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Exact Philosophy of Space-Time *

In honour of Mario Castagnino

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Abstract

Starting from Bunge's (1977) scientific ontology, we expose a materialistic relational theory of space-time, that carries out the program initiated by Leibniz, and provides a protophysical basis consistent with any rigorous formulation of General Relativity. Space-time is constructed from general concepts which are common to any consistent scientific theory and they are interpreted as emergent properties of the greatest assembly of things, namely, the world.

1 Introduction

All disciplines in modern science take the notions of space and time for granted: physics describes elementary particles as objects with wave functions of space and time, chemistry deals with flows of reactants, ecology studies the wandering of plankton in the multitudinous sea and sociology describes the interactions of neighboring cultures along their history.

But for all science space and time are primitive concepts, even for General Relativity that associates the metric structure of space-time with the gravitational field. Indeed, the question "What is space-time?" belongs to protophysics: the branch of ontology dealing with the basic assumptions in physics.

The ontological status of space and time has been a subject of debate for physicists and philosophers during the last 400 years. The kernel of this debate has been the confrontation between two antagonic positions: absolutism and relationalism. The former, held by Newton in his famous discussion with Leibniz (mediated by S. Clarke) [1] was stated by him in his *Principia* [2]

> Absolute time, true and mathematical, in itself and by its own nature, flows evenly without relation to any external thing.
>
> Absolute space, by its own nature, without relation with any external thing, stays always identical and motionless.

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Thus, for absolutists, space-time is the stage where the drama of nature is enacted; i.e. the absolutist position considers space-time as much a thing as planets or electrons are: a physical entity endowed with concrete properties. A modern version of the absolutist position has been held by by J. Wheeler in his geometrodynamical approach to physics [3].

The relationalist position instead asserts that space-time is not a thing but a complex of relations among physical things. In Leibniz's words [1]:

> I have said more than once that I hold space to be something merely relative, as time is; that I hold it to be an order of coexistents, as time is an order of successions.

In our theatrical analogy, relationalists consider space-time as a pattern weaved by the actors.

An important consequence of Leibniz's ideas is that if space-time is not an ontological primitive, then it should be possible to construct it starting from a deeper ontological level. That is to say, the spatiotemporal relations should be definable from more fundamental relations. There have been several attempts to analyze the relational nature of space-time, both subjective and phenomenological (e.g. [4, 5]) and objective and realistic [6, 7].

In this paper we paper I shall present a simplification and streamlining of a relational theory of space-time [8], based on the scientific and realistic ontology of Bunge [7, 9].[1]

The choice of Bunge's approach, which only assumes hypothesis common to all science, is because a deductive theory of space-time cannot be built with blocks alien to the physical science (such as cognoscent subjects or sensorial fields) in order to be compatible with contemporary physical theories.

The theory is presented in an axiomatic way although we shall limit ourselves in this paper to an informal presentation[2].

The ontological theory of space-time is a nice example of the interaction of science (mainly physics) and philosophy. Indeed, the hypothesis used to build space-time will be suggested by scientific observation, leading to a consolidation of its foundations.

2 Ontological summary

In this section we give a brief synopsis of the ontological presuppositions that we take for granted in our theory. For greater detail see [7, 9, 11]. The basic statements of the ontology can be formulated as follows:

1. There exist concrete objects x, named *things.* The set of all the things is denoted by Θ.

[1] The main simplifications with respect to [8] are the introduction of Axiom 5, the use of the notion of simultaneity to analyze clocks and of the axiom of distances to build uniformities. Also, a general rearranging of the theory of space has shortened the overall presentation.

[2] On the advantages of the axiomatic method see [10] and references therein.

2. Things can juxtapose ($\dot{+}$) and superimpose ($\dot{\times}$) to give new things. Juxtaposition as superimposition satisfy a Boolean algebra structure.

