

Dispersive analysis of $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ Belle data

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Abstract

We analyse Belle data on the decay $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ using a dispersive representation of the vector form factor which is consistent with chiral symmetry and preserves analyticity and unitarity exactly. We fit the unknown theoretical parameters from the data, determining the values of the related low-energy observables $\langle r^2 \rangle_V^\pi$ and c_V^π . The implementation of isospin breaking effects is also discussed [1].

Keywords: Tau decays, dispersion relations, chiral Lagrangian, hadronic form factors, meson resonances

1. Introduction

The vector form factor of the pion, $F_V^\pi(s)$, encodes all unknown strong interaction dynamics in $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ decays [2]. It is defined as

$$\langle \pi^-(p_\pi) \pi^0(p_{\pi^0}) | \bar{d} \gamma^\mu u | 0 \rangle = \sqrt{2} (p_\pi - p_{\pi^0})^\mu F_V^\pi(s), \quad (1)$$

with $s = (p_\pi + p_{\pi^0})^2$. This form factor is not only relevant for the understanding of the hadronization of QCD currents at low energies (see [3]), but also represents a crucial ingredient for the evaluation of the leading order (LO) hadronic contribution to the anomalous magnetic moment of the muon, which provides a very stringent probe of new physics [4]. In addition, from the high-energy perspective, the $\pi\pi$ channel is (together with the three pion channel) essential to follow the spin in the Higgs di-tau decay channels at LHC [5], and thus to determine its spin and CP properties with the help of TAUOLA [6].

2. Theoretical setting

In $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ decays the electroweak part of the process is theoretically under control, while the hadronization of the quark currents is more involved since in the spanned energy region QCD is essentially non perturbative. The approximate chiral symmetry of

light-flavoured QCD allows to build an effective field theory, known as Chiral Perturbation Theory (χ PT) [7, 8, 9, 10], which is able to describe accurately the low-energy part of the spectrum, but fails at larger invariant masses [11]. In this region, new particles (ρ , K^* , a_1 , ...) are excited and their momenta and masses are large enough to prevent their use in the expansion parameters of the effective theory. Therefore these new active degrees of freedom have to be included in the action, and a new expansion parameter is needed. In this respect, the inverse of the number of colours of the QCD gauge group has proven to be a useful quantity to build the expansion upon [12, 13, 14]. A modelization of this idea in the meson sector for three flavours is provided by Resonance Chiral Theory ($R\chi T$) [15, 16, 17, 18], which recovers the χ PT results at next-to-leading order (NLO). Since it is known that the lightest resonances dominate the dynamics, the infinite tower of states predicted in the $N_C \rightarrow \infty$ limit can be restricted to the first excitations, taking into account as many states as required by the data. In addition, it is seen that the inclusion of resonance widths is essential to describe the observed phenomenology, although widths arise at NLO in the $1/N_C$ expansion. In principle, one can take into account this effect by computing off-shell widths consistently within $R\chi T$ [19, 20]. On the other hand, as in any effective theory, the symmetry properties determine the operators

allowed in the Lagrangian, but leave the corresponding coupling constants unknown. However, the QCD short-distance behaviour of the Green functions and related form factors [15, 16, 17, 18, 21, 22, 23, 24, 25, 26] provide a set of relations among these coefficients that renders R χ T more predictive.

3. Vector form factor of the pion and fits to data

Different approaches have been developed to deal with the diverse energy regimes. For $s < M_\rho^2$, the computation of $F_V^\pi(s)$ at NNLO in χ PT [27, 28, 29, 30] proves useful. In order to enlarge the domain of applicability up to 1 GeV, unitarization techniques [31, 32] and the Omnès solution to the dispersion relation have been employed [33, 34, 35]. Finally, in order to reach energies up to M_τ , the inclusion of the ρ' resonance [36] and even a tower of resonances, inspired in the $N_C \rightarrow \infty$ limit [37, 38], have been proposed. From the R χ T Lagrangian, including only the $\rho(770)$ multiplet one obtains

