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Physics Letters B 541 (2002) 298–306

PHYSICS LETTERS B

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RPA puzzle in ^{12}C weak decay processes

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Received 13 May 2002; received in revised form 18 June 2002; accepted 1 July 2002

Editor: W. Haxton

Abstract

We explain the origin of the difficulties that appear in a straightforward application of the QRPA in ^{12}C, and we demonstrate that it is imperative to use the projected QRPA (PQRPA). Satisfactory results, not only for the weak processes among the ground states of the triad {^{12}B, ^{12}C, ^{12}N}, but also for the inclusive ones are obtained. We sketch as well a new formalism for the neutrino–nucleus interaction that furnishes very simple final formulae for the muon capture rate and neutrino induced cross sections. © 2002 Elsevier Science B.V. All rights reserved.

PACS: 13.75.Gx; 14.20.Gk; 13.40.Em

Keywords: Neutrino–nucleus cross section; Muon capture; Beta decay; Projected QRPA

New types of nuclear weak processes have been measured in recent years. They are based on neutrino and antineutrino interactions with complex nuclei and, rather than being used to study the corresponding cross sections, they are mainly aimed to inquire on possible exotic properties of neutrino themselves, such as neutrino oscillations and the associate neutrino massiveness, which are not contained in the Standard Model (SM) of elementary particles.

So, in neutrino oscillation experiments with liquid scintillators, the charge-exchange reactions $^{12}\mathrm{C}(\nu_e, e^-)^{12}\mathrm{N}$ and $^{12}\mathrm{C}(\nu_\mu, \mu^-)^{12}\mathrm{N}$, both *exclusive* (to the 1^+ ground state) and *inclusive* (to all final states), are just tools. As such, and to be useful, the corresponding cross sections $\sigma^{\mathrm{exc}}_{e,\mu}$ and $\sigma^{\mathrm{inc}}_{e,\mu}$ must be accurately accounted for by nuclear structure models.

From the recent works [1–4] we have learned, however, that neither RPA nor QRPA are able to explain the weak processes (β-decays, μ-capture, and neutrino induced reactions) among the ground states of the triad {^{12}B, ^{12}C, ^{12}N}. In fact, in the RPA a rescaling factor of the order of 4 is needed to bring the calculations and data in agreement [1], and a subsequent ad hoc inclusion of partial occupancy of the $p_{1/2}$ subshell reduces this factor to less than 2 [2,3]. But, when the RPA is supplemented with the pairing correlations in a self-consistent way, i.e., in the framework of a full QRPA [4], the exclusive cross sections turn out to be again overestimated by a factor of $\cong 4$. Moreover, Volpe et al. [4] have called attention to "*difficulties in choosing the ground state of* ^{12}N *because the lowest state is not the most collective one*" when the QRPA is used.

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PII: S0370-2693(02)02231-1

In the present Letter we explain the origin of the difficulties that appear in a straightforward application of the BCS approximation in a light nucleus such as ^{12}C, and we demonstrate that the problem is circumvented by the employment of the particle number projected BCS (PBCS). We show simultaneously that the proton–neutron QRPA is not a recommended approach, and that the aforementioned RPA puzzle is solved within the projected QRPA (PQRPA) for the charge-exchange excitations [5]. The later approach furnishes satisfactory results not only for the weak processes among the ground states of the triad $\{^{12}$B, ^{12}C, ^{12}N$\}$, but also for the inclusive weak processes. For numerical evaluation of the weak decay observables we have found it suitable to develop a new theoretical framework, which is similar to that build up by Barbero et al. [6] for the neutrinoless double beta decay. The motivation for that and the complete formulation will be exposed elsewhere. Here we just explain the notation and exhibit the final formulae.

The weak Hamiltonian is expressed in the form [6–8]

$$H_{\rm W}(\mathbf{r}) = \frac{G}{\sqrt{2}} J^\dagger_\mu L^\mu(\mathbf{r}) + \text{h.c.}, \tag{1}$$

where

$$J_\mu = \gamma_0\left[g_{\rm V}\gamma_\mu + \frac{g_{\rm M}}{2M} i\sigma_{\mu\nu}k_\nu - g_{\rm A}\gamma_\mu\gamma_5 + \frac{g_{\rm P}}{m_\ell}k_\mu\gamma_5\right], \tag{2}$$

is the hadronic current operator, and

$$L_\mu(\mathbf{r}) = \bar{u}_{s_\ell}(\mathbf{p}, E_\ell)\gamma_\mu(1-\gamma_5)u_{s_\nu}(\mathbf{q}, E_\nu)e^{i\mathbf{r}\cdot\mathbf{k}} \tag{3}$$

is the plane waves approximation for the matrix element of the leptonic current; $G = (3.04545 \pm 0.00006) \times 10^{-12}$ is the Fermi coupling constant (in natural units) [9],

$$k = P_i - P_f \equiv \{k_0, \mathbf{k}\} \tag{4}$$

is the momentum transfer (P_i and P_f are momenta of the initial and final nucleon (nucleus)), M is the nucleon mass, m_ℓ is the mass of the charged lepton, and $g_{\rm V}$, $g_{\rm A}$, $g_{\rm M}$ and $g_{\rm P}$ are, respectively, the vector, axial-vector, weak-magnetism and pseudoscalar effective dimensionless coupling constants. Their numerical values are [7–9]

$$g_{\rm V} = 1, \qquad g_{\rm A} = 1.26, \qquad g_{\rm M} = \kappa_p - \kappa_n = 3.70,$$
$$g_{\rm P} = g_{\rm A}\frac{2Mm_\ell}{k^2 + m_\pi^2}. \tag{5}$$

