GNSS Multi-constellation Positioning Problem: A Numerical Optimization Approach

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Abstract. In this work we analyze the integration of multiple Global Navigation Satellite Systems to get the user position solution. We describe and analyze the different ways that the problem can arise and propose an approach to generalize them onto the same formulation. To solve the resulting problem, we propose two different numerical optimization techniques, the steepest descent method and the Quasi-Newton method.
We validate and asses the performance of the proposed technique by means of simulation. Extensive simulations are also employed to analyze the solution availability and performance that the multi-constellation integration can offer. The enhancement obtained justifies the use of the proposed solution method.

Keywords: GNSS, Multi-Constellation, Positioning, DOP.

1 Introduction

Global navigation satellite systems (GNSS) are constellations of satellites designed to provide positioning and timing information for users on Earth and space [1]. There are several systems that are either fully operational today, such as GPS (USA) and GLONASS (Russia), or in the development stage, as Galileo (European Union) and Compass (China) [2].

The concept of GNSS receiver is evolving from the old conception of a stand-alone GNSS system receiver (e.g. GPS receiver or GLONASS receiver) to a multi-constellation receiver that is able to process signals from multiple GNSS systems. These receivers provide several advantages on obtaining user position and time. Firstly, since each GNSS system is independent from the others, this provides redundancy against the failure of one system. Secondly, there is an increase in the availability of signals, which is critical to maintain the coverage in applications where visibility is severely restricted, e.g. in urban environments. Thirdly, this increase in availability enables an increase in performance against

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the stand-alone solution. The multi-constellation integration, however, presents several challenges to be overcome.

The operation of a GNSS system is based on the measurement of the time of arrival (TOA for short) of the signal from the different satellites that form the system constellation. This TOA is related to the user-to-satellite range by the velocity of propagation of the electromagnetic wave in the medium.[1] Once the range to at least three satellites is known, the three position coordinates of the user can be obtained by a technique called trilateration [2]. This assumes that the receiver was able to measure TOAs in the GNSS time reference system, i.e. it has a clock that keeps perfect synchronism with the satellites' atomic clocks. Since this is not generally true, the measurements are equal to the geometric range plus a term proportional to this non synchronism (referred as user clock bias) and are called pseudo-range measurements. To solve the problem, an extra unknown that takes into account the user clock bias must be incorporated, and so at least four satellites are necessary to solve it.

The problem takes the aforementioned form only when we use one GNSS system. Several formulations to obtain a close solution to it are presented in [3][4]. In case we want to use measurements from multiple constellations we need to account for the differences in the space-time frame of reference of each system. Each GNSS system keeps its own time frame, thus providing independence of operation among them, so the user clock bias is different for each system, and needs to be incorporated as an extra unknown in the problem. On the other hand, the spatial frame differences remain the same along time and can be easily corrected.[2] In summary when we incorporate measurements for a system we add a new unknown into the problem. The exact analytic solution of the two-constellation GNSS navigation problem is exposed in [5].

In this paper, we focus on the multi-constellation integration problem. In Sect. 2, we formulate the problem of positioning and timing in this particular situation. We analyze different possible cases, and propose an approach to generalize them by the same formulation. We solve this problem by means of numerical optimization techniques, described in Sect. 3. In Sect. 4, we analyze how the measurement's uncertainty affect the performance of the solution through the geometry of the problem, an effect called dilution of precision. In Sect. 5, we present simulation results. Based on them we analyze the aforementioned facts of increment in availability and performance that multi-constellation integration can offer. We also verify the applicability of the proposed numerical optimization approach to solve the described problem. Finally, in Sect. 6 conclusions are drawn.

[1] In general it is assumed that this velocity corresponds to the speed of light in vacuum, and some corrections (estimated) are made to incorporate ionospheric and tropospheric delay effects.

[2] They depend on the different geodetic frames adopted by each system.

