DETERMINATION OF SEISMOGRAPH CONSTANTS FROM THE CALIBRATION CURVE

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ABSTRACT

The explicit expression of the long-period WWSSN calibration curve can be obtained solving the differential equations of the seismograph system in terms of $e^{\omega t}$ and identifying the characteristic equation to a product of polynomials of the second degree. The coefficients of these polynomials are the equivalent constants. These and a scale factor can be obtained from the calibration curve. The frequency response of the seismograph system is fully determined by these elements provided that the reduced motor constant and the calibration current are known. The explicit knowledge of the seismograph partial constants is not necessary.

Newton-Raphson's method is generally efficient to obtain the elements from the calibration curve, but it may fail some times. Levenberg's method (slightly modified) and also simple step reduction can be helpful in such cases. The unknowns can be obtained either freely or under the constraint that some of the partial constants have the values assigned to them. The latter may be convenient when the behavior of parts of the system is affected by some trouble.

INTRODUCTION

Without much error, a seismograph may be assumed to be a linear system. Consequently, being $f(t)$ the Earth acceleration, $s(t)$ the function recorded, and representing these functions by the Fourier integrals

$$s(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} S(\omega) e^{j\omega t} d\omega; \quad f(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(\omega) e^{j\omega t} d\omega$$

we may write

$$S(\omega) = F(\omega) H(\omega) e^{j\nu(\omega)}$$

where $H(\omega)$ is the dynamic magnification and $\nu(\omega)$ the phase delay of the system. Expressions for $F(\omega)$ and $\nu(\omega)$ as depending on the partial instrumental constants were given first by Coulomb and Grenet (1935) and later by Hagiwara (1958).

Since the partial constants do not keep their values, and this affects the values of $H(\omega)$ and $\nu(\omega)$, some time ago it was proposed at Lamont Geological Observatory to do a daily calibration signal on the seismograph from which changes in the response can be detected. A step excitation was found to be convenient because it is easily achieved and the Fourier transform of the corresponding response is proportional to $H(\omega) e^{j\nu(\omega)} / j\omega$.

Espinosa *et al.* (1965) gave a set of curves from which some of the constants can approximately be obtained, and J. Brune, in an Appendix to their paper, showed how to acquire them numerically. Teng and Ben Menahem (1965) tried to obtain $H(\omega)$ by calculating numerically the Fourier transform of the calibration curve. They found that, for high frequencies, the result becomes inaccurate. To improve this method, Mitchell and Landisman (1969) proposed another one where a set of

parameters introduced in $H(\omega)e^{j\nu(\omega)}$ minimizes the squared difference between the theoretical and the actual calibration curve. The authors pointed out that the method is unable to yield all the constants, and since the coupling factor is small, they assumed it to be null. A variation to this method, based upon explicit expressions for the calibration curve still assuming zero for the coupling factor, has been given by Jarosch and Curtis (1973) who obtained their results applying the Laplace transform to the seismometer equations.

In this paper, we are intending to do a further step in the same sense. We first show that the explicit expressions can be easily obtained by means of the characteristic equation method, and second that, although the partial instrumental constants cannot be obtained separately, it is possible to obtain other parameters from which $H(\omega)e^{j\nu(\omega)}$ is determined without any assumptions. We also refer to the method of obtaining them, indicating how to improve the Newton-Raphson method (which is generally suitable to this end) when it fails to be convergent.

THE SEISMOMETER EQUATIONS

After Chakrabarty and Chudhury (1964) or Morencos (1972), the response of an electromagnetic seismograph to a ground acceleration, $f(t)$, is described by the following system of equations

$$\begin{aligned} L_1 \dot{I}_1 + q_1 I_1 - q_2 I_2 &= c_1 \dot{x} \\ L_2 \dot{I}_2 + q_3 I_2 - q_2 I_1 &= c_2 \dot{s} \\ \ddot{x} + 2\epsilon_1' \dot{x} + n_1^2 x + c_3 I_1 &= -f \\ s + 2\epsilon_2' \dot{s} + n_2^2 s - c_4 I_2 &= 0 \\ n_1 &= \frac{2\pi}{T_1}, \quad n_2 = \frac{2\pi}{T_2}. \end{aligned} \quad (1)$$

