In-medium covariant propagator of baryons under a strong magnetic field: effect of the intrinsic magnetic moments

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Abstract

We obtain the covariant propagator at finite temperature for interacting baryons immersed in a strong magnetic field. The effect of the intrinsic magnetic moments on the Green function are fully taken into account. We make an expansion in terms of eigenfunctions of a Dirac field, which leads us to a compact form of its propagator. We present some simple applications of these propagators, where the statistical averages of nuclear currents and energy density are evaluated.

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1 Introduction

The dynamics of matter subject to strong magnetic fields has been widely studied in the past [1], and it has received renewed interest due to the analysis of different experimental situations. For instance, some investigations of the last decade[2, 3, 4, 5] have pointed out that matter created in heavy ion collisions could be subject to very intense magnetic fields. As a consequence the particle production can exhibit a distinguishable anisotropy. A preferential emission of charged particles along the direction of the magnetic field is predicted in [2, 3] for noncentral heavy ion collisions, due to magnetic intensities $e\,B \sim 10^2$ MeV^2. Improved calculations taking care of the mass distribution of the colliding ions [4], does not modify essentially the magnitude of the produced fields. Furthermore, the numerical simulations performed by [5] predict larger values $e\,B \sim m_\pi^2 \sim 2 \times 10^4$ MeV^2.
In a very different scenario, the presence of strong magnetic fields is the key issue that distinguishes a kind of astronomical compact objects. The analysis of the observational data in the range from soft X to soft gamma radiation, has showed the features of a class of neutron stars named Soft Gamma Repeaters and Anomalous X Ray Pulsars. These isolated stars are characterized by a sustained X-ray luminosity with energy in the soft (0.5-10 keV) or hard (50-200 keV) spectrum. They can show a time variability, with pulsations at relatively long spin periods. In particular, the Soft Gamma Repeaters exhibit a bursting activity which includes giant flares as a rare manifestation. Both cases can be described within the magnetar model [6, 7, 8], where the X-ray emission as well as bursting are attributed to the dissipation and decay of very strong magnetic fields. Their intensity has been estimated around 10^{15} G at the star surface, and could reach much higher values in the dense interior of the star. The availability of an increasing amount of precision data opens the question on how well the current theoretical description of nuclear matter can fit this empirical evidence.

The properties of the dense hadronic medium have been properly described within a covariant model of the hadronic interaction known as Quantum Hadro-Dynamics (QHD) [9]. It has been used to study the structure of neutron stars and particularly to analyze hadronic matter in the presence of an external magnetic field [10, 11, 12, 13, 14, 15, 16]. The versatility of this formulation allows the inclusion of the intrinsic magnetic moments in a covariant way. Due to the strength of the baryon-meson couplings, the mean field approximation (MFA) is usually employed. Within this approach the meson fields are replaced by their expectation values and assimilated to a quasi-particle picture of the baryons. Finally the meson mean values are obtained by solving the classical meson equations taking as sources the baryonic currents. This scheme is conceptually clear and easy to implement, however it is not evident at all how to include further corrections if they were needed.
In recent years several publications have stressed the role of the intrinsic magnetic moments on the statistical properties of hadronic systems such as the matter susceptibility and magnetization [13, 14, 15], the rise in the population of hyperons in stellar matter [13, 14], and the saturation properties of nuclear

matter [16]. The variation of the magnetic moments of hadrons within the nuclear environment has been pointed out in recent investigations [17].

The purpose of this work is twofold. In first place we construct the covariant propagator of fermions, both neutral and charged, in the presence of an external magnetic field. We give a full treatment including their intrinsic magnetic moments.
Expressions for the covariant propagator of a charged particle subject to an external magnetic field have been presented long time ago [18, 19], and this is a subject of continuous development [20, 21]. However, the effects of the magnetic moment have been neglected assuming its smallness. Exceptionally, in Ref. [22] the proton propagator in-vacuum has been presented.
For magnetic intensities greater than 5×10^{17} G, the influence of the magnetic moments must be taken into account in the determination of the stable configuration of matter [23], and the evaluation of thermodynamical properties [13, 14, 15, 16].
On the other hand, we give here an extension of the Dirac field propagator appropriate to include density and temperature effects in the study of hadronic systems subject to very strong magnetic fields. It is shown that the mean values of the particle densities and currents, agree with the results obtained for nuclear matter within the QHD model in the MFA.
We present a detailed derivation, using a clearly stated notation. Our results open the possibility of using the diagrammatic techniques of the field theory to study quantum corrections and statistical averages of physical processes developing under strong magnetism, including the effects of the magnetic moments of baryons. Thus we propose a complementary tool to extend the analysis of related investigations [24, 25, 22].

The organization of this work is as follows. In the next section we summarize the classical solutions for a Dirac field in the presence of an external magnetic field, considering the intrinsic magnetic moments. For this purpose we follow the general guidelines of [11]. This complete set of solutions is used to make an expansion of the quantum fields, including the appropriate measure of integration in the phase space. Following the standard prescriptions we evaluate the in-medium nucleon propagator in sect. 3. These results are interpreted within the context of the QHD model, and we evaluate nucleon densities and energy densities in sect. 4. Finally, in the last section we present a summary of our results.