2 Problem Formulation

If we denote with $\mathbf{u}$ the user's position, and $\mathbf{s}_j^\alpha$ the position of the j-th satellite, corresponding to the α GNSS system, the geometric range results

$$\mathrm{d}(\mathbf{u}, \mathbf{s}_j^\alpha) \triangleq \|\mathbf{u} - \mathbf{s}_j^\alpha\| \tag{1}$$

where $\|\cdot\|$ is the Euclidian's norm. The pseudo-range measurements then result

$$\rho_j = \mathrm{d}(\mathbf{u}, \mathbf{s}_j^\alpha) + \varrho^\alpha + \nu_j \tag{2}$$

where ϱ^α is the user clock bias in the system α translated into distance and ν_j is noise that corrupts the measurement. We assume that the latter is uncorrelated and identically distributed with zero mean and variance σ_ρ^2. In absence of multipath we can consider it as additive white gaussian noise (AWGN).

As stated before, when we incorporate measurements from a given system, we also incorporate an extra unknown to the problem. Therefore, it doesn't make sense to incorporate only one measurement for such a system, because we increase both, the number of equations and the number of unknowns. On the other hand, if we have four or more measurements from one of the GNSS systems (e.g. system a) we can solve the positioning problem using only the measurements of that system, and then solve for the other systems clock bias unknowns replacing in (2) (e.g. system b).

Based on the previous analysis, we can say that there are three problems really worth solving: when we have four measurements from the same system (four unknowns), when we have three and two measurements from two different systems (five unknowns) and, when we have three pairs of measurements, each pair from a different system (six unknowns). We want to use a single technique to solve the problem arising in all three different cases. To do that, we can eliminate the bias unknowns from the measurements of the form (2) choosing one measurement of each system involved and taking the difference between it and the other measurements of the same system. The three different cases are

- Case I: Only one system, four measurements:

$$\rho_{12} = \mathrm{d}(\mathbf{u}, \mathbf{s}_1^a) - \mathrm{d}(\mathbf{u}, \mathbf{s}_2^a) + \nu_{12} \tag{3}$$
$$\rho_{13} = \mathrm{d}(\mathbf{u}, \mathbf{s}_1^a) - \mathrm{d}(\mathbf{u}, \mathbf{s}_3^a) + \nu_{13} \tag{4}$$
$$\rho_{14} = \mathrm{d}(\mathbf{u}, \mathbf{s}_1^a) - \mathrm{d}(\mathbf{u}, \mathbf{s}_4^a) + \nu_{14}. \tag{5}$$

- Case II: Two Systems, five measurements (three for one system and two for the other):

$$\rho_{12} = \mathrm{d}(\mathbf{u}, \mathbf{s}_1^a) - \mathrm{d}(\mathbf{u}, \mathbf{s}_2^a) + \nu_{12} \tag{6}$$
$$\rho_{13} = \mathrm{d}(\mathbf{u}, \mathbf{s}_1^a) - \mathrm{d}(\mathbf{u}, \mathbf{s}_3^a) + \nu_{13} \tag{7}$$
$$\rho_{45} = \mathrm{d}(\mathbf{u}, \mathbf{s}_4^b) - \mathrm{d}(\mathbf{u}, \mathbf{s}_5^b) + \nu_{45}. \tag{8}$$

– Case III: Three Systems, six measurements (two measurements in each system):

$$\rho_{12} = \mathrm{d}(\mathbf{u}, \mathbf{s}_1^a) - \mathrm{d}(\mathbf{u}, \mathbf{s}_2^a) + \nu_{12} \tag{9}$$

$$\rho_{34} = \mathrm{d}(\mathbf{u}, \mathbf{s}_3^b) - \mathrm{d}(\mathbf{u}, \mathbf{s}_4^b) + \nu_{34} \tag{10}$$

$$\rho_{56} = \mathrm{d}(\mathbf{u}, \mathbf{s}_5^c) - \mathrm{d}(\mathbf{u}, \mathbf{s}_6^c) + \nu_{56} \ . \tag{11}$$