The values T_1 , ϵ_1' , I_1 , L_1 , correspond to the sensor and are, respectively, the period, the damping coefficient, the circulating current, and the self-inductance. T_2 , ϵ_2' , I_2 , L_2 are similar values corresponding to the galvanometer; s represents, as before, the ordinates of the record, x , the displacement of the oscillating center of the sensor, and q_1 , q_2 , c_1 , c_2 , c_3 , and c_4 are constants of the system defined by its electric and electromagnetic characteristics.

Since the case $L_1 \neq 0$, $L_2 \neq 0$ is cumbersome and can be treated in a similar way to the case $L_1 = 0$, $L_2 = 0$, we shall restrict ourselves, for the sake of simplicity, to the latter. Such assumption can be accepted for the long-period instruments of the WWSSN where the influence of the self-inductances has proved to be relatively small. As was pointed out by Salvill *et al.* (1962), the same, instead, is not valid for the short-period instruments. A detailed treatment corresponding to these instruments with the method given here is left to a future paper.

Putting $L_1 = 0$, $L_2 = 0$, the system (1) becomes

$$\begin{aligned} x + 2\epsilon_1 \dot{x} + n_1^2 x - \sigma_1 \dot{s} &= -f \\ \ddot{s} + 2\epsilon_2 \dot{s} + n_2^2 s - \sigma_2 \dot{x} &= 0. \end{aligned} \quad (2)$$

σ_1 and σ_2 are transmission coefficients, their product

$$\sigma^2 = \sigma_1 \sigma_2$$

measures the coupling between the sensor and the galvanometer, and ϵ_1, ϵ_2 are their damping coefficients.

The systems (1) and (2) are also valid when the sensor, being of the rotating type, is acted by a moment or when, being of the displacement type, is acted by a force, except for the meaning of f which is different. Taking this into account, let us consider that being at rest, the sensor is acted by a step current i_p . Then

$$f = -\frac{Gi_p}{M}$$

where G is the reduced motor constant and M the mass of the sensor.

Defining an auxiliary function

$$y = x - \frac{Gi_p}{n_1^2 M},$$

we obtain by replacing it into (2)

$$\begin{aligned} y + 2\epsilon_1 \dot{y} + n_1^2 y - \sigma_1 \dot{s} &= 0 \\ \dot{s} + 2\epsilon_2 \dot{s} + n_2^2 s - \sigma_2 \dot{y} &= 0 \end{aligned} \quad (3)$$

and the following initial conditions

$$t = 0; \quad y = y_0 = -\frac{Gi_p}{n_1^2 M}; \quad \dot{y} = 0. \quad (4)$$

$Gi_p/n_1^2 M$ is the deflection of the sensor caused by i_p and equations (3) correspond to the case in which the sensor is free from external actions. Thus, the galvanometer response to the step current is identical to the one resulting after releasing the sensor, and the expression of the calibration curve can be obtained from either equations (3) or (2). Here, we shall obtain it using equations (3).

THE EXPRESSION OF THE CALIBRATION CURVE

To solve (3), we assume that

$$y = Ae^{wt} \quad s = Be^{wt}.$$

We then obtain the characteristic equation

$$\Phi(w) = (w^2 + 2\epsilon_1 w + n_1^2)(w^2 + 2\epsilon_2 w + n_2^2) - \sigma^2 w = 0 \quad (5)$$

and

$$\frac{A}{B} = \frac{w^2 + 2\epsilon_2 w + n_2^2}{\sigma_2 w}. \quad (6)$$

To get to the roots of (5), we replace $\Phi(w)$ by the product of the following two quadratic polynomials

$$\begin{aligned} \lambda_1(w) &= w^2 + 2a_1 w + p_1 \\ \lambda_2(w) &= w^2 + 2a_2 w + p_2. \end{aligned}$$