2 Dirac solutions for nucleons with magnetic moment

The Lagrangian density for Dirac particles of mass m_b , with anomalous magnetic moments κ_b, interacting through scalar σ and vector ω mesons, and under

the influence of an external electromagnetic field A, is given by ($\hbar = 1$, $c = 1$) [26]

$$\begin{aligned}\mathcal{L} &= \sum_b \bar{\Psi}^{(b)} \left[\gamma_\mu (i\,\partial^\mu - q_b\,A^\mu - g_\omega\,\omega^\mu) + g_\sigma\,\sigma - m_b - \frac{\kappa_b}{2}\,\sigma^{\mu\nu}\,\mathcal{F}_{\mu\nu}\right]\Psi^{(b)} \\ &- \frac{1}{4}\,\mathcal{F}_{\mu\nu}\,\mathcal{F}^{\mu\nu} + \frac{1}{2}\,(\partial_\mu\sigma\,\partial^\mu\sigma - m_\sigma^2\,\sigma^2) - \frac{1}{4}\,\Omega_{\mu\nu}\,\Omega^{\mu\nu} + \frac{1}{2}\,m_\omega^2\,\omega_\mu\,\omega^\mu \qquad (1)\end{aligned}$$

where $\mathcal{F}^{\mu\nu} = \partial^\mu\,A^\nu - \partial^\nu\,A^\mu$ and $\Omega^{\mu\nu} = \partial^\mu\,\omega^\nu - \partial^\nu\,\omega^\mu$ are the electromagnetic and vector meson field strength tensors, q_b denotes the electric charge, and $g_{\sigma,\omega}$ the strong coupling constants, and $\sigma^{\mu\nu} = i/2\,[\gamma^\mu,\gamma^\nu]$. We study the case of a constant external magnetic field B applied along the z axis. In order to fix ideas and facilitate the comparison with previous results [11], we choose the gauge $A^\mu = (0, 0, Bx, 0)$. To simplify the discussion, we consider first baryons interacting only with B, and mesons will be included later.
It must be mentioned that the remaining content of this section, has been studied long time ago, see for instance [11]. But we present here a summary in order to state clearly the notation used.
In this approach the classical eigenstates $\psi^{(b)} = \phi^{(b)}\,e^{-iE^{(b)}\,t}$ of the Dirac equation satisfy

$$[\vec{\alpha}.\vec{\pi} + \gamma^0\,m_b - i\gamma^0\,\gamma^1\gamma^2\,\kappa_b\,B]\phi^{(b)} = E^{(b)}\,\phi^{(b)} \qquad (2)$$

with $\vec{\alpha} = \gamma^0\,\vec{\gamma}$ and $\vec{\pi} = -i\vec{\nabla} - q_b\,\vec{A}$. The label s indicates the alignment of the magnetic moment with the external field. It must be borne in mind that when considering nuclear particles, we can write $\kappa_b = \chi_b\mu_N$, with the anomalous moments $\chi_p = 2.79$ for protons, $\chi_n = -1.91$ for neutrons, and μ_N the nuclear magneton.

2.1 Charged states

The particle solutions for energies E_{ns} are given by
$\phi^{(+)(p)}_{nsp_yp_z}(\xi, y, z) = e^{i(p_y y + p_z z)}\,e^{-\xi^2/2}\,u_{nsp_z}(\xi)$ with

$$u_{nsp_z}(\xi) = N_{ns}\begin{pmatrix} H_n(\xi) \\ \frac{2\,n\,s\,p_z\,\sqrt{qB}\,i}{(\Delta_n + s\,m_p)\,(E_{ns} + s\,\Delta_n - \kappa_p B)}\,H_{n-1}(\xi) \\ \frac{p_z}{E_{ns} + s\,\Delta_n - \kappa_p B}\,H_n(\xi) \\ -\frac{2\,n\,s\,\sqrt{qB}\,i}{\Delta_n + s\,m_p}\,H_{n-1}(\xi) \end{pmatrix} \qquad (3)$$

and,

$$\xi = (-p_y + qBx)/\sqrt{qB} \tag{4}$$

$$\Delta_n = \sqrt{m_p^2 + 2nqB} \tag{5}$$

$$E_{ns} = \sqrt{p_z^2 + (\Delta_n - s\,\kappa_p B)^2} \tag{6}$$

$$N_{ns}^2 = \frac{\sqrt{qB}}{4\,\sqrt{\pi}(2\pi)^2\,2^n\,n!}\frac{(\Delta_n + s\,m_p)\,(E_{ns} + s\,\Delta_n - \kappa_p B)}{m_p\,(\Delta_n - s\,\kappa_p B)} \tag{7}$$

H_n stands for the Hermite polynomials, and $n \geq 1$.
In the case $n = 0$ the physical eigenstate corresponds to $\phi^{(+)(p)}_{0p_yp_z}(\xi, y, z) = e^{i(p_y y + p_z z)}\,e^{-\xi^2/2}\,u_{0p_z}$, with

$$u_{0p_z} = N_0 \begin{pmatrix} 1 \\ 0 \\ \frac{p_z}{E_0 + m_p - \kappa_p B} \\ 0 \end{pmatrix} \tag{8}$$

and

$$E_0 = \sqrt{p_z^2 + (m_p - \kappa_p B)^2} \tag{9}$$

$$N_0^2 = \frac{\sqrt{qB}}{2\,\sqrt{\pi}(2\pi)^2}\frac{E_0 + m_p - \kappa_p B}{m_p - \kappa_p B} \tag{10}$$