Where we have defined

$$\rho_{ij} \triangleq \rho_i - \rho_j \tag{12}$$

$$\nu_{ij} \triangleq \nu_i - \nu_j \ . \tag{13}$$

It can be observed from (3) to (11), that the three cases can be stated in the same way as the intersection of three hyperboloids. *Case I* can be solved in closed form as is presented in [4], but this solution cannot be extended to the other cases because it is based on the fact that the three hyperboloids share a common focus. We propose to solve the presented problem by means of numerical optimization.

Once the vector $\mathbf{u}$ is obtained from these equations, the clock bias for each system can be obtained from any of the equations of the form (2). It is worth noting that once we have a good estimate of the different biases, and therefore of their differences, we can think that these differences will not change very much in time. Then we could put all measurements on the same space-time frame and the problem reduces to the classical one presented in [3], and can be solved with at least four satellites in view.

3 Optimization Problem Formulation

From (3) to (11), we define the k-th element of the measurement residual column vector as

$$[\epsilon(\mathbf{u})]_k \triangleq \mathrm{d}(\mathbf{u}, \mathbf{s}_{i(k)}) - \mathrm{d}(\mathbf{u}, \mathbf{s}_{j(k)}) - \rho_k^* \tag{14}$$

where the system reference superscript was dropped to simplify the notation, we denoted the subscript of the satellite position vector as $i(k)$ and $j(k)$, and we redefined the measurement ρ_k^*, which incorporates the effect of additive noise and whose subscript has been renumbered by k. We define the cost function to be minimized from these measurement residuals as

$$\mathrm{J}(\mathbf{u}) = \epsilon^T(\mathbf{u}) \cdot \mathbf{C}_\nu^{-1} \cdot \epsilon(\mathbf{u}) \tag{15}$$

where $\mathbf{C}_\nu$ is the covariance matrix of the measurement noises.

The covariance matrix depends on the case. If all measurements of the form (2) have noise with the same variance, σ_ρ^2, this matrix results

$$\mathbf{C}_\nu^{(I)} = \sigma_\rho^2 \cdot \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} , \ \mathbf{C}_\nu^{(II)} = \sigma_\rho^2 \cdot \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} , \ \mathbf{C}_\nu^{(III)} = \sigma_\rho^2 \cdot \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \tag{16}$$

where the superscripts stand for the case I, II or III, respectively. The twos in the main diagonal appear because each ν_{ij} in (13) is the difference of two non-correlated noises. The off-diagonal elements that are equal to one consider the correlation between measurements that have a common noise.

Although the problem was formulated in the three "minimal measurements cases", it can can also be applied when the number of measurements exceeds the minimum required by each particular case. We only need to expand the corresponding case in (3) to (11) to add more measurements, and rewrite the rest of the formulation respectively. We then analyze the advantages that this extension can bring.

3.1 Gradient Calculation

For the implementation of the various numerical optimization methods we require the analytical expression of the cost function gradient, whose m-th component is

$$[\nabla \mathrm{J}(\mathbf{u})]_m = 2\,\epsilon^T(\mathbf{u}) \cdot \mathbf{C}_\nu^{-1} \cdot \frac{\partial\,\epsilon(\mathbf{u})}{\partial\,[\mathbf{u}]_m} \tag{17}$$

$$\begin{aligned}\left[\frac{\partial\,\epsilon(\mathbf{u})}{\partial\,[\mathbf{u}]_m}\right]_k &= \frac{\partial\,\mathrm{d}(\mathbf{u},\mathbf{s}_{i(k)})}{\partial\,[\mathbf{u}]_m} - \frac{\partial\,\mathrm{d}(\mathbf{u},\mathbf{s}_{j(k)})}{\partial\,[\mathbf{u}]_m} \\ &= \frac{[\mathbf{u}]_m - \left[\mathbf{s}_{i(k)}\right]_m}{\mathrm{d}(\mathbf{u},\mathbf{s}_{i(k)})} - \frac{[\mathbf{u}]_m - \left[\mathbf{s}_{j(k)}\right]_m}{\mathrm{d}(\mathbf{u},\mathbf{s}_{j(k)})}\end{aligned} \tag{18}$$