λ_1 and λ_2 contain four constants called by Tobyáš and Hubernova (1969) equivalent constants related to the five partial constants contained in (5) as follows

$$\begin{aligned} a_1 + a_2 &= \epsilon_1 + \epsilon_2 \\ 4a_1a_2 + p_1 + p_2 &= 4\epsilon_1\epsilon_2 + n_1^2 + n_2^2 - \sigma^2 \\ a_1p_2 + a_2p_1 &= \epsilon_1n_2^2 + \epsilon_2n_1^2 \\ p_1p_2 &= n_1^2n_2^2. \end{aligned} \quad (7)$$

The roots of (5) are obviously the ones of λ_1 and λ_2 , viz.

$$w_{1,2} = -a \pm \sqrt{a_1^2 - p_1} \quad w_{3,4} = -a_2 \pm \sqrt{a_2^2 - p_2}$$

and the solution of (3) becomes

$$y = \sum_1^4 A_i e^{w_i t} \quad s = \sum_1^4 B_i e^{w_i t};$$

A_i and B_i being determined from the initial conditions and from equations like (6). Assuming that the galvanometer is at rest when $t = 0$, we obtain after some algebra

$$s = \frac{K}{g} [e^{-a_1 t} (\cosh b_1 t + \frac{\kappa_1}{b_1} \sinh b_1 t) - e^{-a_2 t} (\cosh b_2 t - \frac{\kappa_2}{b_2} \sinh b_2 t)] \quad (8)$$

TABLE 1
CASES TAKEN FOR COMPARISON

Case	T_1	T_2	h_1	h_2
a	15.0	100.0	1.0	1.0
b	15.0	100.0	1.5	1.0
c	30.0	75.0	0.8	0.8
d	30.0	75.0	1.5	6.0

where

$$\begin{aligned} K &= \frac{Gi_p \sigma_2}{M}; \quad g = (a_1 + \kappa_1)p_2 - (a_2 - \kappa_2)p_1 \\ \kappa_1 &= a_1 + \frac{p_2 - p_1}{2(a_1 - a_2)}; \quad \kappa_2 = -\left(a_2 + \frac{p_2 - p_1}{2(a_1 - a_2)}\right) \\ b_1 &= \sqrt{a_1^2 - p_1}; \quad b_2 = \sqrt{a_2^2 - p_2}. \end{aligned} \quad (9)$$

In (8), b_1 and b_2 are supposed to be real. Being imaginary, they should be replaced by their modulus and the hyperbolic by the corresponding trigonometric functions. When $b_1 = 0$, the first parenthesis becomes $1 + \kappa_1 t$. Similarly when $b_2 = 0$, the second parenthesis becomes $1 - \kappa_2 t$.

When $\sigma_1 = 0$, expression (8) coincides with the expression obtained by Jarosch and Curtis (1973) and represents the curves given by Espinosa *et al.* (1965). The values of these curves taken from their maximum onward and the values obtained from (8) for four cases indicated in Table 1, are compared in Table 2.

The values obtained from (8), putting $K = 5.0$, were reduced by multiplying them by a factor m given by

$$m = \frac{\sum s_1 s_2}{\sum s_1^2}.$$

s_1 is given by (8) and s_2 is taken from the curves. Both are in millimeters. It appears as though the differences are small.

