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Inverse magnetic catalysis in nonlocal chiral quark models

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Abstract. We study the behavior of strongly interacting matter under an external magnetic field in the context of nonlocal Polyakov–Nambu–Jona-Lasinio (PNJL) -like models. We find that at zero temperature the condensates display the well-known Magnetic Catalysis effect, showing a good quantitative agreement with lattice QCD results. Moreover, when extended to finite temperature we find that the Inverse Magnetic Catalysis effect is naturally incorporated.

1. Introduction

Over the last years the understanding of the behavior of strongly interacting matter under extremely intense magnetic fields has attracted increasing attention, due to its relevance for subjects such as the physics of magnetars [1], the analysis of heavy ion collisions at very high energies [2] or the study of the first phases of the Universe [3]. Consequently, considerable work has been devoted to study the structure of the QCD phase diagram in the presence of an external magnetic field [4]. From most low-energy effective models of QCD it was generally expected that, at zero chemical potential, the magnetic field would lead to an enhancement of the chiral condensate ("magnetic catalysis"), independently of the temperature of the system. However, LQCD calculations [5, 6] show that, whereas at low temperatures one finds indeed such an enhancement, close to the critical chiral restoration temperature light quark condensates exhibit a nonmonotonic behavior as functions of the external magnetic field, which results in a decrease of the transition temperature when the magnetic field is increased. This effect is known as inverse magnetic catalysis (IMC). Although many scenarios have been considered in the last few years to account for the IMC [7–22], the mechanism behind this effect is not yet fully understood. With this motivation, we study the behavior of strongly interacting matter under an external magnetic field in the framework of nonlocal chiral quark models, theories that are proposed as nonlocal extensions of the well-known (P)NJL model, intending to go a step further towards a more realistic effective approach to QCD [23–30].

2. Theoretical formalism

Let us start by stating the Euclidean action for our nonlocal NJL-like two-flavor quark model,

$$S_E = \int d^4x \, \left\{ \bar\psi(x) \left(-i\partial\!\!\!/ + m_c\right) \psi(x) - \frac{G}{2} j_a(x) j_a(x) \right\} \ . \tag{1}$$

Here m_c is the current quark mass, equal for u and d quarks. The nonlocal currents are given by $j_a(x) = \int d^4z\, \mathcal{G}(z)\, \bar\psi(x+\frac{z}{2})\, \Gamma_a\, \psi(x-\frac{z}{2})$, where $\Gamma_a = (\mathbb{1}, i\gamma_5\vec\tau)$, and $\mathcal{G}(z)$ is a nonlocal form factor. In order to study the influence of an external magnetic field we introduce in Eq. (1) a coupling to an electromagnetic gauge field $\mathcal{A}_\mu$. This can be done by performing appropriate changes in the covariant derivative and in the nonlocal currents (see Refs. [32, 33] for details). We restrict to the case of a constant and homogeneous magnetic field along the 3-axis. Next we perform a standard bosonization, introducing scalar and pseudoscalar fields $\sigma(x)$ and $\vec\pi(x)$. Within the mean field approximation (MFA), pseudoscalar field vacuum expectation values (VEVs) vanish, and we assume the VEV of the scalar field, $\bar\sigma$, to be homogeneous in coordinate space. In this way, following the Ritus eigenfunction method [31] we find the corresponding action, $S^{\rm MFA}_{\rm bos}$ [32].

We extend the analysis of the model to a system at finite T by using the standard Matsubara formalism. In order to account for confinement effects, we include the coupling of fermions to the Polyakov loop (PL) assuming that quarks move on a constant color background field $\phi = ig\,\delta_{\mu 0}\, G^\mu_a \lambda^a/2$, where G^μ_a are the SU(3) color gauge fields. We will work in the so-called Polyakov gauge, in which the matrix ϕ is given a diagonal representation $\phi = \phi_3\lambda_3 + \phi_8\lambda_8$, taking the traced Polyakov loop $\Phi = \frac{1}{3}{\rm Tr}\,\exp(i\phi/T)$ as an order parameter of the deconfinement transition. We include effective gauge field self-interactions through a PL potential. In this work we consider two alternative forms proposed in the literature: a potential given by a polynomial function based on a Ginzburg-Landau ansatz (Poly) [35], and the so-called "improved" PL potential proposed in Ref. [36], in which the full QCD potential is related to a Yang-Mills potential (Poly Imp). Then, the MFA thermodynamical potential $\Omega^{\rm MFA}_{B,T}$ and the associated gap equations can be obtained. It is seen that $\Omega^{\rm MFA}_{B,T}$ turns out to be divergent, we regularize it following Ref. [26]. By deriving $\Omega^{\rm MFA}_{B,T}$ with respect to m_c we get the magnetic field dependent quark condensate for each flavor

