

Observational Bounds on Quantum Gravity Signals using Existing Data

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We consider a new set of effects arising from the quantum gravity corrections to the propagation of fields, associated with fluctuations of the spacetime geometry. Using already existing experimental data, we can put bounds on these effects that are more stringent by several orders of magnitude than those expected to be obtained in astrophysical observations. In fact, these results can already be interpreted as questioning the whole scenario of linear (in ℓ_P) corrections to the dispersion relations for free fields in Lorentz violating theories.

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The search for experimental clues about the nature of quantum gravity has been dismissed as unpractical for a long time by the simplistic argument that such effects should appear only when the energy scales of the interaction reaches the Planck scale, a realm far beyond our experimental possibilities. Recently there has been a revolutionary change in this conception originated in [1,2] (see, however, [3]). The former propose a spontaneous violation of Lorentz and *CPT* symmetries occurring at Planck scale, motivated by string theory and parametrized by an extension of the standard model including all possible Lorentz and *CPT* violating interactions. This approach has sparked a number of experimental studies of such violations. The latter generically point out that quantum gravity should predict slight deviations in the laws describing the propagation of photons in vacuum and that the cosmological distances traveled by these gamma rays could amplify such effects, making them observable. Such modifications have been found within two currently popular approaches to quantum gravity: loop quantum gravity [4] and string theory [5]. These effects predict a change in the dispersion relations of photons that leads to their velocity of propagation becoming energy-dependent via corrections of the type $(E\ell_P)^n$, where ℓ_P is the Planck length. Observational bounds upon some of the corresponding parameters have been settled in Refs. [6–8]. For example, by considering the change in the arrival time of these gamma rays, induced by such energy dependence, together with the intrinsic time structure of the corresponding gamma-ray bursts [6], it is possible to find by a simple order of magnitude estimate that one is bringing quantum gravity to the realm of experimental physics.

The basic point of this Letter is that if a theory predicts that photons propagate with an energy-dependent velocity $v(E)$ rather than with the universal speed of light c , this implies a breakdown of Lorentz invariance, either fundamental or spontaneous, since such a statement can be at best valid in one specific inertial frame. This selects a

preferred frame of reference, where the particular form of the corrected equations of motion are valid, and one should then be able to detect the laboratory velocity with respect to that frame. It should not be at all surprising that on the rebirth of an ether-like concept requiring a privileged rest frame—paradoxically inspired by current attempts to obtain a quantum description of general relativity—the ghost of Michelson-Morley’s search should be coming back for revenge. Furthermore, we have today, in contrast with the situation at the end of the 19th century, a rather unique choice for that “preferred inertial frame”: the frame where the cosmic microwave background (CMB) looks isotropic. Our velocity \mathbf{w} with respect to that frame has already been determined to be $w/c \approx 1.23 \times 10^{-3}$ by the measurement of the dipole term in the CMB by COBE, for instance [9]. From the above discussion, it follows that the quantum gravity corrections to the corresponding particle field theory (photons, fermions, and others) should contain \mathbf{w} -dependent terms when described in our laboratory reference frame. These would lead to a breakdown of isotropy in the measurements carried out on earth. Thus, high precision tests of rotational symmetry, using atomic and nuclear systems, should serve to test some of the quantum gravity corrections. The purpose of this Letter is to point out that such types of experiments [10,11] are sufficiently accurate to establish bounds upon the above-mentioned quantum gravity effects, due to the very high degree of sensitivities that have been achieved. We should point out that although the present analysis will focus specifically in loop quantum gravity inspired scenarios the same considerations should apply *mutatis mutandis* to string inspired models of such effects.

The method of analysis can be thought to correspond to the application of the general framework described in Refs. [1(e), 1(f)], to the specific scenarios arising from the quantum gravity motivated effects. In Refs. [4], inspired on the loop quantum gravity approach, the effects of the quantum fluctuations of the “spacetime

metric" in a semiclassical state of the geometry leave their mark on the effective Hamiltonian of the Maxwell field, which propagates in the corresponding spacetime. Such quantum gravity modifications to Maxwell's equations have been extended also to two-component spin-1/2 massive particles which can be physically realized as neutrinos [12].