where $[\mathbf{u}]_m$ and $\left[\mathbf{s}_{j(k)}\right]_m$ stand for the m-th component of the user's position vector, and the m-th component of the j-th satellite position, respectively.

3.2 Steepest Descent Method

The simplest method to solve the optimization problem is the so called steepest descent method [6],

$$\mathbf{u}^{(l+1)} = \mathbf{u}^{(l)} - \alpha_l \cdot \nabla \mathrm{J}_l \tag{19}$$

$$\nabla \mathrm{J}_l \triangleq \nabla \mathrm{J}(\mathbf{u}^{(l)}) \tag{20}$$

where $\mathbf{u}^{(l)}$ refers to the l-th iteration step of the vector $\mathbf{u}$. The α_l parameter is obtained by means of some inexact search algorithm. Its value must satisfy the Wolfe conditions [6],

$$\mathrm{J}(\mathbf{u}^{(l)} + \alpha_l \cdot \nabla \mathrm{J}_l) \leq \mathrm{J}(\mathbf{u}^{(l)}) - c_1 \cdot \alpha_l \cdot \nabla \mathrm{J}_l^T\, \nabla \mathrm{J}_l \tag{21}$$

$$\nabla \mathrm{J}(\mathbf{u}^{(l)} + \alpha_l \cdot \nabla \mathrm{J}_l)^T\, \nabla \mathrm{J}_l \leq c_2\, \nabla \mathrm{J}_l^T\, \nabla \mathrm{J}_l\,. \tag{22}$$

This is the simplest of the analyzed methods (it requires only gradient evaluation), but it has a very slow rate of convergence as we will see in Sect. 5.3.

3.3 Quasi-Newton Optimization

A way to achieve a better rate of convergence is by means of the so called Quasi-Newton methods. Like the steepest descent method, they requiere only gradient evaluation of the objective function at each iteration step. By measuring the changes in gradients, they construct a model of the objective function that produces superlinear convergence. We implement the *Quasi-Newton SR1* method with a *Trust-Region* approach. To solve the subproblem, we implement the so called *Dogleg Path* [6].

3.4 Initialization

In these iterative schemes we need to provide an initial point, $\mathbf{u}^{(0)}$, to start the optimization process. If the differences in system biases are not very large, we can solve the original system of equations (2), by the closed form proposed on [3], obtaining a solution perhaps not so far from the real one, that can be used to initialize the algorithm. Another way to obtain the initial point could be projecting the position of one satellite (or some point in the middle of the positions of available satellites) on Earth's surface, or take the center of Earth (i.e. point $\mathbf{p}_0 = (0, 0, 0)$).

In the former case, we need to be careful because the closed form solution provides a dummy solution in addition to the real one, that must be discerned using additional information (e.g. the user position should be on the Earth's surface).

4 Dilution of Precision

To analyze how the measurement errors affect the computed user position we use the concept of dilution of precision (DOP). Following the derivation made in [2], if we linearize equations (3) to (11) around the correct solution, we can write

$$\mathbf{H} \cdot \boldsymbol{\Delta}\mathbf{u} = \boldsymbol{\Delta}\rho \tag{23}$$

where we define $\boldsymbol{\Delta}\rho$ the error in the measurements and $\boldsymbol{\Delta}\mathbf{u}$ the error in the solution. Then, the error covariance matrix can be obtained as

