

Hubbard ladders in a magnetic field

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The behavior of a two leg Hubbard ladder in the presence of a magnetic field is studied by means of Abelian bosonization. We predict the appearance of a new (doping dependent) plateau in the magnetization curve of a doped 2–leg spin ladder in a wide range of couplings. We also discuss the extension to N –leg Hubbard ladders.

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In the past few years there has been intense research activity, both experimental and theoretical, on ladder systems [1]. These systems interpolate between 1d and 2d and, due to the strong quantum fluctuations enhanced by the low dimensionality, exhibit surprising and exotic behaviors, such as, for instance, the so-called even-odd effect.

A feature of spin ladders which attracted considerable interest is the occurrence of plateaux in their magnetization curves. One of the main observations has been the *rationality* of the value of the magnetization at which plateaux can appear [2–4]. However, it has been recently pointed out that plateaux can also appear at *irrational* values of the magnetization. This happens for example in periodically modulated doped Hubbard chains [5], but this scenario is believed to be generic in doped systems. A novel and interesting feature of the plateaux predicted in [5] is the fact that their position depends on the filling and therefore can be moved down to low magnetization values by means of doping. This makes doping dependent plateaux potentially observable in experiments at low magnetic fields. Previous studies revealed doping dependent plateaux in other systems, such as the one-dimensional Kondo lattice model [6] and an integrable spin- S doped $t - J$ chain [7].

The purpose of this letter is to investigate the conditions of occurrence of magnetization plateaux in doped ladder systems. We have focussed on the case of coupled Hubbard chains, the spin ladders being particular limits of Hubbard ladders at half filling. Various models of coupled chains have been thoroughly investigated in the last decade, but comparatively little attention has been devoted to the effects of a magnetic field on their properties. In [8] (see also [9]) it has been proven that in a N -leg Hubbard ladder gapless excitations exist at momentum $k = 2\pi N n_{\uparrow, \downarrow}$ if the commensurability conditions $N n_{\uparrow, \downarrow} \in \mathbb{Z}$ are not satisfied [10]. This holds even if the low energy excitations cannot be identified as the usual separated charge and spin excitations. According to this result, magnetization plateaux at $m \neq 0$ can only occur

if at least one of the conditions $N n_{\uparrow, \downarrow} = \frac{N}{2}(n \pm m) \in \mathbb{Z}$ is satisfied, which considerably restricts the window to find new plateaux. In order to prove the presence of a plateau one has to show that there is a finite gap in the total spin sector, since the above requirement just provides a necessary condition for the occurrence of a non trivial $m \neq 0$ plateau. We have found that when the conditions

$$N n_{\uparrow, \downarrow} \equiv \frac{N}{2}(n \pm m) \in \mathbb{Z} \quad (1)$$

are simultaneously satisfied, or when only one of them is satisfied and the doping kept fixed, a plateau can indeed occur for some range of parameters. In the last situation, one has a doping dependent plateau which, for the specific case of $N = 2$, is predicted to be present for a wide range of couplings.

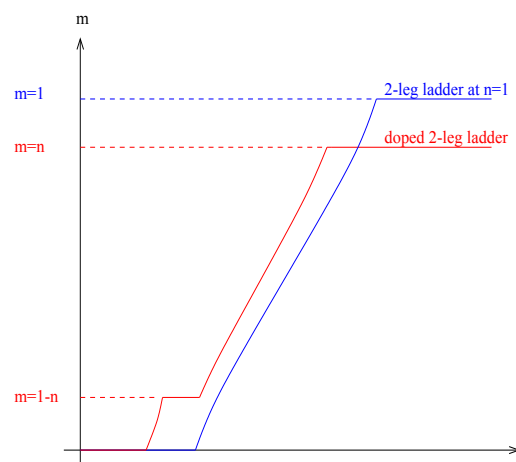


FIG. 1. Schematic magnetization curve of a 2-leg Hubbard ladder at $n = 1$ (half-filling) and $n \neq 1$. The width of the plateaux depends on $t_{\perp}/t, U/t, n$.