The above estimates for $g_{\rm M}$ and $g_{\rm P}$ come from the (well tested) conserved vector current (CVC) hypothesis, and from the partially conserved axial vector current (PCAC) hypothesis, respectively. In the numerical calculation we will use an effective axial-vector coupling $g_{\rm A} = 1$ [10–12]. The finite nuclear size (FNS) effect is incorporated via the dipole form factor with a cutoff $\Lambda = 850$ MeV, i.e., as [13]

$$g \to g\left(\frac{\Lambda^2}{\Lambda^2 + k^2}\right)^2. \tag{6}$$

To use (1) with the non-relativistic nuclear wave functions, the Foldy–Wouthuysen transformation has to be performed on the hadronic current (2). When the velocity dependent terms are neglected,[1] this yields [16]

$$J_0 = g_{\rm V} - (\bar{g}_{\rm A} + \bar{g}_{\rm P1})\boldsymbol{\sigma}\cdot\hat{\mathbf{k}},$$
$$\mathbf{J} = g_{\rm A}\boldsymbol{\sigma} - i\bar{g}_{\rm W}\boldsymbol{\sigma}\times\hat{\mathbf{k}} - \bar{g}_{\rm V}\hat{\mathbf{k}} - \bar{g}_{\rm P2}(\boldsymbol{\sigma}\cdot\hat{\mathbf{k}})\hat{\mathbf{k}}, \tag{7}$$

where the following short notation has been introduced

$$\bar{g}_{\rm V} = g_{\rm V}\frac{|\mathbf{k}|}{2M}, \qquad \bar{g}_{\rm A} = g_{\rm A}\frac{|\mathbf{k}|}{2M},$$
$$\bar{g}_{\rm W} = (g_{\rm V} + g_{\rm M})\frac{|\mathbf{k}|}{2M},$$
$$\bar{g}_{\rm P1} = g_{\rm P}\frac{|\mathbf{k}|}{2M}\frac{k_0}{m_\ell}, \qquad \bar{g}_{\rm P2} = g_{\rm P}\frac{|\mathbf{k}|}{2M}\frac{|\mathbf{k}|}{m_\ell}. \tag{8}$$

For the neutrino–nucleus reaction $k = p - q$, with $p \equiv \{E_\ell, \mathbf{p}\}$ and $q \equiv \{E_\nu, \mathbf{q}\}$, and the corresponding cross section from the initial state $|J_i\rangle$ to the final state $|J_f\rangle$ reads

$$\sigma(E_\ell, J_f) = \frac{|\mathbf{p}|E_\ell}{2\pi}F(Z+1, E_\ell) \times \int_{-1}^{1} d(\cos\theta)\, \mathcal{T}_\sigma\big(|\mathbf{k}|, J_f\big), \tag{9}$$

[1] The effect of the nucleon-velocity terms is of the order of a few per cent, in both the neutrino–nucleus scattering [13] and in the muon capture [14,15].

where $F(Z+1, E_\ell)$ is the Fermi function, $\theta \equiv \hat{\mathbf{q}} \cdot \hat{\mathbf{p}}$, and

$$\mathcal{T}_\sigma\big(|\mathbf{k}|, J_f\big) \equiv \frac{1}{2J_i+1} \sum_{s_\ell, M_i} \sum_{s_\nu, M_f} \left| \int d\mathbf{r}\, \psi_f^*(\mathbf{r}) H_{\mathrm{W}}(\mathbf{r}) \psi_i(\mathbf{r}) \right|^2, \tag{10}$$

with $\psi_i(\mathbf{r}) \equiv \langle \mathbf{r}|J_i M_i\rangle$ and $\psi_f(\mathbf{r}) \equiv \langle \mathbf{r}|J_f M_f\rangle$ being the nuclear wave functions. The transition amplitude can be cast in the form

$$\mathcal{T}_\sigma\big(|\mathbf{k}|, J_f\big) = G^2\Big(\mathcal{M}_{\mathrm{V}} K_{\mathrm{V}} + \sum_{\mu=-1,0,+1} \mathcal{M}_{\mathrm{A}}^\mu K_{\mathrm{A}}^\mu\Big), \tag{11}$$

where

$$\mathcal{M}_{\mathrm{V}} = \frac{4\pi}{2J_i+1} \sum_J \big|\langle J_f \big\| i^J j_J(|\mathbf{k}|r) Y_J(\hat{r}) \big\| J_i\rangle\big|^2,$$
$$\mathcal{M}_{\mathrm{A}}^\mu = \frac{4\pi}{2J_i+1} \times \sum_J \bigg| \sum_L \sqrt{2L+1} \begin{pmatrix} L & 1 & J \\ 0 & -\mu & \mu \end{pmatrix} \times \langle J_f \big\| i^L j_L(|\mathbf{k}|r) \big[Y_L(\hat{r}) \otimes \boldsymbol{\sigma}\big]_J \big\| J_i\rangle \bigg|^2 \tag{12}$$

