ΔL = 2 hyperon semileptonic decays

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We compute the rates of semileptonic $B_A \to B_B l^- l^-$ ($l = e$ or μ) hyperon transitions in a model where intermediate states involve loops of baryons and a Majorana neutrino. These rates turn out to be well below present experimental bounds and other theoretical estimates. From the experimental upper limit on the $\Xi^- \to p\mu^- \mu^-$ decay, we derive the bound $\langle m_{\mu\mu} \rangle \leq 22$ TeV for the effective Majorana mass of the muon neutrino. Also, an estimate of background contributions for these decays due to the allowed $B_A \to B_B l^- l^- \bar{\nu} \bar{\nu}$ decays is provided.

I. INTRODUCTION

Nondegenerated neutrino masses provide at present the most accepted explanation for the well-established experimental results on neutrino oscillations [1]. Nowadays, strong experimental and theoretical efforts are focused on trying to determine the absolute values of neutrinos masses [2]. Of particular interest in this regard is the question about whether neutrinos are Dirac or Majorana particles. A crucial role to address this question is being played by several experiments looking for the possible existence of $|\Delta L| = 2$ transitions [2].

In this paper we focus on the $\Delta L = 2$, $\Delta S = 1$ and 2 transitions between spin-1/2 hyperons, $B_A \to B_B l^- l^-$, following a procedure discussed before in Ref. [3] in the case of the $\Delta S = 0$ $\Sigma^- \to \Sigma^+ e^- e^-$ decays. Up to now, little attention has been paid to these decays because searches on neutrinoless double beta decays of nuclei are far more sensitive probes to effects of Majorana electron neutrinos. Even though experiments on hyperon decays cannot reach very small branching fractions as in nuclear decays, it is worth mentioning that there channels involving muons which are not available in nuclear decays.

Nevertheless, a few experimental bounds on hyperon $|\Delta L| = 2$ transitions have become available recently [4]. From a data analysis of an old BNL experiment, authors of Ref. [5] have derived $B(\Xi^- \to p\mu^- \mu^-) < 3.7 \times 10^{-4}$ at 90% C.L. More recently, the HyperCP Collaboration has reported an improved bound $B(\Xi^- \to p\mu^- \mu^-) < 4.0 \times 10^{-8}$ at 90% c.l. on this decay mode [6]. Besides this, the other experimental bound reported so far is the $\Delta L = -2$ decay mode of the charmed baryon Λ_c^+ with a branching fraction of $B(\Lambda_c^+ \to \Sigma^- \mu^+ \mu^+) < 7 \times 10^{-4}$ also at 90% c.l. [7]. The above decays can be useful in bounding the effective Majorana mass of the muon neutrino, which present bounds come from the upper experimental constraint on $K^+ \to \pi^- \mu^+ \mu^+$ decay [8] $\langle m_{\mu\mu} \rangle < 0.04$ TeV [9] and from $e-p$ scattering at HERA with same-sign dileptons in the final state $\langle m_{\mu\mu} \rangle < 2.9$ TeV [10].

On the theoretical side, the only available studies about $\Delta L = 2$ hyperon decays have been reported in Refs. [3,11]. Based on the dynamics of weak interactions for these processes and phase-space considerations, Ref. [11] suggests that a branching ratio of about 10^{-10} may be possible for such decays in one of the most optimistic new physics scenarios. Just for comparison, let us mention that an explicit calculation done in Ref. [3] for the $\Delta S = 0$ hyperon decay gives $B(\Sigma^- \to \Sigma^+ e^- e^-) \approx 1.49 \times 10^{-35}$ for an effective electron neutrino mass of about $\langle m_{\nu e} \rangle = 10$ eV. Thus, we may expect a large suppression of $\Delta L = 2$ decay rates in a light Majorana neutrino scenario and it is the purpose of our paper to explore in further detail this possibility through an explicit calculation.

II. HYPERON ΔL = 2 DECAYS

In this paper we will consider the $\Delta L = 2$ hyperon decays listed in Table I. We use a model where the dominant contributions are given by loops involving virtual baryon and Majorana neutrino states (see Fig. 1) [3]. The properly antisymmetrized decay amplitude for this process is given by

$$\mathcal{M}_{0\nu} = G^2(\mathcal{M}_1 - \mathcal{M}_2), \qquad (1)$$

where G^2 is the effective weak coupling (G_F is the Fermi constant, and V_{ij} the ij entry of the Cabibbo-Kobayashi-Maskawa matrix):

TABLE I. $\Delta L = 2$ modes of spin-1/2 hyperon semileptonic transitions.