Another solution exists for $n = 0$ and $s = -1$, with eigenvalue $\sqrt{p_z^2 + (m_p + \kappa_p B)^2}$, but it is asymptotically divergent.
The antiparticle states $\phi^{(-)(p)}_{ns}$ correspond to the negative eigenvalues $-E_{ns}$ and have the eigenfunctions $\phi^{(-)(p)}_{nsp_yp_z}(\xi, y, z) = e^{-i(p_y y + p_z z)}\,e^{-\eta^2/2}\,v_{nsp_z}(\eta)$ with

$$v_{nsp_z}(\eta) = N_{ns} \begin{pmatrix} \frac{p_z}{E_{ns} + s\,\Delta_n - \kappa_p B}\,H_n(\eta) \\ \frac{2\,n\,s\,\sqrt{qB}\,i}{\Delta_n + s\,m_p}\,H_{n-1}(\eta) \\ H_n(\eta) \\ \frac{-2\,n\,s\,p_z\,\sqrt{qB}\,i}{(\Delta_n + s\,m_p)\,(E_{ns} + s\,\Delta_n - \kappa_p B)}\,H_{n-1}(\eta) \end{pmatrix} \tag{11}$$

where $\eta = (p_y + qBx)/\sqrt{qB}$ and $n \geq 1$. While for $n = 0$ the antiparticle state $\phi_0^{(-)(p)}$ has negative energy $-E_0$ and its wave function reads $\phi_{0p_yp_z}^{(-)(p)}(\eta, y, z) = e^{-i(p_y y + p_z z)}\, e^{-\eta^2/2}\, v_{0p_z}$ with

$$v_{0p_z} = N_0 \begin{pmatrix} \frac{p_z}{E_0 + m_p - \kappa_p B} \\ 0 \\ 1 \\ 0 \end{pmatrix} \tag{12}$$

The eigenstates are normalized according to [27]

$$< \bar{\phi}_{nsp'_yp'_z}^{(\pm)(p)} | \phi_{nsp_yp_z}^{(\pm)(p)} > \;\; = \;\; \pm\delta(p'_y - p_y)\, \delta(p'_z - p_z) \tag{13}$$

and therefore satisfy the covariant orthogonal conditions

$$\begin{aligned} < \phi_{n's'p'_yp'_z}^{(\pm)(p)\dagger} | \phi_{nsp_yp_z}^{(\pm)(p)} > \;\; &= \;\; \frac{E_{ns}\, \Delta_n}{m_p\, (\Delta_n - s\, \kappa_p\, B)}\, \delta_{n'n}\, \delta_{s's}\, \delta(p'_y - p_y)\, \delta(p'_z - p_z) \\ < \phi_{n's'p'_yp'_z}^{(+)(p)\dagger} | \phi_{nsp_yp_z}^{(-)(p)} > \;\; &= \;\; < \phi_{n's'p'_yp'_z}^{(-)(p)\dagger} | \phi_{nsp_yp_z}^{(+)(p)} >= 0 \end{aligned} \tag{14}$$

These conditions also include the case $n = 0, s = 1$, if $\Delta_0 = m_p$ is assumed.

2.2 Neutral states

The positive energy eigenstates have wave functions $\phi_{\vec{p}s}^{(+)(n)}(\vec{r}) = e^{i\vec{p}.\vec{r}}\, u_{\vec{p}s}$, with

$$u_{\vec{p}s} = N_{\vec{p}s} \begin{pmatrix} 1 \\ \frac{-s\,(p_x + ip_y)\, p_z}{(\Delta + s\, m_n)\, (E_{\vec{p}s} + s\, \Delta - \kappa_n B)} \\ \frac{p_z}{E_{\vec{p}s} + s\, \Delta - \kappa_n B} \\ \frac{s\,(p_x + ip_y)}{\Delta + s\, m_n} \end{pmatrix} \tag{15}$$

and

$$E_{\vec{p}s} \;\; = \;\; \sqrt{p_z^2 + (\Delta - s\, \kappa_n B)^2} \tag{16}$$

$$\Delta \;\; = \;\; \sqrt{m_n^2 + p_x^2 + p_y^2} \tag{17}$$

$$N_{\vec{p}s}^2 \;\; = \;\; \frac{1}{4\,(2\pi)^3} \frac{(\Delta + s\, m_n)\, (E_{\vec{p}s} + s\, \Delta - \kappa_n B)}{m_n\, (\Delta - s\, \kappa_n B)}. \tag{18}$$

On the other hand, the antiparticle states, of energy $-E_{\vec{p}s}$, are $\phi^{(-)(n)}_{\vec{p}s}(\vec{r}) = e^{-i\vec{p}.\vec{r}}\, v_{\vec{p}s}$ with

$$v_{\vec{p}s} = N_{\vec{p}s} \begin{pmatrix} \frac{p_z}{E_{\vec{p}s}+s\,\Delta-\kappa_n B} \\ \\ \frac{s\,(p_x+ip_y)}{\Delta+s\,m_n} \\ \\ 1 \\ \\ \frac{-s\,(p_x+ip_y)\,p_z}{(\Delta+s\,m_n)\,(E_{\vec{p}s}+s\,\Delta-\kappa_n B)} \end{pmatrix} \tag{19}$$

