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Chern–Simons action in noncommutative space

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Abstract

We derive the noncommutative Chern–Simons action induced by Dirac fermions coupled to a background gauge field, for the fundamental, antifundamental, and the adjoint representation. We discuss properties of the noncommutative Chern–Simons action showing in particular that the Seiberg–Witten formula maps it into the standard commutative Chern–Simons action. © 2001 Elsevier Science B.V. All rights reserved.

1. Introduction

Recent results in noncommutative geometry and string theory [1–3], revealed the interest on its own right for studying different field theories like Yang–Mills, $\lambda\phi^4$, QED, Chern–Simons, Wess–Zumino theories and two-dimensional models, in noncommutative space [4–23]. In this respect, it is the purpose of this Letter to analyse different aspects of the noncommutative Chern–Simons (CS) action. First, we discuss how the parity anomaly in a $2 + 1$ massive fermion model induces a Chern–Simons term (as originally observed in [11] for the massless case). Then, we discuss relevant properties of the non-commutative CS action, its relation with the chiral Wess–Zumino–Witten model and its dependence on the noncommutative parameter $\theta_{\mu\nu}$.

Let us start by establishing our conventions. The $*$ -product for fields is defined by

$$(\hat{f} * \hat{g})(x) = e^{\frac{i}{2}\theta_{\mu\nu}\partial_{\xi\mu}\partial_{\zeta\nu}} \hat{f}(x + \xi)\hat{g}(x + \zeta)|_{\xi=\zeta=0} \quad (1)$$

and the Moyal brackets as

$$\{\hat{f}(x), \hat{g}(x)\} = \hat{f}(x) * \hat{g}(x) - \hat{g}(x) * \hat{f}(x). \quad (2)$$

We indicate with a hat functions which have to be multiplied using the $*$ -product. The $U(N)$ gauge-group elements are defined by

$$\hat{g}(x) = e_*^{i\hat{\alpha}(x)} = 1 + i\hat{\alpha}(x) - \frac{1}{2}\hat{\alpha}(x) * \hat{\alpha}(x) + \dots, \quad (3)$$

where $\hat{\alpha}(x)$ is a Lie-algebra valued function of space-time. Gauge fields in the Lie algebras of $U(N)$ transform according to

$$\begin{aligned} \hat{A}_\mu(x) &\rightarrow \hat{g}(x) * \hat{A}_\mu(x) * \hat{g}^{-1}(x) \\ &\quad - \frac{i}{e} \hat{g}(x) * \partial_\mu \hat{g}^{-1}(x) \end{aligned} \quad (4)$$

with the field strength defined as

$$\hat{F}_{\mu\nu} = \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu + ie\{\hat{A}_\mu, \hat{A}_\nu\}. \quad (5)$$

When acting over fermion fields $\hat{\psi}$, even in the $U(1)$ case, there are three possible representations of the

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gauge group action

$$\hat{\psi}(x) \rightarrow \begin{cases} \hat{g}(x) * \hat{\psi}(x), & \text{fundamental representation "f",} \\ \hat{\psi}(x) * \hat{g}^{-1}(x), & \text{anti-fundamental representation "\bar{f}",} \\ \hat{g}(x) * \hat{\psi}(x) * \hat{g}^{-1}(x), & \text{adjoint representation "ad".} \end{cases} \quad (6)$$

Accordingly, the covariant derivative acting on ψ is defined as

$$\widehat{D}_\mu \hat{\psi}(\tau, x) = \begin{cases} \partial_\mu \hat{\psi} + ie \hat{A}_\mu * \hat{\psi}, & \text{"f",} \\ \partial_\mu \hat{\psi} - ie \hat{\psi} * \hat{A}_\mu, & \text{"\bar{f}",} \\ \partial_\mu \hat{\psi} + ie \{ \hat{A}_\mu, \hat{\psi} \}, & \text{"ad".} \end{cases} \quad (7)$$

We write the action for massive fermions, coupled to a gauge field, in $2 + 1$ non-commutative space as

$$S(\hat{A}; m) = \int d^3x \hat{\bar{\psi}}(x) * (i\widehat{\mathcal{D}} - m) \hat{\psi}(x), \quad (8)$$

and define the effective action $\Gamma(\hat{A}; m)$ through

$$e^{i\Gamma(\hat{A}; m)} = Z(\hat{A}; m) = \int \mathcal{D}\hat{\psi} \mathcal{D}\hat{\bar{\psi}} e^{iS(\hat{A}; m)}. \quad (9)$$

2. Induced Chern–Simons term

Before studying some specific properties of non-commutative Chern–Simons action, let us describe how a parity violating Chern–Simons term is induced by fluctuations of massive non-commutative fermions fields, exactly as it happens in the commutative case [20]. We shall just concentrate in the parity odd part of the effective action Γ_{odd} , thus disregarding parity conserving contributions.