3. The *null thing* is a fiction equal to the superimposition of all things.

$$\diamond = \dot{\prod_{x \in \Theta}} x$$

4. Two things are separated if they do not superimpose:

$$x \wr y \Leftrightarrow x \dot{\times} y = \diamond$$

   Non-separated things are called united.

5. Let T a set of things. The *aggregation* of T (denoted $[T]$) is the supremum of T with respect to the operation $\dot{+}$.

6. The world ($\Box$) is the aggregation of all things:

$$\Box = [\Theta] \Leftrightarrow (x \sqsubset \Box \Leftrightarrow x \in \Theta)$$

   where the symbol '$\sqsubset$' means 'to be part of'.

7. All things are made out of basic things $x \in \Xi \subset \Theta$ by means of juxtaposition or superimposition. The basic things are elementary or primitive:

$$(x, y \in \Xi) \wedge (x \sqsubset y) \Rightarrow x = y$$

8. Things x have *properties* $P(x)$. These properties can be intrinsic or relational.

9. A property $p \in P(x)$ of a thing x is called *hereditary* if some of the components of x posses p:

$$\text{Her}\ p =_{\text{Df}} (\exists y)[y \sqsubset x \wedge p \in P(y)]$$

   A non hereditary property is called an *emergent* property.

10. The *state* of a thing x is a set of functions from a domain of reference M to the set of properties $\mathcal{P}$. The set of the accessible states of a thing x is the *lawful state space* of x: $S_{\text{L}}(x)$. The state of a thing is represented by a point in $S_{\text{L}}(x)$.

11. A *legal statement* is a restriction upon the state functions of a given class of things. A *natural law* is a property represented by an empirically corroborated legal statement.

12. The *ontological history* $h(x)$ of a thing x is a part of $S_{\text{L}}(x)$ defined by

$$h(x) = \{\langle t, F(t)\rangle | t \in M\}$$

   where t is an element of some auxiliary set M, and F are the functions that represent the properties of x.

13. There is a single *universal property* of material things called *energy* such that the energy of an isolated thing is unchanged during its ontological history.

14. Two things *interact* if each of them modifies the history of the other:

$$x \bowtie y \Leftrightarrow h(x \dot{+} y) \neq h(x) \cup h(y)$$

15. A thing x_{f} is a *reference frame* for x iff

    (a) M equals the state space of x_{f}, and

    (b) $h(x \dot{+} f) = h(x) \cup h(f)$

16. A *change* of a thing x is an ordered pair of states:

$$(s_1, s_2) \in E_{\mathrm{L}}(x) = S_{\mathrm{L}}(x) \times S_{\mathrm{L}}(x)$$

    A change is called an *event*, and the space $E_{\mathrm{L}}(x)$ is called the *event space* of x.

17. An event e_1 *precedes* another event e_2 if they compose to give $e_3 \in E_{\mathrm{L}}(x)$:

$$e_1 = (s_1, s_2) \ \wedge \ e_2 = (s_2, s_3) \Rightarrow e_3 = (s_1, s_3)$$

The ontology sketched here (due mainly to M. Bunge [7]. See also [12]) is realistic, because it assumes the existence of things endowed with properties, and objective, because it is free of any reference to cognoscent subjects. We will base the axiomatic formulation of the pregeometry of space-time on this ontology.

3 Local Time

Let us state now the set of hypothesis that introduce the notion of *local time*. First we assume the existence of an *order relation* between states of a given basic thing.[3]

Axiom 1 (Existence of temporal order (o)) *For each concrete basic thing $x \in \Theta$ there exist a single ordering relation between their states $\leq$.*

We now give a name to this ordering relation

Axiom 2 (Denotation of temporal order (s)) *The set of lawful states of x is* temporally ordered *by the $\leq$ relation.*

[3]We present an informal classification of axioms in mathematical (m), ontological (o), semantic (s) and physical (f). This classification is non exclusive, i.e. an axiom can belong to more than one class.

