Lorentzian AdS geometries, wormholes, and holography

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We investigate the structure of two-point functions for the quantum field theory dual to an asymptotically Lorentzian Anti de Sitter (AdS) wormhole. The bulk geometry is a solution of five-dimensional second-order Einstein-Gauss-Bonnet gravity and causally connects two asymptotically AdS spacetimes. We revisit the Gubser-Klebanov-Polyakov-Witten prescription for computing two-point correlation functions for dual quantum field theories operators $\mathcal{O}$ in Lorentzian signature and we propose to express the bulk fields in terms of the independent boundary values $\phi_0^{\pm}$ at each of the two asymptotic AdS regions; along the way we exhibit how the ambiguity of normalizable modes in the bulk, related to initial and final states, show up in the computations. The independent boundary values are interpreted as sources for dual operators $\mathcal{O}^{\pm}$ and we argue that, apart from the possibility of entanglement, there exists a coupling between the degrees of freedom living at each boundary. The AdS_{1+1} geometry is also discussed in view of its similar boundary structure. Based on the analysis, we propose a very simple geometric criterion to distinguish coupling from entanglement effects among two sets of degrees of freedom associated with each of the disconnected parts of the boundary.

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I. INTRODUCTION

Asymptotically Anti de Sitter (AdS) geometries play an important role in the gauge/gravity correspondence [1–3], since they provide gravity duals to quantum field theories (QFT) with uv conformal fixed points. There is a general consensus, based on several checks, for the dual interpretation of various asymptotically AdS geometries: a big black hole solution is supposed to describe a thermal QFT state [4], a bulk solution interpolating between an AdS horizon (corresponding to an ir conformal field fixed point) and an AdS geometry at infinity of different radii realizes the renormalization group flow between two conformal fixed points [5]. As a third possibility, certain regular (solitonic) charged AdS solutions are interpreted as excited QFT coherent states [6].

We would like to discuss in this work the more intriguing situation that appears when a wormhole in the bulk causally connects two asymptotic AdS_{d+1} (Lorentzian) boundaries. Holography and the AdS/CFT correspondence in the presence of multiple boundaries is less understood. The implementation of the AdS/CFT paradigm for such cases suggests that the dual field theory lives on the union of the disjoint boundaries, and therefore to be the product of field theories on the different boundaries (see [7]). We will revisit this statement and discuss the issue of whether the two dual theories are independent, decoupled or not.

For the Lorentzian signature, wormhole geometries are ruled out for $d \geq 2$ dimensions as a solution of an Einstein-Hilbert action satisfying natural causality conditions: disconnected boundaries must be separated by horizons [8] (see [9] for recent work in $2 + 1$ dimensions). Studies of wormholes in string theory and in the context of the AdS/CFT correspondence have therefore concentrated on Euclidean signature spaces particularly motivated from applications to cosmology (see references to [10,11]). For completeness, we quote that in the Euclidean context a theorem states that disconnected positive scalar curvature boundaries are also ruled for complete Einstein manifolds of negative curvature [12] (see also [13]). Moreover, [12] proves that for negative curvature boundaries the holographic theory living on them would be unstable for $d \geq 3$ (see also [14]). The wormholes studied in [10] avoided the theorem in [12] since they were constructed as hyperbolic slicings of AdS and supported by extra supergravity fields.

The canonical Lorentzian example of two boundaries separated by a horizon is the Eternal black hole geometry, and the proposal put forward in [15] makes contact with the thermo field dynamics (TFD) formulation of QFT at finite temperature [16]: the two disconnected boundaries amount to two decoupled copies $\mathcal{H}_{\pm}$ of the dual field theory and nonvanishing correlators $\langle \mathcal{O}^{+}(\mathbf{x})\mathcal{O}^{-}(\mathbf{x}')\rangle$ are interpreted as being averaged over an entangled state encoding the statistical/thermal information of the bulk geometry (see also [17,18] for recent work). An interesting second Lorentzian example with two disconnected boundaries was constructed in [19] by performing a nonsingular orbifold of AdS_3. The result of the construction led to two causally connected cylindrical boundaries with the dual field theory involving the discrete light cone quantization (DLCQ) limit of the D1-D5 conformal field theory, but the coupling between the different boundaries degrees of freedom was not clarified. The main difference between these two examples is that in the last case causal contact exists between the conformal boundaries. The no-go theorem [8] is bypassed in the second case because the performed quotient results in the presence of compact direction with the geometry effectively being a S^1 fibration over AdS_2 where the aforementioned theorem does not apply.

The no-go Lorentzian wormholes theorem [8] is also bypassed when working with a higher order gravity theory; moreover, higher order curvature corrections to standard Einstein gravity are generically expected for any quantum theory of gravity. However, not much is known about the precise forms of the higher derivative corrections, other than for a few maximally supersymmetric cases. Since from the pure gravity point of view the most general theory that leads to second-order field equations for the metric is of the Lovelock type [20], we will choose to work with the simplest among them known as Einstein-Gauss-Bonnet theory. The action for this theory only contains terms up to quadratic order in the curvature and our interest in the wormhole solution, found in [21], is that its simplicity permits an analytic treatment. The geometry corresponds to a static wormhole connecting two asymptotically locally AdS regions with base manifold $\tilde{\Sigma}$, which in $d + 1 = 5$ takes the form $\tilde{\Sigma} = H^3$ or $S^1 \times H^2$, where H^2 and H^3 are two- and three-dimensional (quotiented) hyperbolic spaces. The resulting geometry is smooth, does not contain horizons anywhere, and the two asymptotic regions turn out to be causally connected. A perturbative stability study for the five-dimensional solution case of [21] was performed in [22].

We will revisit in the present paper the Gubser-Klebanov-Polyakov-Witten (GKPW) prescription [2,3] for extracting QFT correlators from gravity computations and discuss its application for the Lorentzian wormhole solution found in [21], mentioning along the way the similarities and differences with the AdS_2 case (see [22,23] for other work on the wormhole background discussed here). We recall that the GKPW prescription in Lorentzian signature involves not only boundary data at the conformal boundary of the spacetime but also the specification of initial and final states, and we will show how these states make their appearance in the computations (see [24–29] for discussions on Lorentzian issues related to the GKWP prescription). It is commonly accepted that the QFT dual to a wormhole geometry should correspond to two independent gauge theories living at each boundary and the wormhole geometry encodes an entangled state among them. On the other hand, the causal connection between the boundaries has been argued to give rise to a nontrivial coupling between the two dual theories [19]. We will argue, by performing an analytic continuation to the Euclidean section of the spacetime, that the nonvanishing result obtained for the correlator $\langle \mathcal{O}^+(\mathbf{x})\mathcal{O}^-(\mathbf{x}')\rangle$ between operators located at opposite boundaries signals the existence of a coupling between the fields associated to each boundary.

The paper is organized as follows: in Sec. II, we review the GKPW prescription for computing QFT correlation functions from gravity computations mentioning the peculiarities of Lorentzian signature. In Sec. III, we extend the GKPW prescription for the case of two asymptotic independent boundary data. We apply it to AdS_2, reproducing the results appearing in the literature, and to the wormhole [21] showing their similarities. In Sec. IV, we discuss several arguments regarding the possibility of entanglement and/or interactions among the two dual QFT. We summarize in Sec. V the results of the paper.

II. GKPW PRESCRIPTION WITH A SINGLE ASYMPTOTIC BOUNDARY

The GKPW prescription [2,3] equates the gravity (bulk) partition function for an asymptotically AdS_{d+1} spacetime $\mathcal{M}$, understood as a functional of boundary data, to the generating functional for correlators of a conformal field theory (CFT) defined on the spacetime conformal boundary $\partial\mathcal{M}$. Explicitly, the prescription is

$$\mathcal{Z}_{\text{gravity}}[\phi(\phi_0)] = \langle e^{i\int_{\partial\mathcal{M}} d^d\mathbf{x}\,\phi_0(\mathbf{x})\mathcal{O}(\mathbf{x})}\rangle. \tag{1}$$

In the left-hand side (lhs), $\phi_0 = \phi_0(\mathbf{x})$ stands for the boundary value of the field ϕ, and the right -hand side (rhs) is the CFT generating functional of correlators of the operator $\mathcal{O}$ dual to the (bulk) field ϕ. In the present paper, we will be working in the semiclassical spacetime limit (large N limit of the CFT) and therefore the lhs in (1) will be approximated by the on-shell action of the field ϕ which for simplicity will be taken to be a scalar field of mass m. We are interested in real time (Lorentzian) geometries and in this case, the prescription (1) is incomplete, since a specification of the initial and final states $\psi_{\mathrm{i,f}}$ on which we compute the correlator on the rhs need to be specified. We will discuss this issue below.