$$\begin{aligned} \langle\bar\psi_f\psi_f\rangle^{\rm reg}_{B,T} \;=\; & -\frac{|q_fB|\,T}{\pi} \sum_c \int \frac{dp_3}{2\pi} \sum_{k=0}^{\infty} \sum_{n=-\infty}^{\infty} \\ & \left\{ \frac{M^{-,f}_{k,p_{\|nc}} \left[p^2_{\|nc} + 2k|q_fB| + (M^{+,f}_{k,p_{\|nc}})^2\right] \;+\; (+\leftrightarrow -)}{\left(2k|q_fB| + p^2_{\|nc} + M^{-,f}_{k,p_{\|nc}} M^{+,f}_{k,p_{\|nc}}\right)^2 + p^2_{\|nc}\left(M^{+,f}_{k,p_{\|nc}} - M^{-,f}_{k,p_{\|nc}}\right)^2} \right. \\ & \left. - \frac{2m_c}{p^2_{\|nc} + 2k|q_fB| + m_c^2} \right\} \\ & - \frac{N_c m_c^3}{4\pi^2} \left[\frac{\ln\Gamma(x_f)}{x_f} - \frac{\ln 2\pi}{2x_f} + 1 - \left(1 - \frac{1}{2x_f}\right)\ln x_f \right] \\ & + \frac{|q_fB|}{\pi} \sum_c \sum_{k=0}^{\infty} \alpha_k \int \frac{dp}{2\pi} \frac{m_c}{\epsilon^f_{kp}\left[1 + \exp(\epsilon^f_{kp}/T + i\phi_c)\right]} \ , \end{aligned} \tag{2}$$

where the color index c runs over r, g, b and the color background fields are $\phi_r = -\phi_g = \phi_3$, $\phi_b = 0$. We have defined $p_{\|nc} = (p_3, [(2n+1)\pi T + \phi_c])$ where n is associated to the Matsubara frequencies, $x_f = m_c^2/(2|q_fB|)$, $\alpha_k = 2 - \delta_{k0}$ and $\epsilon^f_{kp} = \sqrt{2k|q_fB| + p^2 + m_c^2}$. We have also defined

$$M^{\lambda,f}_{k,p_{\|nc}} = (-1)^{k-\frac{1-\lambda s_f}{2}} \int_0^\infty dr\, r\, \exp(-r^2/2) \left[m_c + \bar\sigma\, g\!\left(\frac{|q_fB|}{2} r^2 + p^2_{\|nc}\right)\right] L_{k-\frac{1-\lambda s_f}{2}}(r^2). \tag{3}$$

Here $s_f = \mathrm{sign}(q_f B)$, while $g(p^2)$ is the Fourier transform of $\mathcal{G}(z)$, $L_k(x)$ are the Laguerre polynomials, and the index k labels the Landau levels.

Finally, to make contact with the LQCD results quoted in Ref. [6] we define the normalized quark condensate

$$\Sigma^f_{B,T} = \frac{2\,m_c}{S^4}\left[\langle\bar\psi_f\psi_f\rangle^{\rm reg}_{B,T} - \langle\bar\psi_f\psi_f\rangle^{\rm reg}_{0,0}\right] + 1\ , \tag{4}$$

where the scale S is given by $S = (135 \times 86)^{1/2}$ MeV. We also introduce the definitions $\Delta\Sigma^f_{B,T} = \Sigma^f_{B,T} - \Sigma^f_{0,T}$ and $\Delta\bar\Sigma_{B,T} = (\Delta\Sigma^u_{B,T} + \Delta\Sigma^d_{B,T})/2$, which correspond to the subtracted normalized flavor condensate and the normalized flavor average condensate, respectively.

For definiteness we consider the case of a Gaussian form factor $g(p^2) = \exp(-p^2/\Lambda^2)$. Thus the model parameters are m_c, G, and Λ, that we fix so as to reproduce empirical values of f_π, m_π and a given value of the quark condensate at zero T and B, $\Phi_0 \equiv (-\langle\bar\psi_f\psi_f\rangle^{\rm reg}_{0,0})^{1/3}$ (details can be found in Ref. [27]), in addition to the parameters related to the PL potentials considered.

