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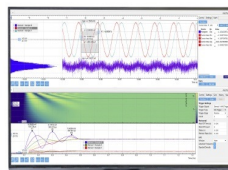
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The Cosmological Slingshot Scenario: a Stringy Proposal for the Early Time Cosmology ¹

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Abstract. In The Cosmological Slingshot Scenario, our Universe is a $D3$ -brane that extends in the $4d$ noncompact directions of a warped Calabi-Yau compactification of IIB Supergravity. Early time cosmology corresponds to a period in which the brane moves inside a warped throat, a non-vanishing angular momentum ensuring that the trajectory of the brane has a turning point. The corresponding induced metric on the $D3$ -brane experiences a cosmological evolution with a bounce. In this framework, the homogeneity, flatness, and isotropy problems of standard cosmology might be avoided. The power spectrum of primordial perturbations of the brane embedding can be found and it is shown to be in agreement to WMAP data.

Keywords: String Theory, D-branes, Mirage Cosmology.

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INTRODUCTION

The Cosmological Slingshot Scenario [1, 2, 3] is a proposal for the cosmic early-time evolution in the String Theory context. According to that, our Universe is a $D3$ -brane moving in a String Theory background of the form $\mathcal{M}^4 \times K^6$. \mathcal{M}^4 is a warped Minkowskian space-time and K^6 is a compact Calabi-Yau (CY) space. The latter includes a throat sourced by a stack of a large number (N) of other $D3$ -branes. The Slingshot is characterized by a non-trivial orbital motion of the Universe in the compact space around the stack of $D3$ -branes. If back-reaction can be neglected (probe brane approximation), a brane observer measures a $4d$ metric induced in terms of the brane embedding, that defines a cosmological brane evolution commonly called Mirage Cosmology [4].

The early-time evolution (*i.e.* well before nucleosynthesis) corresponds to the motion of the $D3$ -brane deep into the throat (Slingshot era) moving towards the hat of the compact space. Since N is taken to be large, close to the stack the probe brane approximation can be used. The late-time cosmology starts when the $D3$ -brane reaches the hat of the CY, the probe brane approximation breaks down and local gravity *à la* Randall-Sundrum [5, 6] dominates the cosmological evolution. Under this approximation, the

¹ Based on [1] and [2].

Slingshot brane observer experiences a non-singular bouncing cosmology. Additionally, as we shall show later on, the Standard Cosmology problems (*i.e.* homogeneity, isotropy and flatness) might be avoided in the brane induced cosmology.

THE COSMOLOGICAL SLINGSHOT SCENARIO

Setup

To make the discussion concrete, we will consider a probe $D3$ -brane moving in a throat of a Calabi-Yau (CY) compact manifold, whose metric and Ramond-Ramond 5-form can be written as

$$ds^2 = h^{-\frac{1}{2}} ds_{\mathcal{M}_4}^2 + h^{\frac{1}{2}} (dr^2 + r^2 ds_5^2) , \quad C_{0123} = 1 - \frac{1}{h} , \quad (1)$$

where h is a function of r only, and ds_5^2 is the base manifold characterizing the remaining part of the transverse space.

The dynamics of a probe brane is governed by the Dirac-Born-Infeld action with the Wess-Zumino coupling

$$S_{DBI} + S_{WZ} = -T_3 \int \sqrt{-g_i} d^4 \xi - T_3 \int C_{(4)} . \quad (2)$$

We assume that all other fields on the brane are switched off and matter is created later. The sign of the Wess-Zumino term has been chosen to represent a $D3$ -brane and $T_3 = 1/(2\pi)^3 g_s l_s^4$ is the tension of the probe. The probe brane is extended along the \mathcal{M}_4 directions, so that it looks like a point particle moving in the transverse space. In the static gauge the resulting induced metric is

$$ds_i^2 = h^{-1/2} [- (1 - h(r'^2 + r^2 \Omega_5'^2)) dt^2 + d\vec{x} \cdot d\vec{x}] , \quad (3)$$

where a prime ($'$) denotes a derivative with respect to t and we have assumed that the only non-vanishing transverse momenta are in the r, Ω_5 directions, Ω_5 representing an angle in the transverse space. Replacing this induced metric into the brane action (2) we get