To set out the notation, we summarize the prescription for massive scalar fields highlighting the points important for our arguments. The AdS_{d+1} metric in Poincaré coordinates reads

$$ds^2 = \frac{R^2}{z^2}(d\mathbf{x}^2 + dz^2), \tag{2}$$

where the term $d\mathbf{x}^2$ stands for $-dt^2 + d\vec{x}^2$. The (conformal) boundary of AdS is located at $z = 0$ and a horizon exists at $z = \infty$.[1] The solution to the ϕ field equation subject to boundary data ϕ_0 set at the conformal boundary is commonly written as

[1]In the Euclidean case $z = \infty$ is just a point, leading to the half plane $z \geq 0$ in (2) being compactified to a sphere.

$$\phi(\mathbf{x}, z) = \int_{\partial\mathcal{M}} d\mathbf{y} \mathsf{K}(\mathbf{x}, z \mid \mathbf{y})\phi_0(\mathbf{y}). \qquad (3)$$

In the free field limit, the Klein-Gordon (KG) equation shows that the asymptotic behavior for ϕ is

$$\phi(\mathbf{x}, z) \sim z^{\Delta_\pm}\phi_0(\mathbf{x}), \qquad z \to 0 \qquad (4)$$

where[2]

$$\Delta_\pm = \frac{d}{2} \pm \mu, \qquad \mu = \sqrt{\frac{d^2}{4} + m^2R^2}. \qquad (5)$$

The bulk-boundary propagator K in (3) is therefore demanded to satisfy [2,3]

$$(\Box - m^2)\mathsf{K}(\mathbf{x}, z \mid \mathbf{y}) = 0 \qquad (6)$$

with boundary condition

$$\mathsf{K}(\mathbf{x}, z \mid \mathbf{y}) \sim z^{\Delta_-}\delta(\mathbf{x} - \mathbf{y}), \qquad z \to 0. \qquad (7)$$

Finally, the bulk-boundary propagator K can be related to the Dirichlet bulk-bulk Green function $\mathsf{G}(\mathbf{x}, z|\mathbf{y}, z')$ through Green's second identity, the result being that K can be obtained from the normal derivative of G evaluated at the spacetime boundary (see [31,32]) as

$$\mathsf{K}(\mathbf{x}, z \mid \mathbf{y}) = \lim_{z'\to 0} \sqrt{-g}\, g^{z'z'}\partial_{z'}\mathsf{G}(\mathbf{x}, z \mid \mathbf{y}, z'). \qquad (8)$$

Two comments are in order: i) the bulk solution for a given boundary data ϕ_0 computed from (3) is not unique, since Lorentzian AdS spaces admit normalizable solutions $\varphi(\mathbf{x}, z)$ that can be added at will to (3) without altering the boundary behavior (7), explicitly

$$\phi(\mathbf{x}, z) = \int_{\partial\mathcal{M}} d\mathbf{y}\mathsf{K}(\mathbf{x}, z \mid \mathbf{y})\phi_0(\mathbf{y}) + \varphi(\mathbf{x}, z). \qquad (9)$$

The consequence of their inclusion on the CFT is interpreted as fixing the initial and final states $|\psi_{\mathrm{i,f}}\rangle$ on which one computes the expectation value on the rhs of (1). Our second observation is ii) in Lorentzian signature, the $z = \infty$ surface is a Killing horizon and therefore an additional boundary where the bulk field needs to be specified for having a well-posed Dirichlet problem [see Fig. 1(a)]. These two observations turn out to be related to the fact that a second condition is required to fully fix the bulk-boundary propagator K (recall that in Euclidean space demanding regularity in the bulk implies $K \to 0$ when $z \to \infty$). The remaining condition on K imposed at the horizon ($z = \infty$) is best expressed in terms of Fourier modes as purely ingoing waves (exponentially decaying) for timelike (spacelike) momenta; this is a well known problem for QFT in curved spacetimes and amounts to the choice of vacuum. The incorporation of normalizable (timelike) modes induces an outgoing component from the horizon which is naturally interpreted as an excitation (see [2,17,25–27,33–35] for related work).

We are interested in computing two-point correlation functions on the dual field theory. To this end we need the on-shell action for a scalar field to quadratic order

$$S = -\frac{1}{2}\int d\mathbf{x}dz\sqrt{-g}(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + m^2\phi^2). \qquad (10)$$

Integrating by parts and evaluating on-shell, the contribution from the conformal boundary is given by (see [34] for a discussion on the horizon contribution)

$$S[\phi_0] = \frac{1}{2}\int d\mathbf{x}[\sqrt{-g}g^{zz}\phi(\mathbf{x}, z)\partial_z\phi(\mathbf{x}, z)]_{z=0}. \qquad (11)$$

Inserting (3) into this expression gives the on-shell action as a functional of the boundary data ϕ_0

$$S[\phi_0] = \frac{1}{2}\int d\mathbf{y}d\mathbf{y}'\phi_0(\mathbf{y})\Delta(\mathbf{y}, \mathbf{y}')\phi_0(\mathbf{y}') \qquad (12)$$

where

$$\Delta(\mathbf{y}, \mathbf{y}') = \int d\mathbf{x}[\sqrt{-g}g^{zz}\mathsf{K}(\mathbf{x}, z \mid \mathbf{y})\partial_z\mathsf{K}(\mathbf{x}, z \mid \mathbf{y}')]_{z=0}. \qquad (13)$$

Taking into account (7) and (8) in (13), one obtains

$$\Delta(\mathbf{y}, \mathbf{y}') \sim [\sqrt{-g}g^{zz}\partial_z\mathsf{K}(\mathbf{y}, z \mid \mathbf{y}')]_{z=0} \qquad (14)$$

$$\sim \lim_{z,z'\to 0}(\sqrt{-g}g^{zz})(\sqrt{-g}g^{z'z'})\frac{\partial^2}{\partial z\partial z'}\mathsf{G}(\mathbf{y}, z \mid \mathbf{y}', z'). \qquad (15)$$

This relation has been used to relate, in the semiclassical limit, the two-point function to the geodesics of the geometry [36].

Summarizing, the two-point function for an operator $\mathcal{O}$ dual to the bulk field ϕ is obtained from the on-shell action as

$$\langle\psi_{\mathrm{f}}|\mathcal{O}(\mathbf{y})\mathcal{O}(\mathbf{y}')|\psi_{\mathrm{i}}\rangle = -i\frac{\delta^2 S[\phi_0]}{\delta\phi_0(\mathbf{y})\delta\phi_0(\mathbf{y}')} = -i\Delta^{\mathrm{i,f}}(\mathbf{y}, \mathbf{y}'). \qquad (16)$$

The initial and final states $\psi_{\mathrm{i,f}}$ on the lhs encode to the ambiguity in adding a normalizable solution to (3); in the following section we will show explicitly how they manifest in (13).

[2] Negative mass scalar fields are allowed in AdS as long as $\mu \geq 0$. The minimum allowed mass for a scalar field in AdS_{d+1} is given by the so-called Breitenlohner-Freedman (BF) bound $\mu_{\mathrm{BF}} = 0$, or equivalently $m^2_{\mathrm{BF}} = -d^2/4$ [30].

AdS global coordinates

The recipe for obtaining QFT correlators from gravity computations involves evaluating bulk quantities at the conformal boundary; as might be suspected from (4), (7), and (13), the evaluation leads to singularities and therefore requires a regularization. We will discuss in what follows how this is done in the AdS global coordinate system since the wormhole case we will discuss later coincides in its asymptotic region with that coordinate system and will therefore be regularized in the same way. Along the way, we will show how the specification of the initial and final states $\psi_{\mathrm{i,f}}$ appear in the computation.

The regularization of (16) is performed imposing the boundary data at some finite distance in the bulk and taking the limit to the boundary at the end of the computations (see [37] for a subtlety when taking the limit). The AdS_{d+1} manifold is fully covered by the so-called global coordinates where the metric takes the form

$$ds^2 = R^2\left[-\frac{dt^2}{1-x^2} + \frac{dx^2}{(1-x^2)^2} + \frac{x^2}{1-x^2}d\Omega^2_{d-1}\right] \tag{17}$$

where we have changed variables to $x = \tanh\rho$ from the standard radial ρ variable to map the conformal boundary to $x = 1$.

We impose the boundary data at a finite distance $x_\epsilon = 1 - \epsilon$; therefore, consistency demands that

$$\lim_{x\to x_\epsilon} \mathsf{K}(t, \Omega, x \mid t', \Omega', x_\epsilon) = \frac{\delta(t-t')\delta(\Omega-\Omega')}{\sqrt{g_\Omega}} \tag{18}$$

and K regular in the interior. The boundary-bulk propagator K satisfying (18) can be obtained from the Klein-Gordon equation solutions $\phi(t, \Omega, x) = e^{-i\omega t}Y_{lm}(\Omega)f_{l\omega}(x)$ as

$$\mathsf{K}(t, \Omega, x \mid t', \Omega', x_\epsilon) = \int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\sum_{lm} e^{-i\omega(t-t')}Y_{lm}(\Omega)Y^*_{lm}(\Omega')f_{l\omega}(x) \tag{19}$$

if we normalize[3] $f_{l\omega}(x_\epsilon) = 1$. For later comparison we quote the differential equation satisfied by $f_{l\omega}(x)$

$$(1-x^2)\frac{d^2f_{l\omega}}{dx^2} + \frac{d-1-x^2}{x}\frac{df_{l\omega}}{dx} + \left(\omega^2 - \frac{q^2}{x^2} - \frac{m^2R^2}{1-x^2}\right)f_{l\omega} = 0. \tag{20}$$

The solution to this equation is a linear combination of two hypergeometric functions, but one of them diverges as $x \to 0$ so regularity in the bulk demands to discard it. The properly normalized regular solution reads

$$f_{l\omega}(x) = \left(\frac{x^{-(d/2)+\nu+1}(1-x^2)^{1/2\Delta_+}}{(1-\epsilon)^{-(d/2)+\nu+1}((2-\epsilon)\epsilon)^{1/2\Delta_+}}\right)\frac{{}_2F_1(\frac{1}{2}(\mu+\nu-\omega+1), \frac{1}{2}(\mu+\nu+\omega+1); \nu+1; x^2)}{{}_2F_1(\frac{1}{2}(\mu+\nu-\omega+1), \frac{1}{2}(\mu+\nu+\omega+1); \nu+1; (1-\epsilon)^2)}. \tag{21}$$

