Quantum Corrections to Lorentz Invariance Violating Theories: Fine-Tuning Problem

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Abstract

It is of general agreement that a quantum gravity theory will most probably mean a breakdown of the standard structure of space-time at the Planck scale. This has motivated the study of Planck-scale Lorentz Invariance Violating (LIV) theories and the search for its observational signals. Yet, it has been recently shown that, in a simple scalar-spinor Yukawa theory, radiative corrections to tree-level Planck-scale LIV theories can induce large Lorentz violations at low energies, in strong contradiction with experiment, unless an unnatural fine-tuning mechanism is present. In this letter, we show the calculation of the electron self-energy in the framework given by the Myers-Pospelov model for a Lorentz Invariance Violating QED. We find a contribution that depends on the prefered's frame four-velocity which is not Planck-scale suppressed, showing that this model suffers from the same *disease*. Comparison with Hughes-Drever experiments requires a fine-tuning of 21 orders of magnitude for this model not to disagree with experiment.

1 Introduction

The two astonishingly successful pillars of contemporary physics, namely Quantum Mechanics and General Relativity have stubbornly resisted their unification. The problem of finding a consistent theory that merges these two aspects of Nature, i.e. a quantum theory of gravitation, is known as the Quantum Gravity (QG) problem. Currently, the two most important approaches to this challenge are String Theory, which also claims to be a complete unification of all forces, and the less ambitious theory, known as Loop Quantum Gravity which is a canonical quantum version of General Relativity. Although based on different approaches and hypotheses, these theories agree in that novel properties of the structure and symmetry of space-time at the Planck-scale must arise, making the experimental exploration of Planck-scale physics of fundamental importance. Quantum Gravity phenomenology had been left aside for a while since directly probing Planck-scale effects on elementary particles would require

energies of the order of $E_{Pl} = \sqrt{\hbar c^5/G} \simeq 10^{19} GeV$, which is currently, and will most probably be for a long time, inaccessible to experimentalists. Nevertheless, Quantum Gravity phenomenology has enjoyed a renewed interest in recent years and has actually become a very active research area by the hand of Amelino Camelia, among others, when it was noticed that some Planck-scale effects may be amplified in several experimental and astrophysical scenarios to the extent of becoming observable with current experimental technology [1],[2].

Lorentz invariance breakdown at the Planck scale, manifested as a modified dispersion relation, is one of the common features that arises in the two mentioned contenders for QG phenomenology (see, for instance, [3], [4]) and one which has attracted considerable attention lately. Lorentz invariance violation (LIV) signifies a modification of the group structure of space-time symmetry and so the usual algebra Casimir $p_\mu \eta^{\mu\nu} p_\nu = m^2$ fails to be an invariant under the set of transformations relating inertial observers in the effective QG theory, i.e. a modified dispersion relation is expected. In fact, the existence of a natural mass scale ($M_{Pl} = \sqrt{\hbar c/G}$) allows the appearance of Planck-scale suppressed terms of powers greater than 2 in momenta in the dispersion relation. These terms can be traced back as having their origin in a modified Lagrangian for fields that contains five(or higher)-dimensional (i.e. non-renormalizable) kinetic terms. This field-point of view suggests that one should consider radiative corrections to the bare vertices of the theory to explore quantum modifications.

The motivation of this letter is the following [5]: The calculation of loop-diagrams requires the integration over arbitrarily high momenta and so the high energy regime of the theory is explored. In the usual Lorentz invariant gauge theories, these diagrams are divergent due to dimension four (or smaller) terms in the Lagrangian, which are made finite by the well-known procedure of renormalization. The incorporation of operators of higher dimension (higher powers in momenta in the dispersion relation) naturally introduces a mass scale (e.g. the Planck scale) and some of the originally divergent diagrams are naturally regularized. The superficial degrees of divergence of the Lorentz invariant diagrams therefore determine the size of the contributions of the Lorentz violating terms, which involve the Planck scale. These contributions are not necessarily Planck-scale suppressed, as frequently stated in the literature. In conclusion, Lorentz violation at the Planck scale is *pulled* down to low energies due to radiative effects, requiring an enormous fine-tuning in order to be consistent with experimental bounds on Lorentz violation.