$$S = -T_3 V_3 \int dt \left[\frac{1}{h} \sqrt{1 - h(r'^2 + r^2 \Omega_5'^2)} + 1 - \frac{1}{h} \right] , \quad (4)$$

where V_3 is the un-warped volume of the directions parallel to the probe. The resulting equations of motion have first integrals provided by

$$U = \frac{1}{h} \left[\frac{1}{\sqrt{1 - h(r'^2 + r^2 \Omega_5'^2)}} - 1 \right] , \quad J = \frac{r^2}{\sqrt{1 - h(r'^2 + r^2 \Omega_5'^2)}} \Omega_5' , \quad (5)$$

that can be inverted to get

$$r^2 = \frac{1}{(1+hU)^2} \left(2U - \frac{J^2}{r^2} + hU^2 \right). \quad (6)$$

The motion will take place at the values of r that make this expression positive. Moreover wherever the expression in parenthesis vanishes, the trajectory will have a turning point.

To interpret the induced metric (3) as the cosmology experienced by an observer on the brane, we need to define the cosmic time according to

$$d\tau = h^{-1/4} \sqrt{1 - h(r^2 + r^2 \Omega_5^2)} dt, \quad (7)$$

in terms of which the metric (3) is identified as a Friedman-Robertson-Walker metric with scale factor

$$a = h^{-1/4}(r). \quad (8)$$

When $r(t)$ has a turning point, it is easy to see that the same happens to $a(\tau)$, generating a nonsingular bouncing cosmology. A Friedmann equation can be written for such cosmology by changing to cosmic time in Eq.(6) and dividing the result by a^2

$$H^2 = \frac{1}{16} a^6 h_r^2(a) \left[2U - \frac{J^2}{r^2(a)} + \frac{U^2}{a^4} \right], \quad (9)$$

where $H = \dot{a}/a$ is the (mirage) Hubble constant (here $\dot{a} = \partial_\tau a$).

The model is completed by smoothly pasting this Mirage era to a local gravity driven late evolution when the brane reaches the top of the CY and gravity becomes localized *à la* Randall-Sundrum [5, 6]. There, the standard late time evolution of the observed Universe is supposed to be well reproduced by the brane dynamics. This assumption involves a transition from a mirage dominated era with a moving brane without any matter, into a local gravity dominated era with an static brane and matter fields excited on it. This transition has to be understood as an analogous of the reheating process in standard inflationary models. It entails a dynamical mechanism under which the kinetic energy of the brane is passed to matter fields. The description of this dynamics as well as the robustness of our predictions for physical observables is an open point that is left for future research.

A concrete example of the above proposed situation is given by a probe moving in the $\text{AdS}_5 \times \text{S}_5$ background. In it, the metric takes the form (1) with

$$h_{\text{AdS}} = \frac{L^4}{r^4}, \quad L^4 = 4\pi l_s^4 N g_s, \quad (10)$$

where l_s is the string length and g_s is the string coupling. The supergravity approximation is valid as long as the curvature radius of the solution is large compared to the string length l_s . String perturbation theory on the other hand requires $g_s \ll 1$.

A close look to Eq.(6) shows that, in the present case, the second factor is a quadratic function of the variable r^{-2} , that will have a root as long as its discriminant is positive

$J^4 - 8U^3L^4 > 0$. For values of r larger than that of the root the function is positive. Then whenever this inequality is satisfied, the probe orbit has an inner turning point.

The resulting scale factor (8) reads

$$a_{\text{AdS}} = \frac{r}{L}. \quad (11)$$

In this background, the induced Friedmann equation (9) becomes

$$H_{\text{AdS}}^2 = \frac{1}{L^2} \left[\frac{2U}{a_{\text{AdS}}^4} - \frac{(J/L)^2}{a_{\text{AdS}}^6} + \frac{U^2}{a_{\text{AdS}}^8} \right], \quad (12)$$

where $H_{\text{AdS}} = \dot{a}_{\text{AdS}}/a_{\text{AdS}}$ is the (mirage) Hubble constant. Since when $J^4 - 8U^3L^4 > 0$ the orbit has a turning point, in that case the corresponding cosmology has a bounce.

