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Mass dependence of the deconfinement and chiral restoration critical temperatures in nonlocal SU(2) Polyakov-Nambu-Jona-Lasinio models

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In the framework of nonlocal SU(2) chiral quark models with Polyakov loop, we analyze the dependence of the deconfinement and chiral restoration critical temperatures on the explicit chiral symmetry breaking driven by the current quark mass. Our results are compared with those obtained within the standard local Polyakov-Nambu-Jona-Lasinio model and with lattice QCD calculations. For a wide range of pion masses, it is found that both deconfinement and chiral restoration critical temperatures turn out to be strongly entangled, in contrast with the corresponding results within the Polyakov-Nambu-Jona-Lasinio model. In addition, it is seen that the growth of the critical temperatures with the pion mass above the physical point is basically linear, with a slope parameter that is close to the existing lattice QCD estimates. On the other hand, we find a tendency to favor an early onset of the first order transition expected in the large quark mass limit.

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I. INTRODUCTION

It is widely believed that as the temperature and/or density increase, strongly interacting matter undergoes some kind of transition from a hadronic phase, in which chiral symmetry is broken and quarks are confined, to a partonic phase, in which chiral symmetry is restored and/or quarks are deconfined. The detailed understanding of this phenomenon is relevant not only in particle physics but also e.g., in the study of the early universe, the interior of neutron stars; therefore it has become an issue of great interest in recent years, both theoretically and experimentally [1]. From the theoretical point of view, one way to address this problem is through lattice QCD calculations [2–4]. However, even if significant improvements have been made in this field in the past few years, this *ab initio* approach is not yet able to provide a full understanding of the QCD phase diagram. One serious difficulty in this sense is given by the so-called sign problem, which prevents straightforward simulations at finite baryon density. In this situation it is worthwhile to develop alternative approaches, such as the study of effective models that show consistency with lattice QCD results and can be extrapolated into regions not accessible by lattice techniques. Here we will concentrate on one particular class of effective theories, namely the so-called nonlocal Polyakov-Nambu-Jona-Lasinio (nlPNJL) models [5–8], in which quarks move in a background color field and interact through covariant nonlocal chirally symmetric four point couplings. Related Polyakov-Dyson-Schwinger equation models have also been recently analyzed [9]. These approaches, which can be considered as an improvement over the (local) PNJL model [10–16], offer a common framework to study both the chiral restoration and deconfinement transitions. In fact, the nonlocal character of the interactions arises naturally in the context of several successful approaches to low-energy quark dynamics [17,18] and leads to a momentum dependence in the quark propagator that can be made consistent [19] with lattice results [20,21]. Moreover, it has been found that, under certain conditions, it is possible to derive the main features of nlPNJL models starting directly from QCD [22]. From the phenomenological side, it has been shown [23–26] that nonlocal models provide a satisfactory description of hadron properties at zero temperature and density.

As mentioned above, it is important to consider situations in which the results obtained within effective models can be compared with available lattice QCD calculations. For example, it is clear that vacuum properties such as the pion mass and decay constant, as well as other features related to the chiral/deconfinement transitions (like e.g., the nature of the transitions, or the critical temperatures) will depend on basic parameters of QCD, such as the number of quark flavors and the values of current quark masses m_q. In particular, for the simplified case of two degenerate flavors with $m_u = m_d = m$, the dependence of several relevant quantities on m has been studied with some detail in lattice QCD. Thus, the corresponding analysis within nlPNJL models can provide an interesting test of the reliability of this effective approach. Actually, it has already been shown that several chiral effective models [27–29] are not able to reproduce the behavior of the critical temperatures observed in lattice QCD when one varies the parameters that explicitly break chiral symmetry (i.e., the current quark masses, or the pion mass in the case of meson models) at vanishing chemical potential.

This fact has been taken as an indication that the transition may be dominated not just by pure chiral dynamics [30]. It is worthwhile to notice that in the framework of the standard (local) NJL model the enhancement of the critical temperature with m is too strong in comparison with lattice QCD estimates. Although the inclusion of confinement effects through the coupling to the Polyakov loop weakens this enhancement, one finds a too large splitting between the chiral restoration and deconfinement transition temperatures [31]. The presence of confinement effects together with a strong entanglement between the chiral restoration and deconfinement transitions is indeed one of the features of nlPNJL models [32].

In view of the above mentioned points, the aim of the present work is to study the effect of explicit chiral symmetry breaking on the deconfinement and chiral restoration critical temperatures within nlPNJL models. This article is organized as follows. In Sec. II we provide a description of the model, proposing two alternative parametrizations. In Sec. III we analyze the m dependence of some pion properties and compare the results with existing lattice calculations. In Sec. IV we analyze the current quark mass dependence of the critical temperatures at vanishing chemical potential, comparing our results with those obtained in alternative models and lattice QCD. Finally in Sec. V we summarize our main results and conclusions.

II. FORMALISM

We consider a nonlocal SU(2) chiral quark model that includes quark couplings to the color gauge fields. The corresponding Euclidean effective action is given by [33]

$$S_E = \int d^4x \bigg\{ \bar{\psi}(x)(-i\gamma_\mu D_\mu + \hat{m})\psi(x) - \frac{G_S}{2}[j_a(x)j_a(x) - j_P(x)j_P(x)] + \mathcal{U}(\Phi[A(x)])\bigg\}, \quad (1)$$

where ψ is the $N_f = 2$ fermion doublet $\psi \equiv (u, d)^T$, and $\hat{m} = \mathrm{diag}(m_u, m_d)$ is the current quark mass matrix. In what follows we consider isospin symmetry, $m_u = m_d = m$. The fermion kinetic term in Eq. (1) includes a covariant derivative $D_\mu \equiv \partial_\mu - iA_\mu$, where A_μ are color gauge fields, and the operator $\gamma_\mu\partial_\mu$ in Euclidean space is defined as $\vec{\gamma}\cdot\vec{\nabla} + \gamma_4\partial/\partial\tau$, with $\gamma_4 = i\gamma_0$. The nonlocal currents $j_a(x)$, $j_P(x)$ are given by

