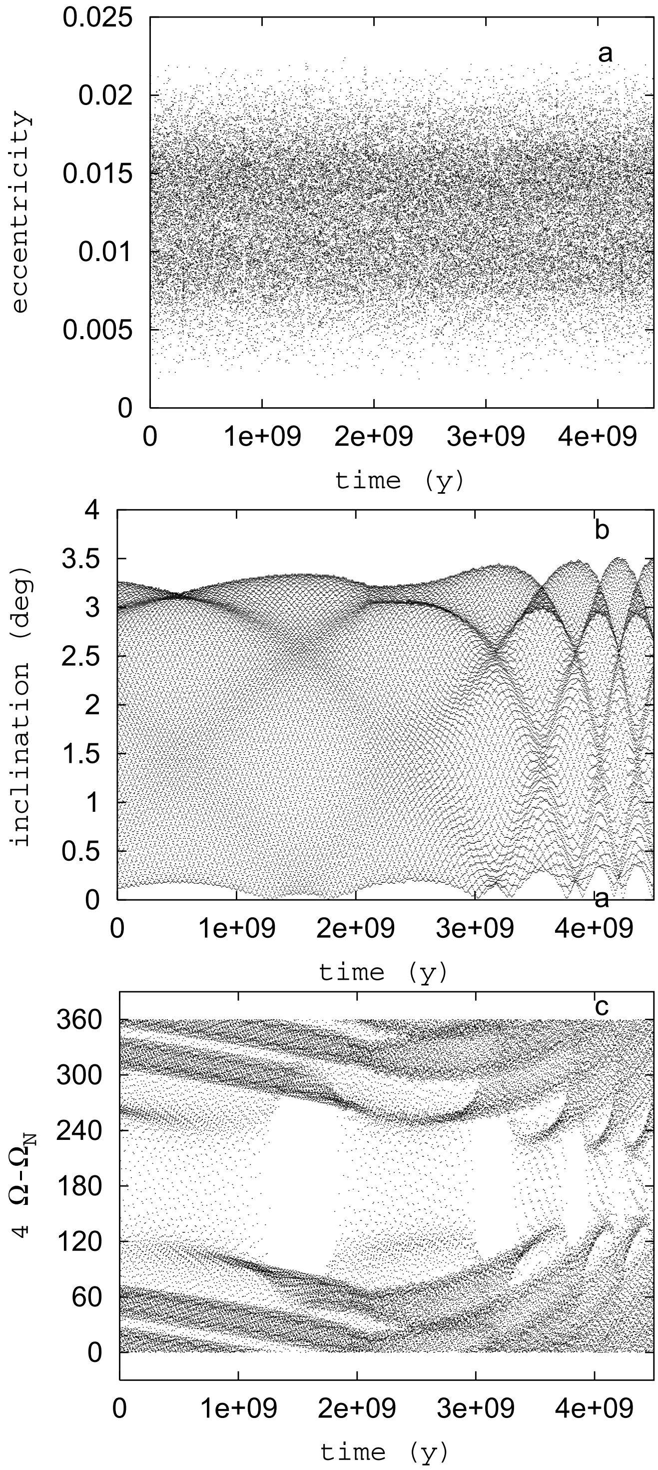

Fig. 5. a) Evolution of the orbital eccentricity of a particle at the 4:1 secular resonance with precession-rate of the node of Neptune (a = 52.07 AU according to the secular linear theory of Brouwer & van Woerkom 1950). **b)** Evolution of the inclination. **c)** Evolution of the angle $4\Omega - \Omega_N$.

could be not as cold as expected. Nevertheless, from the point of view of the detection, a distribution with initial i in the range of 1° is already within the region of coverage of several published surveys (see below).

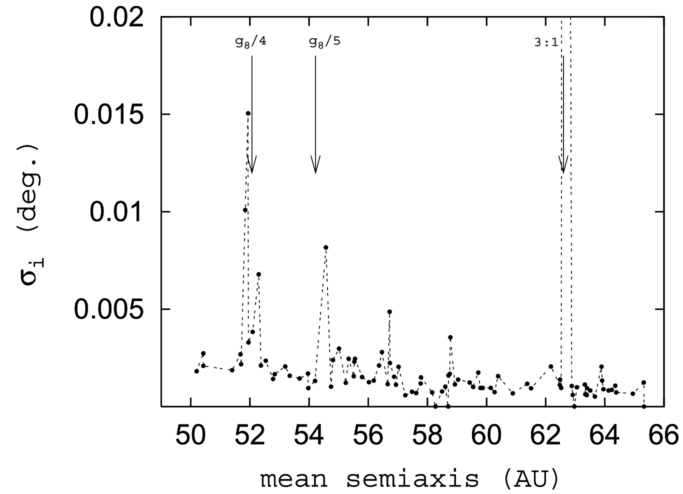

Fig. 6. Dispersion of the maximum value of the orbital inclinations for the sample with $i_0 \leq 0.2°$ from the Invariant Plane.

