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q-Gamow states as continuous linear functionals on analytical test functions

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Abstract

We define here q-Gamow states corresponding to Tsallis' q-statistics. We compute for them their norm, mean energy value and the q-analogue of the Breit–Wigner distribution (a q-Breit–Wigner).
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1. Introduction

In four previous papers [1–4] we have shown that Gamow-states [5,6] can be interpreted as Sebastiao e Silva's Ultradistributions [7–9], whose proper treatment appeals to Rigged Hilbert Space [10–13].

There exist a large number indeed of high energy experiments amenable to successful interpretation through Tsallis' q-statistics [14]. In particular, this happens for LHC experiments in what respects to distributions linked to stationary states. The q-statistics appears to adequately describe the transverse momentum distributions of different hadrons. The 4 LHC experiments yielded publications involving such distributions that seem to be properly fitted using the q-exponential function. The associated q-value is of the order of 1.15, clearly a departure from the Gibbs–Boltzmann's $q = 1-$ value. Accordingly, stationary states before hadronization are not

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thermal equilibrium-ones. Remark that measuring the p_T distribution over a logarithmic range of fourteen decades shows that $q = 1.15$ fits the data over such immense range [15,16].

These circumstances strongly motivate us to investigate complex energy states related to the q-exponential distributions, that is, q-Gamow states (or q-resonances), and establish their relation with Gamow-states. *These q-Gamow states are NOT solutions of Schroedinger's equation but of its non-linear q-version*, proposed by F.D. Nobre, M.A. Rego-Monteiro, and C. Tsallis in [17] and deduced in [21]. (For a brief review about usual Gamow states see [22].) We focus attention then on decay states at a great distance from the dispersion center and ascertain that a q-Gamow representation is adequate. **The reader should note that the use of q-exponentials (respectively, exponentials) to represent a q-resonance (respectively, resonance) is an approximation valid only at a great distance from the dispersion center.**

We begin our considerations with a review of the work of Refs. [1–3] on Gamow states. *The reader is strongly urged to peruse these papers.*

2. Review of the work of Refs. [1–3] on Gamow states

A brief look at is indispensable in order to properly follow our developments. Indeed, following [1–3], and using the notation of [7], we define a Gamow-state in a free space [this means at a great distance from the dispersion center], as

$$|\psi_G\rangle = \int\limits_{-\infty}^{\infty} \{\mathcal{H}[\Im(p)]\mathcal{H}(x) - \mathcal{H}[-\Im(p)]\mathcal{H}(-x)\}\, e^{\frac{ipx}{\hbar}} |x\rangle\, dx, \tag{2.1}$$

or

$$\psi_G(x) = \{\mathcal{H}[\Im(p)]\mathcal{H}(x) - \mathcal{H}[-\Im(p)]\mathcal{H}(-x)\}\, e^{\frac{ipx}{\hbar}}, \tag{2.2}$$

where $\mathcal{H}(x)$ is the Heaviside's step function.

The norm-squared for such a state reads [6]

$$\langle\psi_G|\psi_G\rangle = \int\limits_{0}^{\infty} \mathcal{H}[\Im(p)] e^{\frac{i(p-p^*)x}{\hbar}}\, dx - \int\limits_{-\infty}^{0} \mathcal{H}[-\Im(p)] e^{\frac{i(p-p^*)x}{\hbar}}\, dx. \tag{2.3}$$

These integrals can be easily evaluated. One finds

$$\langle\psi_G|\psi_G\rangle = \{\mathcal{H}[\Im(p)] - \mathcal{H}[-\Im(p)]\}\, \frac{\hbar}{i(p^* - p)} = \frac{\hbar}{2|\Im(p)|}. \tag{2.4}$$

Accordingly, the normalized Gamow-state ϕ_G becomes [6]

$$|\phi_G\rangle = \sqrt{\frac{2|\Im(p)|}{\hbar}}\, |\psi_G\rangle. \tag{2.5}$$

Since [we are using here a special notation employed in the classical text-book by Messiah on quantum mechanics [25]]

$$\langle\phi_G|(H|\phi_G\rangle) = \frac{p^2}{2m}, \tag{2.6}$$

$$(\langle\phi_G|H)|\phi_G\rangle = \frac{p^{*2}}{2m}, \tag{2.7}$$

one encounters, for the mean-energy, a real value that agrees with experiment [1–3] and reads

$$\langle H\rangle = \frac{1}{2}\left[\langle\phi_G|(H|\phi_G\rangle) + (\langle\phi_G|H)|\phi_G\rangle\right] = \frac{p^2+p^{*2}}{4m} = \frac{\Re(p^2)}{2m}. \tag{2.8}$$