Even though Maxwell theory can be considered as the paradigm for studying such quantum gravity corrections, it turns out that those which affect Dirac particles are the ones that in this case produce experimentally interesting effects, as we will see in the sequel. The starting point are the modified equations for a two-component spinor ξ with positive chirality ($\gamma_5 = +1$) derived in [12]. In the nuclear case, which is of interest for our purposes, the scale \mathcal{L} of that reference has the natural choice $\mathcal{L} = 1/m$, where m is typically the particle mass. Also the kinetic energies involved are small compared to the mass so that we can safely set $\nabla^2 \ll m^2$. In this way the relevant equations reduce to

$$\begin{cases} \left[i\frac{\partial}{\partial t} - iA\boldsymbol{\sigma} \cdot \nabla + \frac{K}{2} \right] \xi - m(\alpha - \beta i\boldsymbol{\sigma} \cdot \nabla)\chi = 0, \\ \left[i\frac{\partial}{\partial t} + iA\boldsymbol{\sigma} \cdot \nabla - \frac{K}{2} \right] \chi - m(\alpha - \beta i\boldsymbol{\sigma} \cdot \nabla)\xi = 0, \end{cases} \quad (1)$$

where

$$\begin{aligned} A &= (1 + \Theta_1 m \ell_P), & \alpha &= (1 + \Theta_3 m \ell_P), \\ K &= m\Theta_4 m \ell_P, & \beta &= \Theta_2 \ell_P, \end{aligned} \quad (2)$$

and $\Theta_1, \Theta_2, \Theta_3, \Theta_4$ are constants. We are interested here in analyzing only the corrections which are linear in ℓ_P . In the two-component case we had $\chi = -i\sigma_2 \xi^*$. From now on we consider ξ and χ to be independent spinors, and we rewrite the above set of equations in terms of the four component spinor $\Psi^T = (\xi^T, \chi^T)$. This leads to a modified Dirac equation

$$\left(i\gamma^\mu \partial_\mu + \Theta_1 m \ell_P i\boldsymbol{\gamma} \cdot \nabla - \frac{K}{2} \gamma_5 \gamma^0 - m(\alpha - i\Theta_2 \ell_P \boldsymbol{\Sigma} \cdot \nabla) \right) \Psi = 0, \quad (3)$$

where we have used the representation in which γ_5 is diagonal and the spin operator is $\Sigma^k = (i/2)\epsilon_{klm}\gamma^l\gamma^m$. The normalization has been chosen so that in the limit ($m\ell_P \rightarrow 0$) we recover the standard massive Dirac equation. The term $m(1 + \Theta_3 m \ell_P)$ can be interpreted as a renormalization of the mass whose physical value is taken to be $M = m(1 + \Theta_3 m \ell_P)$. After this modification the effective Lagrangian is

$$L_D = \frac{1}{2} i\bar{\Psi} \gamma^0 (\partial_0 \Psi) + \frac{1}{2} i\bar{\Psi} [(1 + \Theta_1 M \ell_P) \gamma^k - \Theta_2 \ell_P M \Sigma^k] \partial_k \Psi - \frac{1}{2} M \bar{\Psi} \Psi - \frac{K}{4} \bar{\Psi} \gamma_5 \gamma^0 \Psi + \text{H.c.} \quad (4)$$

This Lagrangian is not Lorentz invariant and thus corresponds to the Lagrangian associated with time evolution as seen in the CMB frame. In order to obtain the Hamiltonian corresponding to time evolution as seen in the laboratory frame, we write (4) in a covariant looking form, by introducing explicitly the CMB frame's four velocity $W^\mu = \gamma(1, \mathbf{w}/c)$. In the metric with signature -2 the result is

$$\begin{aligned} L_D &= \frac{1}{2} i\bar{\Psi} \gamma^\mu \partial_\mu \Psi - \frac{1}{2} M \bar{\Psi} \Psi + \frac{1}{2} i(\Theta_1 M \ell_P) \bar{\Psi} \gamma_\mu (g^{\mu\nu} - W^\mu W^\nu) \partial_\nu \Psi + \frac{1}{4} (\Theta_2 M \ell_P) \bar{\Psi} \epsilon_{\mu\nu\alpha\beta} W^\mu \gamma^\nu \gamma^\alpha \partial^\beta \Psi \\ &\quad - \frac{1}{4} (\Theta_4 M \ell_P) M W_\mu \bar{\Psi} \gamma_5 \gamma^\mu \Psi + \text{H.c.} \end{aligned} \quad (5)$$

