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Diffraction efficiency in quasi-degenerate coupling in photorefractive materials

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Abstract

The aim of this work is to present theoretical analysis of the diffraction efficiency in sillenite BTO crystal, for mobile transmission gratings, where the effects of coupling beams are significant [1-5]. In case of the degenerate coupling in photorefractive materials, the efficiency depends of the crystal thickness, coupling coefficient and the input intensity ratio. In the non-degenerate coupling case, the diffraction efficiency also depends on the response time of the photorefractive medium [6]. This response time is the function of concentration ratio [7], i.e., of the relation between acceptor numbers and donor numbers. The diffraction efficiency is obtained by using coupled wave equations in on-Bragg regime using Method Runge-Kutta of order 4 in Matlab. The analysis considers the non-uniformity of the gratings inside the crystal, the concentration ratio of the material and detuning frequency.

Key words: Photorefractive crystal, diffraction efficiency, case non-degenerate, concentration ratio

Resumen

En este trabajo se presenta un análisis teórico de la eficiencia de difracción en cristales silenitas tipo BTO, para redes móviles de transmisión, teniendo en cuenta los efectos de acoplamiento [1-5]. En el caso del acoplamiento degenerado en los materiales fotorrefractivos, la eficiencia de difracción depende del espesor del cristal, el coeficiente de acoplamiento y la razón de intensidad de los haces de registro. Por otra parte, en el caso cuasi-degenerado la eficiencia además de depender de los parámetros ya mencionados también depende del tiempo de respuesta del medio fotorrefractivos [6]. Este tiempo de respuesta es función de la razón de concentración del material[7], es decir de la relación entre el número de aceptores y donores. La eficiencia de difracción surge de la solución de las ecuaciones acopladas en el régimen on-Bragg empleando el método de Runge-Kutta, implementado en Matlab. En el análisis se considera la no uniformidad de las redes dentro del cristal, la razón de concentración del material y la diferencia de frecuencia de registro.

Palabras clave: Cristales fotorrefractivos, eficiencia de difracción, caso cuasi-degenerado, razón de concentración

1. Introduction

Photorefractive materials unlike other nonlinear medium, show a notorious index of refraction variation when it is illuminated for a beam to non-uniform light. The photorefractive sillenite crystals (BTO, BSO and BGO) present a high sensitivity for holographic volume gratings formation, rapid response, long holographic storage times under dark conditions and essentially unlimited recyclability. These crystals are potentially useful for dynamic real-time interferometry applications, optical signal processing, amplification arrangements, optical interconnections, spatial optical switching and self-pumped laser resonators [8-13].

The holographic information storage and retrieval has been realized in several photorefractive materials [14,15]. As the photorefractive crystal is a volume medium, it is used for greatly increased information storage capacity through volume holography. The information storage capacity can be greatly increased by volume holography in photorefractive crystals. In fact, the photorefractive crystal is a volume medium. Kogelnik first developed the coupled-wave theory for volume holograms, and also predicted the diffraction efficiency and Bragg selectivity for thick gratings [16]. The diffraction efficiency is an important parameter to be analyzed in order to characterize the performance of the stored holographic gratings. The diffraction efficiency parameter is defined as the ratio between in the intensity of the diffracted beam and the intensity of the reading beam. In this paper, our specific interest is the diffraction efficiency analysis at Bragg incidence and in the non-degenerate case. Only holographic configuration in which the applied electric field is parallel to the grating vector is considered. Parameters that are controlled for optimizing the diffraction efficiency performance include the crystal absorption, the input beams frequency detuning and the donors density.

2. Theorical Description

Many of the most interesting and practical applications of photorefractive effect arise due to optical wave mixing. In two-wave mixing, a pair of coherent laser beams intersects inside the volume of the photorefractive crystal. If two beams are of same frequency, a stationary interference pattern is formed. In our case, we consider a moving grating by means of two wave mixing in a photorefractive crystal. The moving interference pattern could be generated by a frequency detuning one of the two input beams. In this case, the interference fringe pattern is no longer stationary. The volume index grating can be induced provided the fringe pattern does not move too fast. The classic formalism for analyzing light diffraction in volume holographic gratings is the coupled-wave theory.

As mentioned above, the coupled-wave equations are used to analyze theoretical the diffraction in the mentioned conditions.

2.a. Set Coupled Wave Equations

The electric fields of the two beam can be written as:

$$\vec{E} = A_1 e^{i\left(\omega_1 t - \overrightarrow{k_1}.\vec{r}\right)} + A_2 e^{i\left(\omega_2 t - \overrightarrow{k_2}.\vec{r}\right)} \tag{1}$$

where A_1 and A_2 are the amplitudes of the input beams, $\overrightarrow{k_1}$ and $\overrightarrow{k_2}$ are the wave vector respectively.