are the nuclear matrix elements, and

$$K_{\mathrm{V}} = g_{\mathrm{V}}^2 L_4 + 2 g_{\mathrm{V}} \bar{g}_{\mathrm{V}} L_{40} + \bar{g}_{\mathrm{V}}^2 L_0,$$
$$K_{\mathrm{A}}^\mu = \begin{cases} (g_{\mathrm{A}} - \bar{g}_{\mathrm{P2}})^2 L_0 + 2(g_{\mathrm{A}} - \bar{g}_{\mathrm{P2}})(\bar{g}_{\mathrm{A}} + \bar{g}_{\mathrm{P1}}) L_{40} \\ \quad + (\bar{g}_{\mathrm{A}} + \bar{g}_{\mathrm{P1}})^2 L_4, & \text{for } \mu = 0, \\ (g_{\mathrm{A}} + \mu \bar{g}_{\mathrm{W}})^2 L_\mu, & \text{for } \mu = \pm 1 \end{cases} \tag{13}$$

are the effective coupling constants, which contain the lepton traces

$$L_4 = 1 + \frac{\mathbf{p}\cdot\mathbf{q}}{E_\ell E_\nu}, \qquad L_{40} = \left(\frac{q_0}{E_\nu} + \frac{p_0}{E_\ell}\right),$$
$$L_0 = 1 + \frac{2q_0 p_0 - \mathbf{p}\cdot\mathbf{q}}{E_\ell E_\nu},$$
$$L_{\pm 1} = 1 - \frac{q_0 p_0}{E_\ell E_\nu} \pm \left(\frac{q_0}{E_\nu} - \frac{p_0}{E_\ell}\right), \tag{14}$$

with

$$q_0 = \hat{k}\cdot\mathbf{q} = \frac{E_\nu(|\mathbf{p}|\cos\theta - E_\nu)}{|\mathbf{k}|},$$
$$p_0 = \hat{k}\cdot\mathbf{p} = \frac{|\mathbf{p}|(|\mathbf{p}| - E_\nu\cos\theta)}{|\mathbf{k}|} \tag{15}$$

and the momentum transfer $\mathbf{k}$ is along the z axis ($\hat{k} \equiv \hat{z} \equiv \epsilon_0$).

In going from the results for the neutrino–nucleus reaction cross section to that for the muon capture rate one should keep in mind that: (i) the roles of p and q are interchanged within the matrix element of the leptonic current, which brings in a minus sign in the last term of $L_{\pm 1}$, (ii) the momentum transfer turns out to be $k = q - p$, and therefore the signs on the right-hand sides of (15) have to be changed, and (iii) the threshold values ($\mathbf{p} \to 0 : \mathbf{k} \to \mathbf{q}$, $k_0 \to E_\nu - m_\ell$) must be used for the lepton traces. All this yields

$$L_4 = L_{40} = L_0 = 1, \qquad L_{\pm 1} = 1 \mp 1. \tag{16}$$

Finally, one should remember that instead of summing over the initial lepton spins s_ℓ, as done in (10), one has now to average on the same quantum number. The resulting transition amplitude $\mathcal{T}_\Lambda(J_f)$ is again of the form (11) but the effective charges are here:

$$K_{\mathrm{V}}(\mathbf{p}\to 0) = (g_{\mathrm{V}} + \bar{g}_{\mathrm{V}})^2,$$
$$K_{\mathrm{A}}^\mu(\mathbf{p}\to 0) = \delta_{|\mu|,1}(g_{\mathrm{A}} - \bar{g}_{\mathrm{W}})^2 + \delta_{\mu,0}(g_{\mathrm{A}} + \bar{g}_{\mathrm{A}} - \bar{g}_{\mathrm{P}})^2, \tag{17}$$

with

$$\bar{g}_{\mathrm{V}} = g_{\mathrm{V}} \frac{E_\nu}{2M}, \qquad \bar{g}_{\mathrm{A}} = g_{\mathrm{A}} \frac{E_\nu}{2M},$$
$$\bar{g}_{\mathrm{W}} = (g_{\mathrm{V}} + g_{\mathrm{M}}) \frac{E_\nu}{2M},$$
$$\bar{g}_{\mathrm{P}} = \bar{g}_{\mathrm{P2}} - \bar{g}_{\mathrm{P1}} = g_{\mathrm{P}} \frac{E_\nu}{2M}. \tag{18}$$

For the capture rate one gets [17]

$$\Lambda(J_f) = \frac{E_\nu^2}{2\pi} |\phi_{1S}|^2 \mathcal{T}_\Lambda(J_f), \tag{19}$$

where ϕ_{1S} is the muonic bound state wave function evaluated at the origin. Note that the neutrino energy is fixed by the energy of the final state through the relation: $E_\nu = m_\mu - (m_n - m_p) - E_B^\mu - E_f + E_i$, where E_B^μ is the binding energy of the muon in the $1S$ orbit.