In order to obtain the probability distribution associated to a Gamow state we start by the looking at scalar product between this state and a free one:

$$\langle\phi|\phi_G\rangle = \frac{1}{\hbar}\sqrt{\frac{|\Im(p)|}{\pi}}\left\{\int\limits_0^\infty \mathcal{H}[\Im(p)]e^{\frac{i(p-k)x}{\hbar}}\,dx - \int\limits_{-\infty}^0 \mathcal{H}[-\Im(p)]e^{\frac{i(p-k)x}{\hbar}}\,dx\right\}. \tag{2.9}$$

Thus,

$$\langle\phi|\phi_G\rangle = \frac{i\sqrt{\frac{|\Im(p)|}{\pi}}}{p-k}. \tag{2.10}$$

The ensuing probability distribution is the Breit–Wigner one [6]

$$|\langle\phi|\phi_G\rangle|^2 = \frac{|\Im(p)|}{\pi\left\{[\Re(p)-k]^2+\Im(p)^2\right\}}. \tag{2.11}$$

3. q-Gamow states

According to the q-statistics strictures (this word exits!) we must replace everywhere ordinary exponentials by so-called q-exponentials $e_q(x)$ [14]

$$e_q(x) = [1+(1-q)x]^{1/1-q};\ q\in\mathcal{R}, \tag{3.1}$$

that becomes the ordinary exponential at $q=1$. Accordingly,

$$|\psi_{qG}\rangle = \int\limits_{-\infty}^{\infty}\{\mathcal{H}[\Im(p)]\mathcal{H}(x) - \mathcal{H}[-\Im(p)]\mathcal{H}(-x)\}\otimes \left[1-\frac{i(q-1)px}{\hbar\sqrt{2(q+1)}}\right]^{\frac{2}{1-q}}|x\rangle\,dx, \tag{3.2}$$

or

$$\psi_{qG}(x) = \{\mathcal{H}[\Im(p)]\mathcal{H}(x) - \mathcal{H}[-\Im(p)]\mathcal{H}(-x)\}\left[1-\frac{i(q-1)px}{\hbar\sqrt{2(q+1)}}\right]^{\frac{2}{1-q}}. \tag{3.3}$$

The norm of a q-Gamow state is

$$\langle\psi_{qG}|\psi_{qG}\rangle = \int\limits_0^\infty \mathcal{H}[\Im(p)]\left[1-\frac{i(q-1)px}{\hbar\sqrt{2(q+1)}}\right]^{\frac{2}{1-q}}\left[1+\frac{i(q-1)p^*x}{\hbar\sqrt{2(q+1)}}\right]^{\frac{2}{1-q}}dx + \int\limits_{-\infty}^0 \mathcal{H}[-\Im(p)]\left[1-\frac{i(q-1)px}{\hbar\sqrt{2(q+1)}}\right]^{\frac{2}{1-q}}\left[1+\frac{i(q-1)p^*x}{\hbar\sqrt{2(q+1)}}\right]^{\frac{2}{1-q}}dx, \tag{3.4}$$

or equivalently,

$$\langle\psi_{qG}|\psi_{qG}\rangle = \int_0^\infty \mathcal{H}[\Im(p)]\left[1+\frac{2(q-1)\Im(p)x}{\hbar\sqrt{2(q+1)}}+\frac{(q-1)^2|p|^2x^2}{\hbar^2 2(q+1)}\right]^{\frac{2}{1-q}} dx + \int_0^\infty \mathcal{H}[-\Im(p)]\left[1-\frac{2(q-1)\Im(p)x}{\hbar\sqrt{2(q+1)}}+\frac{(q-1)^2|p|^2x^2}{\hbar^2 2(q+1)}\right]^{\frac{2}{1-q}} dx, \tag{3.5}$$

that can be recast as

$$\langle\psi_{qG}|\psi_{qG}\rangle = \int_0^\infty \left[1+\frac{2(q-1)|\Im(p)|x}{\hbar\sqrt{2(q+1)}}+\frac{(q-1)^2|p|^2x^2}{\hbar^2 2(q+1)}\right]^{\frac{2}{1-q}} dx. \tag{3.6}$$

We effect now the change of variables $y=\frac{(q-1)|p|x}{\hbar\sqrt{2(q+1)}}$ and obtain

$$\langle\psi_{qG}|\psi_{qG}\rangle = \frac{\hbar\sqrt{2(q+1)}}{(q-1)|p|}\int_0^\infty \left[1+\frac{2|\Im(p)|y}{|p|^2}+y^2\right]^{\frac{2}{1-q}} dy. \tag{3.7}$$