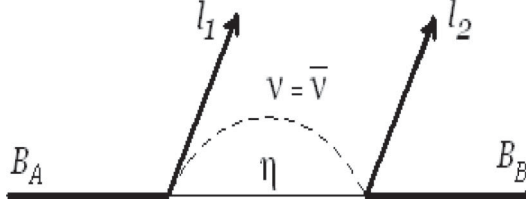| $\Delta S = 0$	$\Delta S = 1$	$\Delta S = 2$
$\Sigma^- \to \Sigma^+ e^- e^-$	$\Sigma^- \to p e^- e^-$	$\Xi^- \to p e^- e^-$
	$\Sigma^- \to p \mu^- \mu^-$	$\Xi^- \to p \mu^- \mu^-$
	$\Xi^- \to \Sigma^+ e^- e^-$	

FIG. 1. Feynman graph for $\Delta L = 2$ hyperon decays. The virtual state η denotes an intermediate hyperon state.

$$G^2 = G_F^2 \times \begin{cases} V_{ud}^2 & \text{for } \Delta S = 0, \\ V_{ud}V_{us} & \text{for } \Delta S = 1, \\ V_{us}^2 & \text{for } \Delta S = 2. \end{cases} \quad (2)$$

The expressions for the decay amplitudes defined in Eq. (1) are [3] ($i = 1, 2$):

$$\mathcal{M}_i = \sum_j m_{\nu j} U_{lj}^2 \int \frac{d^4q}{(2\pi)^4}$$

$$\times \frac{1}{q^2 - m_{\nu j}^2} L^{\alpha\beta}(p_i, p_{3-i}) H_{\alpha\beta}(Q_i(q)), \quad (3)$$

where $Q_i(q) \equiv p_B + p_i - q = p_A - p_{3-i} - q$, U_{lj} are the mixing matrix elements relating the flavor and mass neutrino eigenstates and $m_{\nu j}$ denotes the Majorana mass of the jth neutrino.

Also, in the above expression we have defined the leptonic current (the superscript c denote the charge conjugated spinor):

$$L^{\alpha\beta}(p_1, p_2) = \bar{u}_l(p_1)\gamma^\alpha(1 - \gamma_5)\gamma^\beta u_l^c(p_2) \quad (4)$$

and the hadronic current

$$H_{\alpha\beta}(Q_i(q)) = \sum_\eta \bar{u}(p_B)\gamma_\alpha(f_{B\eta} + g_{B\eta}\gamma_5)$$

$$\times \frac{\not{Q}_i + m_\eta}{Q_i^2 - m_\eta^2} \gamma_\beta(f_{A\eta} + g_{A\eta}\gamma_5)u(p_A), \quad (5)$$

where $f_{A\eta, B\eta}$ and $g_{A\eta, B\eta}$ are the vector and axial-vector form factors for the single weak transitions of hyperons. The subscript η denotes the intermediate states that are allowed in each specific transition (see Table II).

Since we will assume that our form factors are constants, the above amplitude can be written in a more compact form as follows:

TABLE II. Vector (f) and axial-vector (g) form factors at zero momentum transfer taken from Ref. [12]. The second column indicates the intermediate states that are allowed for each transition.

transition	η	$f_{A\eta}$	$g_{A\eta}$	$f_{B\eta}$	$g_{B\eta}$
$\Sigma^- \to \Sigma^+$	Λ	0	0.656	0	0.656
	Σ^0	$\sqrt{2}$	0.655	$\sqrt{2}$	-0.656
$\Sigma^- \to p$	n	-1	0.341	1	1.2670
	Σ^0	$\sqrt{2}$	0.655	$-1/\sqrt{2}$	0.241
	Λ	0	0.656	$-\sqrt{3/2}$	-0.895
$\Xi^- \to \Sigma^+$	Ξ^0	-1	0.341	1	1.267
	Σ^0	$1/\sqrt{2}$	0.896	$\sqrt{2}$	-0.655
	Λ	$\sqrt{3/2}$	0.239	0	0.656
$\Xi^- \to p$	Σ^0	$1/\sqrt{2}$	0.896	$-1/\sqrt{2}$	0.241
	Λ	$\sqrt{3/2}$	0.239	$-\sqrt{3/2}$	-0.895