Lastly, we mention that the B-values for the GT beta transitions are defined and related to the ft-values as [9]

$$\frac{|g_{\mathrm{A}}\langle J_f||\sigma||J_i\rangle|^2}{2J_i+1} \equiv B(\mathrm{GT}) = \frac{6146}{ft}\ \mathrm{s}. \tag{20}$$

To start the discussion on the difficulties found by Volpe et al. [4], it should be remembered that, the pn-QRPA yields the same energy spectra for the four $(Z \pm 1, N \mp 1)$ and $(Z \pm 1, N \pm 1)$ nuclei, when the BCS equations are solved in the parent (Z, N) nucleus under the constraint

$$\sum_{k=n(p)} (2j_k + 1)v_{j_k}^2 = N(Z). \tag{21}$$

This is a physically sound zero order approximation when the nuclei in question are far from the closed shells and possess a significant neutron excess. Yet, as we show below, the use of the QRPA in $N = Z$ nuclei is not free from care.

Let us define the quasiparticle energies relative to the Fermi levels:

$$E_{j_k}^{(\pm)} = \pm E_{j_k} + \lambda_k, \qquad k = \mathrm{p}, \mathrm{n}, \tag{22}$$

where E_{j_k} and λ_k are the BCS quasiparticle energies and chemical potentials, respectively. In the particle–hole limit the energies $E_{j_k}^{(+)}$ ($E_{j_k}^{(-)}$) correspond to the single-particle(–hole) excitations for the levels above (below) the Fermi surface [18], and to the 2p1h (1p2h) seniority-one excitations for levels below (above) the Fermi surface. In nuclei with large neutron excess $E_{j_p}^{(\pm)}$ and $E_{j_n}^{(\pm)}$ are in general quite different, but in $N = Z$ nuclei the proton and neutron spectra are almost equal, except for the Coulomb energy displacement. As a consequence the unperturbed QRPA energies:

$$\mathcal{E}_{j_p j'_n} = \begin{cases} E_{j_p}^{(+)} - E_{j'_n}^{(-)}, & \text{for } (Z+1, N-1), \\ -E_{j_p}^{(+)} + E_{j'_n}^{(-)}, & \text{for } (Z-1, N+1), \\ E_{j_p}^{(+)} + E_{j'_n}^{(-)}, & \text{for } (Z+1, N+1), \\ -E_{j_p}^{(+)} - E_{j'_n}^{(-)}, & \text{for } (Z-1, N-1) \end{cases} \tag{23}$$

are almost degenerate with $\mathcal{E}_{j'_p j_n}$, i.e., $\mathcal{E}_{j_p j'_n} \cong \mathcal{E}_{j'_p j_n}$, for all four odd-odd $(Z \pm 1, N \mp 1)$ and $(Z \pm 1, N \pm 1)$ nuclei. Moreover, in the case of ^{12}C, both the proton and the neutron Fermi energies are placed almost in the middle between the $1p_{3/2}$ and $1p_{1/2}$ shells. This causes an additional degeneracy, namely $E_{1p_{3/2}} \cong E_{1p_{1/2}}$, resulting in

$$\mathcal{E}_{3/2,1/2} \cong \mathcal{E}_{1/2,3/2} \cong \mathcal{E}_{3/2,3/2} \cong \mathcal{E}_{1/2,1/2}, \tag{24}$$

for ^{12}N, ^{12}B, ^{14}N and ^{10}B. But we know that the physically sound energy sequences are:

$$\begin{aligned} {}^{12}\mathrm{N}: \quad & \mathcal{E}_{1/2,3/2}(1\mathrm{p}1\mathrm{h}) < \mathcal{E}_{3/2,3/2}(2\mathrm{p}2\mathrm{h}) \\ & \quad \simeq \mathcal{E}_{1/2,1/2}(2\mathrm{p}2\mathrm{h}) < \mathcal{E}_{3/2,1/2}(3\mathrm{p}3\mathrm{h}), \\ {}^{12}\mathrm{B}: \quad & \mathcal{E}_{3/2,1/2}(1\mathrm{p}1\mathrm{h}) < \mathcal{E}_{3/2,3/2}(2\mathrm{p}2\mathrm{h}) \\ & \quad \simeq \mathcal{E}_{1/2,1/2}(2\mathrm{p}2\mathrm{h}) < \mathcal{E}_{1/2,3/2}(3\mathrm{p}3\mathrm{h}), \\ {}^{14}\mathrm{N}: \quad & \mathcal{E}_{1/2,1/2}(2\mathrm{p}) < \mathcal{E}_{1/2,3/2}(3\mathrm{p}1\mathrm{h}) \\ & \quad \simeq \mathcal{E}_{3/2,1/2}(3\mathrm{p}1\mathrm{h}) < \mathcal{E}_{3/2,3/2}(4\mathrm{p}2\mathrm{h}), \\ {}^{10}\mathrm{B}: \quad & \mathcal{E}_{3/2,3/2}(2\mathrm{h}) < \mathcal{E}_{3/2,1/2}(1\mathrm{p}3\mathrm{h}) \\ & \quad \simeq \mathcal{E}_{1/2,3/2}(1\mathrm{p}3\mathrm{h}) < \mathcal{E}_{1/2,1/2}(2\mathrm{p}4\mathrm{h}), \end{aligned} \tag{25}$$

as can be easily seen from the scrutiny of the particle–hole limits for the seniority-two pn-states, which are indicated parenthetically in (25). The RPA correlations are unable to remedy the situation and the degeneracy in (24) among four lowest $\mathcal{E}_{\mathrm{pn}}$ is the cause for the problems found in Ref. [4] regarding the ground state of ^{12}N.