Another example is that of a probe brane motion in a Klebanov-Strassler (KS) throat [7]. In the region far from the tip of the throat, KS geometry can be well approximated by the Klebanov-Tseytlin (KT) metric [8] that takes the form (5). The warp factor reads

$$h_{\text{KS}} = \frac{L^4}{r^4} \ln\left(\frac{r}{r_s}\right), \quad L^2 = \frac{9}{\sqrt{8}} l_s^2 M g_s, \quad (13)$$

where r_s is proportional to the radius of the blown up sphere at the tip of the cone. To trust the KT approximation we need to ensure that the probe brane will never reach $r \simeq r_s$. This will be the case iff r_s lies inside the forbidden region $r'^2 < 0$. Going back to equation (6) and evaluating it at r_s , we see that this is true when $2r_s U - J^2 < 0$, the probe motion having a turning point at some value of r larger than r_s .

The resulting scale factor is

$$a_{\text{KS}} = \frac{r}{L} \ln^{-1/4}\left(\frac{r}{r_s}\right), \quad (14)$$

and, under the assumption $2r_s U - J^2 < 0$, it corresponds to a bouncing cosmology.

An important ingredient in our argument is that the scale factor for a brane moving in a KS throat (14) can be rewritten as a conformal re-scaling of the corresponding scale factor for a brane moving in AdS_5 . Indeed, when written in conformal time, the induced metric on the brane reads $ds_i^2 = a^{-2} ds_{\text{AdS}}^2$ and we can write

$$a_{\text{KS}} = \Omega(a_{\text{AdS}}) a_{\text{AdS}}, \quad \text{with } \Omega(a_{\text{AdS}}) = \log^{-1/4}\left(a_{\text{AdS}} \frac{L}{r_s}\right). \quad (15)$$

It should be kept in mind that our approximations are valid whenever $a_{\text{AdS}} \gg r_s/L$. Under such a re-scaling, the Hubble constant changes as

$$H_{\text{KS}} = \left(1 + a_{\text{AdS}} \frac{d \ln \Omega}{d a_{\text{AdS}}}\right) H_{\text{AdS}}. \quad (16)$$

The Problems of Standard non-Inflationary Cosmology

We are now ready to study how standard cosmological problems are solved in the Slingshot scenario.

A. Homogeneity. As explained above, in both the AdS case and the KS throat the probe brane experiences a bounce in the String frame. This immediately ensures that homogeneity problem is solved. To check this explicitly, we write the co-moving horizon

$$\Delta\eta = \int_{\eta_i}^{\eta_0} d\eta, \quad (17)$$

where η is conformal time and η_i is its smallest value. To solve homogeneity problem it is required that $\Delta\eta > H_0^{-1}$. Since we have $\eta_i \rightarrow -\infty$ due to the absence of a cosmological singularity, this condition is trivially satisfied.

B. Isotropy. In the AdS case, mirage matter contributes to Friedmann equation (12) with a term $\rho \sim a_{\text{AdS}}^{-8}$. This term dominates over the shear $\rho_{\text{shear}} \sim a_{\text{AdS}}^{-6}$ at early times, avoiding the chaotic behavior [9]. To check whether this is true in the KS case, we should verify that the corresponding mirage contribution dominates over the shear. The form of this contribution can be read from (16), and we can write the quotient

$$\sqrt{\frac{\rho_{\text{shear}}}{\rho}} \propto \left(1 + a_{\text{AdS}} \frac{d \ln \Omega}{d a_{\text{AdS}}}\right)^{-1} \frac{a_{\text{AdS}}}{\Omega^3}. \quad (18)$$

The proportionality constant in (18) parameterizes the anisotropic perturbations in the pre-bounce era. It is simple to check that (18) is an increasing function of a_{AdS} in the region $a \gtrsim e^{((2\sqrt{2}-1)/4)} r_s/L \simeq 1.57 r_s/L$. As we assumed that the Slingshot brane never approaches the tip of the KS throat, this condition is automatically satisfied. Therefore, ρ_{shear}/ρ decreases very rapidly close to the bouncing point in the pre-bounce era, solving the isotropy problem.

