

# The Wess-Zumino-Witten term in non-commutative two-dimensional fermion models

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**ABSTRACT:** We study the effective action associated to the Dirac operator in two dimensional non-commutative Field Theory. Starting from the axial anomaly, we compute the determinant of the Dirac operator and we find that even in the U(1) theory, a Wess-Zumino-Witten like term arises.

**KEYWORDS:** Brane Dynamics in Gauge Theories, Gauge Symmetry, Anomalies in Field and String Theories.

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## 1. Introduction

Interest in non-commutative spaces has been renewed after the discovery that non-commutative gauge theories naturally arise when D-branes with constant B fields are considered [1, 2]. These works as well as that in [3] prompted many investigations both in field theory and in string theory (see references in [3]). Concerning gauge field theories, recent results on chiral and gauge anomalies [4]–[6] have shown that well-known results on “ordinary” models extend naturally and interestingly to the case in which non-commutative spaces are considered. In this work we consider a problem which can be seen as closely related to that of anomalies, namely the evaluation of the two-dimensional fermion determinant in non-commutative space-time. This problem is of interest not only for the analysis of two-dimensional QED and QCD in non-commutative space, but also in connection with abelian and non-abelian bosonization since, as it is well-known, the knowledge of the fermion determinant leads more or less directly to the bosonization rules.

We start by evaluating in section 2 the chiral anomaly in two-dimensional non-commutative space-time in a way adapted to the calculation of fermion determinants through integration of the anomaly. This last is done in section 3 where both the abelian and ( $U(N)$ ) non-abelian fermion determinant is calculated exactly. In both cases we obtain for the determinant a Wess-Zumino-Witten term. Consequences of our results and possible extensions are discussed in section 4.

## 2. The chiral anomaly

### 2.1 Conventions

As usual, we define the  $*$ -product between a pair of functions  $\phi(x), \chi(x)$  as

$$\begin{aligned} \phi * \chi(x) &\equiv \exp\left(\frac{i}{2}\theta^{\mu\nu}\partial_{x_\mu}\partial_{y_\nu}\right)\phi(x)\chi(y)|_{x=y} \\ &= \phi(x)\chi(x) + \frac{i}{2}\theta^{\mu\nu}\partial_\mu\phi\partial_\nu\chi(x) + O(\theta^2), \end{aligned} \tag{2.1}$$

and the (Moyal) bracket in the form

$$\{\phi, \chi\}(x) \equiv \phi(x) * \chi(x) - \chi(x) * \phi(x), \tag{2.2}$$

so that, when applied to (euclidean) space-time coordinates  $x^\mu, x^\nu$ , one has

$$\{x^\mu, x^\nu\} = i\theta^{\mu\nu}, \tag{2.3}$$

which is why one refers to non-commutative spaces. Here  $\theta^{\mu\nu}$  is a real, anti-symmetric constant tensor. Since we shall be interested in two dimensional space-time, one necessarily has  $\theta^{\mu\nu} = \theta\varepsilon^{\mu\nu}$  with  $\varepsilon^{\mu\nu}$  the completely anti-symmetric tensor and  $\theta$  a real constant. In the context of string theory, non-commutative spaces are believed to be relevant to the quantization of D-branes in background Neveu-Schwarz constant B-field  $B_{\mu\nu}$  [1]–[3]. In this context  $\theta^{\mu\nu}$  is related to the inverse of  $B^{\mu\nu}$ . Afterwards, this original interest was extended to the analysis of field theories in non-commutative space and then, as signaled in [6] it becomes relevant to know to what extent old problems and solutions in standard field theory fit in the new non-commutative framework.