Here, ${}_2F_1$ is Gauss hypergeometric function with μ given by (5) and $\nu = \sqrt{(\frac{d-2}{2})^2 + q^2}$, the symmetry of the hypergeometric function in its first two arguments implies that $f_{l\omega}(x) = f_{l-\omega}(x)$. The asymptotic behavior of the solution (21) near the boundary looks like

$$f_{l\omega}(x) \sim C_+(1-x)^{1/2\Delta_+} + C_-(1-x)^{1/2\Delta_-}, \tag{22}$$

where $\Delta_\pm$ are given by (5) and $C_\pm = C_\pm(\mu, \nu, \omega)$. In Lorentzian signature, the KG operator possesses normalizable solutions; these appear for particular values of ω given by [25,30,38]

$$\begin{aligned}\omega_{nl} &= \pm(2n+\nu+\mu+1)\\ &= \pm(2n+l+\Delta_+), \qquad n, l = 0, 1, 2\ldots,\end{aligned} \tag{23}$$

or stated otherwise, these are the frequencies for which[4] $C_- = 0$. The discreteness of the spectrum manifests the "box" character of AdS, and from the dual perspective arises from the compactness of S^3. The quantization of the states (23) in the bulk is interpreted as dual to the QFT states defined on the $S^3 \times \mathbb{R}$ conformal boundary of AdS.

The two-point correlation functions for the dual QFT operators are obtained by plugging

$$\phi(t, \Omega, x) = \int dt'd\Omega'\sqrt{g_{\Omega'}}\mathsf{K}(t, \Omega, x \mid t', \Omega', x_\epsilon)\phi_0(t', \Omega') \tag{24}$$

[3]The spherical harmonics Y_{lm} on the $d-1$ sphere satisfy $\nabla^2Y_{lm} = -q^2Y_{lm}$ with $q^2 = l(l+d-2)$, $l = 0, 1, \ldots$

[4]See [30] for an alternative quantization condition for $0 \le \mu \le 1$ and [39] for its interpretation in the AdS/CFT context.

into the action (10); note that we have not included any normalizable solution to (24) (see next paragraph). The on-shell action leads to a boundary term evaluated at x_ϵ (see (11)–(13)) and the regularized expression for $\Delta(t, \Omega \mid t', \Omega')$ in (12) is therefore written as[5]

$$\begin{aligned}\Delta_{\rm reg}(t, \Omega|t', \Omega') &= -\frac{1}{\sqrt{g_\Omega}}[\sqrt{-g}g^{xx}\partial_x \mathsf{K}(t, \Omega, x \mid t', \Omega', x_\epsilon)]_{x=x_\epsilon}\\ &= -\int \frac{d\omega}{2\pi} e^{-i\omega(t-t')}\sum_{lm} Y_{lm}(\Omega)Y^*_{lm}(\Omega')\left[\frac{x^{d-1}}{(1-x^2)^{(d-2)/2}}\partial_x f_{l\omega}(x)\right]_{x=x_\epsilon}\\ &= -\sum_{lm} Y_{lm}(\Omega)Y^*_{lm}(\Omega')\int \frac{d\omega}{2\pi} e^{-i\omega(t-t')}\left[\frac{x^{d-1}}{(1-x^2)^{(d-2)/2}}\partial_x f_{l\omega}(x)\right]_{x=x_\epsilon}, \end{aligned} \tag{25}$$

where the first line comes from (13) taking into account (18).

Some comments regarding (25): when taking the $\epsilon \to 0$ limit, the expression in the last line turns out to be ambiguous due to the existence of simple poles, located at (23), along the ω-integration contour.[6] These poles manifest the existence of normalizable solutions in the bulk (see (9)) and therefore, in order to define the ω-integration, we need to give a prescription for bypassing the poles. The choice of contour is traditionally understood as the choice between advanced/retarded/Feynman Green function; we will choose to work with the Feynman one in the following. We now call attention to the observation, pointed out in [29], about the relation between contours in the complex ω-plane and choices of normalizable solutions. The observation is simple: any particular choice of contour is equivalent by deformation to choosing the Feynman contour plus contributions from encircling the poles (23). Therefore, the ambiguity in the expression (24) arising from the addition of arbitrary normalizable modes translates into a choice of contour in the complex ω-plane (see Fig. 2). The Feynman contour choice naturally leads to time ordered correlators, and the encircling of positive (negative) normalizable modes fix the initial (final) state $\psi_{\rm i,f}$ in the lhs of (16). Choosing the retarded contour as reference should be interpreted as giving rise to response functions instead of correlation functions. Summarizing, the states are interpreted as created from a single fundamental one $|\psi_0\rangle$ associated to the reference integration contour chosen.

The $\epsilon \to 0$ limit of the expression inside the brackets in (25) also shows several poles in ϵ both analytic and non-analytic. The physical result is obtained by renormalizing the boundary data taking into account the asymptotic behavior in the radial direction (see (22)); in the present case it amounts to rescale ϕ_0 as (see [3,32,37])

$$\phi_0(t, \Omega) = \epsilon^{1/2\Delta_-}\phi_{\rm ren}(t, \Omega). \tag{26}$$

Moreover, since eventually we are interested in correlation functions for separated points, (contact) terms proportional to positive integer powers of q^2 are dropped. The finite term in the $\epsilon \to 0$ limit reads

$$\begin{aligned}\Delta_{\rm ren}(t, \Omega|t', \Omega') &\equiv \lim_{\epsilon\to 0}\epsilon^{\Delta_-}\Delta_{\rm reg}(t, \Omega|t', \Omega')\\ &= \sum_{lm} Y_{lm}(\Omega)Y^*_{lm}(\Omega')\int\frac{d\omega}{2\pi}e^{-i\omega(t-t')}\times\frac{\Delta_+}{2^{\Delta_-}}\frac{\Gamma(1-\mu)}{\Gamma(1+\mu)}\frac{\Gamma(\frac{1}{2}(-\omega+\nu+\mu+1))\Gamma(\frac{1}{2}(\omega+\nu+\mu+1))}{\Gamma(\frac{1}{2}(-\omega+\nu-\mu+1))\Gamma(\frac{1}{2}(\omega+\nu-\mu+1))}. \end{aligned}\tag{27}$$

The numerator of this last expression shows explicitly the appearance of poles along the integration contour precisely at frequencies ω_{nl} given by (23). The specification of a contour $\mathcal{C}$ in the complex ω plane fixes the initial and final states $\psi_{\rm i,f}$ when compared to the standard Feynman one (see Fig. 2).

From the discussion following (25), it should be clear that correlation functions computed on the QFT vacuum state are obtained by choosing the standard Feynman contour for the ω integration in (27). Performing the ω integration we obtain[7]

$$\Delta^F_{\rm ren}(t, \Omega|t', \Omega') = 2i\frac{\Delta_+\Gamma(1-\mu)}{2^{\Delta_-}\Gamma(1+\mu)}\sum_{lm} Y_{lm}(\Omega)Y^*_{lm}(\Omega')\times\left[\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}\frac{\Gamma(n+l+\Delta_+)}{\Gamma(n+l+\frac{d}{2})\Gamma(-(n+\mu))}e^{-i|t-t'|(2n+l+\Delta_+)}\right]. \tag{28}$$

[5] Following [37], when calculating (25) we first compute the x-derivative and afterwards we take the $\epsilon \to 0$ limit.

[6] Their origin can be traced to having normalized $f_{l\omega}(x_\epsilon) = 1$.

[7] Generically, (23) are the only divergencies in (27), and special care must be taken for integer values of μ. We will not discuss the details of this in the present work since experience with the AdS/CFT correspondence has shown that correlation functions do not change qualitatively in the integer limit.

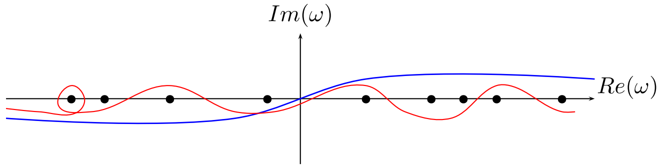

FIG. 2 (color online). *Contours in ω complex plane:* when performing the ω integration in (27) any arbitrary contour can be deformed to be the Feynman contour plus contributions from encircling the poles (23). The encircling of positive (negative) frequency poles fix the initial (final) states $\psi_{\rm i,f}$ in (16).

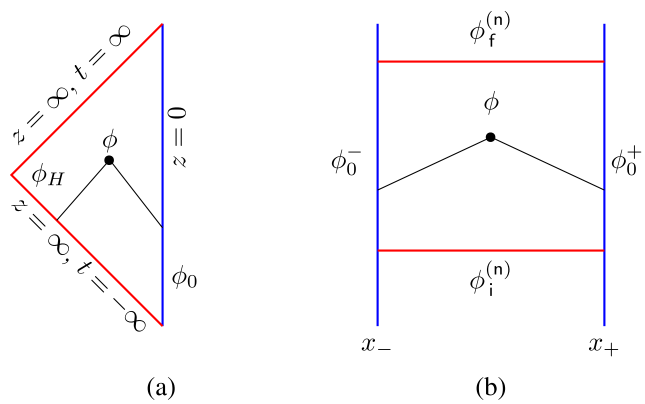

FIG. 1 (color online). (a) Poincare patch of Lorentzian AdS: the value of the scalar field ϕ at any point in the bulk depends not only on the boundary value ϕ_0 but also on the value of the field at the future and past horizons ϕ_H. (b) Lorentzian wormhole geometry: the value of the scalar field ϕ at any point in the bulk depends not only on two boundary values $\phi_0^{\pm}$ but also on normalizable modes in the bulk $\phi^{(\rm n)}$. The dependence, in both pictures, on the normalizable modes correlates to a choice of the initial and final states $|\psi_{\rm i,f}\rangle$ and manifests as a choice of contour in the complex ω plane when computing the correlator (27).