$$j_a(x) = \int d^4z\, \mathcal{G}(z)\bar{\psi}\Big(x+\frac{z}{2}\Big)\Gamma_a\psi\Big(x-\frac{z}{2}\Big),$$
$$j_P(x) = \int d^4z\, \mathcal{F}(z)\bar{\psi}\Big(x+\frac{z}{2}\Big)\frac{i\overleftrightarrow{\not\partial}}{2\kappa_p}\psi\Big(x-\frac{z}{2}\Big), \quad (2)$$

where $\Gamma_a = (\mathbb{1}, i\gamma_5\vec{\tau})$ and $u(x')\overleftrightarrow{\partial}v(x) = u(x')\partial_x v(x) - \partial_{x'}u(x')v(x)$. The functions $\mathcal{G}(z)$ and $\mathcal{F}(z)$ in Eq. (2) are nonlocal covariant form factors characterizing the corresponding interactions. Notice that the four currents $j_a(x)$ require a common form factor $\mathcal{G}(z)$ to guarantee chiral invariance, while the coupling $j_P(x)j_P(x)$ is self-invariant under chiral transformations. The scalar-isoscalar component of the $j_a(x)$ current will generate a momentum dependent quark mass in the quark propagator, while the "momentum" current $j_P(x)$ will be responsible for a momentum dependent quark wave function renormalization. Now we perform a bosonization of the theory, introducing bosonic fields $\sigma_{1,2}(x)$ and $\pi_a(x)$, and integrating out the quark fields. Details of this procedure as well as of the determination of vacuum and meson properties at vanishing temperature in this framework can be found e.g., in Ref. [19].

Since we are interested in the deconfinement and chiral restoration critical temperatures, we extend the bosonized effective action to finite temperature T. This can be done by using the standard Matsubara formalism. Concerning the gauge fields A_μ, we assume that quarks move on a constant background field $\phi = A_4 = iA_0 = ig\delta_{\mu 0}G^\mu_a\lambda^a/2$, where G^μ_a are SU(3) color gauge fields. Then the traced Polyakov loop, which in the infinite quark mass limit can be taken as an order parameter of confinement, is given by $\Phi = \frac{1}{3}\,\mathrm{Tr}\exp(i\phi/T)$. We work in the so-called Polyakov gauge, in which the matrix ϕ is given a diagonal representation $\phi = \phi_3\lambda_3 + \phi_8\lambda_8$. This leaves only two independent variables, ϕ_3 and ϕ_8. In the case of vanishing chemical potential, owing to the charge conjugation properties of the QCD Lagrangian, the mean field traced Polyakov loop is expected to be a real quantity. Since ϕ_3 and ϕ_8 have to be real valued, this condition implies $\phi_8 = 0$. The mean field traced Polyakov loop reads then $\Phi = \Phi^* = [1 + 2\cos(\phi_3/T)]/3$. Thus in the mean field approximation, which will be used throughout this work, the thermodynamical potential Ω^{MFA} at finite temperature and zero chemical potential is given by

$$\Omega^{\mathrm{MFA}} = -4T\sum_{c=r,g,b}\sum_{n=-\infty}^{\infty}\int\frac{d^3\vec{p}}{(2\pi)^3}\log\bigg[\frac{(\rho^c_{n,\vec{p}})^2 + M^2(\rho^c_{n,\vec{p}})}{Z^2(\rho^c_{n,\vec{p}})}\bigg] + \frac{\bar{\sigma}_1^2 + \kappa_p^2\bar{\sigma}_2^2}{2G_S} + \mathcal{U}(\Phi,\Phi^*,T), \quad (3)$$

where $M(p)$ and $Z(p)$ are given by

$$M(p) = Z(p)[m_q + \bar{\sigma}_1 g(p)],$$
$$Z(p) = [1 - \bar{\sigma}_2 f(p)]^{-1}. \quad (4)$$

Here $\bar{\sigma}_{1,2}$ are the mean field values of the scalar fields (note that $\bar{\pi}_a = 0$), while $f(p)$ and $g(p)$ are Fourier transforms of $\mathcal{F}(z)$ and $\mathcal{G}(z)$, respectively. We have also defined

$$(\rho^c_{n,\vec{p}})^2 = [(2n+1)\pi T + \phi_c]^2 + \vec{p}^2, \quad (5)$$

where the quantities ϕ_c are given by the relation $\phi = \mathrm{diag}(\phi_r, \phi_g, \phi_b) = \mathrm{diag}(\phi_3, -\phi_3, 0)$.

To proceed we need to specify the explicit form of the Polyakov loop effective potential $\mathcal{U}(\Phi,\Phi^*,T)$.

We consider two alternative functional forms commonly used in the literature. The first one, based on a Ginzburg-Landau ansatz, reads [13]

$$\mathcal{U}_{\rm poly}(\Phi,\Phi^*,T) = T^4\Big[-\frac{b_2(T)}{4}(|\Phi|^2+|\Phi^*|^2) - \frac{b_3}{6}(\Phi^3+(\Phi^*)^3)+\frac{b_4}{16}(|\Phi|^2+|\Phi^*|^2)^2\Big], \tag{6}$$

where

$$b_2(T) = a_0 + a_1\Big(\frac{T_0}{T}\Big) + a_2\Big(\frac{T_0}{T}\Big)^2 + a_3\Big(\frac{T_0}{T}\Big)^3. \tag{7}$$