In this letter we show that a modified model of QED, known as the Myers-Pospelov model, suffers this *disease* and estimate the order of magnitude of the fine-tuning by comparison with Hughes-Drever measurements [6], which is in agreement with the general conclusions of reference [5]. The organization of the paper is as follows: presentation of the Myers-Pospelov model, calculation of the electron's self energy diagram and finally the discussion of experimental bounds and the fine-tuning problem that this arises.

2 The Myers-Pospelov Model

The Myers-Pospelov (MP) model, which incorporates cubic modification terms in the dispersion relation by introducing a background four-vector w_μ, has received considerable attention lately and exhibits more appealing properties regarding causality conditions than other LIV models [7]. It has been studied in the language of effective field theory by introducing dimension-five operators into the lagrangian. More precisely, the MP's low energy effective field theory introduces Planck's scale-suppressed operators which satisfy the following general constraints: Quadratic in the same field, one more derivative than the usual kinetic term, gauge invariance, Lorentz invariant except for the appearance of w^μ, not reducible to lower dimension by the equations of motion and not reducible to a total derivative. The MP Lagrangian describing the electromagnetic interactions reads [8]:

$$\mathcal{L}_{QED}^{MP} = \mathcal{L}_{QED} + \delta\mathcal{L}_\gamma + \delta\mathcal{L}_\Psi, \tag{1}$$

wherein $\mathcal{L}_{QED} = -\frac{1}{4}F^2 + \bar{\Psi}(\not{p} - e\ \not{A} - m)\Psi$ is the usual QED Lagrangian and

$$\delta\mathcal{L}_\gamma = \frac{\xi}{M_{Pl}} w^a F_{ad} w \cdot \partial (w_b \tilde{F}^{bd}), \tag{2}$$

$$\delta\mathcal{L}_\Psi = \frac{i}{M_{Pl}} \bar{\Psi}(\eta_1\ \not{w} + \eta_2\ \not{w})(n \cdot \partial)^2 \Psi. \tag{3}$$

The photon dispersion relation due to this modified Lagrangian is

$$\left(E^2 - |\boldsymbol{p}|^2 \pm \frac{2\xi}{M_{Pl}} |\boldsymbol{p}|^3\right)(\varepsilon_x \pm i\varepsilon_y) = 0, \tag{4}$$

while

$$\left(E^2 - |\boldsymbol{p}|^2 - m^2 + \frac{1}{M_{Pl}} |\boldsymbol{p}|^3 (\eta_1 + \eta_2 \gamma_5)\right)\Psi = 0 \tag{5}$$

is the modified dispersion relation for electrons in the privileged system of reference, i.e., the one in which $w^\mu = (1, 0, 0, 0)$ meaning that Poincare's symmetry has been broken to $O(3)$ in this frame. Note that in this frame the modifications to the usual dispersion relation are related to the space-components of p while the energy dependence is the usual one. From these expressions, we can perform the calculation of the electron self-energy diagram in which we are interested now.

3 Electron self-energy

Since it is expected that departures from the (incredibly accurate) usual QED theory manifest themselves at high energies, due to it being a quantum gravity effect, we shall consider QED as a good enough approximation for *energies*

smaller than a given energy scale which we shall call Λ, while being the MP theory the accurate description for high energies compared to Λ. Then, we shall express the quantum correction to a given diagram by the symbolical expression

$$\Sigma = \int_0^{\Lambda} d\Sigma_{QED} + \int_{\Lambda}^{\infty} d\Sigma_{MP}. \tag{6}$$

That is to say that we may think of $\delta\Sigma(\xi,\Lambda) \equiv \int_{\Lambda}^{\infty} d\Sigma_{MP}$ as being a *correction* term to the (regularized) self-energy diagram in QED due to quantum gravity effects of the MP model. We wish to stress the fact that the Λ scale introduced here has a two-fold function: It regularizes the usual divergent integral of QED and it allows the calculation of $\delta\Sigma(\xi,\Lambda)$ in the high-energy regime[1]. This *cut-off* is of a physical character and unnecessary at the fundamental level, but it enormously simplifies the calculations and it allows us to identify more clearly the modification the MP model introduces. As we shall see, this correction term is not suppressed by any powers of $1/M_{Pl}$, as could of been naively expected, giving the finite contribution to the Lorentz violation at low energies to which we have referred above.