$$\mathbf{C}_{\boldsymbol{\Delta}\mathbf{u}} = \mathbf{H}^{\sharp} \cdot \mathbf{C}_{\nu} \cdot {\mathbf{H}^{\sharp}}^{T} \tag{24}$$

where

$$\mathbf{H}^{\sharp} = \left(\mathbf{H}^{T} \cdot \mathbf{H}\right)^{-1} \cdot \mathbf{H}^{T} \tag{25}$$

is a generalized inverse of the matrix $\mathbf{H}$. Then, the DOP can be obtained from the elements of the matrix $\mathbf{C}_{\boldsymbol{\Delta}\mathbf{u}}$. The position DOP (PDOP), represents the amplification factor of the standard deviation of the measurement errors into the position solution [2], and is given by

$$PDOP = \sqrt{[\mathbf{C}_{\boldsymbol{\Delta}\mathbf{u}}]_{11} + [\mathbf{C}_{\boldsymbol{\Delta}\mathbf{u}}]_{22} + [\mathbf{C}_{\boldsymbol{\Delta}\mathbf{u}}]_{33}}. \tag{26}$$

5 Simulation Results

5.1 Simulation Scenario

To analyze the multi-constellation user position solution performance, we propose a simulation scenario based on the next considerations:

- Nominal system constellations for GPS, GLONASS and GALILEO. In case of GPS we consider 30 satellites distributed in 6 orbital planes (5 satellites per plane) with an inclination of 55°. The ascending nodes are distributed every 60°. In case of GLONASS we consider 24 satellites distributed in 3 orbital planes (8 satellites per plane) with an inclination of 64.8°. The ascending nodes are distributed every 120°. In case of GALILEO we consider 27 satellites distributed in 3 orbital planes (9 satellites per plane) with an inclination of 56° The ascending nodes are distributed every 120°. In all three cases, we consider that satellites have a uniform distribution on their plane (the mean anomaly). For simplicity, we consider circular orbits. Here, the coarsest approximation is a uniform distribution of satellites per plane. The actual GPS constellation has a spacing that has been optimized to minimize the effects of a single satellite failure on system degradation [7].
- A user position on the Earth surface with a randomly generated longitude with a uniform distribution between $-180°$ and $180°$, and a randomly generated latitude with a uniform distribution between $-75°$ and $75°$, which are sufficient for almost every Earth application.
- A time of evaluation randomly generated with a uniform distribution within a week. Time is important because the constellations distribution depends on it.
- Different elevation masks. These masks determine which satellites are visible from those that are on sky in a given time and in a given user location. The elevation mask is an oversimplified way to take into account the effects of obstacles (e.g. buildings, trees) in the signal path. In a more realistic scenario the interference on signals would depend on both azimuth and elevation rather than only on elevation.
- A bias to the measurements of each system randomly generated with a uniform distribution between 0 an 1000 meters.

With these considerations we run a set of 10000 simulation trials.

5.2 Measurements Availability

One of the facts to analyze by means of simulation is measurements availability to solve the problems of positioning and time, in one of the three cases mentioned in Sect. 2. In Fig. 1 we can see the relative frequency (over the simulation runs) of the event “measurements are sufficient to solve the problem” (availability), with different elevation masks. In the sub-figure labeled as *case I*, we can see the availability of the positioning solution (also known as position fix) using *case I*, if we consider only GPS system, GPS and GALILEO systems or GPS, GLONASS