The potential parameters can be fitted to pure gauge lattice QCD data so as to properly reproduce the corresponding equation of state and Polyakov loop behavior. This yields [13]

$$a_0 = 6.75, \quad a_1 = -1.95, \quad a_2 = 2.625, \quad a_3 = -7.44, \quad b_3 = 0.75, \quad b_4 = 7.5. \tag{8}$$

A second usual form is based on the logarithmic expression of the Haar measure associated with the SU(3) color group integration. The potential reads in this case [14]

$$\mathcal{U}_{\rm log}(\Phi,\Phi^*,T) = \Big\{-\frac{1}{2}a(T)\Phi\Phi^* + b(T)\log[1-6\Phi\Phi^* + 4\Phi^3 + 4(\Phi^*)^3 - 3(\Phi\Phi^*)^2]\Big\}T^4, \tag{9}$$

where the coefficients are parametrized as

$$a(T) = a_0 + a_1\Big(\frac{T_0}{T}\Big) + a_2\Big(\frac{T_0}{T}\Big)^2, \qquad b(T) = b_3\Big(\frac{T_0}{T}\Big)^3. \tag{10}$$

Once again the values of the constants can be fitted to pure gauge lattice QCD results. This leads to [14]

$$a_0 = 3.51, \quad a_1 = -2.47, \quad a_2 = 15.2, \quad b_3 = -1.75. \tag{11}$$

The dimensionful parameter T_0 in Eqs. (7) and (10) corresponds in principle to the deconfinement transition temperature in the pure Yang-Mills theory, $T_0 = 270$ MeV. However, it has been argued that in the presence of light dynamical quarks this temperature scale should be adequately reduced [34,35]. Recent work on Polyakov loop potentials can be found in Refs. [36,37].

Finally, one has to take into account that $\Omega^{\rm MFA}$ turns out to be divergent; thus it has to be regularized. Here we use the prescription described e.g., in Ref. [38], namely

$$\Omega^{\rm MFA}_{\rm reg} = \Omega^{\rm MFA} - \Omega^{\rm free} + \Omega^{\rm free}_{\rm reg} + \Omega_0, \tag{12}$$

where $\Omega^{\rm free}$ is obtained from Eq. (3) by setting $\bar\sigma_1 = \bar\sigma_2 = 0$, and $\Omega^{\rm free}_{\rm reg}$ is the regularized expression for the quark thermodynamical potential in the absence of the four point fermion interaction,

$$\Omega^{\rm free}_{\rm reg} = -4T\int\frac{d^3\vec p}{(2\pi)^3}\sum_{c=r,g,b}\sum_{s=\pm1} \times {\rm Re}\,\ln\Big[1+\exp\Big(-\frac{\epsilon_p + is\phi_c}{T}\Big)\Big], \tag{13}$$

with $\epsilon_p = \sqrt{\vec p^2 + m^2}$. The last term in Eq. (12) is just a constant fixed by the condition that $\Omega^{\rm MFA}_{\rm reg}$ vanishes at $T=0$.

Given the full form of the thermodynamical potential, the mean field values $\bar\sigma_{1,2}$ and ϕ_3 can be obtained as solutions of the coupled set of "gap equations"

$$\frac{\partial\Omega^{\rm MFA}_{\rm reg}}{(\partial\sigma_1,\partial\sigma_2,\partial\phi_3)} = 0. \tag{14}$$

Once these mean field values are obtained, the behavior of other relevant quantities as functions of the temperature and chemical potential can be determined. We concentrate in particular on the chiral quark condensate $\langle\bar q q\rangle = \partial\Omega^{\rm MFA}_{\rm reg}/\partial m$ and the traced Polyakov loop Φ, which will be taken as order parameters of the chiral restoration and deconfinement transitions, respectively. The associated susceptibilities will be defined as $\chi_{\rm ch} = \partial\langle\bar q q\rangle/\partial m$ and $\chi_{\rm PL} = d\Phi/dT$.

To fully specify the model under consideration we proceed to fix the model parameters as well as the nonlocal form factors $g(q)$ and $f(q)$ at the physical point $m_\pi = m_\pi^{\rm phys} = 139$ MeV. We consider two different functional dependences for the form factors. The first one corresponds to the often used exponential functions

$$g(q) = \exp(-q^2/\Lambda_0^2), \qquad f(q) = \exp(-q^2/\Lambda_1^2), \tag{15}$$

which guarantee a fast ultraviolet convergence of the loop integrals. Note that the range (in momentum space) of the nonlocality in each channel is determined by the parameters Λ_0 and Λ_1, respectively. Fixing the current quark mass and chiral quark condensate at $T=\mu=0$ to the phenomenologically adequate values $m = 5.7$ MeV and $\langle\bar q q\rangle^{1/3} = 240$ MeV, the rest of the parameters can be determined so as to reproduce the physical values of f_π and m_π, and by requiring $Z(0) = 0.7$, which is within the range of values suggested by recent lattice calculations [20,21]. In what follows this choice of model parameters and form factors will be referred to as S1. The second type of form factor functional forms considered here is given by

$$g(q) = \frac{1+\alpha_z}{1+\alpha_z f_z(q)}\,\frac{\alpha_m f_m(q) - m\alpha_z f_z(q)}{\alpha_m - m\alpha_z}, \qquad f(q) = \frac{1+\alpha_z}{1+\alpha_z f_z(q)} f_z(q), \tag{16}$$

where

$$f_m(q) = [1 + (q^2/\Lambda_0^2)^{3/2}]^{-1},$$
$$f_z(q) = [1 + (q^2/\Lambda_1^2)]^{-5/2}. \quad (17)$$

As shown in Ref. [19], taking $m = 2.37$ MeV, $\alpha_m = 309$ MeV, $\alpha_z = -0.3$, $\Lambda_0 = 850$ MeV and $\Lambda_1 = 1400$ MeV one can very well reproduce the momentum dependence of mass and wave function renormalization obtained in lattice calculations, as well as the physical values of m_π and f_π. In what follows this choice of model parameters and form factors will be referred to as S2. Details on the model parameters and the predictions for several meson properties in vacuum can be found in Ref. [19]. In principle, both the model parameters and the functional form of the form factors could get modified at nonzero temperatures. Here these effects will be neglected, as it is usually done in the framework of both nonlocal models [5–8,39,40] and Dyson-Schwinger calculations [9].