After a new change of variables $z=y+\frac{|\Im(p)|}{|p|}$ we find

$$\langle\psi_{qG}|\psi_{qG}\rangle = \frac{\hbar\sqrt{2(q+1)}}{(q-1)|p|}\int_{\frac{|\Im(p)|}{|p|}}^\infty \left\{z^2+\frac{[\Re(p)]^2}{|p|^2}\right\}^{\frac{2}{1-q}} dz. \tag{3.8}$$

Finally, after a third change of variables $s=z^2$ we get for our norm

$$\langle\psi_{qG}|\psi_{qG}\rangle = \frac{\hbar\sqrt{2(q+1)}}{2(q-1)|p|}\int_{\frac{|\Im(p)|^2}{|p|^2}}^\infty s^{-\frac{1}{2}}\left\{s+\frac{[\Re(p)]^2}{|p|^2}\right\}^{\frac{2}{1-q}} ds. \tag{3.9}$$

Using the result given in [19] we arrive at:

$$\langle\psi_{qG}|\psi_{qG}\rangle = \frac{\hbar}{5-q}\frac{\sqrt{2(q+1)}}{|p|}\left\{\frac{[\Im(p)]^2}{|p|^2}\right\}^{\frac{q-5}{2(q-1)}} \otimes F\left(\frac{2}{q-1},\frac{5-q}{2(q-1)};\frac{3+q}{2(q-1)};-\frac{[\Re(p)]^2}{[\Im(p)]^2}\right). \tag{3.10}$$

It is shown in [18] that

$$F\left(\frac{2}{q-1},\frac{5-q}{2(q-1)};\frac{3+q}{2(q-1)};-\frac{[\Re(p)]^2}{[\Im(p)]^2}\right)= \left\{\frac{|p|^2}{[\Im(p)]^2}\right\}^{\frac{q-5}{2(q-1)}} F\left(\frac{1}{2},\frac{5-q}{2(q-1)};\frac{3+q}{2(q-1)};\frac{[\Re(p)]^2}{|p|^2}\right), \tag{3.11}$$

which yields for the norm the expression

$$\langle\psi_{qG}|\psi_{qG}\rangle = \frac{\hbar}{5-q}\frac{\sqrt{2(q+1)}}{|p|}\otimes$$
$$F\left(\frac{1}{2},\frac{5-q}{2(q-1)};\frac{3+q}{2(q-1)};\frac{[\Re(p)]^2}{|p|^2}\right) = [A(q,p)]^2, \tag{3.12}$$

so that the normalized q-Gamow state becomes

$$|\phi_{qG}\rangle = [A(q,p)]^{-1}|\psi_{qG}\rangle. \tag{3.13}$$

Noticing that

$$\lim_{q\to 1} F\left(\frac{1}{2},\frac{5-q}{2(q-1)};\frac{3+q}{2(q-1)};\frac{[\Re(p)]^2}{|p|^2}\right) = F\left(\frac{1}{2},4;4;\frac{[\Re(p)]^2}{|p|^2}\right) \tag{3.14}$$

and using a result of [20] one has

$$F\left(\frac{1}{2},4;4;\frac{[\Re(p)]^2}{|p|^2}\right) = \left[\frac{[\Im(p)]^2}{|p|^2}\right]^{-\frac{1}{2}}, \tag{3.15}$$

and

$$\lim_{q\to 1}[A(q,p)]^2 = \frac{\hbar}{2|\Im(p)|}. \tag{3.16}$$

Using now a result derived in [21] for the q-generalization of Schroedinger's equation of Ref. [17], i.e.,

$$H\phi_q(x) = \frac{p^2}{2m}[\phi_q(x)]^q, \tag{3.17}$$

we encounter

$$\langle\phi_{qG}|(H|\phi_{qG}\rangle) = [A(p,q)]^{-2}\frac{p^2}{2m}\otimes$$
$$\left\{\int_0^\infty \mathcal{H}[\Im(p)]\left[1-\frac{i(q-1)px}{\hbar\sqrt{2(q+1)}}\right]^{\frac{2q}{1-q}}\left[1+\frac{i(q-1)p^*x}{\hbar\sqrt{2(q+1)}}\right]^{\frac{2}{1-q}}dx\right.$$
$$\left.+\int_{-\infty}^0 \mathcal{H}[-\Im(p)]\left[1-\frac{i(q-1)px}{\hbar\sqrt{2(q+1)}}\right]^{\frac{2q}{1-q}}\left[1+\frac{i(q-1)p^*x}{\hbar\sqrt{2(q+1)}}\right]^{\frac{2}{1-q}}dx\right\}, \tag{3.18}$$