Similarly to the previous case, these eigenstates are normalized according to

$$< \bar{\phi}^{(\pm)(n)}_{\vec{p}'s} | \phi^{(\pm)(n)}_{\vec{p}s} > \quad = \quad \pm\delta^3(\vec{p}' - \vec{p}) \tag{20}$$

and therefore satisfy the covariant orthogonal conditions

$$\begin{aligned} < \phi^{(\pm)(n)\dagger}_{\vec{p}'s'} | \phi^{(\pm)(n)}_{\vec{p}s} > \quad &= \quad \frac{E_{\vec{p}s}\,\Delta}{m_n\,(\Delta - s\,\kappa_n\,B)}\,\delta_{s's}\,\delta^3(\vec{p}' - \vec{p}) \\ < \phi^{(+)(n)\dagger}_{\vec{p}'s'} | \phi^{(-)(n)}_{\vec{p}s} > \quad &= \quad < \phi^{(-)(n)\dagger}_{\vec{p}'s'} | \phi^{(+)(n)}_{\vec{p}s} >= 0 \end{aligned} \tag{21}$$

3 Dirac fields and Green functions

We propose an expansion of the fields in terms of creation and destruction operators for states with the quantum numbers specified in the previous section. Hence, the coefficients in this expansion correspond to the wave functions previously described [19].
Thus, we obtain for the charged field

$$\begin{aligned} &\Psi^{(p)}(t,\vec{r}) = \int dp_y\, dp_z \sqrt{\tfrac{m_p - \kappa_p\, B}{E_0}} \quad . \\ &\Big[e^{-iE_0\,t}\, e^{i(p_y y + p_z z)}\, e^{-\xi^2/2}\, u_{0p_z}(\xi)\, a^{(p)}_{0p_y p_z} + e^{iE_0\,t}\, e^{-i(p_y y + p_z z)}\, e^{-\eta^2/2}\, v_{0p_z}(\eta)\, d^{\dagger(p)}_{0p_y p_z} \Big] \\ &+ \textstyle\sum_{n=1} \sum_{s=\pm 1} \int dp_y\, dp_z \sqrt{\frac{m_p\,(\Delta_n - s\,\kappa_p\,B)}{E_{ns}\,\Delta_n}} \quad . \\ &\Big[e^{-iE_{ns}\,t}\, e^{i(p_y y + p_z z)}\, e^{-\xi^2/2}\, u_{nsp_z}(\xi)\, a^{(p)}_{nsp_y p_z} + e^{iE_{ns}\,t}\, e^{-i(p_y y + p_z z)}\, e^{-\eta^2/2}\, v_{nsp_z}(\eta)\, d^{\dagger(p)}_{nsp_y p_z} \Big] \end{aligned} \tag{22}$$

and for the neutral field

$$\Psi^{(n)}(t,\vec{r}) = \textstyle\sum_{s=\pm 1} \int d\vec{p} \sqrt{\frac{m_n\,(\Delta - s\,\kappa_n\,B)}{E_{\vec{p}s}\,\Delta}} \Big[e^{-iE_{\vec{p}s}\,t}\, e^{i\vec{p}.\vec{r}}\, u_{\vec{p}s}\, a^{(n)}_{\vec{p}s} + e^{iE_{\vec{p}s}\,t}\, e^{-i\vec{p}.\vec{r}}\, v_{\vec{p}s}\, d^{\dagger(n)}_{\vec{p}s} \Big] \tag{23}$$

where we have introduced an appropriate measure of integration in the phase space [26]. Written in this form, these expressions clearly reduce to the more familiar ones when $\kappa_b = 0$.

These fields satisfy the anti-commutation relations

$$\{\Psi_\alpha^{\dagger(b)}(t,\vec{r}\,'),\Psi_\beta^{(b)}(t,\vec{r})\}=\delta_{\alpha\beta}\,\delta^3(\vec{r}\,'-\vec{r}) \tag{24}$$

if the standard anti-commutation relations are assumed for the creation and annihilation operators, *i.e.*

$$\begin{array}{l}\{a_j^{\dagger(b)},a_{j'}^{(b)}\}=\{d_j^{\dagger(b)},d_{j'}^{(b)}\}=\delta_{jj'}\\ \{d_j^{\dagger(b)},a_{j'}^{(b)}\}=\{a_j^{\dagger(b)},d_{j'}^{(b)}\}=0\end{array} \tag{25}$$

where the indices j,j' stand for a full set of quantum numbers, either discrete or continuum.
These fields are used to evaluate the in-medium causal propagator

$$\begin{array}{l}iG_{\alpha\beta}^{(b)}(t',\vec{r}\,',t,\vec{r})=<T[\Psi_\alpha^{(b)}(t',\vec{r}\,')\bar{\Psi}_\beta^{(b)}(t,\vec{r})]>=\\ \Theta(t'-t)<\Psi_\alpha^{(b)}(t',\vec{r}\,')\bar{\Psi}_\beta^{(b)}(t,\vec{r})>-\Theta(t-t')<\bar{\Psi}_\beta^{(b)}(t,\vec{r})\Psi_\alpha^{(b)}(t',\vec{r}\,')>\end{array} \tag{26}$$

where Θ denotes the Heaviside step function. Here the angular brackets must be regarded as an statistical mean value, as obtained for instance, by evaluating the trace with the density matrix of the system. The same average acting on the products of a pair of creation and/or destruction operators produce the well known results [28]