The above is a partial order relation: there are pairs of states that are not ordered by $\leq$; e.g. given an initial condition (x_0, v_0) for a moving particle, there are states (x_1, v_1) that are not visited by the particle.

Definition 1 (Proper history) *A totally order set of states of x is called a* proper history *of x.*

The above axioms do not guarantee the existence of a single proper history: they allow many of them, as in "The garden of forking paths" [13]. The following axiom forbids such possibility

Axiom 3 (Unicity of proper history (o)) *Each thing has one and only one proper history.*

Remark 1 ("Arrow of time") *The above axioms describe a kind of "arrow of time", although it is not related to irreversibility [14].*

A proper history is also an ontological history. The parameter $t \in M$ has not to be continuous. The following axiom, a very strong version of Heraclitus' hypothesis *Panta rhei*, states that every thing is changing continuously:

Axiom 4 (Continuity (o)) *If the entire set of states of an ontological history is divided in two subsets h_p and h_f such that every state in h_p temporally precedes any state in h_f, then there exists one and only one state s_0 such that $s_1 \leq s_0 \leq s_2$, where $s_1 \in h_\mathrm{p}$ and $s_2 \in h_\mathrm{f}$.*

Remark 2 *The axiom of continuity is stated in the Dedekind form [15].*

Remark 3 (Continuity in quantum mechanics) *Although quantum mechanical "changes of state" are usually considered "instantaneous", theory shows that probabilities change in a continuous way. The finite width of spectral lines also shows a continuous change in time [16].*

The following theorem can be proved with the standard methods of analysis [15]

Theorem 1 (Real representation) *Given a* unit change (s_0, s_1) *there exists a bijection $\mathcal{T} : h \leftrightarrow \Re$ such that*

$$h_1 = \{s(\tau) \mid \tau \in \Re\} \tag{1}$$

$$s_0 = s(0) \tag{2}$$

$$s_1 = s(1) \tag{3}$$

Definition 2 (Local time) *The function $\mathcal{T}$ is called* local time.

Remark 4 *The unit change (s_0, s_1) is arbitary. It defines an arbitrary "unit of local time" [14].*

The above theory of local time has an important philosophical consequence: *becoming*, which is usually conceived as evolution in time, is here more fundamental than time. The latter is constructed as an emergent property of a changing (i.e. a *becoming*) thing.

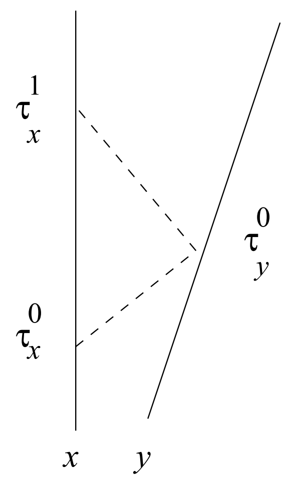

Figure 1: Scheme of a reflex action

4 Simultaneity

In order to introduce the concept of space we shall use the notion of *reflexive action* (or *reflex action*) between two things. Intuitively, a thing x acts on another thing y if the presence of x disturbs the history of y. Events in the real world seem to happen in such a way that it takes some time for the action of x to propagate up to y. This fact can be used to construct a relational theory of space *à la* Leibniz, that is, by taking space as a set of equitemporal things. It is necessary then to define the relation of simultaneity between states of things.

Let x and y be two things with histories $h(x_\tau)$ and $h(y_\tau)$, respectively, and let us suppose that the action of x on y starts at τ_x^0 (See figure 1). The history of y will be modified starting from τ_y^0. The proper times are still not related but we can introduce the reflex action to define the notion of simultaneity. The action of y on x, started at τ_y^0, will modify x from τ_x^1 on. The relation "the action of x on y is reflected to x" is the reflex action. Historically, G. Galilei [17] introduced the reflection of a light pulse on a mirror to measure the speed of light. With this relation we will define the concept of simultaneity of events that happen on different basic things (see also [18]).