3. Numerical Results

Numerical results at $T = 0$ are shown in Fig. 1. In the left panel we quote the predictions for $\Delta\bar\Sigma_{B,0}$ as function of eB for various model parametrizations, found to be very similar for all cases considered and in very good agreement with LQCD results from Ref. [6]. The results for $\Sigma^u_{B,0}-\Sigma^d_{B,0}$, shown in the right panel, present an overall good agreement with LQCD calculations, however, the dependence on the model parametrization is somewhat larger.

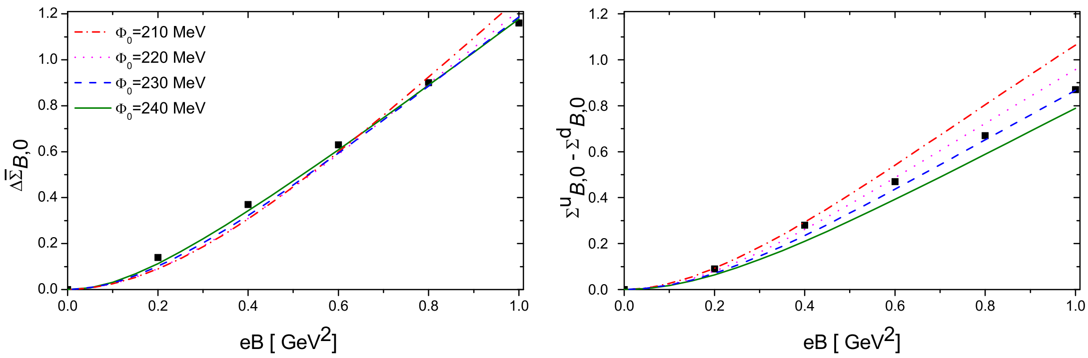

Figure 1. Normalized condensates as functions of eB at $T = 0$. The curves correspond to different parametrizations identified by Φ_0. Full square symbols correspond to LQCD results of Ref. [6]. Left panel: subtracted flavor average; right panel: flavor difference.

We turn now to our results for the case of finite temperature. In Fig. 2 we quote the values obtained for $\Delta\bar\Sigma_{B,T}$ as a function of eB, for some representative values of T. All values correspond to the parametrization leading to $\Phi_0 = 230$ MeV considering the polynomial PL potential, yet qualitatively similar results are found for the other parametrizations. In contrast to what happens at zero temperature, the quantity $\Delta\bar\Sigma_{B,T}$ does not display a monotonous increase with eB when one approaches the chiral transition temperature [for this parameter set one has $T_c(eB = 0) = 179$ MeV]. In fact, the curves reach a maximum after which $\Delta\bar\Sigma_{B,T}$ starts to decrease with increasing eB, implying that the present nonlocal model naturally exhibits the IMC effect found in LQCD. This feature can also be seen from the results displayed in Fig. 3. In the upper pannel we show $(\Sigma^u_{B,T} + \Sigma^d_{B,T})/2$ and Φ as a functions of T for some selected values of eB, while in the lower pannel we present the associated susceptibilities defined as $\chi_{cond} = \partial[(\Sigma^u_{B,T} + \Sigma^d_{B,T})/2]/\partial T$ and $\chi_\Phi = \partial\Phi/\partial T$. As expected, for all values of eB it is found a crossover transition from the chiral symmetry broken phase to the (partially) restored one as the temperature increases. However, contrary to what happens e.g. in the standard local NJL model [4], it is seen that within the present model the transition temperature decreases as the magnetic field increases. Moreover, both chiral restoration and deconfinement transitions are observed simultaneously, as predicted by PNJL-like models at $eB = 0$ [28].

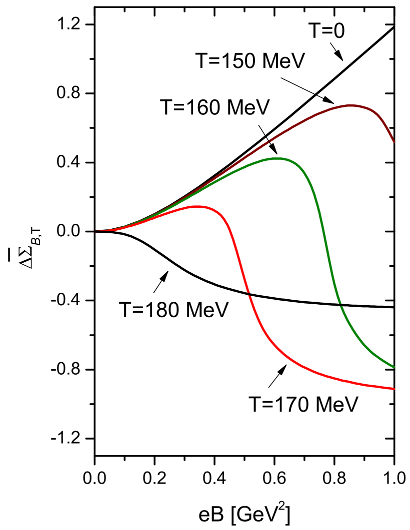

Figure 2. Subtracted normalized flavor average condensate as a function of eB for different representative temperatures. All results correspond to $\Phi_0 = 230$ MeV and polynomial PL potential, also in Fig. 3.