or equivalently,

$$\langle\phi_{qG}|(H|\phi_{qG}\rangle) = [A(p,q)]^{-2}\frac{p^2}{2m}\otimes$$
$$\left\{\int_0^\infty \mathcal{H}[\Im(p)]\left[1-\frac{i(q-1)px}{\hbar\sqrt{2(q+1)}}\right]^{\frac{2q}{1-q}}\left[1+\frac{i(q-1)p^*x}{\hbar\sqrt{2(q+1)}}\right]^{\frac{2}{1-q}}dx\right.$$
$$\left.+\int_0^\infty \mathcal{H}[-\Im(p)]\left[1+\frac{i(q-1)px}{\hbar\sqrt{2(q+1)}}\right]^{\frac{2q}{1-q}}\left[1-\frac{i(q-1)p^*x}{\hbar\sqrt{2(q+1)}}\right]^{\frac{2}{1-q}}dx\right\}. \tag{3.19}$$

We can recast (3.19) as

$$\langle \phi_{qG}|(H|\phi_{qG}\rangle) = [A(p,q)]^{-2}\frac{p^2}{2m} \otimes \left\{ \mathcal{H}[\Im(p)] \left[\frac{i(q-1)p^*}{\hbar\sqrt{2(q+1)}}\right]^{\frac{2}{1-q}} \left[\frac{-i(q-1)p}{\hbar\sqrt{2(q+1)}}\right]^{\frac{2q}{1-q}} \otimes \int\limits_0^\infty \left[1 - \frac{i(q-1)px}{\hbar\sqrt{2(q+1)}}\right]^{\frac{2q}{1-q}} \left[1 + \frac{i(q-1)p^*x}{\hbar\sqrt{2(q+1)}}\right]^{\frac{2}{1-q}} dx + \mathcal{H}[-\Im(p)] \left[\frac{-i(q-1)p^*}{\hbar\sqrt{2(q+1)}}\right]^{\frac{2}{1-q}} \left[\frac{i(q-1)p}{\hbar\sqrt{2(q+1)}}\right]^{\frac{2q}{1-q}} \otimes \int\limits_0^\infty \left[1 + \frac{i(q-1)px}{\hbar\sqrt{2(q+1)}}\right]^{\frac{2q}{1-q}} \left[1 - \frac{i(q-1)p^*x}{\hbar\sqrt{2(q+1)}}\right]^{\frac{2}{1-q}} dx \right\}. \tag{3.20}$$

We use now a result from [23] and obtain

$$\langle \phi_{qG}|(H|\phi_{qG}\rangle) = -\frac{\hbar}{i[A(p,q)]^2}\frac{p}{2m}\mathcal{B}\left(1, \frac{3+q}{q-1}\right)\frac{\sqrt{2(q+1)}}{(q-1)} \otimes \{\mathcal{H}[\Im(p)] - \mathcal{H}[-\Im(p)]\}\, F\left(1, \frac{2}{q-1}; \frac{2(q+1)}{q-1}; 1 + \frac{p^*}{p}\right), \tag{3.21}$$

or equivalently,

$$\langle \phi_{qG}|(H|\phi_{qG}\rangle) = -\frac{\hbar}{i[A(p,q)]^2}\frac{p}{2m}\frac{\sqrt{2(q+1)}}{(3+q)} \otimes \{\mathcal{H}[\Im(p)] - \mathcal{H}[-\Im(p)]\}\, F\left(1, \frac{2}{q-1}; \frac{2(q+1)}{q-1}; 1 + \frac{p^*}{p}\right). \tag{3.22}$$

In analogous fashion we find

$$(\langle \phi_{qG}|H)|\phi_{qG}\rangle = \frac{\hbar}{i[A(p,q)]^2}\frac{p^*}{2m}\frac{\sqrt{2(q+1)}}{(3+q)} \otimes \{\mathcal{H}[\Im(p)] - \mathcal{H}[-\Im(p)]\}\, F\left(1, \frac{2}{q-1}; \frac{2(q+1)}{q-1}; 1 + \frac{p}{p^*}\right). \tag{3.23}$$

Thus, according to [1–3] we obtain for the mean energy value

$$\langle H\rangle_q = \frac{1}{2}\left[\langle \phi_{qG}|(H|\phi_{qG}\rangle) + (\langle \phi_{qG}|H)|\phi_{qG}\rangle\right]. \tag{3.24}$$