The sum over the residues can be computed analytically, giving

$$\begin{aligned}&\langle 0|T\mathcal{O}(t,\Omega)\mathcal{O}(t',\Omega')|0\rangle\\ &\quad = -i\Delta^F_{\rm ren}(t,\Omega|t',\Omega')\\ &\quad = -\frac{2\Delta_+}{2^{\Delta_-}\Gamma(\mu)}\sum_{lm}Y_{lm}(\Omega)Y^*_{lm}(\Omega')\frac{\Gamma(l+\Delta_+)}{\Gamma(l+\frac{d}{2})}\\ &\qquad \times e^{-i|t-t'|(l+\Delta_+)}{}_2F_1\left(1+\mu,l+\Delta_+,l+\frac{d}{2};e^{-2i|t-t'|}\right).\end{aligned} \tag{29}$$

III. GKPW PRESCRIPTION FOR LORENTZIAN WORMHOLES

Our goal in this section will be to extend the GKPW prescription to the case of multiple timelike boundaries; we will discuss the two boundaries case for simplicity. On general grounds, AdS/CFT suggests that the presence of two timelike boundaries should be associated with the existence of two sets $\mathcal{O}^{\pm}$ of dual operators corresponding to the two independent boundary conditions $\phi_0^{\pm}$ that must be imposed on the field ϕ when solving the wave equation.[8]

We consider wormholes with (conformal) boundary topology of the form $\mathbb{R}\times\Sigma$, with $\mathbb{R}$ representing time and $\Sigma = \Sigma_+ + \Sigma_-$ the union of two (spatial) compact disjoint copies $\Sigma_\pm$. The wormholes can be covered by a single coordinate system (x, t, θ) where x is the radial holographic coordinate in the bulk and $(x_\pm, t, \theta)$ the coordinates parametrizing the two boundaries $\mathbb{R}\times\Sigma_\pm$. In the presence of two disconnected conformal boundaries we propose to write the bulk field in terms of the boundary data $\phi_0^{\pm}(\mathbf{y})$ on each of the boundaries as (see Fig. 1(b))

$$\begin{aligned}\phi(\mathbf{y},x) &= \int d\mathbf{y}'\mathsf{K}^i(\mathbf{y},x|\mathbf{y}')\phi_0^i(\mathbf{y}')\\ &= \int d\mathbf{y}'[\mathsf{K}^+(\mathbf{y},x|\mathbf{y}')\phi_0^+(\mathbf{y}') + \mathsf{K}^-(\mathbf{y},x|\mathbf{y}')\phi_0^-(\mathbf{y}')].\end{aligned} \tag{30}$$

Here, $\mathbf{y} = (t,\theta)$; note that the solution for given boundary data is not unique since in Lorentzian signature normalizable solutions can be added to (30). This ambiguity is resolved, as discussed in Sec. II, when a choice of contour in the frequency space ω of the kernel K is given. Our method differs from the proposal developed in [19]: in that work only the K^+ bulk-boundary propagator was discussed and its form was determined by demanding the absence of cuts when extending the radial coordinate to complex values. The prescription led to the conclusion that $\phi_0^{\pm}$ were not independent.

Consistency demands the bulk-boundary propagators $\mathsf{K}^{\pm}(\mathbf{y},x|\mathbf{y}')$ to solve the Klein-Gordon equation (6) with the following boundary conditions:

$$\begin{aligned}&\mathsf{K}^+(\mathbf{y},x|\mathbf{y}')|_{x=x_+} = \delta(\mathbf{y}-\mathbf{y}'), \qquad \mathsf{K}^+(\mathbf{y},x|\mathbf{y}')|_{x=x_-} = 0\\ &\mathsf{K}^-(\mathbf{y},x|\mathbf{y}')|_{x=x_-} = \delta(\mathbf{y}-\mathbf{y}'), \qquad \mathsf{K}^-(\mathbf{y},x|\mathbf{y}')|_{x=x_+} = 0.\end{aligned} \tag{31}$$

These expressions completely determine the bulk-boundary propagators $\mathsf{K}^{\pm}$. The on-shell action (10) results in two terms arising from the boundaries which take the form

$$\begin{aligned}S = -\frac{1}{2}\int d\mathbf{y}([\sqrt{-g}g^{xx}\phi(\mathbf{y},x)\partial_x\phi(\mathbf{y},x)]_{x=x_+}\\ - [\sqrt{-g}g^{xx}\phi(\mathbf{y},x)\partial_x\phi(\mathbf{y},x)]_{x=x_-}).\end{aligned} \tag{32}$$

[8] Some authors have assumed than the two dual field theories are decoupled because of the disconnected structure of the boundary [18].

Inserting the solution (30) into (32), one obtains

$$S[\phi_0] = -\frac{1}{2}\int d\mathbf{y}d\mathbf{y}'\phi_0^i(\mathbf{y})\Delta_{ij}(\mathbf{y},\mathbf{y}')\phi_0^j(\mathbf{y}') \tag{33}$$

with $i, j = +, -$ denoting the two boundaries and Δ_{ij} the generalization of (13). Their explicit forms are

$$\begin{aligned}\Delta_{+i}(\mathbf{y},\mathbf{y}') &= [\sqrt{-g}g^{xx}\partial_x \mathsf{K}^i(\mathbf{y},x|\mathbf{y}')]_{x=x_+},\\ \Delta_{-i}(\mathbf{y},\mathbf{y}') &= -[\sqrt{-g}g^{xx}\partial_x \mathsf{K}^i(\mathbf{y},x|\mathbf{y}')]_{x=x_-}.\end{aligned} \tag{34}$$

As in Sec. II the two-point functions of operators on the same boundary result

$$\langle\psi_{\rm f}|\mathcal{O}^{\pm}(\mathbf{y})\mathcal{O}^{\pm}(\mathbf{y}')|\psi_{\rm i}\rangle \sim -i\Delta_{\pm\pm}(\mathbf{y},\mathbf{y}') \tag{35}$$

and the correlators between operators on opposite boundaries read

$$\langle\psi_{\rm f}|\mathcal{O}^{\pm}(\mathbf{y})\mathcal{O}^{\mp}(\mathbf{y}')|\psi_{\rm i}\rangle \sim -i\Delta_{\pm\mp}(\mathbf{y},\mathbf{y}'). \tag{36}$$

The generalization of the expressions (8) and (15) to backgrounds with two boundaries take the form

$$\mathsf{K}^i(\mathbf{y},x\mid\mathbf{y}') = \lim_{x'\to x^i}\sqrt{-g}\,g^{x'x'}\partial_{x'}\mathsf{G}(\mathbf{y},x\mid\mathbf{y}',x'), \tag{37}$$

which gives

$$\Delta_{ij}(\mathbf{y},\mathbf{y}') \sim \lim_{x\to x^i,x'\to x^j}(\sqrt{-g}g^{xx})(\sqrt{-g}g^{x'x'}) \times\frac{\partial^2}{\partial_x\partial_{x'}}\mathsf{G}(\mathbf{y},x\mid\mathbf{y}',x'). \tag{38}$$

Note that the $\Delta_{\pm\mp}$ correlation function involves in the semiclassical limit a geodesic through the bulk connecting two points, one at each boundary.

A. AdS_2 Lorentzian strip

We will apply in this section the prescription developed above to Lorentzian AdS_2 reobtaining previous results [18]. The AdS_2 Lorentzian metric can be written as

$$ds^2 = R^2\left[-\frac{dt^2}{1-x^2}+\frac{dx^2}{(1-x^2)^2}\right], \tag{39}$$

the timelike boundaries are located at $x=\pm1$ and the (t,x) coordinate system covers the whole spacetime. To find the bulk-boundary propagators $\mathsf{K}^{\pm}$ in (30), we propose

$$\mathsf{K}^{\pm}(t,x) = \int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{-i\omega t}f_\omega^{\pm}(x). \tag{40}$$

Inserting into the KG Eq. (6), we obtain the following differential equation for f_ω

$$(1-x^2)\frac{d^2f_\omega^{\pm}(x)}{dx^2} - x\frac{df_\omega^{\pm}(x)}{dx} + \left(\omega^2-\frac{m^2R^2}{1-x^2}\right)f_\omega^{\pm}(x) = 0. \tag{41}$$

The solution to (41) can be written in terms of generalized Legendre Polynomials as

$$f_\omega^{\pm}(x) = (1-x^2)^{1/4}[a_\omega^{\pm}P_\nu^\mu(x) + b_\omega^{\pm}Q_\nu^\mu(x)] \tag{42}$$

with $\mu = \sqrt{\frac{1}{4}+m^2R^2}$, $\nu = \omega - \frac{1}{2}$ and $a_\omega^{\pm}$, $b_\omega^{\pm}$ arbitrary constants which get fixed when we impose the conditions (31). The conditions translate into[9]

$$f_\omega^{\pm}(\pm x_\epsilon) = 1, \qquad f_\omega^{\pm}(\mp x_\epsilon) = 0. \tag{43}$$