Well aware of all these difficulties, Cha [19], in his study of the Gamow–Teller (GT) resonances, has solved the BCS equations in the daughter nuclei under the constraints

$$\sum_{k=n(p)} (2j_k + 1)\tilde{v}_{j_k}^2 = N \pm 1 \ (Z \mp 1), \tag{26}$$

which gives way to the energy orderings (25). Thus, the problem risen by Volpe et al. [4] can, in principle, be solved by using the Cha's recipe. However, the price to pay is that a different QRPA equation has to be worked out for each nucleus separately, i.e., one for the $(Z + 1, N - 1)$ nucleus and the other for the $(Z - 1, N + 1)$ nucleus, being in each case significant only the positive energy frequencies. This means that we have to abandon the nice properties of the particle–hole charge-exchange RPA, where: (1) only one RPA equation is solved for the $(Z \pm 1, N \mp 1)$ nuclei, and (2) both the positive and negative solutions are physically meaningful, with the β^+ spectrum viewed as an extension of the β^- spectrum to negative energies [20–22]. Note also that, in order to fulfill the GT sum rule, Cha has evaluated the transition probabilities with the usual pairing factors u's and v's, obtained from (21). None of the undesirable features of the Cha's method appear within the charge-exchange PQRPA. This approach has been presented in detail in Ref. [5], and we just mention here that the

PBCS quasiparticle energies read:

$$E_j^{(+)} = \frac{R_0^K(j) + R_{11}^K(jj)}{I^K(j)} - \frac{R_0^K}{I^K},$$
$$E_j^{(-)} = -\frac{R_0^{K-2}(j) + R_{11}^{K-2}(jj)}{I^{K-2}(j)} + \frac{R_0^{K-2}}{I^{K-2}}, \qquad (27)$$

where $K = N, Z$ and the quantities R^K and I^K are defined in [5].

The numerical calculations were performed within the $nl = (1s, 1p, 1d, 2s, 1f, 2p)$ configuration space, and for the residual interaction we adopted the delta force

$$V = -4\pi(v_s P_s + v_t P_t)\delta(r), \qquad (28)$$

where v_s and v_t are given in units of MeV fm^3.

The bare single-particle energies (s.p.e.) e_j were fixed from the experimental energies of the odd-mass nuclei ^{11}C, ^{11}B, ^{13}C and ^{13}N. That is, we assume that the ground states in ^{11}C and ^{11}B are pure quasi-hole excitations $E_{1p_{3/2}}^{(-)}$, and that the lowest observed $1/2^-, 5/2^+, 1/2^+, 3/2^+, 7/2^-$ and $3/2^-$ states in ^{13}C and ^{13}N are pure quasi-particle excitations $E_j^{(+)}$ with $j = (1p_{1/2}, 1d_{5/2}, 2s_{1/2}, 1d_{3/2}, 1f_{7/2}, 2p_{3/2})$. This is in essence the idea of the inverse-gap-equation (IGE) method [23], which also fixes the value of the singlet strength within the pairing channel (v_s^{pair}). We have considered the faraway orbitals $1s_{1/2}, 2p_{1/2}$ and $1f_{5/2}$ as well. Their s.p.e. were taken to by that of a harmonic oscillator (HO) with standard parametrization. The single-particle wave functions were also approximated with that of the HO with the length parameter $b = 1.67$ fm, which corresponds to the estimate $\hbar\omega = 45\,A^{-1/3} - 25\,A^{-2/3}$ MeV for the oscillator energy.

The BCS and PBCS results for neutrons are displayed in Table 1. The underlined quasiparticle energies correspond to single-hole and single-particle excitations, while the non-underlined ones are basically 2h1p and 2p1h excitations. Note that, while the first ones are fairly similar within the BCS and PBCS approaches, the last ones are quite different. (The resulting s.p.e. are also quite similar.) Analogous results are obtained for protons, with the same value of v_s^{pair}.

The unperturbed energies $\mathcal{E}_{j_p j'_n}$ of lowest four pn quasiparticle states within the BCS and PBCS approximations are shown in Table 2. For comparison, the results obtained with the Cha's method are also displayed in the same table. It is easy to see that, while the standard BCS approximation exhibits the degeneracy (24), the Cha's recipe and the PBCS approach produce the energy sequences (25), being the energy separations between the 1p1h, 2p2h and 3p3h-like states significantly larger in the later case. This does not take us by surprise since the role of the number projection is precisely that of restoring the correct number of particles and holes.