TABLE 2
THEORETICAL AND EXPERIMENTAL VALUES COMPARED

t (sec)	Case a		Case b		Case c		Case d		
	s_1^*	$s_2^\dagger$	s_1^*	$s_2^\dagger$	s_1^*	$s_2^\dagger$	t (sec)	s_1^*	$s_2^\dagger$
							36	85.8	87.4
22	157.0	159.5			174.5	178.0	40	85.5	86.5
24	155.3	156.0	146.7	150.0	173.0	175.0	60	78.4	78.0
32	134.7	135.0	136.2	134.0	137.4	135.0	80	69.0	68.0
40	106.2	104.0	112.4	111.0	87.3	83.0	100	60.1	59.5
48	79.2	78.0	86.5	85.0	46.4	43.0	120	52.2	51.5
56	57.0	55.0	63.6	64.0	20.0	16.0	140	45.4	45.0
64	39.9	38.0	45.3	46.0	5.7	4.0	160	39.4	39.0
72	27.5	27.0	31.5	32.0	-0.7	-2.0	180	34.2	35.0
80	18.6	18.5	21.6	22.5	-2.7	-3.5	200	29.8	29.4
88	12.5	12.5	14.6	15.0	-2.7	-3.0	220	25.9	25.5
96	8.3	8.0	9.7	10.2	-2.0	-2.3	240	22.4	22.0
104	5.4	5.5	6.4	6.5	-1.2	-1.5	260	19.5	19.2
112	3.6	2.8	4.2	4.5	-0.6	-0.8	280	16.9	16.9
120	2.3	1.8	2.8	3.0	-0.3	-0.4	300	14.7	14.5
128	1.5	1.2	1.8	2.0	-0.1	-0.1	320	12.8	12.5
136	1.0	0.8	1.2	1.2	-0.1	-0.1	340	11.1	11.0
144	0.6	0.5	0.8	0.8	0	-0.1	360	9.7	9.6
152	0.4	0.2	0.5	0.6			420	6.4	6.2
160	0.3	0.1	0.3	0.3			480	4.2	4.3
168	0.2	0.1	0.2	0.2			540	2.8	3.0
176	0.1	0.1	0.1	0.1			600	1.8	2.0
184	0.1	0.1	0.1	0.1			660	1.1	1.5
$t_{\max}^\ddagger$	21.5	21.0	24.5	23.5	22.3	22.0		36.4	36.0
$t_0^\S$					70.7	70.0			

* Values obtained from equation (8).

† Values given graphically by Espinosa *et al.* (1965).

‡ Values of t corresponding to a maximum of s .

§ Values of t corresponding to $s = 0$

CONSTANTS OBTAINED FROM THE CALIBRATION CURVE

The values of a_1 , a_2 , p_1 , p_2 , and K can be obtained from a convenient set of equations (8) corresponding to different ordinates of the calibration curve. Although these unknowns are not sufficient to calculate the partial constants, they enable us to calculate $\nu(\omega)$ and also $H(\omega)$, provided that $\frac{Gi_p}{M}$ is known. In fact, from the expression resulting for f when $s = e^{j\omega t}$, it is easily obtained

$$H(\omega)e^{j\nu(\omega)} = \frac{j\sigma_2\omega}{\Phi(j\omega)}$$

and hereof

$$H(\omega) = \frac{\sigma_2 \omega}{[((p_1 - \omega^2)^2 + 4a_1^2 \omega^2)((p_2 - \omega^2)^2 + 4a_2^2 \omega^2)]^{1/2}}; \quad \nu = \frac{\pi}{2} - (\nu_1 + \nu_2)$$
$$\sin \nu_1 = \frac{2a_1 \omega}{[(p_1 - \omega^2)^2 + 4a_1^2 \omega^2]^{1/2}}; \quad \cos \nu_1 = \frac{p_1 - \omega^2}{[(p_1 - \omega^2)^2 + 4a_1^2 \omega^2]^{1/2}}$$
$$\sin \nu_2 = \frac{2a_2 \omega}{[(p_2 - \omega^2)^2 + 4a_2^2 \omega^2]^{1/2}}; \quad \cos \nu_2 = \frac{p_2 - \omega^2}{[(p_2 - \omega^2)^2 + 4a_2^2 \omega^2]^{1/2}}.$$

It is seen that ν is thoroughly determined from a_1, a_2, p_1, p_2 , whereas $H(\omega)$ still needs the factor σ_2 to be known, and this is obtained from

$$\sigma_2 = \frac{KM}{l_p G}.$$

It is worthwhile to note that eliminating x from equations (2) and since s and $\dot{s}$ are causal functions, we get

$$\dot{s} + 2\alpha s + \beta \dot{s} + 2\gamma s + \delta \int_0^t s \, dt = -\sigma_2 f.$$