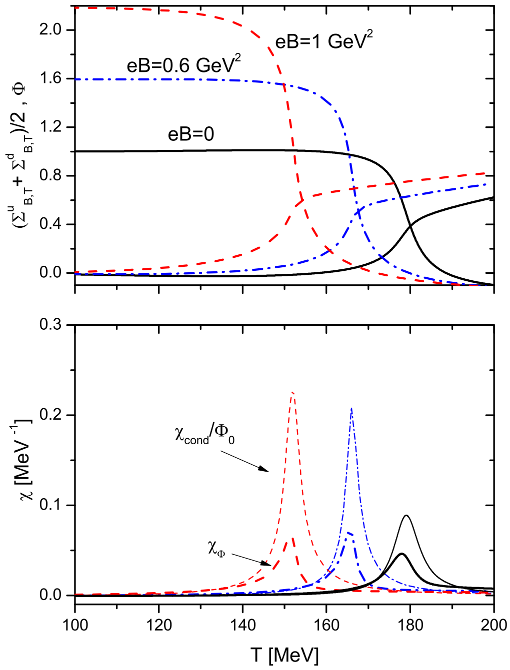

Figure 3. Normalized flavor average condensate and PL (up) and corresponding susceptibilities (down) as functions of T for representative values of eB.

To be more specific, let us define the critical temperature as the value of T at which χ_{cond} reaches a maximum. In Fig. 4, the relative quantity $T_c(B)/T_c(0)$ is displayed as a function of eB together with LQCD results from Ref. [6]. It is seen that all parameter sets considered here lead to a decrease of the critical temperature when eB gets increased, i.e. in all cases the IMC effect is observed. On the other hand, the strength of the IMC effect is rather sensitive to the parametrization. In the case whitout coupling to the PL the decrease of the T_c is small compared with LQCD estimates, however, this is cured once the PL is included.

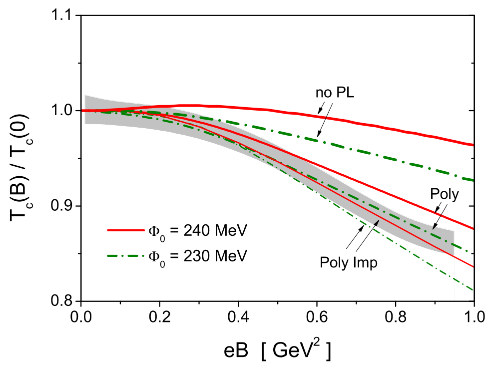

Figure 4. Normalized chiral restoration temperatures as functions of eB for various model parametrizations for the two PL potentials considered. For comparison results obtained excluding the coupling to the PL are also displayed. LQCD results of Ref. [6] are indicated by the grey band.