III. ZERO TEMPERATURE PSEUDOSCALAR MASS AND DECAY CONSTANT AWAY FROM THE PHYSICAL POINT

As stated, we want to study the dependence of nlPNJL model predictions on the amount of explicit chiral symmetry breaking. This can be addressed by varying the current quark mass m, while keeping the rest of the model parameters fixed at their values at the physical point. As a first step we analyze in this section the corresponding behavior of the pion mass and decay constant at vanishing temperature, in comparison with that obtained in the (local) NJL model and in lattice QCD. Our results are shown in Fig. 1. As it is usual in lattice QCD literature, we choose to take m_π instead of m as the independent variable in the plots. The main reason for this is that m_π is an observable, i.e., a scale independent quantity, whereas m is scale dependent; hence its value is subject to possible ambiguities related to the choice of the renormalization point. Dashed and solid lines correspond to parameter sets S1 and S2, respectively, while dotted lines correspond to the curves obtained within the NJL model using the parameter set in Refs. [14,31]. Solid dots stand for lattice QCD results from Ref. [41]. The upper panel shows the behavior of the ratio m_π^2/m as a function of m_π. To account for the above mentioned renormalization point ambiguities, the corresponding quark masses have been normalized so as to yield the lattice value $m_{u,d}^{\overline{\rm MS}} \simeq 4.452$ MeV at the physical point [41]. From the figure one observes that both NJL and nlPNJL models reproduce qualitatively the results from lattice QCD, showing a particularly good agreement in the case of the nlPNJL model for parameter set S2. However, the situation is different in the case of f_π (lower panel in Fig. 1): while the predictions from nonlocal models follow a steady increase with m_π, in agreement with lattice results, the local NJL model in general fails to reproduce this behavior.

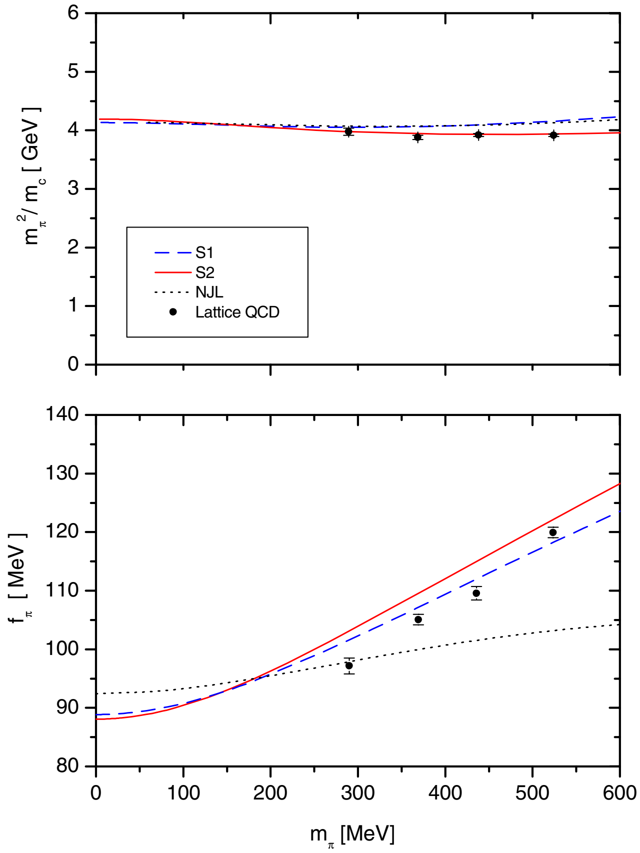

FIG. 1 (color online). Pion properties at $T = 0$ as functions of the pion mass in local and nonlocal chiral quark models. Upper and lower panels correspond to the ratio m_π^2/m_c and the pion decay constant f_π, respectively. Lattice results are taken from Ref. [41], and the NJL model parametrization is that in Refs. [14,31].

Moreover, it can be seen that the discrepancy cannot be cured even if one allows the coupling G_S to depend on the current quark mass [31]. Here we have taken G_S as a constant in both local and nonlocal PNJL models.

We have further analyzed this discrepancy by considering different NJL model parametrizations and both 3D and 4D momentum cutoff regularizations. In all cases the parameters have been determined so as to obtain at the physical point the empirical values of the pion mass and decay constant, as well as an effective quark mass M within the phenomenologically adequate range of 300–600 MeV (see e.g., Ref. [42]). We have checked that (i) for a given value of M, the behavior of f_π as a function of m_π is similar for both 3D and 4D cutoff regularizations, and (ii) for lower values of M (and thus higher cutoffs [42]) the discrepancy between lattice and NJL results gets reduced. However, it is already found that for $M = 300$ MeV the departure from lattice results is basically as large as that shown in Fig. 1 (the corresponding NJL parametrization leads to $M = 325$ MeV [14,31]). We find that M has to be lowered up to values of about 250 MeV—i.e., too small from the phenomenological point of view—to get an acceptable agreement with lattice data. In this way, our results can be considered as a further indication in favor of the inclusion of nonlocal interactions as a step toward a more realistic description of low momenta QCD dynamics. Our agreement with lattice results also indicates that the model parameters should not change significantly with m. Concerning the explicit dependence appearing in Eq. (16) (corresponding to S2), although in our calculations we have considered for consistency that m varies as the current quark mass, the effect of this change is found to be completely negligible.