Let us consider the case in which the departure from the usual dispersion relation is mostly dominated by the ξ parameter in the photon dispersion relation, while keeping the electron propagator unmodified (i.e. $\eta_1 = \eta_2 = 0$). We wish to explore the effect of 1-loop corrections in this scenario, in particular the electron self-energy to first order in $\hbar$ given by

$$\begin{aligned} -i\Sigma_{MP}(p) =& (-ie)^2 \frac{1}{(2\pi)^4} \int d^3k \int dk_0 \gamma^\mu \frac{(\not{p} - \not{k} + m)}{(p_0 - k_0)^2 - [(\boldsymbol{p}-\boldsymbol{k})^2 + m^2]} \gamma_\mu \\ & \frac{1}{k_0^2 - [\boldsymbol{k}^2 \mp \frac{2\xi}{M_{Pl}} |\boldsymbol{k}|^3 + \lambda^2]}. \end{aligned} \tag{7}$$

We recall that the superficial degree of divergence of the electron self-energy diagram in QED is 1 and so a linear divergence is to be expected, but because of Ward's identities this divergence is smoothed into a logarithmic one to match that of the interaction vertex diagram. Since Ward's identities are a consequence of gauge invariance, the same can be said here and it's required that this integral be convergent, as the interaction vertex is finite in the MP model. Indeed, note that by regularizing the integral in such a way that the symmetry $k^\mu \to -k^\mu$ at high energies is preserved and writing the integrand as $I(k) = \frac{1}{2}[I(k) + I(-k)]$, one gets a finite result. That is, the MP model predicts a finite correction to the self-energy diagram. After using the standard Feynman parameterization and performing some integrals, we get (see Appendix)

$$-i\Sigma_{MP}(p) \simeq -i\frac{e^2}{8\pi^2}\left(-\not{p} - \gamma^i p_i + 4m\right) \ln \frac{\xi\Lambda}{2M_{PL}}. \tag{8}$$

[1] More precisely, the Λ scale marks the begining of the regime in which the quantum gravity effects are important. In the privileged reference frame this occurs for big values of $|\boldsymbol{p}|$. Also, observe that Λ acts as an infrared regulator.

Notice that the contribution

$$\frac{e^2}{8\pi^2}\,(-\not{p}+4m)\ln\frac{\xi\Lambda}{2M_{PL}} \tag{9}$$

can be absorbed by the regularized self-energy term coming from QED, leaving

$$\delta\Sigma_{MP}=\frac{e^2}{8\pi^2}\gamma^i p_i\ln\frac{\xi\Lambda}{2M_{PL}} \tag{10}$$

as a new and finite term, due to high energy corrections in the MP model.

Note that (10) has the asymptotic form $f(m/M_P)\ln\xi$ for small ξ and the form $g(\xi)\ln(m/M_P)$ for small m/M_P. Since the latter quantity is $m/M_P\sim 10^{-22}$ we shall have the behaviour $\delta\Sigma_{MP}\propto g(\xi)\ln(m/M_P)$. Thus, the very small quantity m/M_P appears as the argument of a logarithm.

Note also that the Λ dependence on equation (10) appears because a cutoff in momentum space does not preserve the symmetry $k\to -k$.

Now, recall that this expression is to be understood in the preferred system of reference in which $w_\mu=(1,0,0,0)$, but in an arbitrary reference frame with relative velocity w^μ, we must replace p^ι by the projection of p^μ onto the transversal direction of w_μ, i.e., $p^T_\mu=\Pi^\nu_\mu(w)p_\nu$, where $\Pi^\nu_\mu(w)\equiv\delta^\nu_\mu-w_\mu w^\nu$ is the usual transversal projector, if the w four-vector is normalized and $g_{\mu\nu}=diag(+,-,-,-)$. Finally

$$\delta\Sigma_{MP}(p,w)=\frac{e^2}{8\pi^2}\gamma^\mu(g_{\mu\nu}-w_\mu w_\nu)p^\nu\ln\frac{\xi\Lambda}{2M_{Pl}}, \tag{11}$$

wherein w^μ represents the 4-velocity of the observer relative to the preferred system of reference. Note that the limit $\xi\to 0$ must be taken simultaneously with the limit $\Lambda\to\infty$ in such a way that $\xi\Lambda$ remains finite in order to recover the low-energy limit. Notice that the parameter ξ, which controls the extent of Lorentz Invariance violation, regularizes the usual logarithmic divergence, as could of been expected from power counting. This slightly resembles what happens in Non-Commutative Field Theory, where the non-commutativity parameter θ measures the extent of Lorentz invariance violation and regularizes some ultravioletly-divergent diagrams as well [9],[10].