Besides we have a second important fact: observation and experiment suggest that gravitation, whose source is energy, is a universal interaction, carried by the gravitational field.

Let us now state the above hypothesis in axiom form. First, we state

Axiom 5 (Universal interaction) *Any pair of basic things interact.*

This extremely strong axiom states not only that there exist no completely isolated things but that all things are interconnected.

Remark 5 (Universal inteconection) *This universal interconnection of things should not be confused with "universal interconnection" claimed by several mystical schools. The present interconnection is possible only through physical agents, with no mystical content.*

Remark 6 ("Accelerated observers") *It is possible to model two noninteracting things in Minkowski space assuming they are accelerated during an infinite proper time. It is easy to see that an infinite energy is necessary to keep a constant acceleration, so the model does not represent real things, with limited energy supply [16].*

Now consider the time interval $(\tau_x^1 - \tau_x^0)$. Special Relativity suggests that it is nonzero, since any action propagates with a finite speed. We then state

Axiom 6 ("Finite speed axiom" (o)) *Given two different and separated basic things x and y, such as in Figure 1, there exists a minimum positive bound for the interval $(\tau_x^1 - \tau_x^0)$ defined by the reflex action.*

Now we can define

Definition 3 (Simultaneity) τ_y^0 *is simultaneous with* $\tau_x^{1/2} =_{\mathrm{Df}} (1/2)(\tau_x^1 + \tau_x^0)$.

The local times on x and y can be synchronized by the simultaneity relation. However, as we know from General Relativity, the simultaneity relation is transitive only in special reference frames called *synchronous* [18]. We then include the following axiom:

Axiom 7 (Synchronizability (f)) *Given a set of separated basic things $\{x_i\}$ there is an assignment of proper times τ_i such that the relation of simultaneity is transitive.*

With this axiom, the simultaneity relation is an equivalence relation. Now we can define a first approximation to physical space:

Definition 4 (Ontic space) *The equivalence class of states defined by the relation of simultaneity on the set of things is the* ontic space E_{O}.

5 Universal Time

The notion of simultaneity allows the analysis of the notion of *clock*.

Definition 5 (Clock) *A thing $y \in \Theta$ is a* clock *for the thing x if there exists an injective function $\psi : S_{\mathrm{L}}(y) \to S_{\mathrm{L}}(x)$, such that $\tau < \tau' \Rightarrow \psi(\tau) < \psi(\tau')$.*

i.e.: the proper time of the clock grows in the same way as the time of things.

Another much more important concept can be analyzed in the same way

Definition 6 (Universal time) *The name* Universal time *applies to the proper time of a reference thing that is also a clock.*

From this definition we see that "universal time" is frame dependent in agreement with the results of Special Relativity.

6 Geometry

The notion of space we have developed up to now may be called "Philosopher's space" [6, 7]: there is room for things there, and separation, but there are no distances. Our next task is to introduce metric ideas.

6.1 Pregeometric space

We shall define distances mimicking the form it is done in relativity theory: the distance between two simultaneous events is equal to the time light takes to travel between them multiplied by the velocity of light. Alas, we do not have space yet, much less electromagnetism or optics, so we have to take some roundabout. We first introduce c through an axiom:

Axiom 8 ("Light speed" (o)) *c is a constant (uninterpreted) with suitable dimensions.*

Remark 7 *There is no ambiguity here! The theory of units and dimensions has been formalized [19] and the dimensions of distance will depend on the choice of those of c. Only with the development of electromagnetism it will be possible to interpret c as the speed of light. This definition is conventionalist, in contrast with the realistic philosophy adopted [20].*

Let us recall the definitions of (pseudo)metric and (pseudo)metric space.

Definition 7 (Metric (or distance)) *A* metric *on a set M is a function $d : M \times M \to \Re$ that satisfies the conditions*

1. $d(x, y) = d(y, x)$
2. $d(x, y) + d(y, z) \geq d(x, z)$
3. $x = y \Rightarrow d(x, y) = 0$
4. $d(y, x) = 0 \Rightarrow x = y$

If only the first three conditions are satisfied $d(x, y)$ is called a *pseudo-metric.*

A set M is a *(pseudo)metric space* if it admits a (pseudo)metric for every pair of points (elements).