The frequency detuning and the grating wave vector are defined as:

$$\Omega = \omega_2 - \omega_1 \tag{2}$$

$$\vec{K} = \vec{k_2} - \vec{k_1} \quad (3)$$

The refractive index change induced by the moving intensity grating can then be written as:

$$n = n_0 + \left[\frac{n_1}{2} e^{i\phi} \frac{A_1{}^* A_2}{I_0} e^{i(\Omega t - \vec{K} - \vec{r})} + c.c\right] \quad (4)$$

where c.c. represents complex conjugation,n_0 is the refractive index in the absence of light, ϕ is photorefractive grating phase shift and it is given by:

$$\phi = \phi_0 + \tan^{-1}(\Omega\tau) \quad (5)$$

where ϕ_0 is a constant phase shifting the steady state situation and the second term is the phase shift induced by the moving grating method. The parameter n_1 is given by:

$$n_1 = \frac{n_0{}^3}{2} r_{41} \left(\frac{E_d E_q}{E_q + E_d}\right) \left(\frac{1}{1+\Omega^2\tau^2}\right)^{1/2} \left(\frac{I_1}{I_0}\right) \quad (6)$$

where $I_1 = 2A_2 \cdot A_1^*$, $I_0 = |A_2|^2 + |A_1|^2$, r_{41} is electrooptic coefficient, t_0 is a characteristic time constant $t_0 = \frac{1}{rSI_0}$ (S is the cross-section of photo-excitation and r is the concentration ratio $r = \frac{N_D}{N_A}$ where N_D is the donors density and N_A is the aceptors density) and τ is the response time, defined as:

$$\tau = t_0 \left(\frac{E_d + E_\mu}{E_d + E_q}\right) \quad (7)$$

The parameters E_μ is the characteristic field, E_d is the diffusion field and $E_q$is the saturation field defined as

$$E_\mu = \frac{\gamma_R N_A}{\mu_e K} \quad (8)$$

$$E_d = \frac{K k_B T}{q} \quad (9)$$

$$E_q = \frac{N_A}{\epsilon_e K} \quad (10)$$

where k_B is the Boltzmann constant, T is the temperatura, q is the electronic charge,ϵ_e is the effective dielectric constant, γ_R is therecombinationconstant and μ_e is the effective mobility.

The field electric is replaced into the wave equation:

$$\left(\nabla^2 + \frac{\omega^2}{c^2} n^2\right) \vec{E} = 0 \quad (11)$$

and by using the slowly varying envelope approximation, the couple wave equations for the case non-degenerate is obtained. The resulting set of coupled wave equations results:

$$\frac{dA_1}{dz} = \frac{(\Omega\tau + i)}{1+(\Omega\tau)^2} \frac{\Gamma}{2I_0} A_1 A_2 A_2^* - \frac{\propto}{2} A_1 \quad (12)$$

$$\frac{dA_2}{dz} = \frac{(\Omega\tau - i)}{1+(\Omega\tau)^2} \frac{\Gamma}{2I_0} A_2 A_1 A_1^* - \frac{\propto}{2} A_2 \quad (13)$$

where $\propto$ as the bulk absorption coefficient and the parameter Γ is,

$$\Gamma = i \frac{\pi n_0{}^3 r_{41}}{\lambda \cos\theta} \left(\frac{E_d E_q}{E_q + E_d}\right) \left(\frac{I_1}{I_0}\right) = i\gamma_0 \quad (14)$$

where γ_0 is the coupling constant for the case degenerate and is defined:

$$\gamma_0 = \frac{\pi n_0{}^3 r_{41}}{\lambda\cos\theta} \left(\frac{E_d E_q}{E_q + E_d}\right) \left(\frac{I_1}{I_0}\right) \quad (15)$$

where λ is the wavelength and θ is the write-in and read-out angle. The diffraction efficiency is defined as:

$$\eta = \frac{I_2(L)}{I_1(0)} \quad (16)$$

3. Results

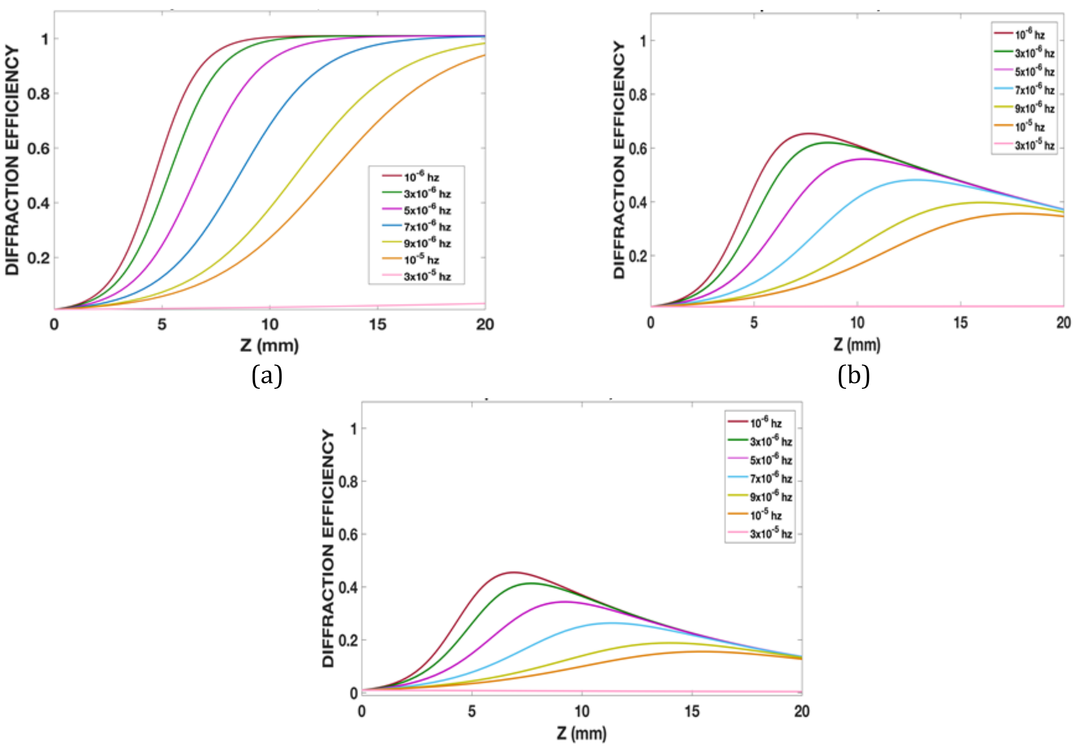
In order to analyze the diffraction efficiency, in this section we shall plot curves obtained by using equations detailed in the previous section and by using the Runge-Kutta method and its algorithm in Matlab. The experimental parameters of the BTO used in simulations are shown in Table 1.

TABLE 1.Parameter for BTO taken for our calculations

Parameter	BTO
ϵ Dielectric constant [1]	47
n_0 Refraction index[1]	2.58
r_{41} Electro-optic coefficient mV^{-1}[1]	$5.1x10^{-12}$
N_A Aceptor Density m^{-3}[4]	$2x10^{22}$
μ_e Mobility lifetime product m^2V^{-1}[1]	$6x10^{-7}$
γ_R Recombination constant m^3J^{-1}[1]	$1.6x10^{-17}$
S Photo ionization cross-section m^2J^{-1}[3]	$1x10^{-5}$
N_D Donor Density m^{-3}[3]	$0.2x10^{25} - 1.6x10^{25}$

The results of Figure 1 show the dependence of the diffraction efficiency in term of the BTO crystal thickness for different detuning frequency (Ω) and different crystal absorption values ((a) $\propto = 0cm^{-1}$ (b) $\propto = 0.5cm^{-1}$(c) $\propto = 1cm^{-1}$). The detuning frequency in our analysis is in the range 10^{-6} Hz - 10^{-5} Hz. We consider in our analysis a BTO crystal in an holographic configuration in which the applied electric field is parallel to the grating vector K_G||<001>. A grating with a frequency of 540 lines/mm is stored in the photorefractive crystal and the coupling constant parameter is γ_0 = 10 cm-1. An He-Ne laser with a wavelength of 632 nm is used in read-out step.

The results of Figure1 show that the maximum diffraction efficiency corresponds to a null absorption parameter. If the absorption is not null the diffraction efficiency presents a maximum for a determined crystal thickness. In our case, the maximum diffraction efficient correspond to the lower detuning frequency Ω=10^{-6} Hz.



(c)

Fig 1. Diffraction efficiency in terms of the crystal thicknesses for a BTO crystal and for different crystal absorption a) $\propto = 0\text{cm}^{-1}$, b) $\propto = 0.5\text{cm}^{-1}$ c) $\propto = 1\text{cm}^{-1}$ and different detuning frequency values (Ω).

