Simulation and Analysis of a Power Law Fluctuation Generator *

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Abstract. Recently, Sato, Takayasu and Sawada (2000) constructed an analog circuit able to generate a signal corresponding to a time series with fluctuations having a probability density function with a power law tail. The exponent of the power law can be arbitrarily fixed by tuning an appropriate resistance. In a sense it is the analog of a differential equation of the Langevin type including both multiplicative and additive noise. The authors claim that their circuit could be used in the near future as a simulator-generator of financial market fluctuations and consequently as a tool for risk estimations and forecasts.

After a discussion on the stability conditions for multiplicative noise, we present an analysis of the power law fluctuation generator in connection with the electronic components and their corresponding parameters. A circuit simulation completes our study.

1 Preliminaries

1.1 Stability

Before entering into the discussion of both the electronic and the simulation analysis, let us briefly review the stability in multiplicative stochastic processes. It is well known that an additive stochastic process is stable whenever the associated deterministic problem is. In fact, given the process defined by

$$\frac{dv}{dt} = L(v) + \eta \tag{1}$$

where η is a standard Gaussian noise, it has a stable stationary behaviour when all its moments $\langle v^n \rangle$ exist up to the n-th order. This imposes a condition on the integral of $L(v)$ Schenzle and Brand (1979).

However, for a multiplicative noise the stability of the deterministic problem is not enough to ensure the stability under fluctuations Schenzle and Brand (1979). Consider now the process

$$\frac{dv}{dt} = -d\,v + v\,\eta \tag{2}$$

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where η is again a standard Gaussian noise and d a parameter. In this case the moments are given by

$$\langle v^n \rangle = \langle v^n \rangle_{t=0} \, exp\,[-n\,t\,(d - n\,D/2)] \tag{3}$$

where D is the measure of the Gaussian fluctuation, namely

$$\langle \eta(t+\tau)\,\eta(t) \rangle = D\,\delta(\tau)$$

The stationary distribution results

$$p_0(v) = v^{-1-2\,d/D} \tag{4}$$

which is not normalizable. It is then clear from Eq. (3) that in this case of multiplicative noise, the condition

$$n \leq \frac{2\,d}{D}$$

has to be fulfilled in order to guarantee the finiteness of the moments $\langle v^n \rangle$ when $t \to \infty$. Consequently, if $d < D/2$, the multiplicative noise overcome the deterministic restoring term $-d\,v$, even if this deterministic problem has a stable solution.

Having these mathematical conditions in mind, we re-analyzed the circuit proposed in Sato, Takayasu and Sawada (2000). The main point is to simulate, electronically or numerically, the equation of Langevin type with multiplicative noise (2), that in the present case reads

$$\frac{dv_0}{dt} = \left(-\frac{1}{R_f\,C_f} + \frac{k}{R_v\,C_f}\,\mu(t) \right)\, v_0 + \psi(t) \tag{5}$$

Notice a change of sign in the first term of the r.h.s of this equation, with respect to Eq. (12) in Sato, Takayasu and Sawada (2000).

1.2 Electronics

The circuit in Fig. 1 is the well-known implementation for an electronic integrator Horowitz and Hill (1980). Considering an ideal amplifier, it means Gain Product Bandwith (GBW)$\to \infty$ and Gain $(A) \to \infty$, the corresponding transfer function is

$$A(s) = \frac{V_0(s)}{V_i(s)} = -\frac{R_f}{R_v}\,\frac{1}{(1 + s\,R_f\,C_f)} \tag{6}$$

with $s = i\,\omega$. Fig. 1 includes also the Bode plot of this expression.