$$\mathcal{M}_i = \sum_j m_{\nu j} U_{lj}^2 \sum_\eta \bar{u}(p_B)\gamma_\nu(f_{B\eta} + g_{B\eta}\gamma_5)$$

$$\times \mathcal{I}_i\gamma_\mu(f_{A\eta} + g_{A\eta}\gamma_5)u(p_A)L^{\nu\mu}(p_i, p_{3-i}), \quad (6)$$

where we have introduced the loop integral:

$$\mathcal{I}_i = \int \frac{d^4q}{(2\pi)^4} \frac{\not{Q}_i(q) + m_\eta}{(q^2 - m_{\nu j}^2)(Q_i^2(q) - m_\eta^2)}$$

$$= \frac{i}{(4\pi)^2}[(\not{p}_A - \not{p}_{3-i})\mathcal{A}_\eta + m_\eta\mathcal{B}_\eta]. \quad (7)$$

The integral $\mathcal{I}_i$ is logarithmically divergent. This divergence can be cured in principle by taking into account the momentum dependence of hyperon form factors, which are expected to fall for large values of momentum transfer. Instead, we chose to introduce a momentum cutoff Λ [3], which can be related to the average distance d between quarks inside hyperons ($\Lambda \sim (2d)^{-1} \approx 1$ GeV, for numerical purposes, as in our previous work [3]). Under this assumption, a straightforward evaluation of the functions $\mathcal{A}_\eta$, $\mathcal{B}_\eta$ gives

$$\begin{aligned} \mathcal{A}_\eta &= \mathcal{C}_1(m_\eta^2/\bar{m}_A^2, \Lambda^2/\bar{m}_A^2) - \mathcal{D}_1(m_\eta^2/\bar{m}_A^2), \\ \mathcal{B}_\eta &= \mathcal{C}_2(m_\eta^2/\bar{m}_A^2, \Lambda^2/\bar{m}_A^2) - \mathcal{D}_2(m_\eta^2/\bar{m}_A^2), \end{aligned} \quad (8)$$

where we have defined $\bar{m}_A^2 = m_A^2 + m_l^2$ and

$$C_1(m, m') = -\frac{1}{2}(2+m) + \frac{1}{4}(1+m^2)\ln\left(1+\frac{m}{m'}\right) + \frac{1}{2}\ln(m') + \frac{2m'-1+2mm'+m+m^2-m^3}{2\sqrt{4m'-(1-m)^2}}$$
$$\times\left[\arctan\left(\frac{1-m}{\sqrt{4m'-(1-m)^2}}\right) + \arctan\left(\frac{1+m}{\sqrt{4m'-(1-m)^2}}\right)\right],$$

$$C_2(m, m') = -2 + \frac{1}{2}(1+m)\ln\left(1+\frac{m}{m'}\right) + \ln(m') + \frac{2m'-(1-m)^2}{\sqrt{4m'-(1-m)^2}}\left[\arctan\left(\frac{1-m}{\sqrt{4m'-(1-m)^2}}\right) + \arctan\left(\frac{1+m}{\sqrt{4m'-(1-m)^2}}\right)\right],$$

$$\mathcal{D}_1(m) = -\frac{1}{2}(2+m) + \frac{1}{2}m^2\ln(m) + \frac{1}{2}(1-m^2)\ln(1-m) + i\frac{\pi}{2}(1+m^2),$$

$$\mathcal{D}_2(m) = -2 + m\ln(m) + (1-m)\ln(1-m) + i\pi(1+m). \tag{9}$$

Since the integral in Eq. (7) (or $C_{1,2}$) diverges logarithmically, the dependence of our results on the cutoff Λ is not very sensitive.

III. RESULTS

In order to evaluate numerically the decay rates, we need as input the values of form factors. In Table II we show the numerical values of vector and axial-vector form factors defined at zero momentum transfer taken from a fit to hyperon semileptonic decays in the limit of SU(3) symmetry [12]. The subscript A (B) refers to the weak transition of initial (final) baryon state and η denote the intermediate states that allowed to contribute in each case. The values of Cabibbo-Kobayashi-Maskawa matrix elements used in our numerical evaluations are $|V_{ud}| = 0.9740$ and $|V_{us}| = 0.2250$ [4].

