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Reply

Reply to the Comment by James F. Lutsko and Jean Pierre Boon

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It is well known that, for obtaining the partition function $\mathcal{Z}$, two alternative routes can be followed:

– the "natural" one, given by $\mathcal{Z}$'s definition in terms of a sum over "un-normalized" probabilities, and

– $\mathcal{Z}$ as the Laplace transform of the energy density.

In the orthodox Boltzmann-Gibbs statistical mechanics, that uses the ordinary exponential function, the two routes yield the same result.

We proved in [1] that such is NOT the case for Tsallis' thermostatistics, for which the first alternative (use of a probability distribution) *diverges* in one or more dimensions, due to the long tail of the q-exponential function. One must necessarily follow the second path (without employing probability distributions), that yields finite results. Thus, the q-Laplace transform is seen to become an indispensable tool for nonextensive statistics.

In their Comment, Lutsko and Boon (LB) raise four interesting points (their essence given below in italics) that certainly deserve detailed debate.

1) LB assert that, *while our procedure yields a finite value for the partition function, the original q-exponential distribution remains un-normalizable.* True, but our whole point is that we do NOT want to employ probability distributions (PD) in out treatment. Our only microscopic input is the energy-density, as discussed, for example, in Reif's text-book [2].

2) *Our formalism would not satisfy the relationship $U = -(\partial\beta F/\partial\beta)_V$.* The F-definition above is wrong. We know that

$$S_q = kZ_q^{q-1}(\ln_q Z_q + \beta \langle U_q\rangle),$$

where k stands for Boltzmann's constant. Using now the prescription

$$F_q = \langle U_q\rangle - TS_q,$$

we find

$$F_q = (1 - Z_q^{q-1}) \langle U_q\rangle - \frac{Z_q^{q-1}}{\beta} \ln_q Z_q,$$

an expression that does not coincide with that of LB for F_q. From our last relation one finds

$$\frac{\partial\beta F_q}{\partial\beta} = (1 - Z_q^{q-1}) \langle U_q\rangle + \beta(1 - Z_q^{q-1}) \frac{\partial \langle U_q\rangle}{\partial\beta} - \beta \langle U_q\rangle \frac{\partial Z_q^{q-1}}{\partial\beta} - \frac{\partial Z_q^{q-1}}{\partial\beta} \ln_q Z_q - Z_q^{q-1} \frac{\partial \ln_q Z_q}{\partial\beta},$$

that, for $q = 1$, reduces to

$$\frac{\partial\beta F}{\partial\beta} = -\frac{1}{Z}\frac{\partial Z}{\partial\beta} = \langle U\rangle ,$$

in full agreement with Tsallis' prescription. The evaluation of $\frac{\partial\beta F_q}{\partial\beta}$ is a function of q that turns out to coincide with $\langle U\rangle$ for $q = 1$, which, in turn, contradicts LB's assertions.

3) *Now the resulting entropy is not equivalent to the original Tsallis entropy evaluated with the q-exponential distribution* (as probability distributions). Of course it is not! We do away with probability distributions in order to avoid the Tsallis divergences.

4) *The expansion used in eq.* (4) of [1] *seems quite arbitrary. One could, for example, replace $a_n x^n$*

by $(2^n a_n)(x/2)^n$ *and thereby obtain an inequivalent form*. Our answer is that the McLaurin expansion is UNIQUE and can not be arbitrarily modified as LB want.

Summing up, the LB comments are interesting and stimulate fertile debate around mathematical niceties that enter Tsallis' theory. These certainly need to be fully explored. Their questions and our answers will hopefully serve such purpose.

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REFERENCES

[1] Plastino A. and Rocca M. C., *EPL*, **104** (2013) 60003.
[2] Reif F., *Fundamentals of Statistical and Thermal Physics* (McGraw-Hill, New York) 1965.