$$F_V^\pi(s) = 1 + \frac{F_V G_V}{F^2} \frac{s}{M_V^2 - s}, \quad (2)$$

where F is the pion decay constant in the chiral limit, $M_V = M_\rho$, and F_V and G_V measure the strength of the $\rho\pi\pi$ and ρV_μ couplings, respectively, and V_μ stands for the quark vector current. If the vanishing of the form factor at large energies is required, the relation $F_V G_V = F^2$ is obtained, yielding $F_V^\pi(s) = \frac{M_V^2}{M_V^2 - s}$. Now one can do better [33] and match this expression to the NLO result in χ PT. Final state interactions are included through the so-called chiral loop functions $A_P(s, \mu^2 = M_\rho^2)$. Then, the unitarity and analyticity constraints determine the Omnès exponentiation of the full loop function, leading to

$$F_V^\pi(s) = \frac{M_V^2}{M_V^2 - s} \exp \left\{ \frac{-s}{96\pi^2 F^2} \left[A_\pi(s) + \frac{1}{2} A_K(s) \right] \right\}. \quad (3)$$

Here one cannot simply include the resonance width by replacing $M_V^2 - s$ by $M_V^2 - s - iM_V \Gamma_V(s)$ in the propagator, since this would double count $\Im m[A_P(s)]$ and analyticity would be violated at NNLO in the chiral expansion. We follow instead a procedure similar to that proposed in Ref. [39] for the $K\pi$ vector form factor, in which unitarity and analyticity are satisfied exactly. The starting point is a form factor as in Eq. (2), where the loop functions are resummed into the denominator:

$$F_V^\pi(s)^{(0)} = \frac{M_V^2}{M_V^2 \left\{ 1 + \frac{s}{96\pi^2 F^2} \left[A_\pi(s) + \frac{1}{2} A_K(s) \right] \right\} - s} \quad (4)$$

$$= \frac{M_V^2}{M_V^2 \left\{ 1 + \frac{s}{96\pi^2 F^2} \Re e \left[A_\pi(s) + \frac{1}{2} A_K(s) \right] \right\} - s - iM_V \Gamma_V(s)}. \quad (5)$$

Thus the relevant ($I = 1, J = 1$) phase shift is taken to be

$$\tan \delta_1^1(s) = \frac{\Im m F_V^\pi(s)^{(0)}}{\Re e F_V^\pi(s)^{(0)}}. \quad (6)$$

This is now used as input for a three-subtracted dispersion relation for the form factor. In this way one gets

$$F_V^\pi(s) = \exp \left\{ \alpha_1 s + \frac{\alpha_2}{2} s^2 + \frac{s^3}{\pi} \int_{4m_\pi^2}^{\infty} ds' \frac{\delta_1^1(s')}{(s')^3 (s' - s - i\epsilon)} \right\}. \quad (7)$$

Kinematical isospin corrections can be easily included in Eq. (7) by considering different masses for the charged and neutral pions and kaons. In addition, at the same order one should also take into account electromagnetic corrections [2, 40], which enter through a local term $f_{\text{local}}^{\text{elm}}$ and a global factor $G_{EM}(s)$ [41]. Thus we consider three possible expressions for the form factor to perform our fits to Belle data:

- Fit I, corresponding to $F_V^\pi(s)$ from Eq. (7).
- Fit II, same as I but with the inclusion of kinematical corrections ($m_{\pi^\pm} \neq m_{\pi^0}$, $m_{K^\pm} \neq m_{K^0}$).
- Fit III, including kinematical and electromagnetic corrections.

Our fitting parameters are M_ρ , F , α_1 and α_2 . It is found that without the inclusion of additional resonances, one can obtain good fits to Belle data for $s \lesssim 1.5 \text{ GeV}^2$. The best fit results to the first 30 points (central value of the bin corresponding to 1.525 GeV^2 , with 0.05 GeV^2 bin width) are shown in Table 1, where we have considered the $1/N_{\pi\pi} dN_{\pi\pi}/ds$ distribution measured by Belle (this includes error correlations). These fits show, firstly, that the dispersive description of the form factor can indeed successfully account for the experimental data up to $s \lesssim 1.5 \text{ GeV}^2$, and secondly, that the approach employed by the Belle Collaboration (named here as II) is indeed an adequate one, as it yields the lowest χ^2/ndf values according to our fits. Notice that, given the low energy threshold for this decay, the subtraction constants are fixed at a relatively low energy scale, and the dispersive representation turns out to be insensitive to the dynamics at large energies. In order to get a result for the form factor that can be valid up to $s = M_\tau^2$, the expression in Eq. (7) can be e.g. matched at intermediate energies to a phenomenologically adequate

function [1] like that given in Ref. [42] (included in the new version of TAUOLA [43]), or the Gounaris-Sakurai parametrization [44] used by Belle [45].