A “non-commutative gauge theory” is defined just by using the  $*$ -product each time the gauge fields have to be multiplied. Then, even in the  $U(1)$  abelian case, the curvature  $F_{\mu\nu}$  has a non-linear term (with the same origin as the usual commutator in non-abelian gauge theories in ordinary space)

$$\begin{aligned} F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu - ie(A_\mu * A_\nu - A_\nu * A_\mu) \\ &= \partial_\mu A_\nu - \partial_\nu A_\mu - ie\{A_\mu, A_\nu\}. \end{aligned} \tag{2.4}$$

This field strength is gauge-covariant (not gauge-invariant, even in the abelian case) under gauge transformations which should be represented by  $U$  elements of the form

$$U(x) = \exp_*(i\lambda) \equiv 1 + i\lambda - \frac{1}{2}\lambda * \lambda + \dots \tag{2.5}$$

The covariant derivative implementing infinitesimal gauge transformations takes the form

$$\mathcal{D}_\mu[A]\lambda = \partial_\mu\lambda + ie(\lambda * A_\mu - A_\mu * \lambda) \tag{2.6}$$

so that an infinitesimal gauge transformation on  $A_\mu$  reads as usual

$$\delta A_\mu = \frac{1}{e} \mathcal{D}_\mu \lambda. \quad (2.7)$$

Concerning finite gauge transformations, one has

$$A_\mu^U = \frac{i}{e} U(x) * \partial_\mu U^{-1}(x) + U(x) * A_\mu * U^{-1}(x). \quad (2.8)$$

Given a fermion field  $\psi$ , one can easily see that the combination

$$\gamma^\mu D_\mu[A] \psi = \gamma^\mu \partial_\mu \psi - ie \gamma^\mu A_\mu * \psi \quad (2.9)$$

transforms covariantly under gauge transformations (2.8),

$$\gamma^\mu D_\mu[A^U] \psi^U = U * \gamma^\mu D_\mu[A] \psi, \quad (2.10)$$

with

$$\psi^U = U(x) * \psi \quad (2.11)$$

and

$$U(x) * U^{-1}(x) = U^{-1} * U(x) = 1. \quad (2.12)$$

A gauge invariant Dirac action can be defined in the form

$$S_f = \int d^d x \bar{\psi}(x) * i \gamma^\mu D_\mu[A] \psi(x). \quad (2.13)$$

## 2.2 The anomaly

Chiral transformations will be written as

$$\psi'(x) = U_5(x) * \psi, \quad (2.14)$$

with

$$U_5(x) = \exp_*(\gamma_5 \alpha(x)) = 1 + \gamma_5 \alpha + \frac{1}{2} \alpha(x) * \alpha(x) + \dots \quad (2.15)$$

The chiral anomaly  $\mathcal{A}_d$  in  $d$ -dimensional space can be calculated from the formula

$$\log J_d[\alpha] = -2 \mathbf{A}_d, \quad (2.16)$$

$$\mathbf{A}_d = \text{Tr} \gamma_5 \delta \alpha(x) |_{\text{reg}} \quad (2.17)$$

here  $J_d[\alpha]$  is the Fujikawa jacobian associated with an infinitesimal chiral transformation  $U = 1 + \gamma_5 \delta \alpha$  and  $\text{Tr}$  includes a matrix and functional space trace.

Let us specialize to the two dimensional case. We shall use the heat-kernel regularization so that (2.17) will be understood as

$$\mathbf{A}_2 = \int d^2 x \mathcal{A}_2(x) * \delta \alpha(x), \quad (2.18)$$

$$\mathcal{A}_2(x) = \lim_{M \rightarrow \infty} \text{Tr} \gamma_5 \exp_* \left( \frac{\mathcal{D} * \mathcal{D}}{M^2} \right). \quad (2.19)$$

After some standard manipulations, (2.19) takes the form

$$\mathcal{A}_2(x) = \frac{1}{4\pi} \text{tr} \gamma_5 \mathcal{D} * \mathcal{D} = \frac{1}{4\pi} \text{tr} (\gamma_5 \gamma^\mu \gamma^\nu) D_\mu * D_\nu. \quad (2.20)$$

Here  $\text{tr}$  is just the matrix trace. Using  $\text{tr}(\gamma_5 \gamma^\mu \gamma^\nu) = 2i \varepsilon^{\mu\nu}$ , eq. (2.20) can be written as

$$\mathcal{A}_2(x) = \frac{e}{2\pi} \varepsilon^{\mu\nu} (\partial_\mu A_\nu - ie A_\mu * A_\nu) = \frac{e}{4\pi} \varepsilon^{\mu\nu} F_{\mu\nu}. \quad (2.21)$$

This result coincides with that first obtained in [4].