Acknowledgments

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References

[1] Duncan R C and Thompson C 1992 *Astrophys. J.* **392** L9
Kouveliotou C *et al* 1998 *Nature* **393** 235
[2] Kharzeev D E, McLerran L D and Warringa H J 2008 *Nucl. Phys.* A **803** 227
Skokov V, Illarionov A Y, and Toneev V 2009 *Int. J. Mod. Phys.* A **24** 5925
Voronyuk V, Toneev V, Cassing W, Bratkovskaya E, Konchakovski V, and Voloshin S 2011 *Phys. Rev.* C **83** 054911
[3] Vachaspati T 1991 *Phys. Lett.* B **265** 258
Enqvist K and Olesen P 1993 *Phys. Lett.* B **319** 178
[4] Kharzeev D E, Landsteiner K, Schmitt A and Yee H U 2013 *Lect. Notes Phys.* **871** 1
Andersen J O, Naylor W R and Tranberg A 2016 *Rev. Mod. Phys.* **88** 025001
Miransky V A and Shovkovy I A 2015 *Phys. Rept.* **576** 1
[5] Bali G S, Bruckmann F, Endrodi G, Fodor Z, Katz S D, Krieg S, Schafer A and Szabo K K 2012 *JHEP* **1202** 044
[6] Bali G S, Bruckmann F, Endrodi G, Fodor Z, Katz S D and Schafer A 2012 *Phys. Rev.* D **86** 071502
[7] Fraga E S, Noronha J and Palhares L F 2013 *Phys. Rev.* D **87** 114014
[8] Bruckmann F, Endrodi G and Kovacs T G 2013 *JHEP* **1304** 112
Bali G S, Bruckmann F, Endrodi G, Gruber F and Schaefer A 2013 *JHEP* **1304** 130
[9] Fukushima K and Hidaka Y 2013 *Phys. Rev. Lett.* **110** 031601
[10] Chao J, Chu P and Huang M 2013 *Phys. Rev.* D **88** 054009
[11] Fraga E S, Mintz B W and Schaffner-Bielich J 2014 *Phys. Lett.* B **731** 154
[12] Ferreira M, Costa P , Menezes D P, Providência C and Scoccola N 2014 *Phys. Rev.* D **89** 016002
[13] Farias R L S, Gomes K P, Krein G I and Pinto M B 2014 *Phys. Rev.* C **90** 025203
[14] Ayala A, Loewe M and Zamora R 2015 *Phys. Rev.* D **91** 016002
Ayala A, Cobos-Martnez J J, Loewe M, Tejeda-Yeomans M E and Zamora R 2015 *Phys. Rev.* D **91** 016007
[15] Fayazbakhsh S and Sadooghi N 2014 *Phys. Rev.* D **90** 105030
[16] Andersen J O, Naylor W R and Tranberg A 2015 *JHEP* **1502** 042
[17] Mueller N and Pawlowski J M 2105 *Phys. Rev.* D **91** 116010
[18] Ferrer E J, de la Incera V and Wen X J 2015 *Phys. Rev.* D **91** 054006
[19] Braun J, Mian W A and Rechenberger S 2016 *Phys. Lett.* B **755** 265
[20] Rougemont R, Critelli R and Noronha J 2016 *Phys. Rev.* D **93** 045013
[21] Ayala A, Dominguez C A, Hernandez L A, Loewe M and Zamora R 2016 *Phys. Lett.* B **759** 99
[22] Mao S 2016 *Phys. Lett.* B **758** 195
[23] Noguera S and Scoccola N N 2008 *Phys. Rev.* D **78** 114002
[24] General I, Gomez Dumm D and Scoccola N N 2001 *Phys. Lett.* B **506** 267
Gomez Dumm D and Scoccola N N 2002 *Phys. Rev.* D **65** 074021
[25] Scarpettini A, Gomez Dumm D and Scoccola N N 2004 *Phys. Rev.* D **69** 114018
[26] Gomez Dumm D and Scoccola N N 2005 *Phys. Rev.* C **72** 014909
[27] Gomez Dumm D, Grunfeld A G and Scoccola N N 2006 *Phys. Rev.* D **74** 054026
[28] Contrera G A, Gomez Dumm D and Scoccola N N 2008 *Phys. Lett.* B **661** 113
Contrera G A, Gomez Dumm D and Scoccola N N 2010 *Phys. Rev.* D **81** 054005
Contrera G A, Orsaria M and Scoccola N N 2010 *Phys. Rev.* D **82** 054026
Carlomagno J P, Gomez Dumm D and Scoccola N N 2013 *Phys. Rev.* D **88** 074034
[29] Gomez Dumm D, Noguera S and Scoccola N N 2011 *Phys. Lett* B **698** 236
Gomez Dumm D, Noguera S and Scoccola N N 2012 *Phys. Rev.* D **86** 074020
[30] Blaschke D, Buballa M, Radzhabov A E and Volkov M K 2008 Yad. Fiz. **71**, 2012 (2008) [Phys. Atom. Nucl. **71**, 1981 (2008)].
[31] Ritus V I 1978 *Sov. Phys. JETP* **48** 788
[32] Pagura V P, Gomez Dumm D, Noguera S and Scoccola N N 2017 *Phys. Rev.* D **95** no. 3 034013
[33] Gomez Dumm D, Izzo Villafañe M F, Noguera S, Pagura V P and Scoccola N N (in press) *Phys. Rev.* D (*Preprint* hep-ph/04742)
[34] Dumitru A, Pisarski R D and Zschiesche D 2005 *Phys. Rev.* D **72** 065008
[35] Ratti C, Thaler M A and Weise W 2006 *Phys. Rev.* D **73** 014019
[36] Haas L M, Stiele R, Braun J, Pawlowski J M and Schaffner-Bielich J 2013 *Phys. Rev.* D **87** 076004