Using the method of [13] we obtain the nonrelativistic limit of the Hamiltonian corresponding to (5), to first order in ℓ_P . To this end we make the identifications $a_\mu = H_{\mu\nu} = d_{\mu\nu} = e_\mu = f_\mu = 0$, $c_{\mu\nu} = \Theta_1 M \ell_P (g_{\mu\nu} - W_\mu W_\nu)$, $g_{\alpha\beta\gamma} = -\Theta_2 M \ell_P W^\rho \epsilon_{\rho\alpha\beta\gamma}$, and $b_\mu = \frac{1}{2} \Theta_4 M^2 \ell_P W_\mu$. From Eq. (26) of [13] we obtain, up to order $(\mathbf{w}/c)^2$, such that $W^\mu = [1 + 1/2(\mathbf{w}/c)^2, \mathbf{w}/c]$,

$$\begin{aligned} \tilde{H} &= \left\{ M c^2 [1 + \Theta_1 M \ell_P (\mathbf{w}/c)^2] + \left[1 + 2\Theta_1 M \ell_P \left(1 + \frac{5}{6} (\mathbf{w}/c)^2 \right) \right] \left(\frac{p^2}{2M} + g\boldsymbol{\mu} \mathbf{s} \cdot \mathbf{B} \right) \right\} \\ &\quad + \left(\Theta_2 + \frac{1}{2} \Theta_4 \right) M \ell_P \left[\left(2M c^2 - \frac{2p^2}{3M} \right) \mathbf{s} \cdot \frac{\mathbf{w}}{c} + \frac{1}{M} \mathbf{s} \cdot \mathbf{Q}_P \cdot \frac{\mathbf{w}}{c} \right] + \Theta_1 M \ell_P \left[\frac{\mathbf{w} \cdot \mathbf{Q}_P \cdot \mathbf{w}}{M c^2} \right], \end{aligned} \quad (6)$$

where $\mathbf{s} = \boldsymbol{\sigma}/2$. Here we have not written the terms linear in the momentum since they average to zero. In (6) g is the standard gyromagnetic factor, and \mathbf{Q}_P is the momentum quadrupole tensor with components $Q_{Pij} = p_i p_j - 1/3 p^2 \delta_{ij}$. The terms in the third square bracket represent a coupling of the spin to the velocity with respect to the

"rest" (privileged) frame. The first one, originally proposed in Ref. [14], has been measured with high accuracy in Refs. [10] where an upper bound for the coefficient has been found. The second term is a small anisotropy contribution and can be neglected. Thus, we find the correction

$$\delta H_S = \left(\Theta_2 + \frac{1}{2} \Theta_4 \right) M \ell_P (2Mc^2) \left[1 + O\left(\frac{p^2}{2M^2 c^2} \right) \right] \mathbf{s} \cdot \frac{\mathbf{w}}{c}. \quad (7)$$

Let us concentrate now on the last term of (6), which represents an anisotropy of the inertial mass, that has been bounded in Hughes-Drever-like experiments. With the approximation $Q_P = -5/3 \langle p^2 \rangle Q/R^2$ for the momentum quadrupole moment, with Q being the electric quadrupole moment and R the nuclear radius, we obtain

$$\delta H_Q = -\Theta_1 M \ell_P \frac{5}{3} \left\langle \frac{p^2}{2M} \right\rangle \left(\frac{Q}{R^2} \right) \left(\frac{w}{c} \right)^2 P_2(\cos\theta), \quad (8)$$

for the quadrupole mass perturbation, where θ is the angle between the quantization axis and \mathbf{w} . Using $\langle p^2/2M \rangle \sim 40$ MeV for the energy of a nucleon in the last shell of a typical heavy nucleus, together with the experimental bounds of Refs. [11] we find

$$\left| \Theta_2 + \frac{1}{2} \Theta_4 \right| < 2 \times 10^{-9}, \quad |\Theta_1| < 3 \times 10^{-5}. \quad (9)$$

Equation (9) is the main result of this paper.