TABLE 3

ORDINATES OF A THEORETICAL CALIBRATION CURVE

<i>t</i> (sec)	<i>y_{nm}</i>	<i>t</i> (sec)	<i>y_{nm}</i>	<i>t</i> (sec)	<i>y_{nm}</i>	<i>t</i> (sec)	<i>y_{nm}</i>
0 0	0.0	28.0	74.937	56 0	33.042	84.0	9 239
4 0	8.529	32.0	70.812	60 0	27.985	88.0	7 581
8 0	29.540	36 0	65.020	64.0	23.546	92 0	6.204
12.0	50.125	40 0	58.411	68 0	19.969	96.0	5.064
16 0	65.033	44.0	51.585	72 0	16.400	100 0	4 124
20.0	73.440	48 0	44.937	76.0	13.595	104.0	3.352
24.0	76 269	52 0	38.712	80 0	11.226	108 0	2.720

α, β, γ , and δ are the second members of (7) and they can be obviously calculated from the first members. Thus, it is seen again that f is fully determined when σ_2 and the four equivalent constants are known.

DETERMINATION OF THE UNKNOWN S

In contrast to the method based on the Fourier transform where a great number of ordinates of the calibration curve are required, the unknowns a_1, a_2, p_1, p_2 , and K can be obtained only from five of them by means of the closed expression (8). However, it is convenient to use many more and to compensate them by the least-squares method. This can be based on the Newton-Raphson method, but since it sometimes becomes divergent, it is advisable to use it with some attenuation devices. To find suitable procedures, we tried: (a) the attenuated least-squares method suggested by Levenberg (1944), slightly modified, and (b) a method which may be called the method of attenuated steps which consists in multiplying the steps by a factor less than 1.

The procedure (a) was tested multiplying by a factor of only the diagonal terms

of the coefficient matrix, and also multiplying by a factor all the terms of the matrix. The procedure (b) was tested combining and not combining it with procedure (a). When combining with procedure (a), all the terms of the coefficient matrix were multiplied. The results obtained are shown in Table 4 where R is the step attenuation factor, Q is the factor applied to the terms of the coefficient matrix, G is given by

$$G = \sum (s - y)^2,$$

y being the ordinates taken from the calibration curve, and

$$D = \left[\left(\frac{\partial G}{\partial a_1} \right)^2 + \left(\frac{\partial G}{\partial a_2} \right)^2 + \left(\frac{\partial G}{\partial p_1} \right)^2 + \left(\frac{\partial G}{\partial p_2} \right)^2 + \left(\frac{\partial G}{\partial K} \right)^2 \right]^{1/2}.$$

TABLE 4
NEWTON-RAPHSON ITERATIONS WITH AND WITHOUT ATTENUATION

Iteration	a_1	a_2	p_1	p_2	K	G	D
1	0.628	0.06	0.140	0.005	2.50	1170.31	203427
2	0.802	0.07479	0.16135	0.005384	3.0796	32.82	206255
3	becomes divergent						
$Q = 1.50$ applied to the diagonal terms							
2	0.670006	0.066058	0.142644	0.004899	2.43921	128.92	290962
75	0.627370	0.072157	0.144305	0.0047108	2.46014	0.98	227
$Q = 1.50$ applied to all the terms							
2	0.744228	0.069861	0.154228	0.005256	2.88613	189.78	166353
9	0.628880	0.062850	0.175531	0.003949	2.50209	3×10^{-5}	9
$R = 0.1$							
2	0.802455	0.074794	0.161354	0.005384	3.07960	32.82	206255
16	0.628446	0.062836	0.175474	0.003948	2.50044	7×10^{-5}	490
$R = 0.1 \quad Q = 1.50$ applied to all the terms							
2	0.802455	0.074794	0.161354	0.005384	3.07960	32.82	206255
15	0.628398	0.062834	0.175471	0.003948	2.50029	5×10^{-5}	966
$R = 0.1 \quad Q = 1.25$ applied to all the terms							
2	0.802455	0.074794	0.161354	0.005384	3.07960	32.82	206255
10	0.628390	0.062832	0.175478	0.003948	2.50030	3×10^{-6}	919

The solution is rigorously obtained when $D = 0$, but it suffices that D be less than a reasonable tolerance τ .