$$<a_{j'}^{(b)}a_j^{\dagger(b)}> \;=\; \delta_{jj'}-<a_j^{\dagger(b)}a_{j'}^{(b)}> \tag{27}$$

$$<d_{j'}^{(b)}d_j^{\dagger(b)}> \;=\; \delta_{jj'}-<d_j^{\dagger(b)}d_{j'}^{(b)}> \tag{28}$$

$$<a_j^{\dagger(b)}a_{j'}^{(b)}> \;=\; \delta_{jj'}\,n_F(T,E_j^{(b)}) \tag{29}$$

$$<d_j^{\dagger(b)}d_{j'}^{(b)}> \;=\; \delta_{jj'}\,n_F(T,-E_j^{(b)}) \tag{30}$$

where n_F denotes the Fermi occupation number

$$n_F(T,p_0)=\frac{\Theta(p_0)}{1+e^{(p_0-\mu_b)/T}}+\frac{\Theta(-p_0)}{1+e^{-(p_0-\mu_b)/T}} \tag{31}$$

at temperature T and chemical potential μ_b associated with the conservation of the baryonic number.
The remaining combination of pairs have null expectation values.
The following expansions of the direct product of the spinors (3),(8),(15) are particularly useful

$$u_{0p_z}(\xi)\otimes\bar{u}_{0p_z}(\xi')=\frac{1}{(2\pi)^2}\frac{\sqrt{qB/\pi}}{4(m_p-\kappa_p\,B)}(E_0\gamma_0-p_z\gamma_3+m_p-\kappa_p\,B)(1+i\gamma_1\gamma_2) \tag{32}$$

$$\begin{array}{l}u_{nsp_z}(\xi)\otimes\bar{u}_{nsp_z}(\xi')=\dfrac{1}{(2\pi)^2}\dfrac{\sqrt{qB/\pi}(\Delta_n+s\,m_p)}{2^{n+3}n!\;m_p(\Delta_n-s\,\kappa_p\,B)}\Big[H_n(\xi)(E_{ns}\gamma_0-p_z\gamma_3+s\,\Delta_n-\kappa_p\,B)\\ \quad+i\,\dfrac{m_p-s\,\Delta_n}{\sqrt{qB}}H_{n-1}(\xi)(E_{ns}\gamma_0-p_z\gamma_3-s\,\Delta_n+\kappa_p\,B)\gamma_1\Big](1+i\,\gamma_1\gamma_2)\\ \qquad\qquad\Big[H_n(\xi')+i\,\dfrac{m_p-s\,\Delta_n}{\sqrt{qB}}H_{n-1}(\xi')\gamma_1\Big]\end{array} \tag{33}$$

$$u_{\vec{p}s}\otimes\bar{u}_{\vec{p}s}=\frac{1}{(2\pi)^3}\frac{i\,s\,\gamma^1\gamma^2}{4m_n(\Delta-\kappa_n sB)}\left[E_{\vec{p}s}\gamma_0-p_z\gamma_3+(s\Delta-\kappa_n\,B)\,i\gamma_1\gamma_2\right]$$
$$(-p_x\gamma_1-p_y\gamma_2+m_n+s\Delta i\gamma_1\gamma_2) \quad (34)$$

A similar result can be obtained for the case of antiparticles.

In order to evaluate (26) we insert the expansions (22) or (23), and we use the expectation values (27)-(30). With the aim of unifying the contributions coming from particles and antiparticles, we apply the following relations

$$\begin{aligned}\frac{i}{2\pi}\int_{-\infty}^{\infty}dp_0\frac{f(p_0)e^{-ip_0(t'-t)}}{p_0^2-E^2+i\epsilon} &= \Theta(t'-t)\frac{f(E)e^{-iE(t'-t)}}{2E}+\Theta(t-t')\frac{f(-E)e^{iE(t'-t)}}{2E}\\ n_F(T,\pm E) &= 2\int_{-\infty}^{\infty}dp_0|p_0|\;\Theta(\pm p_0)\;\delta(p_0^2-E^2)\;n_F(T,p_0)\end{aligned}$$

($E>0$). Finally we make use of Eqs. (32)-(34) and similar relations for antiparticles, to obtain the following results

$$\begin{aligned}G^{(p)}_{\alpha\beta}(t',\vec{r}\,',t,\vec{r}) &= \frac{1}{2}\sqrt{\frac{qB}{\pi}}\int\frac{dp_0\,dp_y\,dp_z}{(2\pi)^3}e^{-ip_0\,(t'-t)}\;e^{i[p_y(y'-y)+p_z(z'-z)]}\;e^{-(\xi'^2+\xi^2)/2}\\ &\left[\Lambda^{0(p)}_{\alpha\beta}\;\Xi(T,E_0)+\sum_{n=1}\sum_{s=\pm1}\frac{\Delta_n+s\,m_p}{2^{n+1}\,n!\,\Delta_n}\Lambda^{ns(p)}_{\alpha\beta}(\xi',\xi)\;\Xi(T,E_{ns})\right] \quad (35)\end{aligned}$$

for charged particles, where

$$\Lambda^{0(p)} = \left(p_0\gamma^0-p_z\gamma^3+m_p-\kappa_p\,B\right)\left(1+i\gamma^1\gamma^2\right) \quad (36)$$