Our main result is schematically summarized in Fig. 1, which shows the expected magnetization curves of a 2-leg Hubbard ladder for both the doped and undoped cases. According to our analysis, apart from the known plateau

at $m = 0$, plateaux should also occur at $m = \pm(1 - n)$. These could be observed experimentally by doping any system described by a 2-leg ladder. Possible candidates are notably the spin- $\frac{1}{2}$ ladder compounds SrCu_2O_3 [11] and $\text{Cu}_2(\text{C}_5\text{H}_{12}\text{N}_2)_2\text{Cl}_4$ (which has been studied under high magnetic fields [12]), or some quasi-one-dimensional spin- $\frac{1}{2}$ organic system. These materials are often difficult to dope. Nevertheless, doping has been achieved for the ladder material $\text{Sr}_{0.4}\text{Ca}_{13.6}\text{Cu}_{24}\text{O}_{41.84}$ under high pressure [13] (in the latter, superconductivity was also observed). Another class of interesting candidates are single wall carbon nanotubes [14], whose low energy physics is also described by a model equivalent to two coupled spin-1/2 fermionic chains away half filling [15].

Let us turn now to our bosonization analysis (see *e.g.* [16]). We start from the 2-leg Hubbard ladder, with the following lattice Hamiltonian:

$$H = H_I + H_{II} + \tilde{\lambda}_c \sum_x n_x^I n_x^{II} + \tilde{\lambda}_s \sum_x \mathbf{S}_x^I \mathbf{S}_x^{II} + \tilde{\lambda} \sum_x (c_{x\uparrow}^{I\dagger} c_{x\downarrow}^{I\dagger} c_{x\uparrow}^{II} c_{x\downarrow}^{II} + c_{x\uparrow}^{II\dagger} c_{x\downarrow}^{II\dagger} c_{x\uparrow}^I c_{x\downarrow}^I), \quad (2)$$

where H_i ($i = I, II$) is the usual Hubbard Hamiltonian:

$$H_i = -t \sum_{x,\alpha} (c_{x+1,\alpha}^{i\dagger} c_{x,\alpha}^i + H.c.) + U \sum_x n_{x,\uparrow}^i n_{x,\downarrow}^i + \mu_i \sum_x (n_{x,\uparrow}^i + n_{x,\downarrow}^i) - \frac{h}{2} \sum_x (n_{x,\uparrow}^i - n_{x,\downarrow}^i), \quad (3)$$

$c_{x,\alpha}^{i\dagger}, c_{x,\alpha}^i$ are electron creation and annihilation operators on site x of chain i , α is the spin index, $n_{x,\alpha}^i = c_{x,\alpha}^{i\dagger} c_{x,\alpha}^i$. The index $i = I, II$ can be considered either as a chain index or as a band index. Accordingly, the Hamiltonian (2) corresponds to different regimes of coupled chains of interacting electrons. These systems have been largely studied analytically, for different range of parameters [17]–[25], and numerically [26–28].

Eq. (2) describes two identical chains with standard Hubbard Hamiltonians $H_{I,2}(t, U_0, \mu_0, h)$ with small intrachain repulsion U_0 , coupled by a direct hopping term:

$$H_{int} = -t_\perp \sum_{x,\alpha} (c_{x,\alpha}^{1\dagger} c_{x,\alpha}^2 + H.c.), \quad (4)$$

where t_\perp is the interchain hopping amplitude. To see this, one simply has to change variables to the bonding and anti-bonding basis:

$$s_x = \frac{c_x^1 + c_x^2}{\sqrt{2}}; \quad a_x = \frac{c_x^1 - c_x^2}{\sqrt{2}}. \quad (5)$$

Rewriting $H_1 + H_2 + H_{int}$ in this basis and identifying the band indices s and a with the indices I and II , one recovers the Hamiltonian (2) with $\mu_{I,II} = \mu \mp t_\perp$, $U = U_0/2$, $4\tilde{\lambda}_c = -\tilde{\lambda}_s = -2\tilde{\lambda} = U_0$. Thus we have to deal with two coupled inequivalent effective Hubbard chains.