4 Lagrangian Counterterm: fine-tuning problem

If the MP model with 1-loop corrections is to be consistent with experimental bounds on low energy Lorentz violation, it is necessary to include a (Lorentz violating) counterterm in the original Lagrangian in the way

$$\mathcal{L}'_{MP}=\mathcal{L}^{1-\text{Loop}}_{MP}-\frac{ie^2}{8\pi^2}\ln(\frac{\xi'\Lambda}{M_{Pl}})\bar{\Psi}\left(\gamma^\mu(g_{\mu\nu}-w_\mu w_\nu)\partial^\nu\right)\Psi. \tag{12}$$

By introducing $\delta \equiv \frac{\xi}{\xi'} - 1$, we have

$$\mathcal{L}'_{MP} = \mathcal{L}^0_{MP} - \frac{ie^2}{8\pi^2}\ln(\delta+1)\bar{\Psi}\left(\gamma^\mu(g_{\mu\nu} - w_\mu w_\nu)\partial^\nu\right)\Psi \tag{13}$$

$$\mathcal{L}'_{MP} \simeq \mathcal{L}^0_{MP} - \frac{i\delta e^2}{8\pi^2}\bar{\Psi}\left(\gamma^\mu(g_{\mu\nu} - w_\mu w_\nu)\partial^\nu\right)\Psi. \tag{14}$$

Notice that the dependence on Λ has disappeared and there is no Planck-scale suppression, as advertised before. The δ parameter therefore measures the extent of Lorentz violation, rising the problem of a fine-tuning of the counterparameter ξ'.

5 Experimental bounds

The associated Hamiltonian in the non-relativistic limit of this Lagrangian, given by the method developed in [11] up to first order in δ, reads

$$\begin{aligned} H = &\left[mc^2\left(1 - \frac{\delta e^2}{4\pi^2}(\mathbf{w}/c)^2\right)\right. \\ &\left. + \left(1 - 2\frac{\delta e^2}{4\pi^2}\left(1 + \frac{5}{6}(\mathbf{w}/c)^2\right)\right)\left(\frac{p^2}{2m} + g\mu\mathbf{s}\cdot\mathbf{B}\right)\right] \\ &- \frac{\delta e^2}{4\pi^2}\left[\frac{\mathbf{w}\cdot Q_P\cdot\mathbf{w}}{mc^2}\right]. \end{aligned} \tag{15}$$

The last term badly breaks Lorentz invariance. It represents an anisotropy of inertial mass and it has been tested in Hughes-Drever like experiments [6]. On account of the approximation $Q_P = -5/3 < p^2 > Q/R^2$ for the momentum quadrupole moment, with Q being the electric quadrupole moment, we get

$$\delta H_Q = -\frac{\delta e^2}{4\pi^2}\frac{5}{3}\left\langle\frac{p^2}{2M}\right\rangle\left(\frac{Q}{R^2}\right)\left(\frac{w}{c}\right)^2 P_2(\cos\theta). \tag{16}$$

for the perturbing Hamiltonian, from which we get a bound for the fine tuning parameter δ

$$|\delta| < 10^{-21}. \tag{17}$$

6 Conclusions

We have explicitly shown the realization, in the framework of the Myers-Pospelov model for a LIV QED of the idea, first exposed in [5], that LIV at the Planck scale at the classical (*i.e.* $\hbar = 0$) level are dragged down to low energies by radiative corrections. This is reflected by the fact, already noted, that the dependence of the LIV term (10) on the very small factor m/M_P is only logarithmic. Thus, the LIV is actually amplified in such a way that a LIV finite counterterm has to be added to suppress the large (order of the QED fine structure constant) term

and an extreme fine-tuning is required in order for this theory to be consistent with present experimental bounds.