$$\begin{aligned}\Lambda^{ns(p)} &= \left[(p_0\gamma^0-p_z\gamma^3+s\Delta_n-\kappa_pB)H_n(\xi')+i\frac{m_p-s\Delta_n}{\sqrt{qB}}(p_0\gamma^0-p_z\gamma^3-s\Delta_n+\kappa_pB)\gamma^1H_{n-1}(\xi')\right]\\ &\left[(1+i\gamma^1\gamma^2)H_n(\xi)+i\frac{m_p-s\Delta_n}{\sqrt{qB}}\gamma^1(1-i\gamma^1\gamma^2)H_{n-1}(\xi)\right] \quad (37)\end{aligned}$$

$$\Xi(T,E) = \frac{1}{p_0^2-E^2+i\epsilon}+2\pi\,i\,n_F(T,p_0)\,\delta(p_0^2-E^2) \quad (38)$$

and $\xi'=(-p_y+qBx')/\sqrt{qB}$.
In addition we have

$$G^{(n)}_{\alpha\beta}(t',\vec{r}\,',t,\vec{r})=\textstyle\sum_{s=\pm1}\int\frac{d^4p}{(2\pi)^4}e^{-ip^\mu\,(x'_\mu-x_\mu)}\Lambda^{s\,(n)}_{\alpha\beta}\;\Xi(T,E_{\vec{p}s}) \quad (39)$$

for neutral particles, where

$$\Lambda^{s\,(n)}=\tfrac{-is\,\gamma^1\gamma^2}{2\Delta}\left[p_0\gamma^0-p_z\gamma^3+i\gamma^1\gamma^2(s\Delta-\kappa_nB)\right]\left(p_x\gamma^1+p_y\gamma^2-m_n-is\Delta\gamma^1\gamma^2\right)$$

eqs. (35) and (39) resume the main findings of this work. Of course, these Green functions satisfy the differential equation

$$\left(i\gamma^\mu D_\mu-m_b+i\gamma^1\gamma^2\,\kappa_b\,B\right)G^{(b)}(x,x')=\delta^4(x-x')$$

where $D_\mu = \partial_\mu + iq_b\, A_\mu$.

It is a well known fact that real time formulations of the thermal field theory [29, 30], like Schwinger-Keldysh theory or Thermo Field Dynamics (TFD), needs to duplicate the degrees of freedom in order to keep the formalism and procedures of the usual field theory. In TFD for instance, to each physical field $\varphi^{(1)}(x)$ there corresponds a dual partner $\varphi^{(2)}(x)$, and they are related by the so called tilde conjugation operation. As a consequence, there is a 2×2 matrix associated to the product of two fields. This is also the case for the one-particle propagators $i\,G^{ab}(x,x') =< T\varphi^{(a)}(x)\varphi^{(b)}(x') >$, and the corresponding self-energies. Within this context the results shown in Eqs.(35) and (39) correspond to the component (1,1) of the TFD representation. However, it suffices to treat the MFA at zero temperature to be developed in the next section.
To evaluate higher order corrections to the finite temperature self energy, the full dependence on the thermal degrees of freedom must be taken into account. This means that for a given perturbative diagram, for each internal line there corresponds a 2×2 propagator and a sum over the thermal index $c = 1, 2$ should be included for each internal vertex [31].
A resume of the Feynman graph rules in TFD for the QHD model are given in Ref.[32].
Within the quasi-particle scheme described at the beginning of this section, it is no difficult to evaluate some thermal expectation values required to complete the TFD propagator. In practice, Eq.(38) must be replaced by the following matrix

$$\Xi(T,E) = \begin{pmatrix} \frac{1}{p_0^2-E^2+i\varepsilon} & 0 \\ 0 & \frac{1}{p_0^2-E^2-i\varepsilon} \end{pmatrix} + 2\pi i\delta(p_0^2-E^2) \begin{pmatrix} n_F(T,p_0) & \bar{n}_F(T,p_0) \\ \bar{n}_F(T,p_0) & -n_F(T,p_0) \end{pmatrix} \quad (40)$$

with n_F as in Eq.(31) and

$$\bar{n}_F(T,p_0) = \frac{e^{(p_0-\mu_b)/2T}}{1+e^{(p_0-\mu_b)/T}}\Theta(p_0) - \frac{e^{-(p_0-\mu_b)/2T}}{1+e^{-(p_0-\mu_b)/T}}\Theta(-p_0)$$

It must be noticed that at zero temperature the matrix in Eq.(40) becomes diagonal, that is, the thermal degrees of freedom are decoupled.

In the next section we study the coherence of these results, by comparing some simple calculations with well established facts of the QHD formalism.