On the other hand, for $U_0 \gg t_\perp$ and close to half filling, (namely, for strong repulsive interactions in the individual chains), the effect of band splitting is suppressed due to the requirement of avoiding double on-site occupancy. In this situation it is more appropriate to consider couplings directly in terms of the original degrees of freedom of the two individual chains; then, I and II have to be identified with the chain index, leading us to two equivalent chains coupled by density-density and spin-spin interactions, as in (2) but with $\tilde{\lambda} = 0$ and $\tilde{\lambda}_{c,s} \sim t_\perp^2/U$.

We are now ready to treat the two different situations at once, by studying the Hamiltonian (2), which contains the two limits described above. The $m = 0$ case has been largely studied in the literature [17]–[28]. The main conclusions in that case are that the spin gap present at half filling survives upon doping, although smaller in magnitude, and the appearance of quasi-long-range superconducting pairing correlations. The main effect of the magnetic field on the single chain is to shift by opposite amounts the up and down Fermi momenta, whose difference is proportional to the magnetization, $k_\perp \equiv k_\uparrow - k_\downarrow = \pi m$. As a first approximation, we can neglect other effects, such as the mixing of spin and charge degrees of freedom, and the dependence of the scaling dimensions of the operators on the magnetic field [29]. Note that the calculations can also be handled straightforwardly without these approximations, but equations become cumbersome (see for example [5]).

In order to write the bosonized expressions for the two cases for $m \neq 0$ we have to define the Fermi momenta $k_{\uparrow,\downarrow}^{I,II}$ separately for each band (or chain) and linearize the dispersion relations around them. We assume that interactions do not change significantly the free Fermi momenta. In the process of bosonization we obtain operators containing oscillating factors made of various combinations of the k_α^i . In the case of inequivalent chains, most of them will be commensurate only if specific conditions depending on t_\perp are satisfied. We will neglect in the following these non-generic situations and will take into account only operators that can be commensurate independently of the value of t_\perp . Moreover, the operator responsible for the $m = 0$ plateau is only commensurate for that value of m and hence it will not enter in the following discussion.

i) Equivalent chains

In this case $k_\alpha^I = k_\alpha^{II}$, I, II represent chain indices and we use the notation $I \equiv 1, II \equiv 2$. The continuum Hamiltonians H_i are written as

$$H_i = \sum_{\nu=c,s} \frac{v_\nu}{2} \int dx [K_\nu (\partial_x \Theta_\nu^i)^2 + K_\nu^{-1} (\partial_x \Phi_\nu^i)^2], \quad (6)$$

where $v_{c,s}, K_{c,s}$ are the velocities and the effective Luttinger parameters for charge and spin degrees of freedom. Away from half-filling ($n \neq 1$) and for non zero magnetization [30]), the effective interaction Hamiltonian reads

$$\begin{aligned}
\mathcal{H}_{int} = & \lambda_c \partial_x \Phi_c^1 \partial_x \Phi_c^2 + \frac{\lambda_s}{4} \partial_x \Phi_s^1 \partial_x \Phi_s^2 + \\
& - \lambda_1 \left(-\cos[\sqrt{4\pi}\Phi_c^-] \cos[\sqrt{4\pi}\Phi_s^-] + \right. \\
& + \cos[2(k_\downarrow^1 + k_\uparrow^2)x - \sqrt{8\pi}\Phi_\uparrow^+] + \\
& + \cos[2(k_\uparrow^1 + k_\downarrow^2)x - \sqrt{8\pi}\Phi_\uparrow^+] \left. \right) + \\
& + \lambda_2 \cos[\sqrt{4\pi}\Phi_c^-] \cos[\sqrt{4\pi}\Theta_s^-], \quad (7)
\end{aligned}$$

where $\Phi_{c/s}^i = \frac{1}{\sqrt{2}}(\Phi_\uparrow^i \pm \Phi_\downarrow^i)$, and $\Phi^\pm = \frac{1}{\sqrt{2}}(\Phi^1 \pm \Phi^2)$ [31]. We have kept only the more relevant operators for the case of large U .