Additionally, since

$$\lim_{q\to 1} F\left(1, \frac{2}{q-1}; \frac{2(q+1)}{q-1}; 1 + \frac{p^*}{p}\right) = \frac{2p}{p-p^*}, \tag{3.25}$$

we have

$$\lim_{q\to 1}\langle H\rangle_q = \frac{\Re(p^2)}{2m} = \langle H\rangle. \tag{3.26}$$

We investigate now the q-analogue of the Breit–Wigner distribution tackling

$$\langle\phi|\phi_{Gq}\rangle = \frac{1}{\sqrt{2\pi\hbar A(q,p)}}\left\{\mathcal{H}[\Im(p)]\int\limits_0^\infty e^{-ikx}\left[1+\frac{i(1-q)px}{\hbar\sqrt{2(q+1)}}\right]^{\frac{2}{1-q}} dx \right.$$
$$\left. -\mathcal{H}[-\Im(p)]\int\limits_0^\infty e^{ikx}\left[1-\frac{i(1-q)px}{\hbar\sqrt{2(q+1)}}\right]^{\frac{2}{1-q}} dx\right\}, \tag{3.27}$$

and rewrite it as

$$\langle\phi|\phi_{Gq}\rangle = \frac{1}{\sqrt{2\pi\hbar A(q,p)}}\left\{H[\Im(p)]\left[\frac{i(1-q)p}{\hbar\sqrt{2(q+1)}}\right]^{\frac{2}{1-q}}\otimes\right.$$
$$\int\limits_0^\infty e^{-ikx}\left[x+\frac{\hbar\sqrt{2(q+1)}}{i(1-q)p}\right]^{\frac{2}{1-q}} dx - H[-\Im(p)]\left[-\frac{i(1-q)p}{\hbar\sqrt{2(q+1)}}\right]^{\frac{2}{1-q}}\otimes$$
$$\left.\int\limits_0^\infty e^{ikx}\left[x-\frac{\hbar\sqrt{2(q+1)}}{i(1-q)p}\right]^{\frac{2}{1-q}} dx\right\}. \tag{3.28}$$

We appeal now to a result of [24] and obtain

$$\langle\phi|\phi_{Gq}\rangle = -i\sqrt{\frac{\hbar}{2\pi A(q,p)}}\left[\frac{\sqrt{2(q+1)}}{(1-q)p}\right]^{\frac{2}{q-1}} k^{\frac{3-q}{q-1}} e^{\frac{\sqrt{2(q+1)}k}{(1-q)p}}\otimes$$
$$\Gamma\left[\frac{3-q}{1-q}, \frac{\sqrt{2(q+1)}\,k}{(1-q)p}\right], \tag{3.29}$$

which leads to the q-Breit–Wigner result

$$|\langle\phi|\phi_{Gq}\rangle|^2 = \frac{\hbar}{2\pi A(q,p)}\left[\frac{2(q+1)}{(1-q)^2|p|^2}\right]^{\frac{2}{q-1}} k^{\frac{2(3-q)}{q-1}} e^{\frac{\sqrt{2(q+1)}k(p+p^*)}{(1-q)|p|^2}}\otimes$$
$$\Gamma\left[\frac{3-q}{1-q}, \frac{\sqrt{2(q+1}\,k}{(1-q)p}\right]\left\{\Gamma\left[\frac{3-q}{1-q}, \frac{\sqrt{2(q+1}\,k}{(1-q)p}\right]\right\}^*. \tag{3.30}$$

Note that (3.27) converges uniformly for $q \to 1$ [since the q-exponential converges in that way to the ordinary one], i.e.,

$$\lim_{q\to 1}|\langle\phi|\phi_{Gq}\rangle|^2 = \frac{|\Im(p)|}{\pi\left\{[\Re(p)-k]^2+\Im(p)^2\right\}}. \tag{3.31}$$

4. Conclusions

The q-statistics appears to adequately describe the transverse momentum distributions of different hadrons. The 4 LHC experiments yielded publications involving such distributions that seem to be properly fitted using the q-exponential function. The associated q-value is of the order of 1.15, clearly a departure from the Gibbs–Boltzmann's $q = 1-$ value. Accordingly, stationary states before hadronization are not thermal equilibrium-ones [15,16].

In this work we gave introduced q-Gamow states, associated to the q-statistics, that are solutions to a non-linear, q-generalization of Schroedinger's equation [17,21].

For such a purpose we have computed their norm, the mean energy value, and the concomitant q-Breit–Wigner distributions. In all instances, results tend to the customary ones when the all important q-parameter of Tsallis' obeys $q \to 1$. It is hoped that these q-Gamow states may be found by recourse to the extant LHC data.

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