Under the condition $s \gg 1/(C_f\,R_f)$, Eq (6) gives rise to

$$V_0(s) = -\frac{R_f}{R_v}\,\frac{1}{(s\,R_f\,C_f)}\,V_i(s) \tag{7}$$

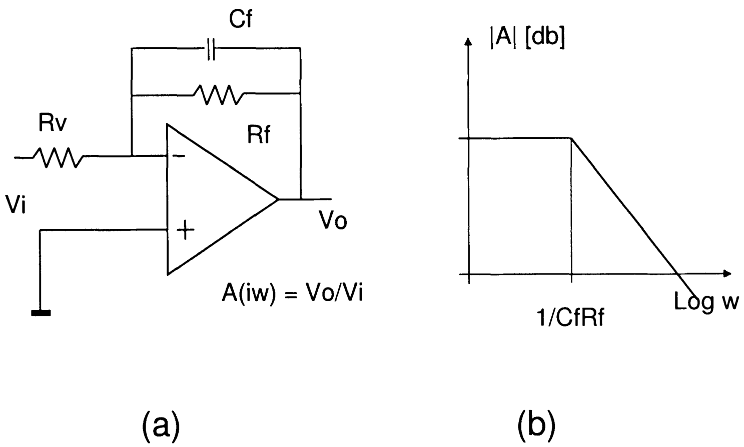

Fig. 1. Electronic integrator and its Bode plot

and

$$v_0(t) = -\frac{1}{R_f C_f} \int v_i(t)\, dt \tag{8}$$

Consequently the above circuit works well as an integrator for frequencies above $\omega_p = 1/(R_f C_f)$. In summary, to implement an integrator using a real operational amplifier (OA) its GBW must be larger than the highest frequency of interest and $A_0 \gg R_f/R_v$.

In order to verify the above conditions, the Bode plot of the desired integrator, should be compared with the Bode plot of the OA. As a rule of thumb, the implementation will be valid if the plot of the OA "contains" the integrator's Bode plot.

2 Analysis

2.1 Circuit

Original parameters The criteria above was applied to the original circuit proposed in Sato, Takayasu and Sawada (2000) that includes the values

$$C_f = 10\,pF \ ; \ R_f = 100\,K\Omega \ ; \ R_v = 5 - 200\,\Omega$$

giving rise to

$$f_p = 160\,KHz \ ; \ A = \frac{R_f}{R_v} = 54 - 86\,db.$$

In Fig. [?], a Bode diagram of Eq. (6) for $R_f = 100\,K\Omega$, $C_f = 10\,pF$ and $R_v = 50\,\Omega$ and $200\,\Omega$ is plotted together with the open loop gain of the

OA $LF157$ obtained from the National Semiconductor Co. Linear Databook (1982). As can easily be seen, the condition that the OA "contains" the desired characteristic is not fulfilled. So the changes of R_v between the specified values don't produce a full change in the "integrator gain" and the observed effects are only marginal ones. In other words, a change in R_v is not fully reflected as a change of the "integrator gain".

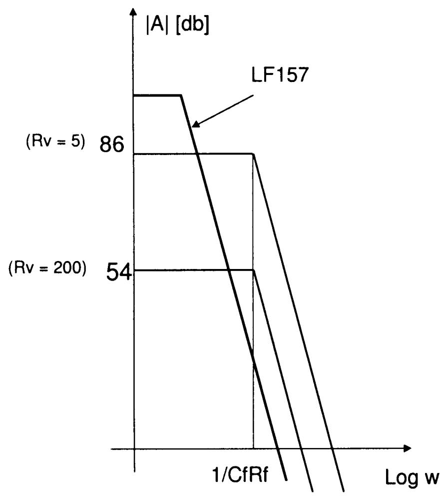

Fig. 2. Comparison between Bode plots for the amplifier and the integrator

On the other hand, when the circuit was tested with the specified values of R_v, the output shown unwanted saturation. Only for values of $R_v > 1.7\,K\Omega$, the amplifier remains almost linear as can be seen in Fig. 3. The reported behaviour in Sato, Takayasu and Sawada (2000) was qualitative seen for $1.7\,K\Omega < R_v < 4\,K\Omega$. For larger values, the electrical noise dominates.

It is important to remark that our measurements were done with a Tek scope (Model TDS 3030) with 8 bits of resolution. Nevertheless we are certain that this technical detail doesn't change our conclusions.