An additional set of 100 particles were generated with $e \leq 0.05$, $i \leq 0.2°$ with respect to the IP. These initial conditions are intended to reproduce a primordial very cold outer disk, as suggested by Hahn (2000). As the region with 50 AU $\leq a \leq$ 65 AU represents the zone where some interesting dynamical evolution could be expected, the initial semi major axes were limited to this region. In this case, the total time span of the numerical integration was 4.5×10^9 y. Figure 6 displays the map of dispersions of orbital inclinations, generated in the same way as for sample II.

Figure 6 shows that σ_i may be as large as 0.002 to 0.005. Using the same estimation as before, we predict that the amplitude of the inclinations may grow by a factor of up to 2.

It is argued that if the distant disk is a very cold disk, it could have evaded detection simply because most searches were performed at the wrong places. Figure 7a shows the sky distribution of the initial conditions of the particles with initial $i \leq 0.2°$. In order to have a better coverage of the sky, we "cloned" each object by repeating the same orbital elements except the mean anomaly, that was uniformly distributed in the interval $(0°, 360°)$. The places where most published searches were performed are also shown. As it is evident, with the exception of two of the surveys by Allen et al. (Allen et al. 2001, surveys labeled D and G in their Table 1), and the survey by Gladman et al. (2000). (The surveys they conducted around Uranus and Neptune in 1999, to look for irregular satellites. At the time of the survey, Uranus was at ecliptic coordinates $\lambda = 315°, \beta = -0.71°$. Thus the invariable plane was covered.) all the deep surveys made so far were not designed to search for a very cold distant disk around the IP.

Figure 7b displays the sky distribution of the same particles after 4.5 Gy of orbital evolution. The same procedure of "cloning" was performed. It is evident that even such an initially very cold disk suffers a spread, in such a way that another survey (Luu & Jewit 1998) falls very close to the region of coverage.

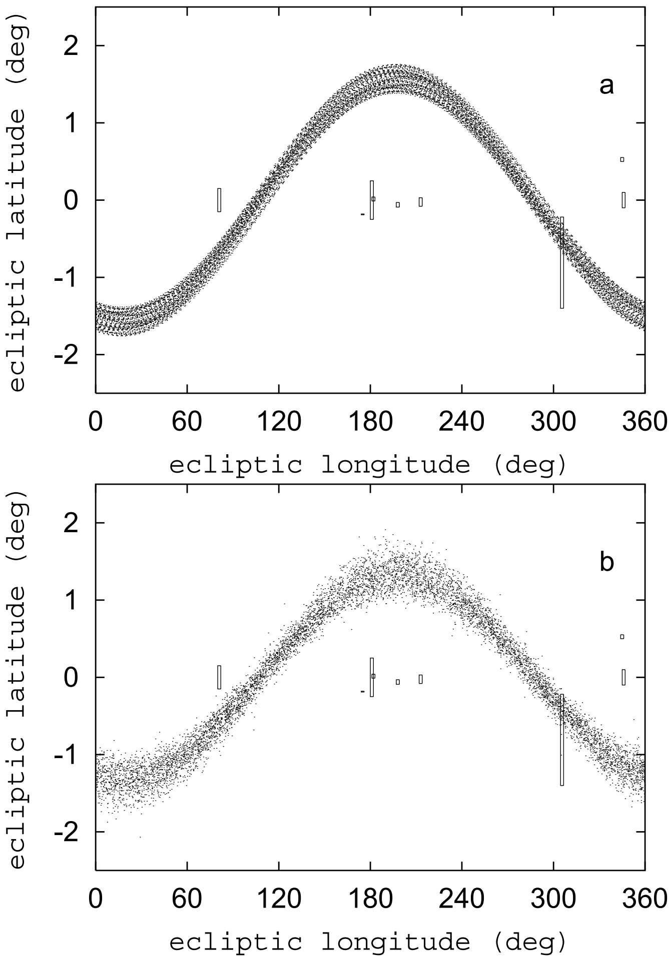

Fig. 7. a) Sky distribution of the initial conditions of the particles with $i_0 \leq 0.2°$ from the Invariant Plane. **b)** Same as **a)**, but for the same particles after 4.5 Gy of evolution. The places where most published surveys were carried out are also shown as open rectangles.

4. Conclusions

The numerical experiments we have carried out have revealed that the region of the solar system with 50 AU $\leq a \leq$ 65 AU may be considered dynamically evolved, whereas beyond 65 AU the disk is completely inert to planetary perturbations. Primordial objects from this "inner" region are subject to several perturbing agents, the most prominent being high-order secular resonances with the node frequency of Neptune. This effect may slowly spread the inclinations of the distant disk by a factor of up to 2.

Another possibility, not addressed in this paper, is if the primordial plane was not the present IP of the solar system. Nevertheless, in this case, the amplitude of oscillations in i would be, at present, higher than if the objects were on the IP.

In any case, if the disk was not initially very cold, the spreading due to the dynamical evolution is much more significant. In this case, we must look for another explanation for the failure of the past surveys, because almost all of them intersect possible populated regions of the distant disk.

In our model, the mass of the distant disk itself was ignored. This would shift the locations of the secular resonances. However, we shouldn't expect a substantial change of the dynamical behaviour of the region $a \geq 50$ AU by this effect, althought it should be investigated in the future.

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