C. Flatness. The curvature contribution to the Hubble equation² can be disregarded if the quantity $|\Omega_{\text{total}} - 1| = 1/a^2 H^2$ passes through a minimum where it satisfies the phenomenological constraint

$$|\Omega_{\text{total}} - 1|_{\text{min}} < 10^{-8}. \quad (19)$$

For the AdS case, the above quantity evaluated at its minimum reads

$$\begin{aligned} |\Omega_{\text{total}} - 1|_{\text{min}} &= \frac{(J^2 + \sqrt{J^4 - 6L^4 U^3})^3}{4L^2 U^2 (J^4 - 4L^4 U^3 + J^2 \sqrt{J^4 - 6L^4 U^3})} \\ &\simeq \left(\frac{J}{2LU}\right)^2 + \mathcal{O}\left(\frac{L^4 U^3}{J^4}\right) < 10^{-8}. \end{aligned} \quad (20)$$

² The details of how to include a curvature term in the mirage Hubble equation can be found in [1].

This condition is not a fine tuning in parameter space, but just a restriction to a two dimensional region. In this sense flatness problem might be alleviated in the Slingshot scenario.

For the KS case we have, after conformal re-scaling

$$|\Omega_{\text{total}} - 1| = \frac{f^2}{a_{\text{AdS}}^2 H_{\text{AdS}}^2}, \quad f = \frac{4 \ln(a_{\text{AdS}} L / r_s)}{4 \ln(a_{\text{AdS}} L / r_s) - 1}. \quad (21)$$

The KT approximation is valid for $r_{\text{min}} \gg r_s$; to fix ideas we will use $r_{\text{min}} > 10^2 r_s$. In this region we have $f = \mathcal{O}(1)$ and decreasing in a_{AdS} . Consequently, the flatness problem in the KS space might, in good approximation, be alleviated by the same choice of parameters used in the AdS case.

PRIMORDIAL PERTURBATIONS

The Hollands-Wald Mechanism

In inflationary scenarios the primordial perturbations are produced by quantum fluctuations of the inflaton field and are codified into its two point correlation function in the vacuum state. However, these fluctuations are over-damped by the expansion of the Universe at super-horizon scales. At these scales then, the quantum state becomes characterized by a large occupation number and the system collapses into a classical state. This classical state represents a random spectrum of perturbations whose variance is given by the quantum correlations evaluated at the quantum-to-classical transition point [10].

Let us now turn our attention into the mechanism proposed by Hollands and Wald in [11]. A perturbation of wavelength λ smaller than a typical quantum scale, say l_c , is in its quantum vacuum. In an expanding background, the wavelength of a perturbation grows in time ($\lambda \propto a$) and whenever $\lambda \sim l_c$, or in other words, as soon as the perturbation becomes macroscopic, wavelengths bigger than the horizon scale collapse into a classical random state. In the proposal of [11], the relevant fluctuations are so continuously “created” at “super-horizon” scales. Thus, a coherent spectrum of classical perturbations is produced with variance given by matching the classical correlations with the quantum correlations at the quantum-to-classical transition point.

It has been suggested [12] that a space-time uncertainty relation $\Delta X \Delta T \gtrsim l_s^2$ should be realized in String Theory, ΔX and ΔT representing the uncertainties in measuring space and time distances. Since the smallest length that can be probed in String Theory is the 11-dimensional Planck length, $\Delta X > l_{p11} \sim g_s^{1/3} l_s$, we obtain that the smallest measurable time is $\Delta T \gtrsim g_s^{-1/3} l_s$. The period of a wave propagating in a D -brane is $2\pi\omega^{-1} \sim \lambda$ and cannot be smaller than that ΔT , implying

$$\lambda > l_s g_s^{-1/3}. \quad (22)$$

We therefore have a minimal wavelength for a perturbation on the brane as in the Hollands-Wald mechanism. This strongly suggests to use such mechanism to study the

cosmological perturbations in the Slingshot model^{3,4}.

Technically, the mechanism explained above introduces a vacuum state in which the perturbation is destroyed (coming from the pre-bounce era) and then created again (after the bounce) at the time η_* in which the proper wavelength of the corresponding quantum mode reaches the value

$$a(\eta_*)/k \equiv a_*/k = l_c. \quad (23)$$

We start by perturbing the embedding of the probe brane by writing $r = r(\eta) + \delta r(\eta, \vec{x})$ and $\Omega_5 = \Omega_5(\eta) + \delta\Omega(\eta, \vec{x})$, where $r(\eta), \Omega_5(\eta)$ are the solutions of the equations of motion obtained from action (2), written as functions of the conformal time η . In the non-relativistic approximation $hU \ll 1$ we have $\eta \equiv t$ and Eqs.(5) are integrated to

$$r(\eta) = \sqrt{2U\eta^2 + \frac{J^2}{2U}}, \quad \Omega_5(\eta) = \arctan\left(\frac{2U}{J}\eta\right). \quad (24)$$

Note that we have a turning point at $r_{min} = J/\sqrt{2U}$.