### 3. The two-dimensional fermion determinant

Let us write the gauge field in the two-dimensional case in the form

$$\mathcal{A} = \frac{1}{e} (i \not{\partial} \exp_*(\gamma_5 \phi + i\eta)) * \exp_*(-\gamma_5 \phi - i\eta). \quad (3.1)$$

Note that in the  $\theta_{\mu\nu} \rightarrow 0$  limit, eq. (3.1) reduces to the usual decomposition of a two-dimensional gauge field in the form

$$eA_\mu = \varepsilon_{\mu\nu} \partial^\nu \phi + \partial_\mu \eta, \quad (3.2)$$

which allows to decouple fermions from the gauge-field and then obtain the fermion determinant as the jacobian associated to this decoupling [8]. Now, the form (3.1) was precisely proposed in [9] to achieve the decoupling in the case of non-abelian gauge field backgrounds, this leading to the calculation of the  $QCD_2$  fermion determinant in a closed form. Afterwards [10], it was shown that writing a two dimensional gauge field as in eq. (3.1) (without the  $*$ -product but in the  $U(N)$  case) does correspond to the choice of a gauge condition. Eq. (3.1) is then the extension of this approach for a case in which non-commutativity arises from the use of the  $*$ -product.

At the classical level, the change of fermionic variables

$$\begin{aligned} \psi &= \exp_*(\gamma_5 \phi + i\eta) * \chi \\ \bar{\psi} &= \bar{\chi} * \exp_*(\gamma_5 \phi - i\eta) \end{aligned} \quad (3.3)$$

completely decouples the gauge field, written as in (3.1), leading to an action of free massless fermions,

$$S_f = \int d^2x \bar{\chi} * i \not{\partial} \chi \quad (3.4)$$

Of course, this is not the whole story: at the quantum level there is a Fujikawa jacobian  $J$  [7] associated to change (3.3). In order to compute this jacobian, we follow the method introduced in [8, 9]. Consider then the change of variables

$$\begin{aligned} \psi &= U_t * \chi_t, \\ \bar{\psi} &= \bar{\chi}_t * U_t^\dagger, \end{aligned} \quad (3.5)$$

where

$$U_t = \exp_*(t(\gamma_5\phi + i\eta)), \quad (3.6)$$

and  $t$  is a real parameter,  $0 \leq t \leq 1$ . Given the fermion determinant defined as

$$\det(\not{\partial} - ie \not{A}) = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp(-S_f[\bar{\psi}, \psi]) \quad (3.7)$$

we proceed to the change of variables (3.5) which leads to

$$\begin{aligned} \det(\not{\partial} - ie \not{A}) &= J[\phi, \eta; t] \int \mathcal{D}\bar{\chi}_t \mathcal{D}\chi_t \exp(-S_f[\bar{\chi}_t, \chi_t]) \\ &= J[\phi, \eta; t] \det D_t, \end{aligned} \quad (3.8)$$

where  $J[\phi, \eta; t]$  stands for the jacobian

$$\mathcal{D}\bar{\psi}\mathcal{D}\psi = J[\phi, \eta; t] \mathcal{D}\bar{\chi}_t \mathcal{D}\chi_t \quad (3.9)$$

and we have defined

$$D_t = U_t^\dagger * (\not{\partial} - ie \not{A} *) U_t. \quad (3.10)$$

Now, since the l.h.s. in (3.8) does not depend on  $t$  we get, after differentiation,

$$\frac{d}{dt} \log \det D_t = -\frac{d}{dt} \log J[\phi, \eta; t] \quad (3.11)$$

or, after integrating on  $t$  and using that  $D_0 = \not{\partial} - ie \not{A}$  and  $D_1 = \not{\partial}$

$$\det(\not{\partial} - ie \not{A}) = \det \not{\partial} \exp\left(-2 \int_0^1 dt \mathbf{A}_2(t)\right), \quad (3.12)$$