The solutions to (43) read

$$f_\omega^{+}(x) = \left(\frac{1-x^2}{1-x_\epsilon^2}\right)^{1/4}\frac{Q_\nu^\mu(x)P_\nu^\mu(-x_\epsilon) - Q_\nu^\mu(-x_\epsilon)P_\nu^\mu(x)}{Q_\nu^\mu(x_\epsilon)P_\nu^\mu(-x_\epsilon) - Q_\nu^\mu(-x_\epsilon)P_\nu^\mu(x_\epsilon)} \tag{44}$$

$$f_\omega^{-}(x) = \left(\frac{1-x^2}{1-x_\epsilon^2}\right)^{1/4} \times\frac{Q_\nu^\mu(x)P_\nu^\mu(x_\epsilon) - Q_\nu^\mu(x_\epsilon)P_\nu^\mu(x)}{Q_\nu^\mu(-x_\epsilon)P_\nu^\mu(x_\epsilon) - Q_\nu^\mu(x_\epsilon)P_\nu^\mu(-x_\epsilon)}. \tag{45}$$

Analyzing the asymptotic behavior near the boundary in (42), one finds normalizable modes for

$$\omega_n = \pm\left(n+\mu+\frac{1}{2}\right), \qquad n = 0,1,2\ldots \quad \text{and} \qquad \frac{b_{\omega Q}}{a_{\omega Q}} = -\frac{2\tan\pi\mu}{\pi}. \tag{46}$$

The renormalized Δ_{ij} functions disregarding contact terms result

$$\Delta_{\mathsf{ren}_{\pm\pm}}(t,t') = \mp\frac{2^{\Delta_-}}{2\pi}\frac{\Gamma(1-\mu)}{\Gamma(1+\mu)}\int\frac{d\omega}{2\pi}e^{-i\omega(t-t')} \times\Gamma\left(\frac{1}{2}+\mu-\omega\right)\Gamma\left(\frac{1}{2}+\mu+\omega\right)\cos(\pi\omega) \tag{47}$$

$$\Delta_{\mathsf{ren}_{\pm\mp}}(t,t') = \mp\frac{2^{\Delta_-}}{\Gamma(\mu)^2}\int\frac{d\omega}{2\pi}e^{-i\omega(t-t')}\Gamma\left(\frac{1}{2}+\mu-\omega\right) \times\Gamma\left(\frac{1}{2}+\mu+\omega\right), \tag{48}$$

as before, the integrands in these expressions show poles in ω at the values given by (46).

The integrals (47) and (48) can be computed using the residue theorem once a contour in the complex plane is chosen. For the Feynman contour, we obtain

[9] As discussed in Sec. II, the boundary data is imposed at a finite distance $x = \pm x_\epsilon$ where $x_\epsilon = 1-\epsilon$.

$$\Delta^F_{\mathrm{ren}_{\pm\pm}}(t,t') = \mp\left(\frac{i^{\Delta_- - \Delta_+}}{8^{\Delta_+}\pi^{1/2}}\right)\frac{\Gamma(\frac{1}{2}+\mu)}{\Gamma(\mu)\sin^{2\Delta_+}(\frac{t-t'}{2})}$$
$$\Delta^F_{\mathrm{ren}_{\pm\mp}}(t,t') = \mp\left(\frac{8^{\Delta_-}i}{4}\right)\frac{\Gamma(1+2\mu)}{\Gamma(\mu)^2\cos^{2\Delta_+}(\frac{t-t'}{2})} \tag{49}$$

The vacuum expectation values between operators on the same and opposite boundaries result (cf. [18])

$$\langle 0|T\mathcal{O}^{\pm}(t)\mathcal{O}^{\pm}(t')|0\rangle = \pm\left(\frac{4^{\Delta_-}i^{2\Delta_-}}{8^{\Delta_+}}\right)\frac{\Gamma(2\mu)}{\Gamma(\mu)^2\sin^{2\Delta_+}(\frac{t-t'}{2})}, \tag{50}$$

$$\langle 0|T\mathcal{O}^{\pm}(t)\mathcal{O}^{\mp}(t')|0\rangle = \mp\left(\frac{8^{\Delta_-}}{4}\right)\frac{\Gamma(1+2\mu)}{\Gamma(\mu)^2\cos^{2\Delta_+}(\frac{t-t'}{2})}. \tag{51}$$

The first line gives the result for operators on the same boundary and has the expected conformal behavior $|t-t'|^{-2\Delta_+}$ when the operators approach each other. The second line corresponding to operators located on different boundaries becomes singular for $t = t' + (2n+1)\pi$, $n \in \mathbb{Z}$; this singularity reflects the existence of causal (null) curves connecting the boundaries and it has been argued that their existence hints at an interaction between the two sets of degrees of freedom $\mathcal{O}^{\pm}$ [19].[10] The observed periodicity in time relates to a peculiar property of AdS; this is the convergence of null geodesics when passing to the universal cover and can be also understood as a consequence of the eigenmodes (46) being equally spaced (see [38–40]).

In the massless case ($\mu = \frac{1}{2}$), the two-point functions take the form

$$\langle 0|T\mathcal{O}^{\pm}(t)\mathcal{O}^{\pm}(t')|0\rangle = \pm\frac{1}{8\pi\sin^2(\frac{t-t'}{2})},$$
$$\langle 0|T\mathcal{O}^{\pm}(t)\mathcal{O}^{\mp}(t')|0\rangle = \mp\frac{1}{4\pi\cos^2(\frac{t-t'}{2})} \tag{52}$$

B. Wormhole

We now turn to the analysis of two-point functions in a wormhole background; this is a spacetime geometry with two conformal boundaries connected through the bulk. We will work with a toy model wormhole, which permits analytical treatment consisting of a static geometry that connects two asymptotically locally AdS regions with base manifolds of the form H^3 or $S^1 \times H^2$, with H^n a n-dimensional (quotiented) hyperbolic space. The geometry does not contain horizons anywhere, and the two asymptotic regions are causally connected. The spacetime was found as a solution of Einstein-Gauss-Bonnet gravity, which in $d+1=5$ dimensions takes the form

$$S_5 = \kappa\int \epsilon_{abcde}\left(R^{ab}R^{cd} + \frac{2}{3l^2}R^{ab}e^c e^d + \frac{1}{5l^4}e^a e^b e^c e^d\right)e^e.$$

Here $R^{ab} = d\omega^{ab} + \omega^a_f\omega^{fb}$ is the curvature two form for the spin connection ω^{ab}, and e^a is the vielbein. The $(d+1)$-dimensional wormhole metric found in [21] reads

$$\begin{aligned} ds^2 &= R^2[-\cosh^2\rho dt^2 + d\rho^2 + \cosh^2\rho d\tilde{\Sigma}^2_{d-1}] \\ &= R^2\left[-\frac{dt^2}{1-x^2} + \frac{dx^2}{(1-x^2)^2} + \frac{d\tilde{\Sigma}^2_{d-1}}{1-x^2}\right] \end{aligned} \tag{53}$$

where $d\tilde{\Sigma}^2_{d-1}$ is a constant negative curvature metric on the compact base manifold $\tilde{\Sigma}_{d-1}$. Note that two disconnected conformal boundaries are located at $x = \pm 1$.

To construct the boundary to bulk propagators $\mathsf{K}^{\pm}$ discussed above, we propose

$$\mathsf{K}^{\pm}(t,x,\theta|t',\theta') = \int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\sum_Q e^{-i\omega(t-t')}Y_Q(\theta)Y^*_Q(\theta')f^{\pm}_{\omega Q}(x) \tag{54}$$

where $Y_Q(\theta)$ are harmonic functions[11] on $\tilde{\Sigma}_{d-1}$. Inserting (54) into the KG Eq. (6), one finds that $f_{\omega Q}$ satisfies

$$(1-x^2)\frac{d^2f^{\pm}_{\omega Q}(x)}{dx^2} + (d-2)x\frac{df^{\pm}_{\omega Q}(x)}{dx} + \left[(\omega^2 - Q^2) - \frac{m^2R^2}{1-x^2}\right]f^{\pm}_{\omega Q}(x) = 0. \tag{55}$$

The solutions to (55) can be written in terms of generalized Legendre polynomials as [22]

$$f^{\pm}_{\omega Q}(x) = (1-x^2)^{d/4}[a^{\pm}_{\omega Q}P^{\mu}_{\nu}(x) + b^{\pm}_{\omega Q}Q^{\mu}_{\nu}(x)] \tag{56}$$

where μ is given by (5) and

$$\nu = \varpi - \frac{1}{2} = \sqrt{\left(\frac{d-1}{2}\right)^2 + \omega^2 - Q^2} - \frac{1}{2}. \tag{57}$$

$a^{\pm}_{\omega Q}$, $b^{\pm}_{\omega Q}$ in (56) are constant coefficients which get fixed when we impose the conditions (31); these are

$$f^{\pm}_{\omega Q}(\pm x_\epsilon) = 1, \qquad f^{\pm}_{\omega Q}(\mp x_\epsilon) = 0. \tag{58}$$

The solutions for (56) satisfying (58) are

[10] This interpretation assumes that the boundary spatial foliations $\Sigma^{(t)}_+$ and $\Sigma^{(t')}_-$ should be identified as being the same Cauchy surface $\tilde{\Sigma}$ of a single base manifold where the dual field theory lives in.