In what follows, we will use as our variable the Bardeen potential $\delta\Phi_k = \delta r_k/r$ [15], in terms of which the action (2) can be expanded to quadratic order in δ 's and their derivatives, getting (in Fourier space)

$$S = T_3 \sum_k \int d\eta \left(\frac{r^2}{2} (\delta\Phi_k'^2 + \delta\Omega_k'^2 - k^2(\delta\Phi_k^2 + \delta\Omega_k^2)) + J\delta\Omega_k' \delta\Phi_k - J\delta\Omega_k \delta\Phi_k' \right). \quad (25)$$

Power Spectrum and Spectral Index

The canonical quantization procedure applied to the action (25) provide the normalized operators

$$\delta\hat{\Phi}_k = u_1 \hat{a}_1 + u_2 \hat{a}_2 + c.c. , \quad \delta\hat{\Omega}_k = v_1 \hat{a}_1 + v_2 \hat{a}_2 + c.c. , \quad (26)$$

where a_i, a_i^\dagger are standard annihilation and creation operators, and

$$u_1 = \sqrt{\frac{U}{kT_3}} \frac{\eta}{r^2} e^{-ik\eta} , \quad u_2 = \sqrt{\frac{1}{UkT_3}} \frac{J}{2r^2} e^{-ik\eta} , \quad (27)$$

$$v_1 = u_2 = \sqrt{\frac{1}{UkT_3}} \frac{J}{2r^2} e^{-ik\eta} , \quad v_2 = -u_1 = -\sqrt{\frac{U}{kT_3}} \frac{\eta}{r^2} e^{-ik\eta} . \quad (28)$$

³ In fact, the complete spectrum will include a blue part [13] coming from the perturbations created in the pre-bounce era, that were not contracted up to the minimal wavelength at the bounce, and an additional red part [3] due to the ones that were. Here we will fix our attention in this second part

⁴ In the original proposal of [11] the perturbation was produced by the same radiation which sets the CMB. However, as pointed out by [14], the perturbation coming out from the horizon today, was necessarily born when the energy density of radiation was much bigger than the Planck energy, which makes the mechanism unreliable. In the Slingshot instead, perturbations are created by brane fluctuations in a regime in which the supergravity approximation is still valid, and no extra quantum effect takes place. Moreover, super-horizon causality is not required, since in the Slingshot perturbations are overdamped at sub-horizon scales $k < J/r^2$.

We are interested in the correlation of the Bardeen potential $\delta\hat{\Phi}$ at the time of creation η_* . Using the above formulas to define $r_* = r(\eta_*)$, it is straightforward to check that

$$\langle \delta\hat{\Phi}_k \delta\hat{\Phi}_{k'} \rangle = \delta_{k,k'} \frac{1}{2kT_3 r_*^2}. \quad (29)$$

We will consider the transition point of the quantum to the classical description in the region in which $k \ll J/r^2$. In this limit, we can discard the k^2 term in the action (25) and write its classical solutions as

$$\delta\Phi_k = \frac{C_k}{2J} + A_k \sin(2\theta + \phi_k), \quad \delta\Omega = -\frac{D_k}{2J} + A_k \cos(2\theta + \phi_k), \quad (30)$$

where $\theta = \Omega_5(\eta) - \Omega_5(\eta_*)$ and ϕ_k, C_k, D_k, A_k are constants of integration, that can be written as

$$\begin{aligned} C_k &= r^2 \delta\Omega'_k + 2J \delta\Phi_k, & D_k &= r^2 \delta\Phi'_k - 2J \delta\Omega_k, \\ A_k &= \frac{r^2}{2J} [\delta\Phi'_k \cos(2\theta + \phi_k) - \delta\Omega'_k \sin(2\theta + \phi_k)]. \end{aligned} \quad (31)$$

We now consider initial conditions arising from the matching of the classical to the quantum system at the time $\eta = \eta_*$. Therefore C_k, D_k, A_k will be taken as Gaussian stochastic variables with correlations $\langle \dots \rangle_c$ matching the quantum correlators $\langle \dots \rangle$ at $\eta = \eta_*$.