where we have used

$$\mathbf{A}_2(t) = \frac{d}{dt} \log J[\phi, \eta; t]. \quad (3.13)$$

Now, it is trivial to identify  $\mathbf{A}_2(t)$  with the two-dimensional chiral anomaly as defined in eq. (2.17), just by writing  $\delta\alpha = \phi dt$ ,

$$\mathbf{A}_2(t) = \text{Tr}(\gamma_5 * \phi)|_{\text{reg}}. \quad (3.14)$$

In order to have a gauge-invariant regularization ensuring that the  $\eta$  part of the transformation does not generate a jacobian, we adopt, in agreement with (2.18) and (2.19),

$$\mathbf{A}_2(t) = \lim_{M \rightarrow \infty} \text{Tr} \left( \gamma_5 \exp\left(\frac{\not{D}_t * \not{D}_t}{M^2}\right) * \phi \right) \quad (3.15)$$

so that finally one has

$$\mathbf{A}_2(t) = \frac{e}{2\pi} \int d^2x \varepsilon^{\mu\nu} (\partial_\mu A_\nu^t - ie A_\mu^t * A_\nu^t) * \phi = \frac{e}{4\pi} \int d^2x \varepsilon^{\mu\nu} F_{\mu\nu}^t * \phi, \quad (3.16)$$

where we have introduced

$$\gamma_\mu A_\mu^t = -\frac{1}{e} (i \not{\partial} U_t) * U_t^{-1} \tag{3.17}$$

and analogously for  $F_{\mu\nu}^t$ . In summary, we can write for the U(1) fermion determinant

$$\det(\not{\partial} - ie \mathcal{A}) = \exp\left(-\frac{e}{2\pi} \int d^2x \int_0^1 dt \varepsilon^{\mu\nu} F_{\mu\nu}^t * \phi\right) \det \not{\partial}. \tag{3.18}$$

It will be convenient to use the relation

$$\gamma^\mu \gamma_5 = -i \varepsilon^{\mu\nu} \gamma_\nu \tag{3.19}$$

to rewrite (3.18) in the form

$$\det(\not{\partial} - ie \mathcal{A}) = \exp\left(\frac{ie}{2\pi} \text{tr} \int d^2x \int_0^1 dt \gamma_5 \phi * (\not{\partial} \mathcal{A}^t - ie \mathcal{A}^t * \mathcal{A}^t)\right) \det \not{\partial}. \tag{3.20}$$

Then, one can exploit the identity

$$\begin{aligned} \text{tr} \int d^2x \frac{1}{2} \frac{d}{dt} \mathcal{A}^t * \mathcal{A}^t &= \frac{1}{e} \text{tr} \int d^2x \gamma_5 i \not{\partial} \phi * \mathcal{A}^t + 2 \text{tr} \int d^2x \gamma_5 \mathcal{A}^t * \phi * \mathcal{A}^t + \\ &+ \frac{1}{e} \text{tr} \int d^2x (\not{D}\eta) * \mathcal{A} \end{aligned} \tag{3.21}$$

and find for (3.20)

$$\begin{aligned} \log \det(\not{\partial} - ie \mathcal{A}) &= -\frac{e^2}{4\pi} \text{tr} \int d^2x \mathcal{A} * \mathcal{A} + \frac{e^2}{2\pi} \text{tr} \int dt \int d^2x \gamma_5 \phi * \mathcal{A} * \mathcal{A} + \\ &+ \frac{e}{2\pi} \int dt \int d^2x (\not{D}\eta) * \mathcal{A} + \log \det \not{\partial}. \end{aligned} \tag{3.22}$$