Following the usual procedure, we can compute the decay rates from the unpolarized probability obtained from the decay amplitudes given in the previous section. In the second column of Table III we show the decay rates normalized to the square of the effective Majorana neutrino mass which is defined as

$$\langle m_{ll}\rangle \equiv \sum_l m_{\nu j}U_{lj}^2. \tag{10}$$

In the third column of Table III, we display the branching ratios for each decay mode. Just for illustrative purposes we have used $\langle m_{ee}\rangle = 10$ eV and $\langle m_{\mu\mu}\rangle = 10$ MeV for

TABLE III. Decay rates (normalized to the effective neutrino mass $\langle m_{ll}\rangle^2$) and branching ratios for $\Delta L = 2$ hyperon decays. We use $\langle m_{ee}\rangle^2 = (10$ eV$)^2$ and $\langle m_{\mu\mu}\rangle^2 = (10$ MeV$)^2$ to evaluate the branching ratios.

	$\Gamma_{0\nu}/\langle m_{ll}\rangle^2$ [sec^{-1}/MeV^2]	$B(B_A \to B_B l^- l^-)$
$\Sigma^- \to \Sigma^+ e^- e^-$	1.000×10^{-15}	1.48×10^{-35}
$\Sigma^- \to pe^- e^-$	0.497×10^{-10}	7.35×10^{-31}
$\Sigma^- \to p\mu^- \mu^-$	0.426×10^{-11}	6.31×10^{-20}
$\Xi^- \to \Sigma^+ e^- e^-$	0.841×10^{-13}	1.38×10^{-33}
$\Xi^- \to pe^- e^-$	1.150×10^{-12}	1.88×10^{-32}
$\Xi^- \to p\mu^- \mu^-$	0.480×10^{-12}	7.87×10^{-21}

the effective Majorana masses of electron and muon neutrinos, respectively.

As we can expect [11], there is a considerable enhancement in the rates of $\Delta S \neq 0$ transitions due mainly to a larger phase space available in such decays. Differences in the form factors for different intermediate states play a less important role. On the other hand, di-muon decays appear to have larger branching ratios because the bounds on the effective Majorana mass of muon neutrinos are rather poor at present. Conversely, if we use the present experimental upper limit on the $\Xi^- \to p\mu^- \mu^-$ decay [6] we derive the following upper bound on the effective muon neutrino mass:

$$\langle m_{\mu\mu}\rangle < 22 \text{ TeV}. \tag{11}$$

Although this bound is loosely compared to $\langle m_{\mu\mu}\rangle \lesssim 0.04$ TeV obtained from $K^+ \to \pi^- \mu^+ \mu^+$ decays [9], it is the first bound derived from $\Delta L = 2$ hyperon decays. An improvement of 5 orders of magnitude on the experimental upper limit of the $\Xi^- \to p\mu^- \mu^-$ branching ratio would be required to produce a similar bound on $\langle m_{\mu\mu}\rangle$. Note also from Table III, that the $\Sigma^- \to p\mu^- \mu^-$ decay offers a good chance to provide an upper limit on this effective Majorana mass parameter, although no experimental bound on this decay has been reported up to now.

IV. BACKGROUND: $\beta\beta$ DECAYS WITH TWO NEUTRINOS

Double beta decays with two neutrinos $B_A \to B_B l^- l'^- \bar{\nu}_l \bar{\nu}_{l'}$, which are allowed in the standard model, can provide the main source of background for $\Delta L = 2$ decays of hyperons. Just for completeness, in this section we provide an estimate of their branching fractions.

Following a model discussed in a previous work [3], we will assume the decays under consideration proceed through the decay chain $B_A^- \to B^* l^- \bar{\nu}_l \to B_B^+ l^- l'^- \bar{\nu}_l \bar{\nu}_{l'}$, where B^* is a neutral baryon intermediate state. We further assume that the dominant contributions are given by the B^* states that belong to the same octet as $B_{A,B}$ [3]. In this scheme, hyperon decays where intermediate states B^* can be on-shell simultaneously for the production and decay subprocesses, will largely dominate the decay rate [3].