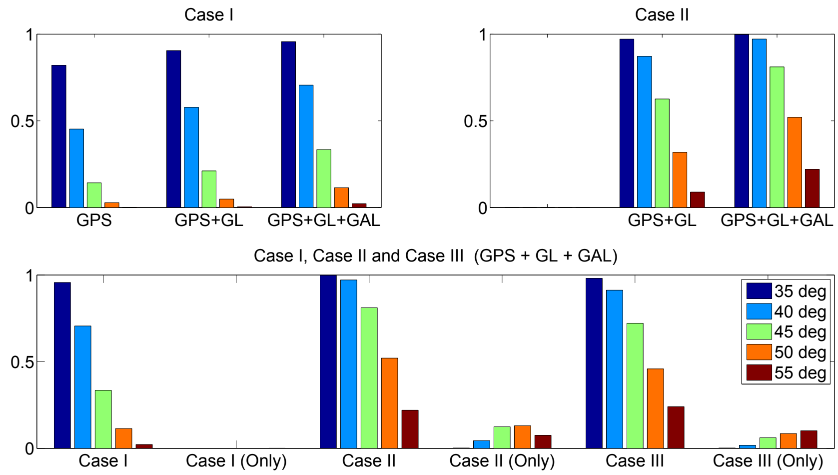

Fig. 1. Probability of having sufficient measurements to solve the problem on different cases and different elevation masks

and GALILEO systems. In each case we solve the problem using only one of them, but we have one, two or three possibilities. As expected, the availability increases with the addition of more systems

In the sub-figure labeled as *case II*, we can see the availability of the positioning fix using *case II*, considering either GPS and GALILEO systems or GPS, GLONASS and GALILEO systems. In each case, we solve the problem using two of them, but we have one, or three possibilities (3 choose 2). As in *case I*, the availability grows when we incorporate a new system. We can also see that the availability also grows related to *case I*, particularly with higher restrictive elevation masks.

On the bottom sub-figure, we compare the availability using the three systems, if we are able to solve using the different cases aforementioned. We can see that in general availability grows when we enable multi-constellation solution. We also see that there are situations where the solution could only be obtained using one of the cases (the situations where only *case I* is applicable are negligible). It is worth mentioning that there are situations where more than one case is possible.

This GPS-centrist analysis is due to the fact that GPS is the system that has been fully operational since its creation, while GLONASS has had ups and downs in coverage and GALILEO is on development stage. So, we take GPS as the primary system. A similar analysis could be done changing the order.

5.3 Numerical Solutions

To verify the correct operation of the proposed algorithms, we analyze the iteration steps obtained in three different cases, using the steepest descent method and the Quasi-Newton method. In Fig. 2, we can see the error in the three coordi-

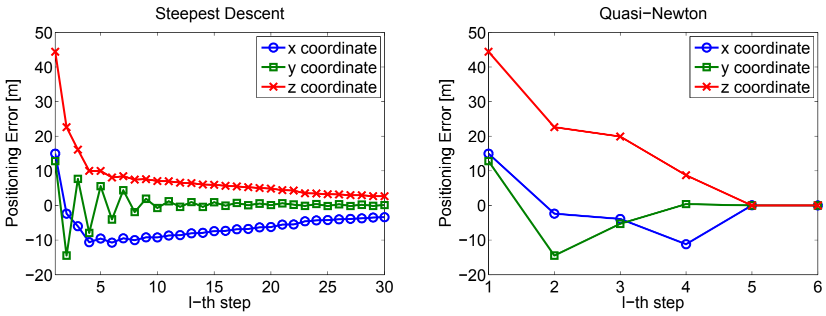

Fig. 2. Iteration steps in case I using steepest descent method and Quasi-Newton method

nates of position through different iteration steps when we consider a simulation trial where there were 4 GPS satellites available, and we solve using *case I*. Here, we haven't yet incorporated the effect of the measurement noise.