IV. DEPENDENCE OF CRITICAL TEMPERATURES ON EXPLICIT CHIRAL SYMMETRY BREAKING

In this section we analyze within our nonlocal models the mass dependence of the critical temperatures for the deconfinement and chiral restoration transitions at vanishing chemical potential. We start by considering the temperature dependence of the chiral and deconfinement order parameters, as well as the corresponding susceptibilities, for some representative values of the pion mass. The corresponding results for the lattice motivated parametrization S2 are shown in Fig. 2, including both the case of the polynomic (left panels) and logarithmic (right panels) Polyakov potentials. Qualitatively similar results are found for the exponential parametrization S1. Let us take the case of the polynomic potential. From the figure it is seen that both transitions proceed as smooth crossovers, as expected from lattice QCD results. Moreover, we observe that as m_π increases, the position of the peaks of the susceptibilities $\chi_{\rm ch}$ and $\chi_{\rm PL}$ (left lower panel) move simultaneously toward higher values of T, the difference between the corresponding critical temperatures being in all cases at the level of a few MeV. It is also seen that as m_π increases the chiral restoration transition tends to be less pronounced, while the

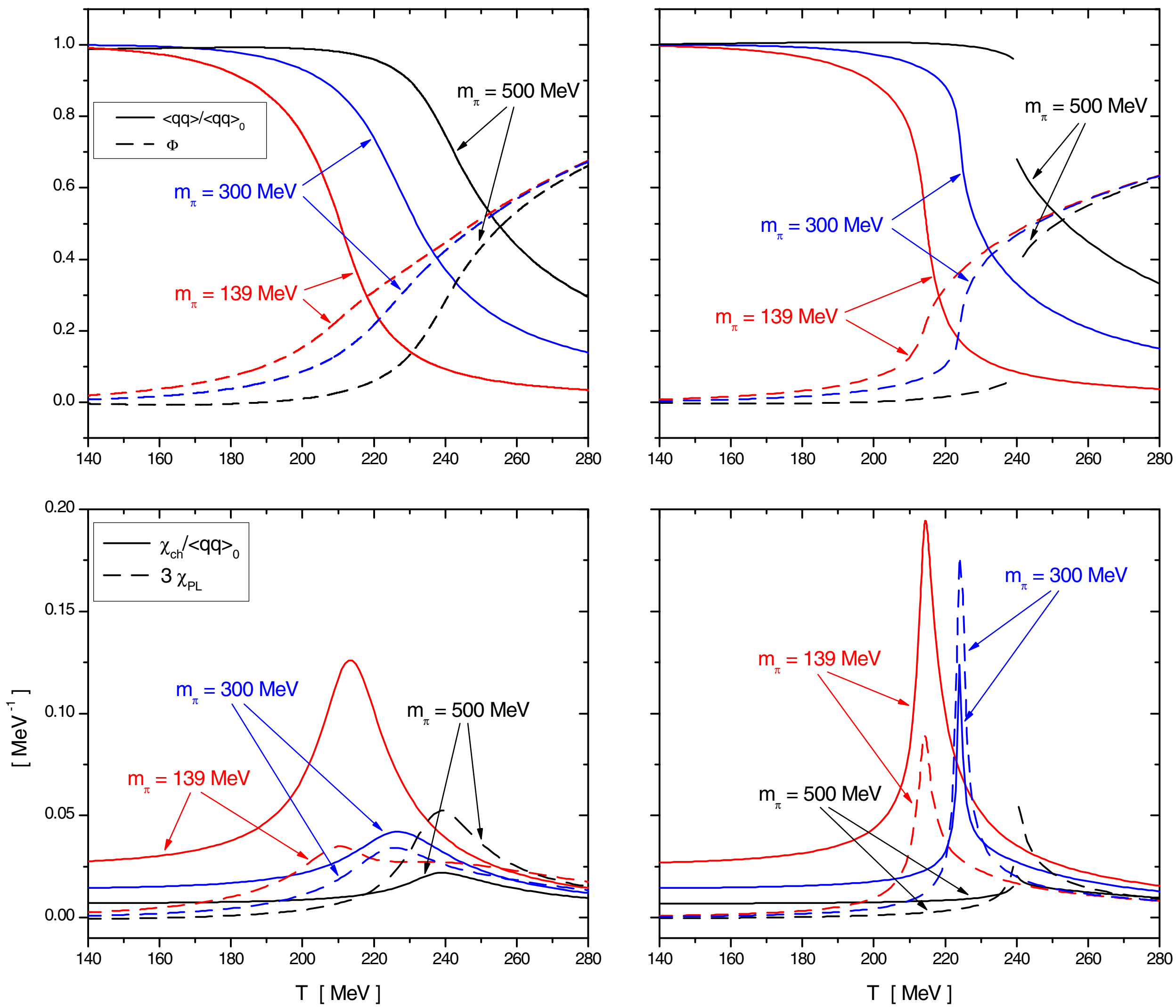

FIG. 2 (color online). Order parameters (upper panels) and the corresponding susceptibilities (lower panels) as functions of the temperature for some representative values of the pion mass. Left (right) panels correspond to the polynomic (logarithmic) Polyakov potential.