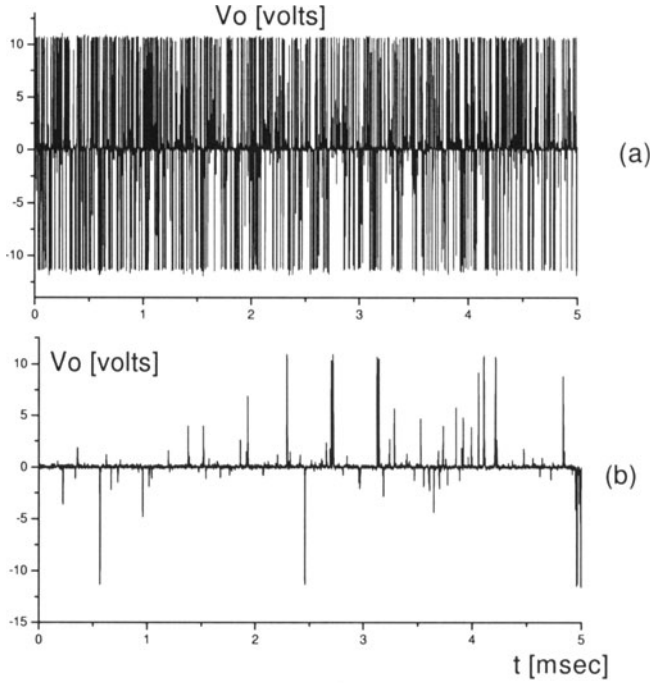

Fig. 3. Output of the circuit with the original parameters for (a) $R_v = 35\,\Omega$ and (b) $R_v = 2200\,\Omega$

Modified parameters To include the integrator characteristic inside the OA open loop, a lower pole frequency (ω_p) is in order. Then, we decided to use the values

$$C_f = 11\,pF \;\; ; \;\; R_f = 800\,K\Omega \;\; ; \;\; R_v = 6 - 12K\Omega.$$

Resulting in

$$f_p = 18\,KHz \;\; ; \;\; A = 35 - 45\,db$$

This is not an arbitrary selection because a value of C_f higher than $15\,pF$ put the system in a heavy oscillation. A careful study of LF157 characteristics shows that its stability conditions are greatly affected if a capacitor larger than $15\,pF$ is connected between the input and the output. Consequently, a different amplifier should be used instead.

The circuit with modified parameters worked as predicted. In Fig. 4 we present the log-log plot of the probability density function for the R_v values specified.

Using the adequate values for $C_f\,R_f$, a tighter control of the integrator gain is obtained. Notice also that for lower values of R_v, that means higher

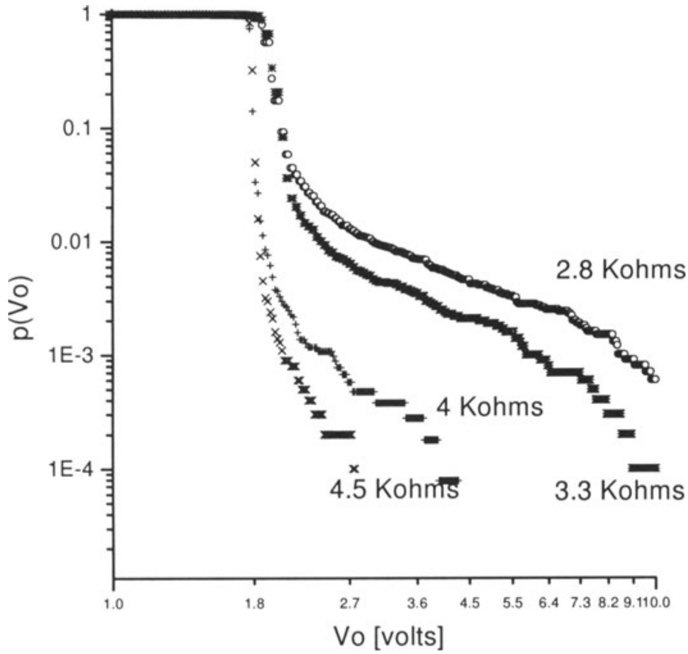

Fig. 4. Probability density function for the modified parameters

gain, the circuit doesn't met the integrator condition, and for R_v higher than $12\,K\Omega$ the electrical noise dominates.

In order to check the influence of the amplifier's offset voltage we performed additional tests. This offset, independently of the circuit values, does not affect the final power law behaviour and can be reduced in the data processing stage.

2.2 Simulation

If the circuit is to be really useful in predicting fluctuations on a quantitative way, it must be necessary to know the meaning of each electrical parameter on the output signal in order to reproduce, for example, the probability distribution of historical data. This work would be tedious, when not impossible using the real circuit.