A second possibility to look for experiments constraining the quantum gravity corrections to particle interactions is provided by the electrodynamics of Gambini-Pullin [4]. The effective Lagrangian density is [15]

$$L = \frac{1}{2} (E_i E_i - B_i B_i) - 4\pi(\phi\rho - J_k A_k) + \theta \ell_P (E_i \epsilon_{ipq} \partial_p E_q - B_i \epsilon_{ijk} \partial_j B_k), \quad (10)$$

which is clearly not Lorentz invariant and thus must correspond to the Lagrangian associated with time evolution as seen in the CMB frame. We rewrite the Lagrangian (10), to order ℓ_P , in a covariant looking form, by introducing explicitly the privileged frame's four velocity W^μ , obtaining

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \theta \ell_P U^{\beta\mu\psi\delta\nu} F_{\beta\mu} \partial_\psi F_{\delta\nu} - 4\pi J^\mu A_\mu, \quad (11)$$

$$U^{\tau\theta\psi\delta\nu} = \left(K^{\tau\theta\psi\delta\nu} + \frac{1}{4} \epsilon_{\beta\mu}^{\tau\theta} K^{\beta\mu\psi\zeta\eta} \epsilon_{\zeta\eta}^{\delta\nu} \right),$$

with

$$K^{\beta\mu\psi\delta\nu} = \frac{1}{4} W_\alpha (\epsilon^{\alpha\beta\psi\delta} W^\mu W^\nu - \epsilon^{\alpha\mu\psi\delta} W^\beta W^\nu + \epsilon^{\alpha\mu\psi\nu} W^\beta W^\delta - \epsilon^{\alpha\beta\psi\nu} W^\mu W^\delta). \quad (12)$$

Following the standard Noether procedure for Lagrangians depending upon the second derivatives of the basic field A_μ we can calculate the modified energy-momentum tensor for the Maxwell field in the laboratory. To first order in ℓ_P and to second order in the velocity \mathbf{w} , the corresponding Hamiltonian density is

$$T^{00} = \frac{1}{2} (E^2 + B^2) - \theta \ell_P \mathbf{E} \cdot \nabla \times \mathbf{E} + \theta \ell_P \mathbf{B} \cdot \nabla \times \mathbf{B} - \frac{2\theta \ell_P}{c} \frac{\mathbf{w}}{c} \cdot \mathbf{E} \times \frac{\partial \mathbf{E}}{\partial t} + \frac{w^i}{c} \ell_P R_{ij}(E, B, \partial_k) \frac{w^j}{c} + O\left(\frac{w}{c} \right)^3. \quad (13)$$

In principle it seems worthwhile to consider the effect of time dependent fields, such as the fourth term above, in experiments designed to test the isotropy of the laws of physics. As far as we know, no experiment has been performed up to this date that could detect such effects, and so it would be very interesting to analyze the degree to which these predictions can be tested with the current available technology. The quadratic piece R_{ij} in (13) includes only parity-violating terms which do not produce additional contributions to the quadrupolar mass modifications (8). Our corrections (11) to the Maxwell action are power counting nonrenormalizable and being considered to be highly suppressed in the standard model extension of Ref. [1], they are left out in Refs. [16]. Recently such terms have been considered in [17] in relation to the problems of stability and microcausality of the theory at high energies. The term proportional to $g_{\alpha\beta\gamma}$ in (5) is also excluded from the standard model extension, because it is incompatible with the electroweak structure, as stated in Ref. [1(f)]. However, it has been subsequently considered in Ref. [8] for the case of protons and neutrons, because of the composite nature of these particles. From our perspective this term should always be present, and in our analysis it is responsible for the correction δH_S , leading to one of the bounds established in this paper.

We have found that after identifying the preferred frame of reference associated with Planck scale physics effects with the CMB frame, existing results of atomic and nuclear physics experiments can be translated into very strict bounds on the quantum gravity induced modifications to the propagation of Dirac fields. This is a remarkable case in which the interplay of cosmology, atomic, and nuclear physics serves to shed light on a field that is usually considered to be beyond the realm of experimental physics, namely, quantum gravity. Moreover, the resulting bounds of order 10^{-5} and 10^{-9} on terms that were formerly expected to be of order unity already call into question the scenarios inspired on the various approaches to quantum gravity, suggesting the existence of Lorentz violating Lagrangian corrections which are linear in Planck's length. This would not apply, however, to ℓ_P -dependent Lorentz covariant theories [18]. Alternatively, we could view the existence of such bounds on the linear corrections as wanting for an explanation for the appearance of yet one more unnaturally small number in the fundamental laws of physics.

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