confinement one becomes steeper. In the case of the logarithmic potential, we also observe that the transition temperatures increase with m_π, as expected. However, for a given value of m_π both the chiral restoration and deconfinement transitions are steeper than in the case of the polynomic potential, and the correlation between them is stronger (e.g., the difference between the transition temperatures for $m_\pi = m_\pi^{\rm phys}$ is now about 0.02 MeV). In fact, it turns out that already for a critical mass $m_\pi^{\rm crit} = 500$ MeV one finds a first order phase transition. In the case of the polynomic potential, the onset of this kind of phase transition occurs at a larger critical mass, $m_\pi^{\rm crit} \gtrsim 700$ MeV. According to lattice QCD results, the onset of a first order phase transition is indeed expected above a certain critical amount of explicit symmetry breaking [43]. However, present estimations [44–46] indicate that the corresponding critical pseudoscalar mass should be in the range of a few GeV (e.g., Ref. [45] quotes $m_\pi^{\rm crit} \sim 40T_c$); therefore the early change in the character of the transition appears as an unrealistic feature of the effective models. In this sense, we point out that our predictions are reliable within a limited range of values of m_π, with an upper bound that we estimate to be of about 600 MeV. Thus, for different forms of the effective Polyakov potential the onset of the first order phase transition can occur or not within the range of validity of the model. As stated, the ansatz for the potential can be taken from theoretical inputs and lattice QCD calculations in the limit of a pure glue theory. A discussion on the related ambiguities can be found e.g., in Ref. [47]. Here we stress that, given these ambiguities, the model is compatible with values of the critical mass beyond the region where the predictions are expected to be reliable. In addition, one can consider the effect of corrections that go beyond the mean field approximation. Although the role of these corrections is expected to be less important as the quark mass increases [5], in the mass range considered here they can be significant enough to soften the transitions and lower the critical temperatures [9]. Even if we do not expect this effect to enhance the values of $m_\pi^{\rm crit}$ up to the GeV range, it should contribute in the right direction to push the critical mass up to values above the upper bound of validity of our approach. Some important steps have been taken to study beyond-mean-field corrections [5,7,39], but a fully nonperturbative scheme to account for meson fluctuations in nonlocal models is still lacking. As it is pointed out in Ref. [40], these fluctuations could also help to avoid thermodynamical instabilities that could arise in these models.

In Fig. 3 we show the results for the mass dependence of the critical transition temperatures within our nonlocal models. For comparison we also quote typical curves obtained in the framework of the local PNJL model (here we have considered the parametrization in Ref. [14]). Upper and lower panels correspond to polynomic and logarithmic Polyakov potentials, respectively, with $T_0 = 270$ MeV.

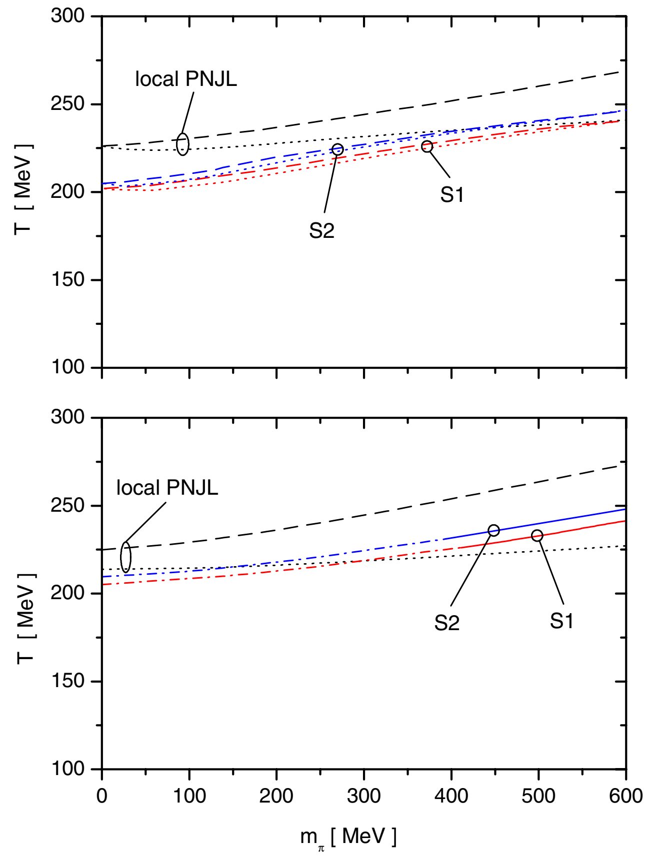

FIG. 3 (color online). Critical temperatures as functions of the pion mass for PNJL and nlPNJL models, considering polynomic (upper panel) and logarithmic (lower panel) Polyakov loop potentials. Dashed and dotted lines correspond to chiral restoration and deconfinement transition temperatures, respectively. For the nlPNJL models with a logarithmic potential (lower panel), both transitions occur at the same temperature, and they can be of first order (solid lines) or proceed as a smooth crossover (dash-dotted lines).

Before discussing in detail the results obtained for the nlPNJL models, let us comment on those corresponding to the PNJL model: from Fig. 3 we observe that already at the physical value $m_\pi = m_\pi^{\rm phys}$ the model predicts a noticeable splitting between the chiral restoration temperature $T_{\rm ch}$ (dashed line) and the deconfinement temperature $T_{\rm PL}$ (dotted line). In addition, it is seen that the growth of $T_{\rm ch}$ with m_π is stronger than that of $T_{\rm PL}$, which implies that the splitting between both critical temperatures becomes larger if m_π is increased. This is not supported by existing lattice results [48,49], which indicate that both transitions take place at approximately the same temperature, up to values of m_π even larger than those considered here. Comparing both panels it is seen that the splitting is more pronounced for the PNJL model that includes a logarithmic Polyakov potential.

We turn now to the curves obtained within nonlocal models. First of all, from the figure it is seen that both parametrizations S1 and S2 lead to qualitatively similar results. Contrary to the situation in the PNJL model, in nlPNJL models both the chiral restoration and deconfinement transitions occur at basically the same temperature for all considered values of m_π. Moreover, comparing the results for the two alternative Polyakov loop potentials we see that the main qualitative difference between them is the already mentioned fact that in the case of the logarithmic potential there is a critical pion mass of about 400 MeV where the character of the transition changes from crossover to first order (dash-dotted and solid lines in the lower panel of Fig. 3, respectively). By analyzing in more detail the pion mass dependence of the critical temperatures, it is seen that for m_π above the physical mass the nlPNJL model results can be accurately adjusted through a linear function

$$T_c(m_\pi) = Am_\pi + B. \tag{18}$$

This is in agreement with the findings of the lattice calculations of Refs. [48,49]. Our results for the slope parameter A for both parametrizations and Polyakov loop potentials are in the range of 0.06–0.07. For comparison, most lattice calculations find $A \lesssim 0.05$ [48,50–52], while according to some recent analyses [49,53] the value could be somewhat above this bound. Thus the slope parameter predicted by the nonlocal PNJL models appears to be compatible with lattice estimates. This can be contrasted with the results obtained within pure chiral models, where one finds a strong increase of the chiral restoration temperature with m_π [27–29]. For example, within the chiral quark model of Ref. [29] one gets a value $A = 0.243$.