Table 1
BCS and PBCS results for neutrons. E_j^{exp} stand for the experimental energies used in the fitting procedure, and e_j are the resulting s.p.e. The underlined quasiparticle energies correspond to single-hole excitations (for $j = 1s_{1/2}, 1p_{3/2}$) and to single-particle excitations (for $j = 1p_{1/2}, 1d_{5/2}, 2s_{1/2}, 1d_{3/2}, 1f_{7/2}, 2p_{3/2}$). The non-underlined energies are mostly two hole–one particle and two particle–one hole excitations. The fitted values of the pairing strengths v_s^{pair} in units of MeV fm^3 are also displayed

Shell	E_j^{exp}	BCS			PBCS		
		$E_j^{(+)}$	$E_j^{(-)}$	e_j	$E_j^{(+)}$	$E_j^{(-)}$	e_j
$1s_{1/2}$		11.34	$\underline{-35.13}$	−23.58	19.93	$\underline{-34.99}$	−22.37
$1p_{3/2}$	−18.72	−5.07	$\underline{-18.72}$	−7.80	−1.28	$\underline{-18.73}$	−7.24
$1p_{1/2}$	−4.94	$\underline{-4.94}$	−18.85	−2.07	$\underline{-4.95}$	−22.33	−1.51
$1d_{5/2}$	−1.09	$\underline{-1.09}$	−22.70	2.12	$\underline{-1.09}$	−26.82	2.16
$2s_{1/2}$	−1.85	$\underline{-1.86}$	−21.93	2.70	$\underline{-1.85}$	−25.98	2.68
$1d_{3/2}$	2.72	$\underline{2.72}$	−26.51	6.24	$\underline{2.73}$	−30.79	6.26
$1f_{7/2}$	5.81	$\underline{5.82}$	−29.61	8.14	$\underline{5.83}$	−33.61	8.17
$2p_{3/2}$	7.17	$\underline{7.18}$	−30.98	11.49	$\underline{7.16}$	−35.23	11.47
$2p_{1/2}$		$\underline{12.89}$	−36.69	17.30	$\underline{12.89}$	−41.01	17.32
$1f_{5/2}$		$\underline{16.72}$	−40.52	19.18	$\underline{16.72}$	−44.58	19.21
v_s^{pair}				23.16			23.92

Table 2
Unperturbed energies $\mathcal{E}_{j_p j_n'}$ (in units of MeV) of lowest four proton–neutron quasiparticle states in the neighborhood of ^{12}C, within the approximations: (a) BCS equations are solved in ^{12}C with the condition (21), (b) BCS equations are solved in daughter nuclei, employing (24) as suggested by Cha [19], and (c) number projected BCS (PBCS) equations are employed. The underlined energies are equal for all three cases, because they are adjusted to the experimental data via the IGP procedure [23]

$j_p j_n'$	$E_{j_p} + E_{j_n'}$	^{12}N			^{12}B			^{14}N			^{10}B		
		(a)	(b)	(c)	(a)	(b)	(c)	(a)	(b)	(c)	(a)	(b)	(c)
1/2, 3/2	14.0	16.8	16.8	16.8	11.2	15.2	18.7	−7.0	−4.8	−3.2	35.0	36.7	38.5
3/2, 1/2	13.8	16.6	20.6	23.8	11.0	11.0	11.0	−7.2	−5.0	−3.5	34.8	36.6	38.3
1/2, 1/2	14.1	16.9	18.7	20.4	11.3	13.1	14.8	−6.9	−6.9	−6.9	35.1	38.7	42.1
3/2, 3/2	13.7	16.5	18.7	20.2	10.9	13.1	14.7	−7.3	−2.9	0.2	34.7	34.7	34.7

Table 3
Results for the energy of the $J^\pi = 1_1^+$ state in ^{12}N in units of MeV, the average B(GT)-value for the β-decay from ^{12}N and ^{12}B, and the μ-capture rates to the ground state (Λ^{exc}) and to all final states (Λ^{inc}) in ^{12}B in units of 10^3 s^{-1}. In the upper part of the table the smallest (largest) estimates obtained in previous RPA calculations are shown. As explained in the text three different PQRPA calculations are presented. The lower and upper experimental B(GT)-value correspond to ^{12}N and ^{12}B, respectively

	$E(1_1^+)$	B(GT)	Λ^{exc}	Λ^{inc}
RPA [1]		1.94 (2.02)	22.8 (25.4)	57 (59)
RPA [2]			32.4 (34.8)	69 (72)
RPA + pair [2]			4.1 (7.3)	31 (36)
CRPA [3]		0.693 (0.776)	8.5 (9.3)	40 (42)
RPA [4]	13.74	2.03	25.4	51
PBCS	16.78	1.063	15.2	66
PQRPA (I)	17.89	0.568	7.8	46
PQRPA (II)	18.14	0.477	6.5	40
PQRPA (III)	18.13	0.480	6.5	42
Expt.	17.34 [27]	0.466–0.526 [28]	6.2 ± 0.3 [29]	38 ± 1 [30]