This is the final form for the fermion determinant in a U(1) gauge theory. In order to write it in a more suggestive way connecting it with the Wess-Zumino-Witten term, let us consider the light-cone gauge  $A_+ = 0$ , Then, one can see after some algebra that [11]

$$\begin{aligned} \log\left(\frac{\det(\not{\partial} - ie \mathcal{A})}{\det \not{\partial}}\right) &= -\frac{1}{8\pi} \int d^2x (\partial_\mu g(x)^{-1}) * (\partial_\mu g(x)) + \\ &+ \frac{i}{12\pi} \epsilon_{ijk} \int_B d^3y g(x, t)^{-1} * (\partial_i g(x, t)) * \\ & * g(x, t)^{-1} * (\partial_j g(x, t)) g^{-1} * (\partial_k g(x, t)) \end{aligned} \tag{3.23}$$

here we have written  $A_- = (i/e)g(x) * \partial_- g^{-1}(x)$  with  $g(x) = \exp_*(2\phi(x))$ ,  $g(x, t) = \exp_*(2\phi(x)t)$  and  $d^3y = d^2x dt$  so that the integral in the second line of eq. (3.24) runs over the three dimensional manifold  $B$ , which in compactified euclidean space

can be identified with a ball with boundary  $S^2$ . Index  $i$  runs from 1 to 3. As in the ordinary commutative case, because the determinant was computed in euclidean space, elements  $g$  should be considered as belonging to  $U(1)_C$  (the complexified  $U(1)$ ) [11, 12].

So, we have found for the two-dimensional non-commutative fermion determinant that, even for a  $U(1)$  gauge field background, a Wess-Zumino-Witten term arises due to non-commutativity of the  $*$ -product. Of course, in the  $\theta^{\mu\nu} \rightarrow 0$  limit in which the  $*$ -product becomes the ordinary one, the  $U(1)$  fermion determinant contribution to the gauge field effective action reduces to  $(-1/2\pi) \int d^2x \phi \partial^\mu \partial_\mu \phi$  which is nothing but the Schwinger determinant result expressed in a gauge-invariant way.

The method we have employed has the advantage that it can be trivially generalized to the case of a  $U(N)$  gauge group. One has just to take into account that in (3.1) one has

$$\phi = \phi^a t^a, \quad \eta = \eta^a t^a, \quad (3.24)$$

with  $t^a$  the  $U(N)$  generators. Then, as originally shown in [9] for the commutative case, the fermion determinant can be seen to be given by

$$\det(\not{\partial} - ie \not{A}) = \exp \left( -\frac{e}{4\pi} \text{tr}^c \int d^2x \int_0^1 dt \varepsilon^{\mu\nu} F_{\mu\nu}^t * \phi \right) \det \not{\partial}, \quad (3.25)$$

where  $\text{tr}^c$  is a trace over the  $U(N)$  algebra. Then, following the same steps leading to (3.24), one gets, in the  $U(N)$  case

$$\begin{aligned} \log \left( \frac{\det(\not{\partial} - ie \not{A})}{\det \not{\partial}} \right) &= -\frac{1}{8\pi} \text{tr}^c \int d^2x (\partial_\mu g^{-1}) * (\partial_\mu g) + \\ &+ \frac{i}{12\pi} \epsilon_{ijk} \text{tr}^c \int_B d^3y g^{-1} * (\partial_i g) * g^{-1} * (\partial_j g) g^{-1} * (\partial_k g), \end{aligned} \quad (3.26)$$

where again, in the light-cone gauge we have written

$$A_- = -\frac{i}{e} g * \partial_- g^{-1}, \quad A_+ = 0 \quad (3.27)$$

$$g = \exp_*(2\phi^a t^a). \quad (3.28)$$

Eq. (3.27) is the generalization of the expression given in [13] for the two-dimensional non-abelian fermion determinant to the case of non commutative space-time.

## 4. Conclusion

We studied in this article the effective action of the gauge degrees of freedom in a two dimensional non-commutative Field Theory of fermions coupled to a gauge field. Using Fujikawa's approach, we computed the chiral anomaly and, from it, the fermionic determinant of the non-commutative Dirac operator.



As it was to be expected, the result for the fermion determinant corresponds to the  $*$ -deformation of the standard result. Now, the fact that a Moyal bracket enters in the field strength curvature even in the abelian case, has important consequences, some of which have already been signaled in [4]–[6] where chiral and gauge anomalies in non-commutative spaces have been analyzed.