[11] These functions satisfy $\nabla^2_{\tilde{\Sigma}}Y_Q = -Q^2Y_Q$ and a compact manifold without boundaries has $Q^2 \geq 0$. The eigenmodes and eigenvalues for the Laplacian on a *smooth* compact hyperbolic manifold cannot be expressed in closed analytic form and depend on the freely acting discrete subgroup of $SO(d-1,1)$ chosen to perform the quotient. See [41] and references therein for a numerical treatment of the problem.

$$f^+_{\omega Q}(x) = \left(\frac{1-x^2}{1-x_\epsilon^2}\right)^{d/4} \times \frac{P^\mu_\nu(x)Q^\mu_\nu(-x_\epsilon) - P^\mu_\nu(-x_\epsilon)Q^\mu_\nu(x)}{P^\mu_\nu(x_\epsilon)Q^\mu_\nu(-x_\epsilon) - P^\mu_\nu(-x_\epsilon)Q^\mu_\nu(x_\epsilon)}$$
$$f^-_{\omega Q}(x) = \left(\frac{1-x^2}{1-x_\epsilon^2}\right)^{d/4} \times \frac{P^\mu_\nu(x)Q^\mu_\nu(x_\epsilon) - P^\mu_\nu(x_\epsilon)Q^\mu_\nu(x)}{P^\mu_\nu(-x_\epsilon)Q^\mu_\nu(x_\epsilon) - P^\mu_\nu(x_\epsilon)Q^\mu_\nu(-x_\epsilon)}. \tag{59}$$

The possibility for two independent bulk-boundary propagators arises from the fact that the base manifold $\tilde{\Sigma}$ never shrinks to zero size inside the bulk (see (53)) and therefore regularity in the interior imposes no constraint in the solutions (56). Normalizable modes appear for

$$\omega_{nQ} = \pm\sqrt{(\mu + \frac{1}{2} + n)^2 + Q^2 - \left(\frac{d-1}{2}\right)^2}, \qquad n = 0, 1, \ldots,$$
$$\text{and} \quad \frac{b_{\omega Q}}{a_{\omega Q}} = -\frac{2\tan\pi\mu}{\pi}. \tag{60}$$

For these frequencies, the index ν in (57) takes the value $\nu_n = \mu + n$ and the resulting solution becomes normalizable. The two-point functions (35) and (36) between operators $\mathcal{O}_\pm$ on the same boundary and opposite boundaries take the form

$$\langle\psi_{\rm f}|T\mathcal{O}^\pm(t,\theta)\mathcal{O}^\pm(t',\theta')|\psi_{\rm i}\rangle = \pm i\frac{2^{\Delta_-}d}{\pi 2^d}\frac{\Gamma(1-\mu)}{\Gamma(1+\mu)}\sum_Q Y_Q(\theta)Y^*_Q(\theta') \times \int\frac{d\omega}{2\pi}e^{-i\omega(t-t')}\Gamma\left(\frac{1}{2}+\mu-\varpi\right) \times\Gamma\left(\frac{1}{2}+\mu+\varpi\right)\cos(\pi\varpi) \tag{61}$$

$$\langle\psi_{\rm f}|T\mathcal{O}^\pm(t,\theta)\mathcal{O}^\mp(t',\theta')|\psi_{\rm i}\rangle = \pm i\frac{2^{\Delta_-}}{2^{d-1}}\frac{1}{\Gamma(\mu)^2}\sum_Q Y_Q(\theta)Y^*_Q(\theta') \times\int\frac{d\omega}{2\pi}e^{-i\omega(t-t')}\Gamma\left(\frac{1}{2}+\mu-\varpi\right)\Gamma\left(\frac{1}{2}+\mu+\varpi\right). \tag{62}$$

A few comments on these expressions: (i) in the large frequency limit $\varpi \sim \omega$ and the integrands in (61) and (62) coincide with those of AdS_2 (cf. (47) and (48)), (ii) when a Feynman contour is chosen, a time ordered product should be understood on the rhs of (61) and (62) and (iii) although the Gamma functions $\Gamma(\frac{1}{2}+\mu\pm\varpi)$ present two branch cuts at $\omega = \pm\sqrt{Q^2 - (\frac{d-1}{2})^2}$ (see (57)), the product in the integrands (61) and (62) is free of them.

The correlation between operators inserted on opposite boundaries is nonvanishing, and this result has been explained in different ways depending on the context: (i) as the result of being computing the correlator (62) on an entangled state of two noninteracting boundary theories black hole (BH) context [15]) or (ii) as due to an interaction between the theories defined on each of the boundaries (D1/D5 orbifold in [19]). The crucial point in both arguments was the absence/existence of causal connection between the asymptotic regions.

IV. ENTANGLEMENT VS. COUPLING

In this section, we will review several thoughts regarding the interpretation of the results (51) and (62). We would like to address the issue of whether the results (51) and (62) are the consequence of: (i) an interaction between the two dual QFT theories, or (ii) due to the correlators being evaluated on an entangled state, or (iii) both.

A. Entanglement entropy

The entanglement entropy S_A is a nonlocal quantity (as opposed to correlation functions) that measures how two subsystems A and B are correlated. For a d-dimensional QFT, it is defined as the von Neumann entropy of the reduced density matrix ρ_A obtained when we trace out the degrees of freedom inside a $(d-1)$-dimensional spacelike submanifold B which is the complement of A (see [42] for a review).

In [43,44], a proposal was made for a holographic formula for the entanglement entropy of a CFT_d dual to an AdS_{d+1} geometry. It reads

$$\mathsf{S}_A = \frac{\text{Area}(\gamma_A)}{4G_N^{(d+1)}}, \tag{63}$$

where γ_A is a $(d-1)$-dimensional minimal surface in AdS_{d+1} whose boundary $\mathcal{S}$, located at AdS infinity, coincides with that of A; this is $\mathcal{S} = \partial\gamma_{\rm A} = \partial{\rm A}$ and $G_N^{(d+1)}$ and is the Newton constant in AdS_{d+1}. This formula, which assumes the supergravity approximation of the full string theory, has been applied in the AdS_3/CFT_2 setup showing agreement with known 2D CFT results [42].

One can imagine applying a generalization of (63) to the wormhole geometry (53) as follows[12]: the wormhole presents two disconnected spatial regions $\Sigma_\pm$ at which two identical degrees of freedom are supposed to be living on. Imagine constructing a codimension 1 closed surface $\mathcal{S}$ bounding a *small* region B on Σ_-; experience from Wilson loops and brane embeddings show that as B remains small, the minimal surface will be located near $x = -1$. As we gradually increase the size of B, the minimal surface anchored on Σ_- dips deeper into the bulk and in the limit when B occupies all space $B \to \Sigma_-$ the boundary $\mathcal{S}$ collapses and the minimal surface gets localized at the throat $x = 0$ of the geometry, giving a nonzero result

[12]See [45] for an generalization of (63) to the Euclidean wormhole constructed in [10].

$$\mathsf{S}_{\tilde{\Sigma}} = \frac{\mathrm{Area}(\tilde{\Sigma})}{4G_N^{(d+1)}}. \tag{64}$$

This result should be understood as indicating that the quantum state in the dual QFT described by the wormhole geometry is not separable; stated otherwise, the result (62) can be attributed to the wormhole geometry realizing an entangled state on the Hilbert space product $\mathcal{H} = \mathcal{H}_+ \otimes \mathcal{H}_-$ with the entropy (64) resulting from integrating out $\mathcal{H}_-$.

B. Interaction between the dual copies

We will now argue that apart from the entanglement discussed above, the causal contact between the wormhole asymptotic boundaries leads to an interaction term between the two sets of degrees of freedom living at each boundary. In particular, we argue that a coupling between the field theories will exist whenever the asymptotic regions are in causal contact.

We start quoting the partition function on the gravity side for the wormhole geometry in the semiclassical limit; its form is

$$\mathcal{Z}_{\mathrm{gravity}}[\phi(\phi_0^+, \phi_0^-, \mathcal{C})] \sim e^{-(i/2)\int d\mathbf{y}d\mathbf{y}' \phi_0^i(\mathbf{y})\Delta_{ij}(\mathbf{y},\mathbf{y}')\phi_0^j(\mathbf{y}')} \tag{65}$$

where $\mathbf{y} = (t, \theta)$ denote the boundary points and $i, j = +, -$ refer to the asymptotic boundaries where prescribed data $\phi_0^\pm$ is given, the contour $\mathcal{C}$ fixes the normalizable solution in the bulk and the expression for Δ_{ij} is given by (34). According to the GKPW prescription, the partition function (65) is the generating functional for $\mathcal{O}^\pm$ correlators; this is

$$\mathcal{Z}_{\mathrm{gravity}}[\phi(\phi_0^+, \phi_0^-, \mathcal{C})] = \langle \psi_f | T e^{i\int d\mathbf{y}\phi_0^+(\mathbf{y})\mathcal{O}^+(\mathbf{y}) + i\int d\mathbf{y}\phi_0^-(\mathbf{y})\mathcal{O}^-(\mathbf{y})} | \psi_i \rangle_{\mathrm{QFT}}, \tag{66}$$

where the observables $\mathcal{O}^\pm$ should be constructed as local functionals of the fundamental fields $\Psi_\pm$ living on each boundary. These fields are assumed to describe independent degrees of freedom: $[\mathcal{O}^+, \mathcal{O}^-] = 0$ on the same spatial slice (see footnote 10).