By comparison with Hughes-Drever measurements[6], we have been able to determine the magnitude of the fine-tuning to be (17), in agreement with the general conclusions of reference [5].

Fine-tuning problems already arise within the Standard model, considered one of the most succesfull theories, but it is expected to be solved at a more fundamental level and has been considered unacceptable in a fundamental theory by some authors [13],[14]. The fine-tuning problem discussed here could also be considered as a sign suggesting, a still unknown, underlying structure.The implications of this study are quite restrictive on possible Lorentz violating scenarios; if the exact Lorentz invariance hypothesis is relaxed at the Planck-scale, then a deeper and precise mechanism for ensuring Lorentz invariance at low energies when quantum corrections are considered, is needed. Some mechanisms to deal with this problem have been proposed, but none of these are, at present, fully satisfactory. The reader is referred to [16] for an updated review and discussion of these proposals and, in general, of LIV's role in quantum gravity phenomenology.

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APPENDIX

We show in this appendix the calculations leading to (8). The Feynman parameterization

$$\frac{1}{ab} = \int_0^1 dz \frac{1}{\left(az + b[1-z]\right)^2} \tag{18}$$

allows us to express the integral in (7) as

$$I = \int dz \int dk_0 \frac{-2\ \not{p} + 4m + 2\ \not{k}}{\left(\left[p^2 - 2pk - m^2\right] z + k^2 + \left[-\lambda^2 + \frac{2\xi}{M_{Pl}}|\boldsymbol{k}|^3\right](1-z)\right)^2}, \tag{19}$$

where the standard Dirac gamma matrices properties $\gamma^\mu\ \not{p}\gamma_\mu = -2\ \not{p}$ and $\gamma^\mu\gamma_\mu = 4$ have been used. By shifting solely the time-like integration variable k_0 to $k^0 - p^0 z$, we get

$$I = \int dz \int dk_0 \frac{-2\ \not{p}(1-z) + 4m + 2\gamma^i(k_i + p_i z)}{\left(k_0^2 + p_0^2 z(1-z) - (\boldsymbol{p}-\boldsymbol{k})^2 z - m^2 z - \left(\boldsymbol{k}^2 + \lambda^2 - \frac{2\xi}{M_{Pl}}|\boldsymbol{k}|^3\right)(1-z)\right)^2}, \tag{20}$$

where we have used the fact that the integral with $\gamma^0 k_0$ is exactly zero, due to the integrand being parity-odd in k_0. The table expression

$$I = \int \frac{d^d k}{(k^2 + 2kq - r^2)^\alpha} = (-1)^{d/2} i\pi^{d/2} \frac{\Gamma(\alpha - \frac{d}{2})}{\Gamma(\alpha)} \frac{1}{[-q^2 - r^2]^{\alpha - d/2}} \tag{21}$$

allows the full computation of the integral in QED. Although not so useful now, it allows the whole integration of the time-like component of k, by taking

$$d = 1, \quad \alpha = 2, \tag{22}$$

$$r^2 = -p_0^2 z(1-z) + (\boldsymbol{p} - \boldsymbol{k})^2 z + m^2 z + \left(\boldsymbol{k}^2 + \lambda^2 - \frac{2\xi}{M_{Pl}} |\boldsymbol{k}|^3\right)(1-z) \tag{23}$$

$$q = 0. \tag{24}$$

This way, the integral to be performed over the spatial components of k is

$$-\frac{\pi}{2} \int d^3k \frac{-2 \not{p}(1-z) + 4m + 2\gamma^i (k_i + p_i z)}{\left[p_0^2 z(1-z) - (\boldsymbol{p} - \boldsymbol{k})^2 z - m^2 z + \left(-\boldsymbol{k}^2 - \lambda^2 + \frac{2\xi}{M_{Pl}} |\boldsymbol{k}|^3\right)(1-z)\right]^{3/2}}. \tag{25}$$

It is important to stress that no shifting in the spatial components of k has been done, but solely in the time component, k_0, which is consistent with the lagrangian's symmetry and the integral being calculated as the energy integral has no intermediate cut-off.