The bosonic bilinear terms can be absorbed in the kinetic part of the Hamiltonian by moving to the \pm basis defined above. As a consequence, the velocities and more importantly the effective Luttinger parameters are renormalized in the following way:

$$K_c \rightarrow K_c^\pm = K_c(1 \pm \lambda_c K_c/v_c)^{-1/2}, \quad (8)$$

$$K_s \rightarrow K_s^\pm = K_s(1 \pm \lambda_s K_s/4v_s)^{-1/2}. \quad (9)$$

From the above equations we see that in the charge sector, the Luttinger parameter for the total charge field (symmetric combination) is slightly reduced with respect to K_c whereas the one for the relative charge (antisymmetric combination) is slightly increased (we assume $\lambda_c > 0$). Let us recall that K_c decreases from 1 for $U = 0$ to 1/2 for infinite repulsion, thus $K_c^+ < 1$. We can repeat the analysis above for the spin sector and the scaling dimensions of the perturbing operators in eq. (7) at zero loop are then given by:

$$\cos[\sqrt{4\pi}\Phi_c^-] \cos[\sqrt{4\pi}\Phi_s^-] \rightarrow K_c^- + K_s^-, \quad (10)$$

$$\cos[\sqrt{8\pi}\Phi_{\uparrow,\downarrow}^+] \rightarrow K_c^+ + K_s^+, \quad (11)$$

$$\cos[\sqrt{4\pi}\Phi_c^-] \cos[\sqrt{4\pi}\Theta_s^-] \rightarrow K_c^- + 1/K_s^-. \quad (12)$$

Due to the double cosine terms in eq. (7) we conclude that both spin and charge antisymmetric sectors are massive. Let us now consider the symmetric sectors, which are affected by the terms in the third and fourth lines of eq. (7). These operators are commensurate only if conditions (1) are satisfied. These appear to be exactly the same conditions we found in the dimerized Hubbard chain for the appearance of plateaux [5]. When the two conditions are simultaneously satisfied we expect both Φ_s^+ and Φ_c^+ to be massive. Indeed, both K_c^+ and K_s^+ are decreased with respect to their values in the absence of interchain coupling, and we can conclude that the operators, when commensurate, are relevant [32]. When instead only one of these conditions is satisfied, let us say the one with the + sign, only the field Φ_\uparrow^+ gets a relevant cosine interaction. Then following the analysis in [5] we can conclude that a magnetization plateau occurs at $m = 1 - n$ if the total density is kept fixed.

ii) Inequivalent chains

In this case, I, II are band indices. When $t_\perp > 2t$, the bands are splitted and the situation is completely analogous to one found in the doped p -merized Hubbard chain we treated in [5]. Following the same lines, we can prove that a plateau occurs when $2n_{\uparrow,\downarrow} \in \mathbb{Z}$. Notice that in this particular regime the plateau at $m = 0$ is no longer present. When $t_\perp \sim t$, non trivial processes between bands are allowed and in this case the effective interaction Hamiltonian away from half-filling (and at $m \neq 0$) reads

$$\begin{aligned}
\mathcal{H}_{int} = & \lambda_c \partial_x \Phi_c^I \partial_x \Phi_c^{II} + \frac{\lambda_s}{4} \partial_x \Phi_s^I \partial_x \Phi_s^{II} + \\
& - \lambda_1 \left(\cos[2(k_\downarrow^I + k_\uparrow^{II})x - \sqrt{8\pi}\Phi_\downarrow^+] + \right. \\
& + \cos[2(k_\uparrow^I + k_\downarrow^{II})x - \sqrt{8\pi}\Phi_\uparrow^+] \left. \right) + \\
& + \lambda_3 \cos[\sqrt{4\pi}\Theta_s^-] \cos[\sqrt{4\pi}\Theta_c^-]. \quad (13)
\end{aligned}$$

In this expression, we have neglected all terms containing combinations of k_α^i depending explicitly on t_\perp . The last marginal term in eq. (13) is generated radiatively. The derivative terms lead to a renormalization of the Luttinger parameters as in eqs. (8-9).