Consider the simplest situation corresponding to choosing $\mathcal{C}$ to be the Feynman contour; that is, we are computing the vacuum to vacuum transition amplitude on the field theory side. The rhs in (66) can be written as

$$\langle \psi_0 | T e^{i\int d\mathbf{y}\phi_0^+(\mathbf{y})\mathcal{O}^+(\mathbf{y}) + i\int d\mathbf{y}\phi_0^-(\mathbf{y})\mathcal{O}^-(\mathbf{y})} | \psi_0 \rangle_{\mathrm{QFT}} = \mathrm{Tr}[\rho_{\psi_0} T e^{i\int d\mathbf{y}\phi_0^+(\mathbf{y})\mathcal{O}^+(\mathbf{y}) + i\int d\mathbf{y}\phi_0^-(\mathbf{y})\mathcal{O}^-(\mathbf{y})}], \tag{67}$$

where the trace operation is performed over a complete set of states of the dual field theory Hilbert space and $\rho_{\psi_0} = |\psi_0\rangle\langle\psi_0|$ is the density matrix associated to the vacuum wormhole state. This vacuum state belongs to the Hilbert space $\mathcal{H} = \mathcal{H}_+ \otimes \mathcal{H}_-$ and according to the arguments reviewed in the last subsection, it is not separable as a single tensor product $|\psi_0\rangle \neq |\psi_+\rangle \otimes |\psi_-\rangle$.

To analyze the possibility of interaction between the fields living at each boundary we consider the system at finite temperature $T = \beta^{-1}$. The absence of singularities in the Euclidean continuation implies that the wormhole spacetime can be in equilibrium with a thermal reservoir of arbitrary temperature, or stated otherwise, the periodicity in Euclidean time is arbitrary. At thermal equilibrium the field theory state is described by the Boltzmann distribution $\rho_\beta = e^{-\beta H}$, with $H = H_+[\Psi_+] + H_-[\Psi_-] + H_{\mathrm{int}}[\Psi_+, \Psi_-]$ the dual field theory Hamiltonian, and H_{int} a possible coupling between the two identical sets of degrees of freedom $\Psi_\pm$.

Let us see now that an interaction term H_{int} should be present in the Hamiltonian in order to avoid a contradiction. The argument goes as follows: finite temperature correlation functions on the field theory side are obtained from the Euclidean rotation of (67), which reads

$$\langle e^{-\int d\mathbf{y}\phi_0^+(\mathbf{y})\mathcal{O}^+(\mathbf{y}) - \int d\mathbf{y}\phi_0^-(\mathbf{y})\mathcal{O}^-(\mathbf{y})} \rangle_\beta = \mathrm{Tr}[\rho_\beta e^{-\int d\mathbf{y}\phi_0^+(\mathbf{y})\mathcal{O}^+(\mathbf{y}) - \int d\mathbf{y}\phi_0^-(\mathbf{y})\mathcal{O}^-(\mathbf{y})}]; \tag{68}$$

the field theory is defined on an Euclidean manifold with $S^1_\beta \times \tilde{\Sigma}$ topology (see footnote 10). The AdS/CFT proposal equates this expression to the Euclidean continuation of the lhs in (66), the result is

$$\mathrm{Tr}[e^{-\beta H} e^{-\int d\mathbf{y}\phi_0^+(\mathbf{y})\mathcal{O}^+(\mathbf{y}) - \int d\mathbf{y}\phi_0^-(\mathbf{y})\mathcal{O}^-(\mathbf{y})}] = \mathcal{Z}_{E\mathrm{gravity}}[\phi(\phi_0^+, \phi_0^-)] \sim e^{-S_E[\phi_0^+, \phi_0^-]}. \tag{69}$$

On the right, $S_E[\phi_0^+, \phi_0^-]$ corresponds to the on-shell scalar field action on the Euclidean wormhole background, with time compactified on a circle of radius β. Upon Euclidean rotation and time compactification, the resulting geometry is cylinder-like with topology $S^1_\beta \times I \times \tilde{\Sigma}$; the two boundaries at which one imposes the boundary data $\phi_0^\pm$ are located at the endpoints $x_\pm$ of the finite interval I.[13] Note that in the Euclidean context the solution in the interior (bulk) is completely specified by the boundary data, no normalizable solutions exist, and rotating the Feynman contour to the imaginary time axis is straightforward and leads to a nonsingular solution. The explicit expression for the rhs of (69) is

$$\mathcal{Z}_{E\mathrm{gravity}}[\phi(\phi_0^+, \phi_0^-)] \sim e^{-(1/2)\int d\mathbf{y}d\mathbf{y}' \phi_0^i(\mathbf{y})\tilde{\Delta}_{ij}(\mathbf{y},\mathbf{y}')\phi_0^j(\mathbf{y}')}, \tag{70}$$

where $\tilde{\Delta}_{ij}$ denotes the Euclidean rotated bulk-boundary propagators (34). Finally, from (69) and (70) we obtain for the gauge/gravity prescription for a wormhole at finite temperature

[13] Note the similarity with the geometry studied in [10].

$$\mathrm{Tr}[e^{-\beta H}e^{-\int d\mathbf{y}\phi_0^+(\mathbf{y})\mathcal{O}^+(\mathbf{y})-\int d\mathbf{y}\phi_0^-(\mathbf{y})\mathcal{O}^-(\mathbf{y})}] \sim e^{-(1/2)\int d\mathbf{y}d\mathbf{y}'\phi_0^i(\mathbf{y})\tilde{\Delta}_{ij}(\mathbf{y},\mathbf{y}')\phi_0^j(\mathbf{y}')}. \tag{71}$$

This is the key formula for our argument because if one now assumes that the degrees of freedom Ψ_+, Ψ_- are decoupled, this is

$$H[\Psi_+,\Psi_-] = H_+[\Psi_+] + H_-[\Psi_-], \tag{72}$$

then $\rho_\beta = e^{-\beta H_+}e^{-\beta H_-}$, and the lhs of (71) factorizes into a product of two quantities: one depending on ϕ_0^+ and one depending on ϕ_0^-.[14] However, the gravity computation does not factorize because of the nonvanishing $\tilde{\Delta}_{\pm\mp}$ terms. We interpret this result as manifesting the existence of a nontrivial coupling $H_{\rm int}$ between the two dual degrees of freedom $\Psi_\pm$: the field theory dual to the wormhole geometry contains a nontrivial coupling term between the two boundary degrees of freedom Ψ_+ and Ψ_-.

C. Highlights and applications

The outcome of the above observations is that the wormhole geometry encodes the description of a dual field theory with two copies of fundamental fields in interaction. Moreover, the quantum state described by the wormhole is entangled. In particular, the Euclidean continuation can be seen as prescription to *separate*, in the dual field theory, entanglement effects from possible interaction terms $H_{\rm int}$. A nonvanishing Euclidean two-point function between operators located at different boundaries must be interpreted as originated from an interaction term $H_{\rm int}$, rather than an entanglement effect. In the following we confront this point of view with two other relevant geometries appearing in the literature: the AdS_{1+1} geometry and the eternal AdS black hole [15].

AdS_{1+1} *geometry*: The analysis of the AdS_{1+1} background (39) is entirely analogous to the one performed for the wormhole case above. The Euclidean section of the global metric (39) has two boundaries upon compactifying the time direction and therefore two boundary conditions $\phi_0^\pm$ are required in the semiclassical limit (see (65)). The arguments leading to (71) apply with the Euclidean correlation functions $\tilde{\Delta}_{ij}$ immediately obtained from the Lorentzian formulas (51). We therefore conclude from the fact that $\tilde{\Delta}_{\pm\mp} \neq 0$ that AdS_{1+1} is dual to a conformal quantum mechanics composed of two interacting sectors.

Eternal AdS black hole: The crucial difference between the wormhole (53) and the maximally extended AdS-BH gravity solution is well known [15]: upon performing the Euclidean continuation of the AdS-BH solution, the existence of a horizon in Lorentzian signature generates a conical singularity in the Euclidean manifold that can only be avoided by demanding a precise periodicity in Euclidean time. The resulting Riemannian geometry has inevitably only one asymptotic boundary, and therefore requires imposing only one asymptotic boundary data ϕ_0; this indicates the existence of a single set of degrees of freedom Ψ and a unique Bulk-Boundary propagator $\tilde{\Delta}$. The system in equilibrium with a thermal bath of fixed temperature (determined by the BH mass) has a generating function that reads

$$\mathrm{Tr}[e^{-\beta H}e^{-\int d\mathbf{y}\phi_0(\mathbf{y})\mathcal{O}(\mathbf{y})}] \sim e^{-(1/2)\int d\mathbf{y}d\mathbf{y}'\phi_0(\mathbf{y})\tilde{\Delta}(\mathbf{y},\mathbf{y}')\phi_0(\mathbf{y}')}. \tag{73}$$

The real time (maximally extended BH) solution was analyzed and interpreted in the AdS/CFT context in [15]. The second *causally disconnected* boundary, present in Lorentzian signature, was understood as supporting the TFD partners needed for obtaining a thermal state upon their integration and lead to a doubling of the Hilbert space as $\mathcal{H} = \mathcal{H}_+ \otimes H_-$. We stress the fact that the second set of degrees of freedom is causally disconnected from the original zero temperature set; although they appear in Lorentzian signature and give rise to a nontrivial Δ_{ij}, they are fictitious from a physical point of view. The causal disconnection between the boundaries relates to the two-point function $\Delta_{\pm\mp}$ never becoming singular.