Being ΔZ , the approximation to the solution, and ϵ_G , the inaccuracy of G , we may put

$$\tau < \frac{\epsilon_G}{\Delta Z}. \quad (10)$$

Since

$$\epsilon_G = 2\mu_g\sqrt{G} < 2\mu_g\delta\sqrt{N}$$

(11)

where N is the number of ordinates, δ , their probable error, and μ_g , the presumable least value of $s - y$, we may assume that $\epsilon_G = 2\mu_g\delta\sqrt{N}$.

Levenberg proposed to use a factor Q which is variable with every iteration. As it may become too big not allowing the iteration process to proceed, we used instead a constant value between 1 and 2. This factor and R were used in procedure (b) only where G increased.

The values y used in our test are given in Table 3. They were obtained from (8) putting $a_1 = 0.628319$, $a_2 = 0.0628319$, $p_1 = 0.175459$, $p_2 = 0.00394784$. These are the values corresponding to case (b) of Table 1. K was assumed to be 2.50 in order that the ordinates be similar to those ordinarily given by the calibration curve.

TABLE 5

RESULTS EXPECTED AND OBTAINED UNDER AND FREE OF CONSTRAINTS

		Constraints					
		Case 1			Case 2		
		$n_1 = 0.418879; T_1 = 15.0 \text{ sec}$			$n_1 = 0.418879; T_1 = 15.0 \text{ sec}$		
		$n_2 = 0.062832; T_2 = 100.0 \text{ sec}$			$\epsilon_1 = 0.628319; h_1 = 1.5$		
		$\epsilon_2 = 0.062832, h_2 = 1.00$					
		Obtained					
Un-knowns	Expected	s_2 case b from Table 2			y from Table 3		
		No constraints	Case 1	Case 2	No constraints	Case 1	Case 2
a_1	0.628319	0.454657	0.598277	0.628715	0.628446	0.627626	0.628218
a_2	0.062832	0.054023	0.061991	0.062832	0.062836	0.062812	0.062832
p_1	0.175459	0.236426	0.178239	0.176868	0.175474	0.175350	0.175459
p_2	0.003948	0.003150	0.003882	0.004062	0.003948	0.00394628	0.003948
K	4.81, 2.50	5.35232	4.81112	4.99997	2.50044	2.49744	2.50
ϵ_1	—	—	0.597435	—	—	0.627606	—
ϵ_2	—	—	—	0.0632287	—	—	0.062832
n_2	—	—	—	0.063993	—	—	0.062832
T_1'	15.00	12.92	14.88	14.94	15.00	15.00	—
T_2'	100.00	111.95	100.84	98.58	100.00	100.02	—
h_1'	1.50	0.94	1.42	1.49	1.50	1.50	—
h_2'	1.00	0.96	0.99	0.99	1.00	1.00	—
G	—	1.73	17.9	54.8	0.00007	0.00001	0.02
D	—	1.04	0.35	2.07	490	0.05	—

To choose a value for τ , we assumed $\Delta Z = 5 \times 10^{-7}$, $\mu_g = 0.1$, and $\delta = 0.0005$. Consequently, we adopted $\tau = 200 \sqrt{N}$, and starting from certain approximate values, we expected to obtain the above values for the unknowns.

From Table 4, it is seen that using the Newton-Raphson method without attenuations, it may become impossible to arrive at a solution; however, Levenberg's modified method and the step attenuation method, can lead to it. It is also seen that the solution differs very little from what was expected and that convergency is better when not only the diagonal terms of the coefficient matrix are multiplied by Q , but all of them.