Now consider the reflex action relation (Figure 1). We shall first define

Definition 8 (Ontic distance) *The* ontic distance *between the simultaneous states at τ_y^0 and $\tau_x^{1/2}$ is the function*

$$d(x, y) = \frac{c}{2} \mid \tau_x^1 - \tau_x^0 \mid$$

We still do not know if $d(x, y)$ is a distance (we have given it a name, that is all). So we state

Axiom 9 (Pseudometric (m)) *The function $d(x, y)$ is a pseudo-metric on the ontic space E_{O}.*

With the former axioms one can prove that E_{O} is a pseudo-metric space and that it can be completed.

Definition 9 (Pregeometric space) Pregeometric space E_{P} *is the completion of E_{O}.*

Remark 8 *The function $d(x, y)$ is nonzero for separated things because of axiom 6.*

The ontic distance $d(x, y)$ is a pseudo-metric because basic things are usually bulky and, in the case of gravitational or electromagnetic fields, they have infinite size. Axiom 9 only guarantees that separated things have non-zero distance.

6.2 Geometric space

To build geometric space we have to introduce point-like constructs.

Definition 10 (Ontic point) *Let $\xi \subset \Theta$ be a family of things. We say that ξ is a* complete family of united things *if it satisfies:*

1. *Any two things of ξ are united.*
2. *For any thing $x \notin \xi$ there is a thing $y \in \xi$ separated of x.*

Now we define a distance between ontic points

Definition 11 (Distance between ontic points) *Let ξ, η be two ontic points. The* distance between ontic points *is*

$$d_{\mathrm{G}}(\xi, \eta) = \sup_{(i,j)} d(x_i, y_j)$$

where $i \in I, j \in J$ belong to the respective index sets.

Figure 3 describes in an intuitive way how the pseudo-metric distances converge to the distance between the two ontic points.

Theorem 2 (Metricity) *The set of ontic points is a metric space with distance d_{G}.*

Proof: The first three distance conditions are satisfied because d is a pseudo-metric. To show that the fourth is satisfied observe that if $\xi \neq \eta$ there are $x_i \wr y_j$ and $d(x_i, y_j) > 0$ by axiom 6. So we find

$$\begin{aligned} \xi \neq \eta \quad &\Rightarrow \quad d_{\mathrm{G}}(\xi, \eta) > 0 \\ d_{\mathrm{G}}(\xi, \eta) > 0 \quad &\Rightarrow \quad \xi = \eta \end{aligned}$$

□

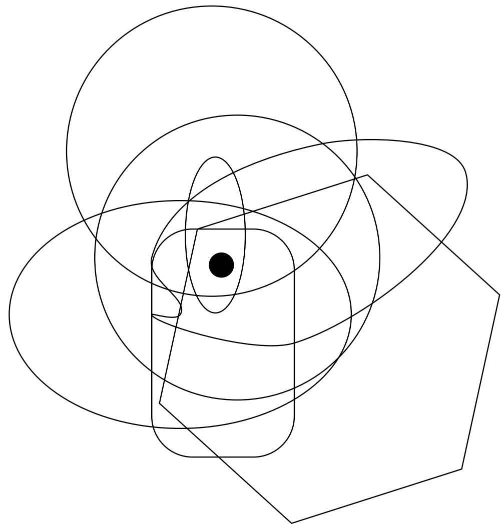

Figure 2: Scheme of an ontic point: The family of things ξ "closes" around the black dot representing the ontic point.

The isometric completion theorem [21, 22, 23] guarantees that the metric space of ontic points has a completion. This justifies the definition

Definition 12 (Geometric space) *The completion of ontic space is the* geometric space E_{G}.