After having established truthful unperturbed PQRPA energies we proceed with full calculations, where the values of v_s and v_t within the particle–particle (pp) and particle–hole (ph) channels are needed. In similar calculations of double beta decaying nuclei [6,24], which possess significant neutron excess, the following procedure has been pursued: (i) v_s^{ph} and v_t^{ph} were taken from the study of energetics of the GT resonances done by Nakayama et al. [25] (see also Ref. [11]), and (ii) the pp strengths were fixed on the basis of the isospin and SU(4) symmetries as: $v_s^{\text{pp}} \equiv v_s^{\text{pair}}$, and $v_t^{\text{pp}} \gtrsim v_s^{\text{pp}}$. Different to what happens in the $N > Z$ nuclei, such a parametrization is not suitable for the $N = Z$ nuclei, and the best agreement with data is obtained when the pp channel is totally switched off. Thus we will exhibit here only the results for $v_s^{\text{pp}} = v_t^{\text{pp}} = 0$, and the next three sets of ph parameters:

Calculation I: $v_s^{\text{ph}} = v_s^{\text{pair}} = 23.92$ MeV fm^3, and $v_t^{\text{ph}} = v_s^{\text{ph}}/0.6 = 39.86$ MeV fm^3. That is, the singlet ph strength is taken to be the same as v_s^{pair} obtained from the gap equation, while the triplet ph depth is estimated from the relation used by Goswami and Pal [26] in the RPA calculation of ^{12}C.

Calculation II: $v_s^{\text{ph}} = 27$ MeV fm^3, and $v_t^{\text{ph}} = 64$ MeV fm^3. These values were suggested in Refs. [11,25] and have been extensively used in the QRPA calculations of ^{48}Ca [6,24].

Calculation III: $v_s^{\text{ph}} = v_t^{\text{ph}} = 45$ MeV fm^3. This parametrization gives fairly good results for the energies of the $J^\pi = 0_1^+$ and 1_1^+ states in ^{12}B and ^{12}N.

In Table 3 we confront our PBCS and PQRPA results with previous RPA and QRPA calculations [1–4], and with experiments [27–30] for: the energy of the $J^\pi = 1_1^+$ state in ^{12}N, the B(GT)-value for the β-decay from ^{12}N and ^{12}B, and the exclusive

Table 4
Results for averaged exclusive and inclusive neutrino–nucleus cross sections $\langle\sigma_e\rangle$ (in units of 10^{-42} cm^2) and $\langle\sigma_\mu\rangle$ (in units of 10^{-40} cm^2). (See caption to Table 1)

	$\langle\sigma_e^{\rm exc}\rangle$	$\langle\sigma_e^{\rm inc}\rangle$	$\langle\sigma_\mu^{\rm exc}\rangle$	$\langle\sigma_\mu^{\rm inc}\rangle$
RPA [1]	36.0 (38.4)	42.3 (44.3)	2.48 (3.11)	21.1 (22.8)
RPA [2]	54.8 (68.2)	63.2 (76.3)	3.35 (3.80)	21.1 (22.4)
RPA + pair [2]	7.1 (16.0)	12.9 (22.7)	0.39 (0.77)	13.5 (15.2)
CRPA [3]	12.5 (13.9)	18.15 (19.28)	1.06 (1.06)	17.8 (18.2)
RPA [4]	50.0	55.1	2.09	19.2
QRPA [4]	42.9	52.0	1.97	20.3
PBCS	21.0	41.2	1.67	19.1
PQRPA (I)	9.9	21.6	0.72	14.6
PQRPA (II)	8.0	18.5	0.56	12.8
PQRPA (III)	8.1	17.4	0.56	13.4
Expt.	$9.1 \pm 0.4 \pm 0.9$ [36]	$14.8 \pm 0.7 \pm 1.4$ [36]	$0.66 \pm 0.1 \pm 0.1$ [37]	$12.4 \pm 0.3 \pm 1.8$ [37]
	$8.9 \pm 0.3 \pm 0.9$ [38]	$13.2 \pm 0.4 \pm 0.6$ [38]	$0.56 \pm 0.08 \pm 0.10$ [39]	$10.6 \pm 0.3 \pm 1.8$ [39]

and inclusive μ-capture rates to ^{12}B: $\Lambda(J_f^\pi = 1_1^+)$ and $\sum_{J_f^\pi} \Lambda(J_f^\pi)$. We do not show our QRPA results because of the above mentioned difficulties with the $J^\pi = 1_1^+$ ground states in ^{12}N and ^{12}B. In comparing the calculations of B(GT) with data it should be remembered that it is still not clear the origin of the observed 10% difference measured ft values for the GT β-decays from the ground states $J^\pi = 1^+$ in ^{12}B and ^{12}N to the ground state $J^\pi = 0^+$ in ^{12}C: $ft(^{12}\text{B}) = (1.1669 \pm 0.0037) \times 10^4$ seg, and $ft(^{12}\text{N}) = (1.3178 \pm 0.0084) \times 10^4$ seg [28]. In the past this difference has been attributed mostly to the violation of charge symmetry in the involved nuclear states, and occasionally also to the second class current (or induced tensor interaction) which violates the G-parity [31–33].[2] As this kind of effects are not considered in the present work the above ft values will be taken as lower and upper experimental limits. The corresponding B-values, obtained from (20) ($B_B(\text{GT}) = 0.526$ and $B_N(\text{GT}) = 0.466$) are shown in Table 3. Due to the same reason, the small difference ($\lesssim 3\%$) between the theoretical results for B_B(GT) and B_N(GT) is not physically relevant and only the mean values $(B_B(\text{GT}) + B_N(\text{GT}))/2$ are exhibited.