4 Mean values of nuclear matter densities

In order to keep the simplicity of the discussion, we have reduced the problem to its bare minimum. Up to this point we have regarded the nucleon as a non-interacting particle. Now, we include the strong interaction between the nucleons and its environment in the MFA of the QHD model. Within this scheme, the lightest mesons dress the nucleon giving rise to a quasi-particle picture. The net effect is to modify the mass of the nucleons by $m_b^* = m_b - g_\sigma\,\sigma_0$, as a consequence the modified quantities Δ_n^*, Δ^* must be introduced. The single particle

spectrum becomes $\epsilon_{b\,j} = E_j + g_\omega\,\omega_0$, where σ_0, ω_0 are the uniform in-medium expectation values of the meson fields $f_0(500)$ and $\omega(782)$, respectively [9].
We can include these modifications into the propagators (35) and (39) by assuming that the masses m_p^* and m_n^* represent the in-medium values described above, while the change in the energy spectrum can be handled by replacing the chemical potentials μ_b, which are implicit in the Fermi distribution functions n_F, by the effective ones $\bar{\mu}_b = \mu_b - g_\omega\,\omega_0$.
In the following we check the validity of the results presented in the previous section by comparing our calculations for the nuclear scalar, baryon and energy densities, with those presented in previous calculations. In the spirit of the MFA we neglect the divergent contributions coming from the Dirac sea. We start examining the average mean field scalar $\rho_s^{(b)}$ and baryon $\rho^{(b)}$ densities, using the general definitions

$$\begin{aligned}\rho_s^{(b)}(t,\vec{r}) = & \quad < \bar{\Psi}^{(b)}(t,\vec{r})\,\Psi^{(b)}(t,\vec{r}) > \\ = & \quad -i \lim_{(t'\to t^+,\vec{r}\,'\to\vec{r})} Tr\{G^{(b)}(t,\vec{r},t',\vec{r}\,')\} \end{aligned} \tag{41}$$

$$\begin{aligned}\rho^{(b)}(t,\vec{r}) = & \quad < \bar{\Psi}^{(b)}(t,\vec{r})\,\gamma^0\,\Psi^{(b)}(t,\vec{r}) > \\ = & \quad -i \lim_{(t'\to t^+,\vec{r}\,'\to\vec{r})} Tr\{\gamma^0\,G^{(b)}(t,\vec{r},t',\vec{r}\,')\} \end{aligned} \tag{42}$$

For the case of protons the results are

$$\begin{aligned}\rho_s^{(p)} = & \quad \frac{qB}{2\pi^2}\int_0^\infty dp_z\{\frac{(m_p^* - \kappa_p\,B)}{\epsilon_0}[n_F(T,\epsilon_0) + n_F(T,-\epsilon_0)] \\ + & \quad m_p^* \sum_{n=1}\sum_{s=\pm 1}\frac{(\Delta_n^* - s\,\kappa_p B)}{\epsilon_{ns}\,\Delta_n^*}[n_F(T,\epsilon_{ns}) + n_F(T,-\epsilon_{ns})]\} \end{aligned} \tag{43}$$

$$\begin{aligned}\rho^{(p)} = & \quad \frac{qB}{2\pi^2}\int_0^\infty dp_z\{[n_F(T,\epsilon_0) - n_F(T,-\epsilon_0)] \\ + & \quad \sum_{n=1}\sum_{s=\pm 1}[n_F(T,\epsilon_{ns}) - n_F(T,-\epsilon_{ns})]\} \end{aligned} \tag{44}$$

Similarly for neutrons

$$\rho_s^{(n)} = \quad \int\frac{d^3p}{(2\pi)^3}\sum_{s=\pm 1}\frac{(\Delta^* - s\,\kappa_n B)}{\epsilon_{\vec{p}s}\,\Delta^*}[n_F(T,\epsilon_{\vec{p}s}) + n_F(T,-\epsilon_{\vec{p}s})] \tag{45}$$

$$\rho^{(n)} = \quad \int\frac{d^3p}{(2\pi)^3}\sum_{s=\pm 1}[n_F(T,\epsilon_{\vec{p}s}) - n_F(T,-\epsilon_{\vec{p}s})] \tag{46}$$

As the temperature tends to zero, $T \to 0$, we have $n_F(T,-\epsilon^{(b)}) \to 0$, $n_F(T,\epsilon^{(b)}) \to \Theta(\bar{\mu}_b - \epsilon^{(b)})$. The last condition defines the Fermi momentum for protons and neutrons. For the first case we have $p_{Fs}^{(p)} = \sqrt{\bar{\mu}_p^2 - (\Delta_{n\,\max}^* - s\kappa_p B)^2}$, with the highest occupied Landau level given by the condition $|\Delta_{n\,\max}^* - s\kappa_p B| = \bar{\mu}_p$.

While for neutrons the Fermi surface is defined by $p_{Fs}^{(n)} = \sqrt{\bar{\mu}_n^2 - (\Delta_{\max}^* - s\kappa_n B)^2}$ along the z-axis and by $|\Delta_{\max}^* - s\kappa_n B| = \bar{\mu}_n$ in the orthogonal plane.

The baryonic contribution to the energy density $\varepsilon^{(b)}$ arises from the mean field value of the Hamiltonian density operator $\mathcal{H}^{(b)}(t,\vec{r}) = \bar{\Psi}^{(b)}(t,\vec{r})\,(i\gamma^0\partial/\partial t)\,\Psi^{(b)}(t,\vec{r})$, *i.e.*

$$\varepsilon^{(b)} =< \mathcal{H}^{(b)} >= -i \lim_{(t'\to t^+,\vec{r}\,'\to\vec{r})} Tr\{(i\gamma^0\partial/\partial t)\,G^{(b)}(t,\vec{r},t',\vec{r}\,')\} \tag{47}$$

Thus, we have

$$\begin{aligned} \varepsilon^{(p)} = \quad & \frac{qB}{2\pi^2}\int_0^\infty dp_z\{\epsilon_0\,[n_F(T,\epsilon_0) + n_F(T,-\epsilon_0)] \\ + \quad & \sum_{n=1}\sum_{s=\pm 1}\epsilon_{ns}\,[n_F(T,\epsilon_{ns}) + n_F(T,-\epsilon_{ns})]\} \end{aligned} \tag{48}$$

$$\varepsilon^{(n)} = \quad \int \frac{d^3p}{(2\pi)^3}\sum_{s=\pm 1}\epsilon_{\vec{p}s}\,[n_F(T,\epsilon_{\vec{p}s}) + n_F(T,-\epsilon_{\vec{p}s})] \tag{49}$$

By taking the limit $T \to 0$ of these results we find a complete agreement with the calculations of [11].