V. DISCUSSION

We have reviewed the GKPW prescription in the Lorentzian context relating the ambiguity in adding normalizable modes to (3) to the integration contour $\mathcal{C}$ required to bypass the integrand singularities in (27) and (61) arising from the existence of normalizable modes. To compute Lorentzian quantities, one needs to fix a reference contour $\mathcal{C}_{\rm ref}$ and the two sensible choice are retarded or Feynman. These choices relate on the QFT side to being computing either response or correlation functions. When choosing the Feynman path as reference contour, any given contour $\mathcal{C}$ differs from $\mathcal{C}^F$ by contributions from encircling poles, these encircled poles fix the initial and final states that appear in the correlation functions (see (16)).

In Sec. III, we extended the GKPW prescription to spacetimes with more than one asymptotic timelike boundaries; in particular, we studied the simplest two boundaries case. We proposed to write the bulk field in terms of the two independent asymptotic boundary values and as a toy example we applied the construction to AdS_{1+1} reobtaining previous results, we afterwards computed the two-point correlation functions for the Einstein-Gauss-Bonnet wormhole (53). At this point, we must emphasize that the principal difference of our method with the one performed in [19], for an AdS_3 orbifold, consists in that we considered the boundary values $\phi_0^\pm$ of

[14] Note that in the absence of interaction $[\mathcal{O}_+,\mathcal{O}_-] = 0$ no matter if the operators insertion points are spacelike/timelike separated.

the bulk scalar field as independent quantities; moreover, we explicitly showed the possibility of constructing two boundary-bulk propagators $\mathsf{K}^{\pm}$ (their boundary conditions were given in (31)). The construction performed in [19] showed a relation between the boundary values $\phi_0^{\pm}$ and this was understood as indicating that the $\pm$ sources for the dual field theory are turned on in a correlated way. A question remained as whether the two sets of data are independent or redundant in the dual formulation, on the other hand another question is the origin of the nonzero result for operators located at different boundaries ($\Delta_{\pm\mp}$) this could either be due to entanglement or interactions or both.

In Sec. IV, we applied the ideas on holographic entanglement entropy developed in[44] to the wormhole geometry. The nonvanishing of $\mathsf{S}_{\tilde{\Sigma}}$ obtained for the degrees of freedom living on opposite boundaries suggests that the wormhole should be understood as representing an entangled state in $\mathcal{H} = \mathcal{H}_+ \otimes \mathcal{H}_-$. On the other hand, the causal connection between the boundaries suggests that a coupling might exist as well. To attack this issue we consider putting the wormhole system in contact with a thermal bath upon Euclideanization; the resulting geometry still has two boundaries connected through the bulk, this indicated, to avoid a contradiction, that the dual QFT consists of two copies $\mathcal{H}_{\pm}$ in interaction. In summary, the number of disconnected boundaries of the Euclidean section determines the number of physical degrees of freedom.

We would like to emphasize finally the implications of this approach on quantum gravity, which could be seen as one of our main motivations. This subject has been discussed in different contexts in the last years and referred to as emergent spacetime [46]. In this sense, we showed how important topological and causal properties (connectivity) of the spacetime geometry are encoded in the QFT action, and that part of this information is not only in the ground state but in its interacting structure. We hope this conclusion might contribute to the construction of rules towards a geometry engineering.

We should mention that in the presence of interactions one needs to address the way the points on opposite boundaries are identified. A first approach to this problem is to identify the points $\mathbf{y}, \mathbf{y}'$ in configuration space at which $\Delta_{\pm\mp}$ diverges and study the consistency of identifying them, this is currently under investigation and will be reported elsewhere.

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[1] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998).
[2] S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, Phys. Lett. B **428**, 105 (1998).
[3] E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998).
[4] E. Witten, Adv. Theor. Math. Phys. **2**, 505 (1998).
[5] D. Z. Freedman, S. S. Gubser, K. Pilch, and N. P. Warner, Adv. Theor. Math. Phys. **3**, 363 (1999).
[6] H. Lin, O. Lunin, and J. M. Maldacena, J. High Energy Phys. 10 (2004) 025.
[7] J. de Boer, L. Maoz, and A. Naqvi, arXiv:hep-th/0407212.
[8] G. J. Galloway, K. Schleich, D. Witt, and E. Woolgar, Phys. Lett. B **505**, 255 (2001).
[9] K. Skenderis and B. C. van Rees, Commun. Math. Phys. **301**, 583 (2010).
[10] J. M. Maldacena and L. Maoz, J. High Energy Phys. 02 (2004) 053.
[11] N. Arkani-Hamed, J. Orgera, and J. Polchinski, J. High Energy Phys. 12 (2007) 018.
[12] E. Witten and S. T. Yau, Adv. Theor. Math. Phys. **3**, 1635 (1999).
[13] M.-l. Cai and G. J. Galloway, Adv. Theor. Math. Phys. **3**, 1769 (1999).
[14] A. Buchel, Phys. Rev. D **70**, 066004 (2004).
[15] J. M. Maldacena, J. High Energy Phys. 04 (2003) 021.
[16] P. Martin and J. Schwinger, Phys. Rev. **115**, 1342 (1959); J. Schwinger, J. Math. Phys. (N.Y.) **2**, 407 (1961); K. T. Mahanthappa, Phys. Rev. **126**, 329 (1962); P. M. Bakshi and K. T. Mahanthappa, J. Math. Phys. (N.Y.) **4**,1 (1963); Keldysh, JETP **47**, 1515 (1964); Y. Takahashi and H. Umezawa, Collective Phenomena **2**, 55 (1975).
[17] E. Barnes, D. Vaman, C. Wu, and P. Arnold, Phys. Rev. D **82**, 125042 (2010).
[18] T. Azeyanagi, T. Nishioka, and T. Takayanagi, Phys. Rev. D **77**, 064005 (2008).
[19] V. Balasubramanian, A. Naqvi, and J. Simon, J. High Energy Phys. 08 (2004) 023.
[20] D. Lovelock, J. Math. Phys. (N.Y.) **12**, 498 (1971).
[21] G. Dotti, J. Oliva, and R. Troncoso, Phys. Rev. D **75**, 024002 (2007); **76**, 064038 (2007).
[22] D. H. Correa, J. Oliva, and R. Troncoso, J. High Energy Phys. 08 (2008) 081.
[23] M. Ali, F. Ruiz, C. Saint-Victor, and J. F. Vazquez-Poritz, Phys. Rev. D **80**, 046002 (2009).
[24] C. P. Herzog and D. T. Son, J. High Energy Phys. 03 (2003) 046.
[25] V. Balasubramanian, P. Kraus, and A. E. Lawrence, Phys. Rev. D **59**, 046003 (1999).
[26] V. Balasubramanian, P. Kraus, A. E. Lawrence, and S. P. Trivedi, Phys. Rev. D **59**, 104021 (1999).

[27] V. Balasubramanian, S. B. Giddings, and A. E. Lawrence, J. High Energy Phys. 03 (1999) 001.
[28] D. Marolf, J. High Energy Phys. 05 (2005) 042.
[29] K. Skenderis and B. C. van Rees, Phys. Rev. Lett. **101**, 081601 (2008); J. High Energy Phys. 05 (2009) 085.
[30] P. Breitenlohner and D. Z. Freedman, Phys. Lett. **115B**, 197 (1982); Ann. Phys. (N.Y.) **144**, 249 (1982); L. Mezincescu and P. K. Townsend, Ann. Phys. (N.Y.) **160**, 406 (1985).
[31] E. D'Hoker and D. Z. Freedman, arXiv:hep-th/0201253.
[32] J. McGreevy, Lectures on String theory, MIT Open Course, http://ocw.mit.edu/courses/physics/8-821-string-theory-fall-2008/lecture-notes/.
[33] U. H. Danielsson, E. Keski-Vakkuri, and M. Kruczenski, J. High Energy Phys. 01 (1999) 002.
[34] D. T. Son and A. O. Starinets, J. High Energy Phys. 09 (2002) 042.
[35] C. P. Herzog and D. T. Son, J. High Energy Phys. 03 (2003) 046.
[36] L. Fidkowski, V. Hubeny, M. Kleban, and S. Shenker, J. High Energy Phys. 02 (2004) 014.
[37] D. Z. Freedman, S. D. Mathur, A. Matusis, and L. Rastelli, Nucl. Phys. **B546**, 96 (1999).
[38] S. J. Avis, C. J. Isham, and D. Storey, Phys. Rev. D **18**, 3565 (1978).
[39] I. R. Klebanov and E. Witten, Nucl. Phys. **B556**, 89 (1999); E. Witten, arXiv:hep-th/0112258.
[40] S. W. Hawking and G. F. R. Ellis, *The Large Scale Structure of Space-Time* (Cambridge University Press, Cambridge, England, 1973).
[41] N. J. Cornish and D. N. Spergel, arXiv:math.DG/9906017.
[42] T. Nishioka, S. Ryu, and T. Takayanagi, J. Phys. A **42**, 504008 (2009).
[43] S. Ryu and T. Takayanagi, Phys. Rev. Lett. **96**, 181602 (2006).
[44] S. Ryu and T. Takayanagi, J. High Energy Phys. 08 (2006) 045.
[45] V. E. Hubeny, M. Rangamani, and T. Takayanagi, J. High Energy Phys. 07 (2007) 062.
[46] N. Seiberg, arXiv:hep-th/0601234.