Using the quantum solutions described above at the matching point $\eta = \eta_*$ after a lengthy but straightforward calculation we have

$$\begin{aligned} \langle C_k C_{k'} \rangle_c &= \delta_{k,k'} \frac{k^2 r_*^2 + 2U}{2kT}, & \langle A_k A_{k'} \rangle_c &= \delta_{k,k'} \frac{k^2 r_*^2 + 2U}{8J^2 kT}, \\ \langle A_k C_{k'} \rangle_c &= \delta_{k,k'} \frac{\sin \phi}{4kT J r_*^2} \{2J^2 - 2U r_*^2 - k^2 r_*^4 - 4JU \eta_* \cot \phi_k\}. \end{aligned} \quad (32)$$

The matching of

$$\langle \delta\Phi_k \delta\Phi_{k'} \rangle_c = \langle \delta\hat{\Phi}_k \delta\hat{\Phi}_{k'} \rangle, \quad (33)$$

requires $\phi_k = \pi/2$; this is the selection of positive frequencies.

In general correlators depend on time through θ . However in the region $k \ll J/r^2$, the oscillation rapidly stabilizes in time when $2U\eta_{\text{asympt.}} > 2\pi J$. We will consider this to happen well before the nucleosynthesis. At this time then

$$\delta\Phi_k = \frac{C_k}{2J} - A_k \cos(2\Omega_5(\eta_*)) = \frac{C_k}{2J} + A_k \left(1 - \frac{2r_*^2}{r_*^2}\right). \quad (34)$$

Using the initial conditions found above we then get in the limit $k \ll J/r_*^2 < J/r_{\text{min}}^2$

$$\langle \delta\Phi_k \delta\Phi_{k'} \rangle_c \Big|_{\eta > \eta_{\text{asympt.}}} \simeq \frac{\delta_{k,k'}}{2kT r_*^2} \left[1 - \left(\frac{r_{\text{min}}}{r_*}\right)^2 \right], \quad (35)$$

so the power spectrum of temperature fluctuations is

$$P(k) \simeq \frac{1}{2kT_3 r_*^2} \left[1 - \left(\frac{r_{min}}{r_*} \right)^2 \right]. \quad (36)$$

A consistency condition for the production of the perturbation is that $r_{min} < r_*$. So we see that in the limit $r_{min} \ll r_*$ we obtain the power spectrum introduced in [1].

Since we assumed that a perturbation is created when its physical wavelength reaches a fixed value l_c , we have from Eq.(23), $kl_c = a_*$. In the AdS metric Eq.(11) implies $kl_c L = r_*$, resulting in the power spectrum

$$P(k) \simeq \frac{1}{2T_3 (l_c L)^2 k^3} \left[1 - \frac{r_{min}^2}{(l_c L)^2 k^2} \right], \quad (37)$$

for which the scalar spectral index $n_s - 1 = d \ln(k^3 P(k)) / d \ln k$ reads

$$n_s \simeq 1 - \frac{2}{1 - \frac{(l_c L)^2}{r_{min}^2} k^2}, \quad (38)$$

and we see that the flat spectrum found in [1] at the matching point is blue-shifted by the subsequent time evolution.

In the Klebanov-Tseytlin (KT) metric on the other hand, the condition $kl_c = a_*$ is solved by $r_* = r_s e^{-W_{-1}(-\zeta)/4}$ where $\zeta = 4(r_s/Ll_c k)^4 \leq e^{-1}$ and $W_{-1}(x)$ is the negative branch of Lambert's W -function. Then the power spectrum (36) is explicitly written as

$$P(k) = \frac{1}{2T_3 k r_s^2} e^{\frac{1}{2}W_{-1}(-\zeta)} \left(1 - \left(\frac{r_{min}}{r_s} \right)^2 e^{\frac{1}{2}W_{-1}(-\zeta)} \right), \quad (39)$$

whereas the scalar spectral index turns out to be

$$\begin{aligned} n_s &= 1 + \frac{2}{1 + W_{-1}(-\zeta)} \left(1 - \frac{W_{-1}(-\zeta)}{1 - (r_s/r_{min})^2 e^{-\frac{1}{2}W_{-1}(-\zeta)}} \right) \\ &\simeq 1 + \frac{2}{\ln(\zeta)} - \frac{2\sqrt{\zeta}}{\sqrt{\zeta} - (r_s/r_{min})^2}, \end{aligned} \quad (40)$$

where the expansion of the Lambert W function for small argument $W(-\zeta) \simeq \ln(\zeta) + \dots$ was used in the second line. Since in this limit $\ln(\zeta) < 0$, the first correction on n_s is negative. On the other hand, the second correction is red or blue according to the sign of its denominator. It will be negative whenever

$$\sqrt{\zeta} > r_s^2 / r_{min}^2, \quad (41)$$

from which we immediately see that long wavelengths are red-shifted.