TABLE IV. Branching ratios for $2\bar{\nu}\beta\beta$ decays of hyperons. Results in second column include all the allowed intermediate baryon states, and in the third column we keep only contributions of the Σ^0 intermediate state (see text).

Decay Mode	BR with all intermediate states	BR with Σ^0 intermediate state
$\Sigma^- \to \Sigma^+ e^- e^- \bar{\nu}\,\bar{\nu}$	8.59×10^{-31}	8.59×10^{-31}
$\to p e^- e^- \bar{\nu}\,\bar{\nu}$	1.02×10^{-3}	2.85×10^{-23}
$\to p e^- \mu^- \bar{\nu}\,\bar{\nu}$	4.5×10^{-4}	1.23×10^{-23}
$\to p \mu^- \mu^- \bar{\nu}\,\bar{\nu}$	0	0
$\Xi^- \to \Sigma^+ e^- e^- \bar{\nu}\,\bar{\nu}$	6.59×10^{-14}	5.57×10^{-25}
$\to \Sigma^+ e^- \mu^- \bar{\nu}\,\bar{\nu}$	1.20×10^{-15}	6.78×10^{-27}
$\to p e^- e^- \bar{\nu}\,\bar{\nu}$	4.68×10^{-7}	1.85×10^{-17}
$\to p e^- \mu^- \bar{\nu}\,\bar{\nu}$	3.80×10^{-7}	8.00×10^{-18}
$\to p \mu^- \mu^- \bar{\nu}\,\bar{\nu}$	5.49×10^{-8}	9.75×10^{-20}

According with the convolution formula (see for example: [13]), the rates for the $2\bar{\nu}\,\beta\beta$ decay processes are given to a good approximation by

$$\Gamma(B_A^- \to B_B^+ l^- l'^- \bar{\nu}_l \bar{\nu}_{l'}) = \sum_{B^*} \Gamma(B_A^- \to B^* l^- \bar{\nu}_l) \times B(B^* \to B_B^+ l'^- \bar{\nu}_{l'}), \quad (12)$$

where, for the decays of our interest, B^* can be any of n, Λ, Σ^0, and Ξ_0 real baryon states that are allowed by the kinematics of the decay process.

The results for the branching fractions are shown in Table IV, where we have used the results of Ref. [14] for the rates of single beta hyperon decays. In the second column of Table IV, we display the results obtained when all the on-shell B^* intermediate states are allowed to contribute. Some branching fractions in column 2 of Table IV appear to be surprisingly large, although they correspond to an unrealistic situation. Indeed, in a real experiment, contributions with n, Λ, and Ξ^0 intermediate states can be discriminated and removed from data due to the large lifetimes of these particles.

A more realistic estimate of the branching ratios are given in the third column of Table IV. These estimates include only the contribution of Σ^0 as an intermediate state. Indeed, the Σ^0 can be considered as an *irreducible* contribution given its very short lifetime ($\tau_{\Sigma^0} = 7.4 \times 10^{-20}$ sec.), which make appear the two charged leptons as emitted from a common primary vertex.

As a validation of our approximated formula in Eq. (12), we observe that the branching fraction for the $\Sigma^- \to \Sigma^+ e^- e^- \bar{\nu}\,\bar{\nu}$ transition (8.59×10^{-31}) is very close to the result of the exact calculation (1.36×10^{-30}) obtained in Ref. [3].

V. CONCLUSIONS

In this paper we have studied the $\Delta L = 2$ transitions in hyperon semileptonic decays. An explicit calculation of the branching ratios using a model where loops are dominated by virtual baryons and Majorana neutrinos shows that such decays are more suppressed than expectations based on dimensional grounds [11] and perhaps beyond the scrutiny of present experiments. Using the present experimental bound on the branching ratio of $\Xi^- \to p \mu^- \mu^-$ decays, we get $\langle m_{\mu\mu} \rangle \lesssim 22$ TeV for the effective Majorana mass of the muon neutrino. This bound is 2 orders of magnitude less restrictive than the present bound on this parameter obtained from $K^+ \to \pi^- \mu^+ \mu^+$ decays. Finally, it is interesting to note that, beyond any bound that can be obtained on the effective Majorana masses, the observation of $\Delta L = 2$ hyperon transitions will signal the presence of new physics.

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