Similarly, in Table 4 are given the results for the exclusive and inclusive flux-averaged neutrino scattering cross sections to ^{12}N: $\langle\sigma_\ell(J_f^\pi = 1_1^+)\rangle$, $\sum_{J_f^\pi} \langle\sigma_\ell(J_f^\pi)\rangle$ with $\ell = e, \mu$. They are defined as

$$\langle\sigma_\ell(J_f)\rangle = \int dE_\nu\, \sigma(E_\ell = E_i - E_f + E_\nu, J_f)\, \bar{f}(E_\nu), \tag{29}$$

where $\bar{f}(E_\nu)$ is the normalized neutrino flux. For electron neutrinos it was approximated by the Michel spectrum, and for the muon neutrinos we used that from Ref. [40].

We wish to restate the ingredients that play a part in the agreement between the data and calculations for the ground state processes within the triad $\{^{12}\text{B}, ^{12}\text{C}, ^{12}\text{N}\}$. They are: (a) the pairing short range correlations, which are added to improve the description of the ^{12}C ground state, (b) the RPA-type correlations, which are repulsive in the particle–hole channel, and (c) the effective axial-vector coupling constant, $g_{\rm A} = 1$, which in principle simulates the removal of the spin strength due to the coupling to the Δ resonance [10–12,42]. For instance, these effects reduce the bare single-particle value $B(\text{GT}) = (16/9)g_{\rm A}^2$ by factors 1.7, 1.8–2.2 and 1.6, respectively. It is worthy of note that the PBCS by itself reproduces better the data than the majority of previous RPA and QRPA calculations [1–4]. We have considered all orbitals from $1s_{1/2}$ up to $1f_{5/2}$, but the valence p-shell correlations (both pairing and RPA like) are definitely the most important ones for the quenching of the $1_1^+ \leftrightarrow 0_1^+$ transition rates. Yet, as discussed by Vogel et al. [3,

[2] Presently, the study of the G-parity irregular weak nucleon current is still of interest [34,35].

Table 5
Results for the energies (in units of MeV) and the partial muon capture rates (in units of 10^3 s^{-1}) the bound excited states in ^{12}B. In the upper part of the table are shown the previous theoretical calculations based on the RPA [1,41] (where only the results for Λ are reported) and the on shell model [42]

Model	J_f^π	1_1^+	2_1^+	2_1^-	1_1^-
RPA [1,41]	Λ	25.4 (22.8)	$\leqslant 10^{-3}$	0.04 (0.02)	0.22 (0.74)
SM [42]	E	0.00	0.76	1.49	1.99
	Λ	6.0	0.25	0.22	1.86
PBCS	E	0.00	0.00	3.10	3.10
	Λ	15.4	0.40	1.70	1.13
PQRPA (I)	E	0.00	0.34	2.83	3.13
	Λ	7.83	0.21	0.34	0.66
PQRPA (II)	E	0.00	0.50	2.82	3.31
	Λ	6.50	0.16	0.18	0.51
PQRPA (III)	E	0.00	0.28	2.82	2.97
	Λ	6.54	0.17	0.18	0.58
Expt [43,44]	E	0.00	0.95	1.67	2.62
	Λ	6.00 ± 0.40	0.21 ± 0.10	0.18 ± 0.10	0.62 ± 0.20

45], the effect of these correlations on the dipole and quadrupole operators is very tiny.

In addition to the total μ-capture rates in Table 3, we show the individual rates to the individual bound states of ^{12}B in Table 5. They represent another test for our calculation and have been derived from the intensities of the observed de-excitation γ rays following the μ-capture [43,44]. The agreement between the experiment and our PQRPA estimate for the energies of the $J_f^\pi = 2_1^+, 2_1^-$ and 1_1^- states is only moderate, but that for the capture rates is as good or even better than in a recent shell model (SM) study [42].

In summary, we have shown that to account for the weak decay observables around ^{12}C in the framework of the RPA, besides including the BCS correlations, it is imperative to perform the particle number projection. Thus, this is the way out of the RPA puzzle in ^{12}C. More, as far as we are acquainted with, such an important effect of the projected linear response theory for charge-exchange excitations has never before been observed, indicating that it could be more relevant in light $N = Z$ nuclei than in heavy nuclei with large neutron excess [5]. Thus, it could be interesting enough to investigate the consequences of the PQRPA in other $N = Z$ and $N \cong Z$ nuclei. On the other hand, the fact that we have been forced to switch off completely the residual interaction in the particle–particle channel could indicate that some relevant piece of physics is still lacking in our approach. In this sense it would be very illuminating to redo the PQRPA calculations with more realistic forces than the one used here.

Acknowledgements

The authors acknowledge the support of ANPCyT (Argentina) under grant BID 1201/OC-AR (PICT 03-04296) and of CONICET under grant PIP 463. F.K. and A.M. are fellows of the CONICET Argentina.

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