DETERMINATION UNDER CONSTRAINTS

The long-period WWSSN instruments allow n_2 , ϵ_2 , and n_1 to be determined quite well separately or to be left adjusted to some standard values of these partial constants; in fact, this is done by the stations from time to time. Therefore, one might wish to obtain the unknowns from the calibration curve under the constraints that some of the values of the partial constants are those assigned to them by the station. Taking this into account, we may consider the following cases

Case	Known Constants		
1	n_1	n_2	ϵ_2
2	n_1	ϵ_1	
3	n_2	ϵ_2	
4	n_1	n_2	

In all these cases, the constraints are defined by equation (7) from which a dependence of some unknowns from the remaining can be obtained.

In case 1, we may calculate a_2 , p_2 , and ϵ_2 from a_1 and p_1 as follows

$$p_2 = \frac{n_1^2 n_2^2}{p_1}; \quad a_2 = \frac{\epsilon_2(n_2^2 - n_1^2) + a_1(p_2 - n_2^2)}{n_2^2 - p_1}; \quad (12)$$

$$\epsilon_1 = \frac{a_1(p_2 - p_1) + \epsilon_2(p_1 - n_1^2)}{n_2^2 - p_1}$$

and a_1 , p_1 , and K should be obtained from the calibration curve. In case 2 we have

$$n_2^2 = \frac{p_1 p_2}{n_1^2}; \quad a_1 = \frac{\epsilon_1(n_1^2 - n_2^2) + a_2(p_1 - n_1^2)}{n_1^2 - p_2}; \quad \epsilon_2 = \frac{a_2(p_1 - p_2) + \epsilon_1(p_2 - n_2^2)}{n_1^2 - p_2}$$

and a_2 , p_1 , p_2 , and K are obtained from the calibration curve. The same is valid for case 3; one must only interchange the subscripts 1 and 2.

Case 1 finds application particularly when one is interested in knowing ϵ_1 whose determination is generally not easy. Case 2 occurs when trouble affects the galvanometer changing its period and damping, a fact that happens frequently due to some fungus developed in moisture existing in its container. Finally, in case 4, we put for p_2 the expression written in (12) and the unknowns to be obtained are a_1 , a_2 , p_1 , and K .

Table 5 compares results obtained under different constraints and others free of them corresponding to the same readings. In it we also give the following additional values

$$T_1' = \frac{2\pi}{\sqrt{p_1}}; \quad T_2' = \frac{2\pi}{\sqrt{p_2}}; \quad h_1' = \frac{a_1}{\sqrt{p_1}}; \quad h_2' = \frac{a_2}{\sqrt{p_2}}$$

which generally should differ little from T_1 , T_2 , h_1 , and h_2 , respectively.

The accuracy of the results is highly dependent on the accuracy to which the readings, y , on the calibration curve are made. This is very low before its maximum; it is, therefore, convenient to use the ordinates from this point onwards.

CONCLUSIONS

From the considerations made, the following conclusions have resulted.

1. The known fact that the release and the step current test produce the same calibration curve is theoretically explained.
2. Its explicit expression can easily be obtained by the method of the characteristic equation.
3. The seismograph frequency response is determined by the equivalent constants when a factor K , the reduced motor constant and the calibration current are known, and there is no need to know explicitly the values of the partial instrumental constants.
4. K and the equivalent constants can be obtained from the calibration curve either freely or under the constraint that some of the partial constants be the ones given by the recording station.
5. Determinations under constraint are to be made when one wishes to obtain the seismometer damping coefficient as well as the equivalent constants when the galvanometer period and damping have been affected by some trouble.
6. The Newton-Raphson iteration method can be used to obtain the unknowns, but since it fails sometimes, becoming divergent, it is convenient to improve it using either the step attenuation procedure or Levenberg's attenuated least-squares procedure modified by putting a reasonable constant instead of his variable factor and applying it to all the terms of the coefficient matrix.

ACKNOWLEDGMENT

The authors are indebted to the Centro Regional de Sismología para America del Sur (CERESIS) for making the publication of the present paper possible.

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Manuscript received October 14, 1977