As in the strong U limit, when $2n_{\uparrow,\downarrow} \in \mathbb{Z}$, the second and third line of (13) become commensurate. We can now repeat the analysis concerning the relevance of these operators. Their dimensions are determined by the forward scattering terms which can be computed at first order in $U \ll t, t_\perp$. It is straightforward to see that $K_{c,s}^+$ is renormalized from 1 to $K_{c,s}^+ < 1$, which makes the term $\cos[\sqrt{8\pi}\Phi_\uparrow^+]$ (respectively $\cos[\sqrt{8\pi}\Phi_\downarrow^+]$) marginally relevant. From this and the previous analysis, we can therefore conclude that both in weak and in strong coupling regime of the 2-leg Hubbard ladder, a magnetization plateau is expected when $2n_{\uparrow,\downarrow} \in \mathbb{Z}$, and the magnetization curve at density $n < 1$ should have the schematic form depicted in Fig. 1.

It is interesting to analyze the behaviour of the diagonal components of the charge density wave (CDW) and superconducting pairing (SC) operators at a doping dependent plateau, as it occurs *e.g.* for $m = 1 - n$. In the case of equivalent chains the behaviour of the correlators of such order parameters depends on which operator, between the first and the last double cosine terms in eq. (7) dominates. When the first term dominates, one could find among diagonal SC and CDW operators [33]

$$\begin{aligned}
\mathcal{O}_{CDW,\downarrow}^\pm & \sim e^{i\sqrt{2\pi}\Phi_\uparrow^+} \left(e^{i\sqrt{2\pi}\Phi_\downarrow^-} \pm e^{-i\sqrt{2\pi}\Phi_\downarrow^-} \right), \\
\mathcal{O}_{SC\downarrow}^\pm & \sim e^{i\sqrt{2\pi}\Theta_\uparrow^+} \left(e^{i\sqrt{2\pi}\Phi_\downarrow^-} \mp e^{-i\sqrt{2\pi}\Phi_\downarrow^-} \right), \quad (14)
\end{aligned}$$

algebraic decaying correlators. On the contrary, when it is the last one which dominates, then all correlators decay exponentially. One can argue that varying the values of λ_s and the magnetization, both situations could be achieved, but this requires a more detailed analysis of

the Bethe Ansatz equations in the presence of a magnetic field. For the case of inequivalent chains the analysis is similar but more involved, and will be discussed elsewhere [34]. However, we find also in this case indications for quasi-long-range order of the pairing order parameter.

The generalization of the above analysis to N -leg Hubbard ladders can be performed along the same lines, and the presence of a non-trivial plateau when $Nn_{\uparrow,\downarrow} \equiv n \pm m \in \mathbb{Z}$ could be predicted. It is important to stress however that for the operator responsible for such a plateau to be relevant a fair amount of interchain coupling would be needed. This situation is reminiscent of what was found in the study of N -leg spin ladders, where the interchain coupling should be strong enough for the appearance of non-trivial plateaux [4]. Details of this analysis will be presented elsewhere [34].

The behavior of a 2-leg doped Hubbard ladder in the presence of a magnetic field generalizes the 2-leg spin ladder case. Our main result is that the magnetization curve should present two plateaux for a non-trivial value of the magnetization (see Fig. 1). The plateau at zero magnetization persists for non zero doping, while the doping dependent one can be obtained only by keeping the doping of the system fixed (as it is the case in most experimental settings). This should be contrasted to the dimerized Hubbard chain, in which the zero magnetization plateau at half filling is shifted to a non-trivial value when the system is doped. More importantly, the doping dependent plateaux are predicted for a wide range of couplings, which should allow for their experimental observation at quite low magnetic fields.

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- [31] Strictly speaking for $h \neq 0$ the fields $\Phi_{c,s}$ do not diagonalize the effective Hubbard Hamiltonian, thus eq. (6) is an approximation.
- [32] The relevance of these operators can be established explicitly for some particular values of the couplings U , t_{\perp} and the magnetization m by solving numerically the Bethe ansatz equations [29].
- [33] We use the following definitions: $\mathcal{O}_{\text{CDW}\downarrow}^{\pm} \sim \psi_{1L\downarrow}^{\dagger} \psi_{1R\downarrow} \pm \psi_{2L\downarrow}^{\dagger} \psi_{2R\downarrow}$, $\mathcal{O}_{\text{SC}\downarrow\downarrow}^{\pm} \sim \psi_{1R\downarrow} \psi_{2L\downarrow} \pm \psi_{2R\downarrow} \psi_{1L\downarrow}$.
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