It is also worthwhile to discuss the effect of considering a value of T_0 that depends on the presence of quark matter, as suggested in Refs. [34,35] (see also Ref. [54]). The lowering of T_0 leads to an overall decrease of the transition temperatures, which keep the rising linear dependence on m_π but with a slope parameter that gets reduced by about 15–20%. The main noticeable difference is that in all cases the transition becomes steeper, which leads to an earlier onset of the first order transition. For example, for the parameter set S2 we find that the transition becomes of first order already at $m_\pi \simeq 500$ MeV in the case of the polynomic Polyakov potential, and about one-half of this value for the logarithmic one. As discussed, these critical masses appear to be too small in comparison with present lattice QCD estimations. These results correspond to a value of T_0 of about 210 MeV, which follows from the approach of Ref. [34] for the case of nonvanishing current quark masses.

V. SUMMARY AND CONCLUSIONS

In this work we have analyzed the dependence of the deconfinement and chiral restoration critical temperatures on the explicit chiral symmetry breaking driven by the current quark mass. We work in the framework of SU(2) nonlocal chiral quark models with Polyakov loop (nlPNJL models), considering two different functional forms of the Polyakov loop effective potential commonly used in the literature, namely a polynomic function and a logarithmic function. As a first step we have considered the mass dependence of the pion mass and decay constant at vanishing temperature, in comparison with that obtained in the local NJL model and in lattice QCD. We have found that, while lattice results for the ratio m_π^2/m are in agreement with both local and nonlocal models, those for f_π show a significant increase with m_π that can be reproduced only by the predictions of nonlocal models. Concerning the deconfinement and chiral restoration critical temperatures, we have found that, contrary to the case of the local PNJL model, in nlPNJL models both critical temperatures turn out to be strongly entangled for the considered range of pion masses. In addition, it is seen that the growth of critical temperatures with the pion mass above the physical point is basically linear, with a slope parameter that is close to existing lattice QCD estimates. On the other hand, it is found that in general the present mean field calculation leads to a too early onset of the first order transition known to exist in the large quark mass limit. However, depending on the ansatz for the Polyakov potential, the corresponding critical mass may lie beyond the upper limit of reliability of our low energy effective models. One also expects this critical mass to be enhanced if the analysis is improved by including a fully nonperturbative treatment of meson fluctuations.

[1] See e.g., D. H. Rischke, Prog. Part. Nucl. Phys. **52**, 197 (2004); P. Jacobs and X. N. Wang, Prog. Part. Nucl. Phys. **54**, 443 (2005); K. Fukushima and T. Hatsuda, Rep. Prog. Phys. **74**, 014001 (2011).
[2] C. R. Allton, S. Ejiri, S. Hands, O. Kaczmarek, F. Karsch, E. Laermann, and C. Schmidt, Phys. Rev. D **68**, 014507 (2003); C. R. Allton, M. Döring, S. Ejiri, S. J. Hands, O. Kaczmarek, F. Karsch, E. Laermann, and K. Redlich, Phys. Rev. D **71**, 054508 (2005).
[3] Z. Fodor and S. D. Katz, J. High Energy Phys. 04 (2004) 050; Y. Aoki, Z. Fodor, S. D. Katz, and K. K. Szabo, J. High Energy Phys. 01 (2006) 089.
[4] F. Karsch and E. Laermann, in *Quark Gluon Plasma III*, edited by R. C. Hwa and X. N. Wang (World Scientific, Singapore, 2004).
[5] D. Blaschke, M. Buballa, A. E. Radzhabov, and M. K. Volkov, Yad. Fiz. **71**, 2012 (2008) [Phys. At. Nucl. **71**, 1981 (2008)].