For the steepest descent method we show only around the first quarter of steps which are representative. We can see that with the second method we achieve a higher rate of convergence (6 against 120 steps, using the same stop criteria). In Fig. 3, we can see the error in the three coordinates of position

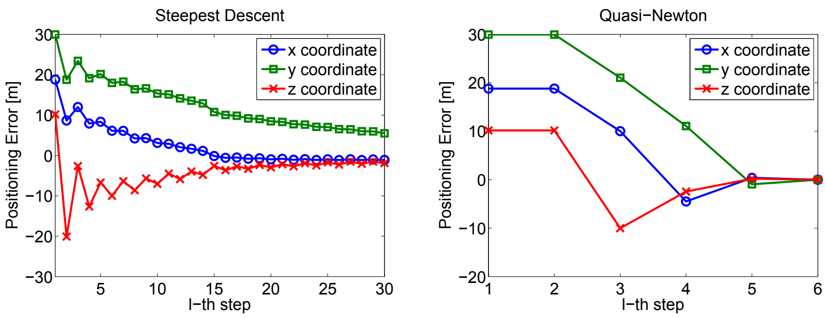

Fig. 3. Iteration steps in case II using steepest descent method and Quasi-Newton method

through different iteration steps when we consider a simulation trial where there were 3 GPS and 2 GLONASS satellites available, and we solve using *case II*. The conclusions are similar to the previous case. In Fig. 4, we can see the error in the three coordinates of position through different iteration steps when we consider a

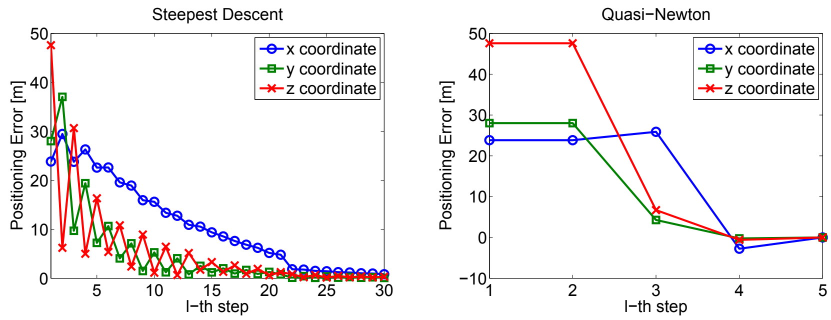

Fig. 4. Iteration steps in case III using steepest descent method and Quasi-Newton method

simulation trial where there were 2 GPS, 2 GLONASS and 2 GALILEO satellites available, and we solve using *case III*. The conclusions are similar to the previous case.

In the three cases we have obviated the clock biases solutions. These can be obtained in a simple way once the position is known.

The results obtained on the other simulation trials are similar, and when we incorporated the effect of the measurement noise, the statistics of the error verify that provided by the DOP. These results verify the operativity of the proposed algorithm in the three cases, and are not shown because of space considerations.

5.4 Performance

We have analyzed the increment in availability that the multi-constellation integration can offer, and a way to solve the positioning problem in the multi-constellation case. A question that arises is whether it is convenient to incorporate measurements of a second system when the solution can be obtained using only one. To answer that, in Fig. 5 we compare the PDOP obtained, in trials when we could get a position solution using both GPS alone, and GPS plus GLONASS. In the left sub-figure we consider the trials when just 4 GPS measurements were available. We can see that adding GLONASS measurements in this case we can considerably improve the performance, reducing the PDOP.

In the right figure, we consider all the trials where both solutions are applicable. We can see that the PDOP also increases, as logical.

In both cases, the PDOP reduction is expected because we increase the number of measurements, but the analysis shows that the effort that the addition of another system requires (we could solve the problem using only GPS), results in a considerable performance enhancement. Also, in Fig. 6 we compare the PDOP obtained, in trials when we could get a position solution using both GPS plus GLONASS and GPS plus GLONASS plus GALILEO. In the left sub-figure we consider the trials when just 3 GPS and 2 GLONASS measurements,