We propose to use some simulation procedures that, without loosing the electrical point of view, put in evidence the influence of each of the parameters. We have tried several configurations using the Spice program (1998). Although this program allows one to use realistic components, by means of transistor- level models, we prefer to use quite ideal representation where the different parameters of the circuit, desirable or not, can be added separately. It can be mentioned that the simulation with real components simulation is also plagued of serious convergence problems, being certainly difficult to make it runs in a fiable way.

Fig. 5 shows the circuit used in the simulation. It reproduces the differential equation of interest. Different gain blocks allows one to change amplitudes, parameters such as offset voltage and saturation limits when added, as well as different noise generators.

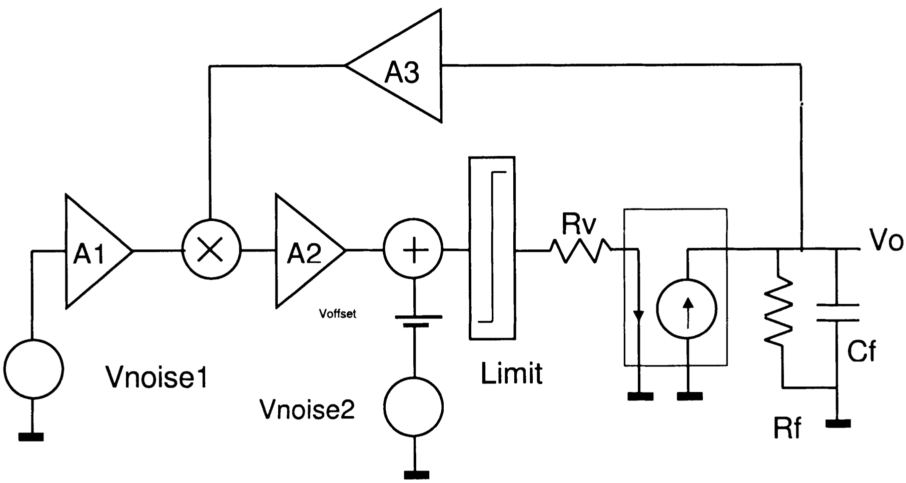

Fig. 5. Simulated circuit

The output of the simulation is summarized in Fig. 6.

3 Conclusions

We have exhaustively analyzed the recently proposed electronic analog circuit that generates fluctuations with a probability density functions having a power law tails.

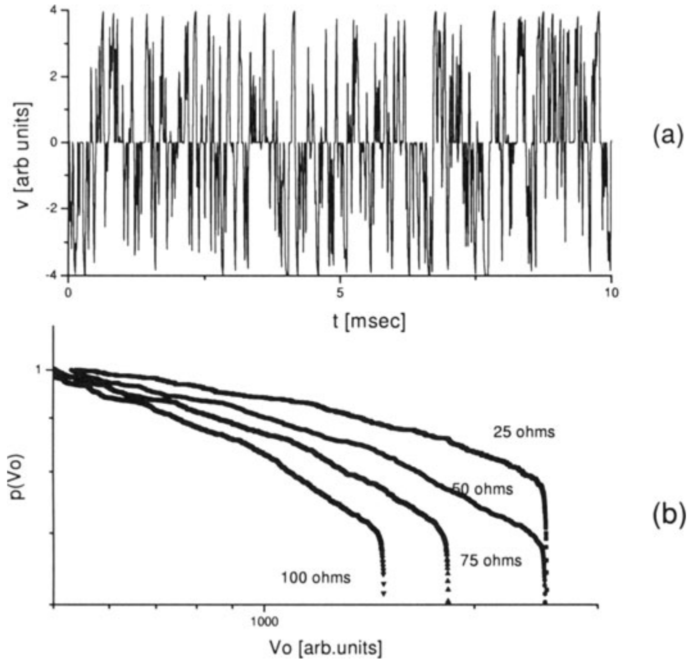

Fig. 6. Output of the simulation. a)signal; b) density of probability

We found that the originally proposed parameters are not the most pertinent ones because they place the circuit in the borderline of the integrator Bode plot. With appropriate parameters, the circuit performance is noteworthily improved. Moreover, by changing the integrated circuit originally chosen, a further improvement can be achieved.

Finally, the numerical simulation, even if carried out in terms of ideal electronic components, reinforces the previous conclusions. It clearly appears to be an alternative way to implement the use of the circuit in the estimation of economy time series behaviour.

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