5 Conclusions

In this work we have evaluated the covariant propagator for nucleons in the presence of a strong magnetic field. We have extended previous results by including the intrinsic magnetic moment and the effects of finite density and temperature. We have performed a detailed derivation with a clear statement of the notation used, a fact that was lacking in the literature.
Furthermore, our results have been interpreted in the context of the Thermo Field Dynamics, and definite expressions for the thermal propagator as 2×2 matrix have been presented.
We have performed some simple calculations which show the coherence with previous results. We consider that the relevance of our findings lies in the fact that it allows the evaluation of in medium nuclear processes by including the full effects of an external magnetic field, and further corrections can be included by using the diagrammatic techniques of relativistic Quantum Field Theory.
Self consistent calculations of corrections to the propagation of hadrons in matter under strong magnetic fields are now in progress.

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References

[1] D. Lai, Rev. Mod. Phys. **73**, 629 (2001).

[2] D. Kharzeev, Phys. Lett. B **633**, 260 (2006).

[3] D. E. Kharzeev, L. D. McLerran, H. J. Warringa, Nucl. Phys. A **803**, 227 (2008).

[4] Y. -J. Mo , S. -Q. Feng, Y. -F. Shi, Phys. Rev. C **88**, 024901 (2013).

[5] V. V. Skokov, A. Y. Illarionov, V. D. Toneev, Int. J. Mod. Phys. A **24**, 5925 (2009).

[6] R. S. Duncan, C. Thompson, Astrophys. J. **392**, L9 (1992).

[7] C. Thompson, R. S. Duncan, Mon. Not. Roy. Astron. Soc. **275**, 255 (1995).

[8] C. Thompson, R. S. Duncan, Astrophys. J. **473**, 322 (1996).

[9] B. D. Serot, J. D. Walecka, *Advan. Nucl. Phys.* **16**, 1 (1986).

[10] S. Chakrabarty, D. Bandyopadhyay, S. Pal, Phys. Rev. Lett. **78**, 2898 (1997).

[11] A. Broderick, M. Prakash, J. M. Lattimer, Astroph. J. **537**, 351 (2000).

[12] R. Mallick, R. Gopal, S. J. Ghosh, S. Raha, S. Roychowdhury, J. Phys. G **39**, 095201 (2012).

[13] J. Dong, W. Zuo, J. Gu, Phys. Rev. D **87**, 103010 (2013).

[14] J. Dong, U. Lombardo, W. Zuo, H. Zhang, Nucl. Phys. A **898**, 32 (2013).

[15] A. Rabhi, M. A. Perez-Garcia, C. Providencia, I. Vidaña Phys. Rev. C **91**, 045803 (2015).

[16] Z. Rezaei, G. H. Bordbar, Eur. Phys. J. A **52**, 132 (2016).

[17] C. Y. Ryu, C. H. Hyun, M.-K. Cheoun, J. Phys. G **37**, 105002 (2010).

[18] J. Schwinger, Phys. Rev. **82**, 664 (1951).

[19] V. I. Ritus, Sov. Phys. JETP **48**, 788 (1978).

[20] A. V. Kuznetsov, A. A. Okrugin, Int. J. Mod. Phys. A **26**, 2725 (2011).

[21] V. A. Miransky, I. A. Shovkovy, Phys. Rept. **576**, 1 (2015).

[22] T. Maruyama, M. Cheoun, T. Kajino, Y. Kwon, G. J. Mathews, Phys. Rev. D **91**, 123007 (2015).

[23] R. Aguirre, E. Bauer, J. Phys. G **42**, 105101 (2015).

[24] A. Ayala, A. Bashir, A. Raya, A. Sanchez, J. Phys. G **37**, 015001 (2010).

[25] E. V. Gorbar, V. A. Miransky, I. A. Shovkovy, X. Wang, Phys. Rev. D **88**, 25043 (2013).

[26] C. Itzykson, J. B. Zuber, *Quantum Field Theory*, (McGraw-Hill, 1980).

[27] J. D. Bjorken, S. D. Drell, *Relativistic Quantum Fields*, (McGraw Hill, 1965).

[28] A. L. Fetter, J. D. Walecka, *Quantum Theory of many-particle systems*, (McGraw Hill, 1971).

[29] K. C. Chou, Z. B. Su, B. L. Hao, L. Yu, Phys. Rep. **118** (1985) 1.

[30] N. P. Landsman, Ch. G. van Weert, Phys. Rep. **145** (1987) 141.

[31] H. Matsumoto, I. Ojima, H, Umezawa, Ann. Phys. **152** (1984) 348.

[32] K. Saito, T. Maruyama, K. Soutome, Phys. Rev. C **40** (1989) 407