In our framework, where the anomaly was integrated in order to obtain the fermion determinant, this reflects in the fact that a Wess-Zumino-Witten like term arises *both* in the abelian and in the non-abelian cases (eqs. (3.24) and (3.27) respectively). This should have, necessarily, implications in relevant aspects of two-dimensional theories since, as it is well-known, bosonization is closely related to the form of the fermion determinant [15]. Indeed, the bosonization rules for fermion currents as well as the resulting current algebra can be easily derived by differentiation of the Dirac operator determinant  $\det(\not{d} - i\not{g})$  with respect to the source  $s_\mu$  (see [16] for a review). Now, as one learns from ordinary non-abelian bosonization, where the Polyakov-Wiegmann identity plays a central rôle in the bosonization recipe, here one should have an analogous identity which will lead to non-trivial changes at the level of currents and, a fortiori, for the current algebra. In view of the relevance of these objects in connection with two-dimensional bosonic and fermionic models, it will be worthwhile to pursue the investigation initiated here in this direction.

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## References

- [1] A. Connes, M.R. Douglas and A. Schwarz, *Noncommutative geometry and matrix theory: compactification on tori*, *J. High Energy Phys.* **02** (1998) 003 [[hep-th/9711162](#)].
- [2] M.R. Douglas and C. Hull, *D-branes and the noncommutative torus*, *J. High Energy Phys.* **02** (1998) 008 [[hep-th/9711165](#)].
- [3] N. Seiberg and E. Witten, *String theory and noncommutative geometry*, *J. High Energy Phys.* **09** (1999) 032 [[hep-th/9908142](#)].
- [4] F. Ardalan and N. Sadooghi, *Axial anomaly in non-commutative QED on  $R^4$* , [hep-th/0002143](#).
- [5] J.M. Gracia-Bondía and C.P. Martin, *Chiral gauge anomalies on noncommutative  $R^4$* , [hep-th/0002171](#).

- [6] L. Bonora, M. Schnabl and A. Tomasiello, *A note on consistent anomalies in non-commutative YM theories*, hep-th/0002210.
- [7] K. Fujikawa, *Path integral measure for gauge invariant fermion theories*, *Phys. Rev. Lett.* **42** (1979) 1195; *Path integral for gauge theories with fermions*, *Phys. Rev. D* **21** (1980) 2848.
- [8] R. Roskies and F. Schaposnik, *Comment on Fujikawa's analysis applied to the Schwinger model*, *Phys. Rev. D* **23** (1981) 558.
- [9] R.E.G. Saraví, F.A. Schaposnik and J.E. Solomin, *Path integral formulation of two-dimensional gauge theories with massless fermions*, *Nucl. Phys. B* **185** (1981) 239.
- [10] R. Roskies in *Symmetries in particle physics*, I. Bars, A. Chodos and C-H. Tze eds., Plenum Press, New York 1984.
- [11] R.E.G. Saraví, F.A. Schaposnik and J.E. Solomin, *Towards a complete solution of two-dimensional quantum chromodynamics*, *Phys. Rev. D* **30** (1984) 1353;  
K.D. Rothe, *On the exact gauge invariant calculation of the fermion determinant in massless two-dimensional QCD*, *Nucl. Phys. B* **269** (1986) 269.
- [12] S.G. Naculich and H.J. Schnitzer, *Constructive methods for higher genus correlation functions of level one simply laced WZW models*, *Nucl. Phys. B* **332** (1990) 583.
- [13] A.M. Polyakov and P.B. Wiegmann, *Theory of nonabelian goldstone bosons in two dimensions*, *Phys. Lett. B* **131** (1983) 121; *Goldstone fields in two-dimensions with multivalued actions*, *Phys. Lett. B* **141** (1984) 223.
- [14] E. Witten, *Nonabelian bosonization in two dimensions*, *Comm. Math. Phys.* **92** (1984) 455.
- [15] P. di Vecchia and P. Rossi, *On the equivalence between the Wess-Zumino action and the free Fermi theory in two-dimensions*, *Phys. Lett. B* **140** (1984) 344.
- [16] J.C.L. Guillou, E. Moreno, C. Núñez and F.A. Schaposnik, *Non abelian bosonization in two and three dimensions*, *Nucl. Phys. B* **484** (1997) 682 [hep-th/9609202].