[6] G. A. Contrera, D. Gómez Dumm, and N. N. Scoccola, Phys. Lett. B **661**, 113 (2008); Phys. Rev. D **81**, 054005 (2010).
[7] T. Hell, S. Roessner, M. Cristoforetti, and W. Weise, Phys. Rev. D **79**, 014022 (2009); **81**, 074034 (2010).
[8] G. A. Contrera, A. G. Grunfeld, and D. B. Blaschke, arXiv:1207.4890.
[9] D. Horvatic, D. Blaschke, D. Klabucar, and O. Kaczmarek, Phys. Rev. D **84**, 016005 (2011).
[10] P. N. Meisinger and M. C. Ogilvie, Phys. Lett. B **379**, 163 (1996).
[11] K. Fukushima, Phys. Lett. B **591**, 277 (2004).
[12] E. Megias, E. R. Arriola, and L. L. Salcedo, Phys. Rev. D **74**, 065005 (2006).
[13] C. Ratti, M. A. Thaler, and W. Weise, Phys. Rev. D **73**, 014019 (2006).
[14] S. Roessner, C. Ratti, and W. Weise, Phys. Rev. D **75**, 034007 (2007).
[15] S. Mukherjee, M. G. Mustafa, and R. Ray, Phys. Rev. D **75**, 094015 (2007).
[16] C. Sasaki, B. Friman, and K. Redlich, Phys. Rev. D **75**, 074013 (2007).
[17] T. Schafer and E. V. Shuryak, Rev. Mod. Phys. **70**, 323 (1998).
[18] C. D. Roberts and A. G. Williams, Prog. Part. Nucl. Phys. **33**, 477 (1994); C. D. Roberts and S. M. Schmidt, Prog. Part. Nucl. Phys. **45**, S1 (2000).
[19] S. Noguera and N. N. Scoccola, Phys. Rev. D **78**, 114002 (2008).
[20] P. O. Bowman, U. M. Heller, and A. G. Williams, Phys. Rev. D **66**, 014505 (2002); P. O. Bowman, U. M. Heller, D. B. Leinweber, and A. G. Williams, Nucl. Phys. B, Proc. Suppl. **119**, 323 (2003); M. B. Parappilly, P. O. Bowman, U. M. Heller, D. B. Leinweber, A. G. Williams, and J. B. Zhang, Phys. Rev. D **73**, 054504 (2006).
[21] S. Furui and H. Nakajima, Phys. Rev. D **73**, 074503 (2006).
[22] K.-I. Kondo, Phys. Rev. D **82**, 065024 (2010).
[23] R. D. Bowler and M. C. Birse, Nucl. Phys. **A582**, 655 (1995); R. S. Plant and M. C. Birse, Nucl. Phys. **A628**, 607 (1998).
[24] W. Broniowski, B. Golli, and G. Ripka, Nucl. Phys. **A703**, 667 (2002); A. H. Rezaeian, N. R. Walet, and M. C. Birse, Phys. Rev. C **70**, 065203 (2004).
[25] A. Scarpettini, D. Gómez Dumm, and N. N. Scoccola, Phys. Rev. D **69**, 114018 (2004).
[26] D. Gómez Dumm, A. G. Grunfeld, and N. N. Scoccola, Phys. Rev. D **74**, 054026 (2006).
[27] J. Berges, D. U. Jungnickel, and C. Wetterich, Phys. Rev. D **59**, 034010 (1999).
[28] A. Dumitru, D. Roder, and J. Ruppert, Phys. Rev. D **70**, 074001 (2004).
[29] J. Braun, B. Klein, H.-J. Pirner, and A. H. Rezaeian, Phys. Rev. D **73**, 074010 (2006).
[30] E. S. Fraga, L. F. Palhares, and C. Villavicencio, Phys. Rev. D **79**, 014021 (2009).
[31] T. Kahara and K. Tuominen, Phys. Rev. D **80**, 114022 (2009); **82**, 114026 (2010).
[32] V. Pagura, D. Gómez Dumm, and N. N. Scoccola, Phys. Lett. B **707**, 76 (2012).
[33] G. A. Contrera, M. Orsaria, and N. N. Scoccola, Phys. Rev. D **82**, 054026 (2010).
[34] B.-J. Schaefer, J. M. Pawlowski, and J. Wambach, Phys. Rev. D **76**, 074023 (2007).
[35] B.-J. Schaefer, M. Wagner, and J. Wambach, Phys. Rev. D **81**, 074013 (2010); T. K. Herbst, J. M. Pawlowski, and B. J. Schaefer, Phys. Lett. B **696**, 58 (2011).
[36] C. Sasaki and K. Redlich, Phys. Rev. D **86**, 014007 (2012).
[37] M. Ruggieri, P. Alba, P. Castorina, S. Plumari, C. Ratti, and V. Greco, Phys. Rev. D **86**, 054007 (2012).
[38] D. Gómez Dumm and N. N. Scoccola, Phys. Rev. C **72**, 014909 (2005).
[39] A. E. Radzhabov, D. Blaschke, M. Buballa, and M. K. Volkov, Phys. Rev. D **83**, 116004 (2011).
[40] S. Benic, D. Blaschke, and M. Buballa, Phys. Rev. D **86**, 074002 (2012).
[41] J. Noaki *et al.* (JLQCD and TWQCD Collaborations), Phys. Rev. Lett. **101**, 202004 (2008).
[42] M. Buballa, Phys. Rep. **407**, 205 (2005).
[43] E. Laermann and O. Philipsen, Annu. Rev. Nucl. Part. Sci. **53**, 163 (2003).
[44] C. Alexandrou, A. Borici, A. Feo, P. de Forcrand, A. Galli, F. Jegerlehner, and T. Takaishi, Phys. Rev. D **60**, 034504 (1999).
[45] H. Saito *et al.* (WHOT-QCD Collaboration), Phys. Rev. D **84**, 054502 (2011); H. Saito, S. Ejiri, S. Aoki, T. Hatsuda, K. Kanaya, Y. Maezawa, H. Ohno, and T. Umeda, Phys. Rev. D **85**, 079902(E) (2012).
[46] M. Fromm, J. Langelage, S. Lottini, and O. Philipsen, J. High Energy Phys. 01 (2012) 042.
[47] K. Kashiwa, R. D. Pisarski, and V. V. Skokov, Phys. Rev. D **85**, 114029 (2012).
[48] F. Karsch, E. Laermann, and A. Peikert, Nucl. Phys. **B605**, 579 (2001).
[49] V. G. Bornyakov, R. Horsley, S. M. Morozov, Y. Nakamura, M. I. Polikarpov, P. E. L. Rakow, G. Schierholz, and T. Suzuki, Phys. Rev. D **82**, 014504 (2010).
[50] F. Karsch, Proc. Sci. LAT2007 (2007) 015.
[51] V. G. Bornyakov *et al.*, Proc. Sci. LAT2005 (2006) 157.
[52] M. Cheng *et al.*, Phys. Rev. D **74**, 054507 (2006).
[53] S. Ejiri, F. Karsch, E. Laermann, C. Miao, S. Mukherjee, P. Petreczky, C. Schmidt, W. Soeldner, and W. Unger, Phys. Rev. D **80**, 094505 (2009).
[54] J. M. Pawlowski, AIP Conf. Proc. **1343**, 75 (2011).