6.3 Euclidean space

Finally we need additional hypotheses implying that the structure of geometric space is euclidean. Blumenthal [24] has given a set of axioms for Euclidean geometry based on the notion of distance. So we shall assume

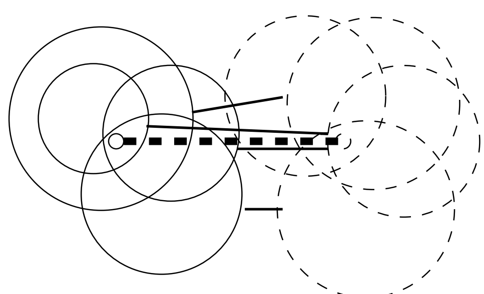

Figure 3: Construction of the distance between ontic points.

Axiom 10 (Euclidean structure (m)) *Geometric space satisfies axioms* $2, 3, 5', 6'$ *and* 7 *of reference* [24]*.*

An informal description of Blumenthal's axioms is as follows

2 and 3: There are at least three aligned points.

5': There are three unaligned points.

6': There are four non coplanar points.

7: There is no fourth dimension.

The exact formulation of these axioms uses only the notion of distance.

On the other hand, the fourth axiom of Blumenthal is a theorem in this formulation:

Theorem 3 (Completeness) *Geometric space* E_{G} *is complete.*

which follows from the isometric completion theorem. From the above, the following result can be derived [24]:

Theorem 4 (Euclidicity) *Geometric space* E_{G} *is euclidean.*

Theorem 3 has a deep ontological consequence. Since ontic space E_{O} is dense in geometrical space E_{G} we derive the

Theorem 5 (Aristotle-Leibniz) *Ontic space is a* plenum*.*

that is: there are concrete things everywhere.

Remark 9 *This theorem is, in spite of appearances, in agreement with modern physics. Indeed, the* plenum *hypothesis (introduced by Aristotle and later supported by Leibniz) is confirmed in Quantum Physics, and it leads to the prediction of a plethora of vacuum phenomena (like the Casimir effect), in good agreement with observation.*

Remark 10 *Let us remark that we have not assumed the existence of a* plenum*: is a consequence that the ontic space is dense in* E_{G}*. Neither we have assumed that ontic space* E_{O} *is euclidean, but that it is dense in an Euclidean space.*

Remark 11 *Remark 9 suggests that quantum mechanics is a necessary extension of classical mechanics to get a* plenum*. This is not true, since it is possible to "fill" the space with fluids, such as* dark matter *or* "dark energy"*, as it is assumed in modern cosmology [14].*

Our final axiom is a semantic one, stating the interpretation of the geometric space

Axiom 11 (Physical space (s)) E_{G} *represents physical space* E_{Ph}*.*

This axiom closes the present theory of space-time.

7 Conclusion

In the present theory, space-time is not a thing but a substantial property of the largest system of things, the world $\Box$, emerging from the set of the relational properties of basic things. Thus, any existential quantification over space-time can be translated into quantification over basic things. This shows that space-time has no ontological independence, but it is the product of the interrelation between basic ontological building blocks. For instance, rather than stating "space-time possesses a metric", it should be said: "the evolution of interacting things can be described attributing a metric tensor to their spatiotemporal relationships". In the present theory, however, space-time is interpreted in an strictly materialistic and Leibnitzian sense: it is an order of successive material coexistents.

We have mentioned above some simple philosophical consequences of this theory: becoming is more fundamental than time, and space (space-time, indeed) is a *plenum*.

We have exposed a materialistic relational theory of space-time, that carries out the program initiated by Leibniz, and provides a protophysical basis consistent with any rigorous formulation of General Relativity. Space-time is constructed from general concepts which are common to any consistent scientific theory. The particular hypothesis used for the construction have been taken from well corroborated scientific facts. It is shown, consequently, that there is no need for positing the independent existence of space-time over the set of individual things.

Acknowledgments

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