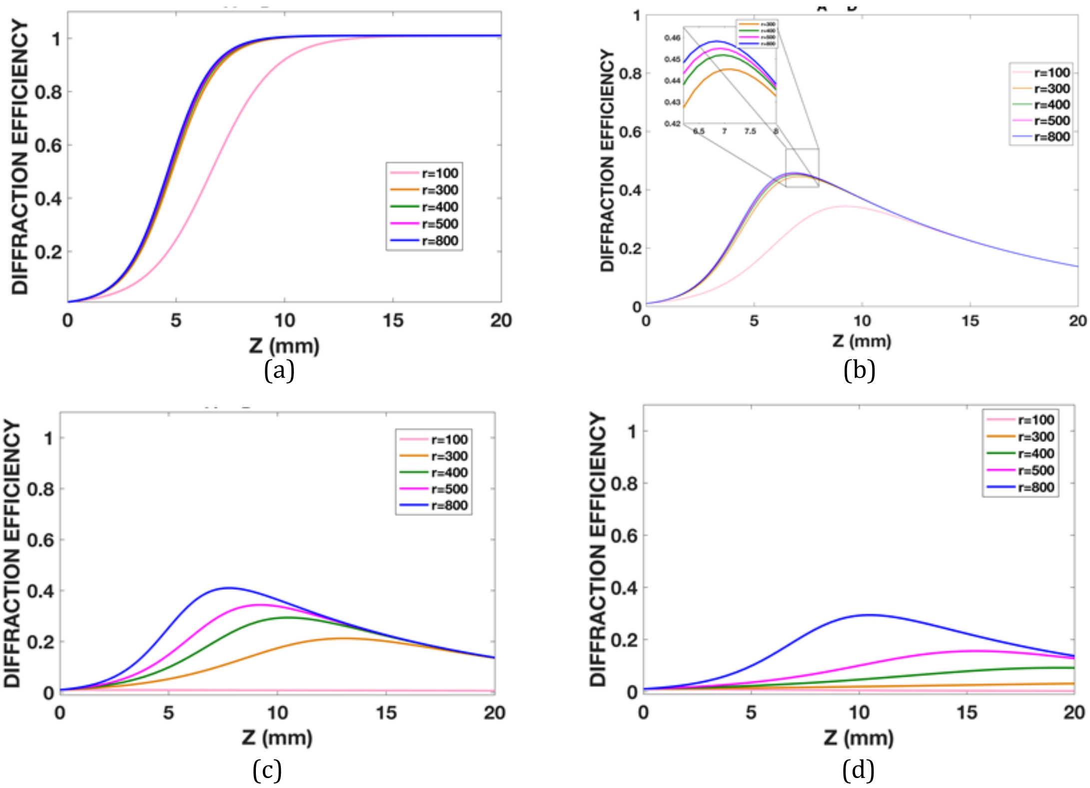


Fig 2. Diffraction efficiency in terms of the crystal thicknesses for a BTO crystal and for different concentration ratio values and crystal absorption (a) $\propto = 0\ \text{cm}^{-1}$, $\Omega = 10^{-6}\text{Hz}$ (b) $\propto = 1\ \text{cm}^{-1}$, $\Omega = 10^{-6}\text{Hz}$ (c) $\propto = 1\ \text{cm}^{-1}$, $\Omega = 5\text{x}10^{-6}\text{Hz}$ and (d) $\propto = 1\ \text{cm}^{-1}$, $\Omega = 10^{-6}\text{Hz}$

The diffraction efficiency with respect to BTO crystal thickness, for different concentration r, is plotted in Figure 2. Note that the diffraction efficiency decreases for lower values of the parameter r and the detuning frequency. By observing Figure 2 a) it is evident that the behavior of the diffraction efficiency curves for r values between 300 and 800 is very similar. This result corresponds to a BTO crystal with absorption α = 0 cm^{-1} and a detuning frequency Ω=10^{-6} Hz. The same similar behavior is observed in Figure 2 b) for r values between 300 and 800. This result corresponds to a BTO crystal with absorption α=1 cm^{-1} and a detuning frequency Ω=10^{-6} Hz.

4. Conclusions

The mechanism for the recording of a hologram by a moving interference pattern in a photorefractive crystal has been studied theoretically. The diffraction efficiency is investigated in sillenite $Bi_{12}TiO_{20}$crystals (BTO). The mobile transmission gratings case is analyzed. In our case, the coupling beams effects are significant. The Runge-Kutta method and its algorithm in Matlab is used to obtain the theoretical curves. It should be pointed out that the diffraction efficiency in a BTO crystal not only depends on the coupling coefficient, crystal thicknesses and absorption coefficient, it depends on the detuning frequency of the input beam in write in step and the donor density. In particular, the maximum diffraction efficiency for 540 lines/mm input grating is obtained for $r \geq 500$, $\Omega = 10^{-6}$hz, $\gamma_0 = 10$ cm^{-1}, $\propto = 1\ \mathrm{cm}^{-1}$. The diffraction efficiency by two-wave mixing in photorefractive BTO under the condition analyzed in our manuscript is very sensitive to the absorption coefficient. Further studies are in preparation and will be reported later.

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