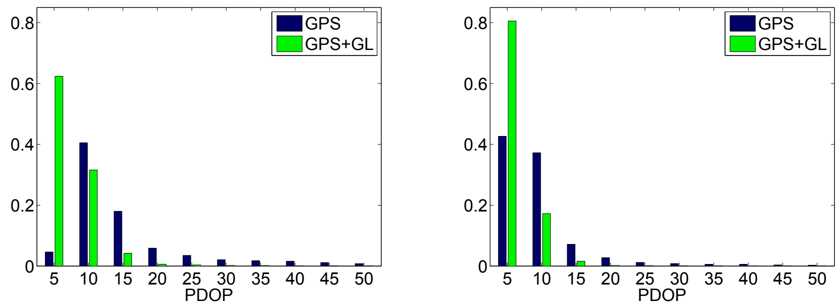

Fig. 5. PDOP enhancement by incorporation of GLONASS system

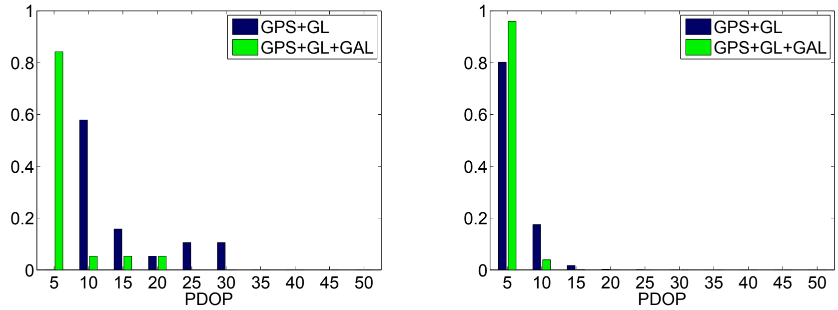

Fig. 6. PDOP enhancement by incorporation of GALILEO system

or viceversa, were available, while in the right one we consider all the trials aforementioned. We can see that the improvement is more appreciable in the former.

6 Conclusions

In this work, we analyzed the problem of solving for user position using multiple GNSS systems. We discriminate three possible cases that could arise: when there are at least 4 measurement from the same GNSS system; when there are at least three measurement from one given GNSS system and at least two measurements from another given GNSS system; and when there are at least two measurements from each of three GNSS systems.

We proposed an approach to integrate the three cases onto the same formulation. In this way, we make this formulation independent of each particular case. We used two numerical optimization techniques to solve this problem. It is worth mentioning that although the problem was formulated in the three "minimal measurements cases", it can also be applied when the number of measurements exceeds the minimum required by each particular case.

By means of simulation we validated the applicability of the proposed techniques. As expected, we obtained a faster rate of convergence when using the Quasi-Newton implementation. Based on simulation results, we also analyzed the availability of user position and the enhancement of performance that can be obtained if multiple GNSS system are integrated.

Issues as the existence and uniqueness of the solution in the multi-constellation case is a matter of discussion [5] and needs to be analyzed in more detail. The feasibility of implementing the proposed algorithms into a real-time GNSS receiver must also be analyzed.

References

1. S. Gleason and D. Gebre-Egziabher. *GNSS Applications and Methods*. GNSS technology and applications series. Artech House, 2009.
2. Elliott D. Kaplan. *Understanding GPS: Principles & Applications*. Artech House, 1996.
3. S. Bancroft. An algebraic solution of the GPS equations. In *IEEE Transactions on Aerospace and Electronic Systems*, volume AES-21, pages 56–59, 1985.
4. J.L. Leva. An alternative closed-form solution to the GPS pseudo-range equations. *Aerospace and Electronic Systems, IEEE Transactions on*, 32(4):1430 –1439, oct 1996.
5. Jyh-Ching Juang and Yung Tsai. On exact solutions of the multi-constellation gnss navigation problem. *GPS Solutions*, 13:57–64, 2009.
6. J. Nocedal and S. J. Wright. *Numerical Optimization*. Springer, New York, second edition, 2007.
7. J.B.Y. Tsui. *Fundamentals Of Global Positioning System Receivers: A Software Approach*. Wiley Series in Microwave and Optical Engineering. John Wiley & Sons, Incorporated, 2004.