In the table, the errors quoted in single brackets are those resulting only from the fit, i.e. neglecting the systematic errors arising from our theoretical approach. These are e.g. given by the energy range to be fitted, the number of subtractions and the value of the upper integration limit in the dispersive integral. In order to estimate the associated total errors we have extended our fit up to energies in the range $[1.325, 1.525]$ GeV², we have taken into account the results for 2 and 4 subtractions, and we have taken s_∞ in the range $[4, \infty]$ GeV². In this way we end up with the numbers quoted in double brackets in Table 1. Notice that the input values for the ρ mass and width still need to be translated to the physical pole values, which are reasonably lower [1].

It is seen that our results show a lower χ^2/ndf for our fitting options I and II, compared to that obtained for option III. Thus, the best agreement with the data is reached by including SU(2) isospin breaking only kinematically, although comparable results are obtained in the isospin symmetric case.

Finally, we have also computed the low-energy observables $\langle r^2 \rangle_V^\pi$ and c_V^π appearing in the low- s expansion of $F_V^\pi(s)$. Our results are quoted in Table 2, together with those obtained in previous analyses. It is seen that the values are entirely compatible, while the errors are found to be slightly reduced thanks to the good quality of present Belle data.

4. Conclusions

We have elaborated a dispersive representation of $F_V^\pi(s)$, which preserves analyticity and unitarity exactly and reproduces the low-energy limit of χ PT up to NLO. We have performed different fits to Belle experimental data, which allow to determine our four input model parameters. The fits show a good agreement with the data, and no significant improvement is found after the inclusion of isospin breaking corrections. In addition, from our fits we have evaluated the low-energy quantities $\langle r^2 \rangle_V^\pi$ and c_V^π , which turn out to be consistent with previous determinations. Our framework is also able to provide good quality fits to $\sigma(e^+e^- \rightarrow \pi^+\pi^-)$ scattering at low energies, which can be used to determine $a_\mu^{\pi\pi,LO}$ from τ decays and e^+e^- scattering consistently.

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Parameter	Fit Value(I)	Fit Value(II)	Fit Value(III)
M_ρ	0.8431(5)((17))	0.8280(4)((14))	0.8276(4)((21))
F_π	0.0901(2)((5))	0.0902(2)((4))	0.0906(2)((4))
α_1	1.87(1)((3))	1.84(1)((3))	1.81(1)((2))
α_2	4.29(1)((7))	4.34(1)((7))	4.40(1)((6))
χ^2/ndf	1.37	1.39	1.56
$\Gamma_\rho(M_\rho^2)$	0.207(1)((3))	0.194(1)((3))	0.192(1)((4))

Table 1: Fit results to the Belle $1/N_{\pi\pi}dN_{\pi\pi}/ds$ distribution, including correlations between errors. The errors in single and double brackets correspond to those arising only from the fit and those obtained after considering theoretical systematics, respectively. Energy units are given in GeV powers. $\Gamma_\rho(M_\rho^2)$ is obtained using the fitted values of M_ρ and F_π and is given only for reference.

Determination	$\langle r^2 \rangle_V^\pi$ (GeV ⁻²)	c_V^π (GeV ⁻⁴)
Our fit	10.86(14)	3.84(3)
Bijnens and Talavera [30]	11.22(41)	3.85(60)
Pich and Portolés [34]	11.04(30)	3.79(4)

Table 2: Low-energy observables of the vector pion form factor up to the quadratic term. We give the results from our fit, the $\mathcal{O}(p^6)$ χ PT analysis in Ref. [30] and the dispersive analysis in Ref. [34].