By taking the external electron to be on-shell we have

$$I = -\frac{\pi}{2} \left(\int d^3k \frac{-2 \not{p}(1-z) + 4m - 2\gamma^i p_i z}{\left[-p_0^2 z^2 - k^2 + 2pkz\cos\theta + \frac{2\xi}{M_{Pl}}(1-z)k^3\right]^{3/2}} \right.$$

$$\left. + \int d^3k \frac{2\gamma^i k_i}{\left[p_0^2 z(1-z) - (\boldsymbol{p} - \boldsymbol{k})^2 z - m^2 z + \left(-\boldsymbol{k}^2 - \lambda^2 + \frac{2\xi}{M_{Pl}} |\boldsymbol{k}|^3\right)(1-z)\right]^{3/2}} \right), \tag{26}$$

$$\equiv -\frac{\pi}{2} \left(I_{Log.} + I_{Lin.} \right), \tag{27}$$

where the infrared regularization parameter λ has been omitted because this expression is valid at high energies. We have already argued that the $I_{Lin.} = 0$ sow we perform the calculation of the (originally) logarithmically divergent integral, I_{Log}. We may first perform the angular integral over $d\Omega = -d(\cos\theta)d\phi$,

getting

$$I_{Log.} = -2\pi\left(-2\not{p}(1-z)+4m-2\gamma^i p_i z\right)\cdot \int dk k \frac{2}{2|\boldsymbol{p}|z}\left(\frac{1}{\sqrt{\frac{2\xi}{M_{Pl}}(1-z)k^3-k^2+2pkz-p_0^2z^2}}\right. \left. -\frac{1}{\sqrt{\frac{2\xi}{M_{Pl}}(1-z)k^3-k^2-2pkz-p_0^2z^2}}\right). \quad (28)$$

By introducing $\varepsilon \equiv 2pkz - p_0^2z^2$ in the first term and $\varepsilon' = 2pkz + p_0^2z^2$ in the second one we have, to first order in ε and ε',

$$\begin{aligned} I_{Log} &\simeq \frac{\pi}{|\boldsymbol{p}|z}\left(-2\not{p}(1-z)+4m-2\gamma^i p_i z\right)\int dk \frac{(\varepsilon+\varepsilon')k}{\left(-k^2+2k^3\frac{\xi}{M_{Pl}}(-z+1)\right)^{\frac{3}{2}}} \\ &\simeq 4\pi\left(-2\not{p}(1-z)+4m-2\gamma^i p_i z\right)\int dk \frac{1}{k\left[\frac{2\xi}{M_{PL}}(1-z)k-1\right]^{3/2}}. \quad (29) \end{aligned}$$

The definite integral between the values Λ and ∞ , in which we are interested in, reads

$$\begin{aligned} I_{Log} &\simeq -8\pi\left(-2\not{p}(1-z)+4m-2\gamma^i p_i z\right)\cdot \left(\frac{\pi}{2}-\frac{1}{\sqrt{\frac{2\xi}{M_{PL}}(1-z)\Lambda-1}}-\arctan\sqrt{\frac{2\xi}{M_{PL}}(1-z)\Lambda-1}\right) \\ I_{Log} &\simeq -4\pi\left(-2\not{p}(1-z)+4m-2\gamma^i p_i z\right)\left(\pi+i\ln\frac{\xi\Lambda(1-z)}{2M_{PL}}\right), \quad (30) \end{aligned}$$

where the standard $\tan^{-1}(ix) = i\tanh^{-1}(x)$ and $\tanh^{-1}(x) = \frac{1}{2}\ln(\frac{1+x}{1-x})$ identities have been used in the last line and the small terms when $\xi\Lambda/M_{PL} \ll 1$ have been neglected. Integrating over the Feynman parameter z,

$$\int_0^1 I_{Log} dz = -4\pi i\left(-\not{p}-\gamma^i p_i+4m\right)\ln\frac{\xi\Lambda}{2M_{PL}} + \text{finite terms}, \quad (31)$$

which finally gives

$$-i\Sigma_{MP}(p) = -i\frac{e^2}{8\pi^2}\left(-\not{p}-\gamma^i p_i+4m\right)\ln\frac{\xi\Lambda}{2M_{PL}}. \quad (32)$$

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