On the other hand, if the last term is positive, then $\sqrt{\xi} < r_s^2/r_{min}^2$ and the overall sign of the correction has to be evaluated taking into account the joint contribution of both terms in (40). After some manipulations we find that the correction is red whenever

$$\sqrt{\xi} \left(1 - 2 \log \sqrt{\xi}\right) < (r_s/r_{min})^2, \quad (42)$$

from which we conclude that short wavelengths are also red-shifted, and there is an intermediate range of wavelengths that is blue-shifted.

BACK-REACTION AND EFFECTIVE 4D THEORY

The 4D effective theory for warped compactifications of IIB supergravity with (static) D-branes has been derived by a perturbative approach in [16], and by a gradient expansion method in [17]. Using these results, in [2] the following effective 4d Lagrangian describing Slingshot cosmology has been found

$$S_{brane} = \frac{L^2}{2} \int d^4x \sqrt{-g} \left[\left(\frac{1}{\kappa^2 r^2} - \frac{T_3}{6N} \right) R + \frac{6}{\kappa^2} \frac{(\nabla r)^2}{r^4} - \frac{T_3}{N} (\nabla \Omega_5)^2 \right]. \quad (43)$$

The resulting equations of motion, specialized to a Friedmann-Robertson-Walker background with scale parameter $a(\eta)$, result into the following set of equations

$$\begin{aligned} \frac{r''}{r} + 2 \frac{r'}{r} \left(\frac{a'}{a} - \frac{r'}{r} \right) + \Omega_5'^2 &= 0, & \frac{d}{d\eta} (a^2 \Omega_5') &= 0, \\ \frac{T_3 \kappa^2}{N} \left(\frac{a'^2}{a^2} + \Omega_5'^2 \right) &= \frac{6}{r^2} \left(\frac{a'}{a} - \frac{r'}{r} \right)^2. \end{aligned} \quad (44)$$

As can be checked by direct substitution, an exact solution of the full system is

$$a = \frac{1}{L} \sqrt{\frac{J^2}{2U} + 2U\eta^2}, \quad \Omega_5' = \frac{J}{L^2} \frac{1}{a^2}, \quad r = \frac{aL}{1 + \kappa \sqrt{\frac{UT_3}{3N}} \eta}. \quad (45)$$

We obtained the same scale factor evolution as in the mirage approximation in [2], but now considering local gravity back-reactions. The only difference from the mirage approximation is on the r evolution, due to the denominator $1 + \kappa \sqrt{UT_3/3N} \eta$. Since $\kappa \sqrt{UT_3/3N}$ is supposed to be small, the mirage approximation breaks down at very late or very early times, *i.e.* when the brane leaves the throat, as expected. When this denominator is taken into account, the system is no longer time symmetric and there is a field singularity at the time in which the denominator vanishes. However, since the extra-dimensional space is compact $r < r_{max}$, where r_{max} defines the cut-off of the compact space, this singularity is just fictitious, and the effective action cannot be trusted when $1 + \kappa \sqrt{UT_3/3N} \eta \simeq 0$. There a description *à la* Randall-Sundrum [6] must be used.

SUMMARY

The Cosmological Slingshot scenario provides an example of a bouncing cosmology in which the dynamics of the bounce is under control. It provides alternative solutions to the problems of standard cosmology, and the resulting perturbations spectrum is in agreement with WMAP data. In the way it has been presented here it still has to be checked that all the constraints, that appeared during our calculations due to our approximations and to phenomenological inputs, are mutually compatible. Even if in [1] some of these cross checks have been performed successfully, a complete cross-checking is still needed, and it may be the case that all the problems of standard cosmology cannot be solved at the same time. For example, to make our solution of flatness problem compatible with perturbation spectrum, a very strong lower bound on the conserved quantity U is needed. The model is currently under research.

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