2.1. Fundamental and anti-fundamental representations

The calculation of the effective action for fermions in the fundamental and the anti-fundamental representations gives the same answer. We shall describe first the case of the fundamental representation. As in the original calculation in [20], one obtains the contribution to $\Gamma_{\text{odd}}(\hat{A}; m)$ from the vacuum polarization and

the triangle graphs

$$\begin{aligned} i\Gamma_{\text{odd}}[\hat{A}; m] &= \left(\frac{1}{2} \text{Tr} \int \frac{d^3p}{(2\pi)^3} \hat{A}_\mu(p) \Pi^{\mu\nu}(p; m) \hat{A}_\nu(-p) \right. \\ &\quad \left. + \frac{1}{3} \text{Tr} \int \frac{d^3p}{(2\pi)^3} \frac{d^3q}{(2\pi)^3} \Gamma^{\mu\nu\rho}(p, q; m) \right. \\ &\quad \left. \times \hat{A}_\mu(p) \hat{A}_\nu(q) \hat{A}_\rho(-p-q) \right) \Big|_{\text{odd}}. \end{aligned} \quad (10)$$

Here Tr represents the trace over the $U(N)$ algebra generators, with

$$\begin{aligned} \Pi^{\mu\nu}(p; m) &= -e^2 \int \frac{d^3k}{(2\pi)^3} \text{tr} \left[\gamma^\mu \frac{\not{k} - m}{k^2 - m^2} \gamma^\nu \frac{\not{k} + \not{p} - m}{(k+p)^2 - m^2} \right], \end{aligned} \quad (11)$$

$$\begin{aligned} \Gamma^{\mu\nu\rho}(p, q; m) &= e^3 \exp\left(-\frac{i}{2} p_\lambda \theta^{\lambda\delta} q_\delta\right) \\ &\quad \times \int \frac{d^3k}{(2\pi)^3} \text{tr} \left[\gamma^\mu \frac{(\not{k} - m)}{k^2 - m^2} \gamma^\nu \frac{(\not{k} - \not{q} - m)}{(k-q)^2 - m^2} \right. \\ &\quad \left. \times \gamma^\rho \frac{(\not{k} + \not{p} - m)}{(k+p)^2 - m^2} \right]. \end{aligned} \quad (12)$$

As first observed in [11] for massless fermions, there are no nonplanar contributions to the parity odd sector of the effective action, the only modification arising from noncommutativity is the θ -dependent phase factor in $\Gamma^{\mu\nu\rho}$, associated to external legs in the cubic term, which is nothing but the star product in configuration space. The result for $\Gamma_{\text{odd}}(\hat{A}; m)$ is analogous to the commutative one except that the star $*$ -product replaces the ordinary product.

Regularization of the divergent integrals (11) and (12) can be achieved by introducing in the original action (8), bosonic-spinor Pauli–Villars fields with mass M . These fields give rise to additional diagrams, identical to those of Eq. (10), except that the regulating mass M appears in place of the physical mass m . Since we are interested in the parity violating part of the effective action, we keep only the parity-odd terms in (11) and (12) (and in the corresponding regulator field graphs). To leading order in ∂/m , the gauge-invariant parity violating part of the effective action is, for the

fundamental representation, given by

$$\begin{aligned}\Gamma_{\text{odd}}^f(\hat{A}, m) &= \frac{1}{2} \left(\frac{m}{|m|} + \frac{M}{|M|} \right) \hat{S}_{\text{CS}}(\hat{A}) + O(\partial^2/m^2) \\ &= \pm \hat{S}_{\text{CS}}(\hat{A}) + O(\partial^2/m^2)\end{aligned}\quad (13)$$

with

$$\begin{aligned}\hat{S}_{\text{CS}}(\hat{A}) &= \frac{e^2}{4\pi} \int d^3x \epsilon^{\mu\nu\rho} \\ &\quad \times \text{tr} \left(\hat{A}_\mu * \partial_\nu \hat{A}_\rho + \frac{2ie}{3} \hat{A}_\mu * \hat{A}_\nu * \hat{A}_\rho \right).\end{aligned}\quad (14)$$

As it is well known, the relative sign of the fermion and regulator contributions depends on the choice of the Pauli–Villars regulating Lagrangian (of course the divergent parts should cancel out independently of this choice). In the first line of (13) we have made a choice such that the two contributions add to give the known Chern–Simons result of the second line. Note that even in the Abelian case, the Chern–Simons action contains a cubic term (analogous to that arising in the ordinary non-Abelian case).

As expected, the effective action is gauge invariant even under large gauge transformations. This is due to the fact that we have taken into account both parity-violating sources: that originated in the (parity non-invariant) fermion mass term and that related to the regularization prescription (which requires the introduction of the mass M).

As advanced, the parity odd part of the effective action for fermions in the anti-fundamental representation gives the same answer. There is a change of sign $e \rightarrow -e$ on each vertex, compensated by a change in the momenta dependence of propagators due to the different ordering of fields in the f and \bar{f} covariant derivatives (see (7)).

2.2. Adjoint representation

The diagrams contributing to $\Pi^{\mu\nu}$ in the adjoint representation are shown in Fig. 1.

Planar diagrams 1a and 1b coincide with those arising in the fundamental and the anti-fundamental representation, thus giving, each one, the previously computed answer (13). Concerning the non-planar

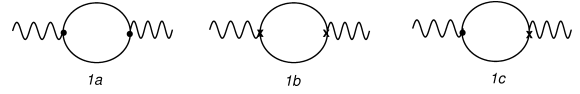


Fig. 1. Diagrams contributing to $\Pi^{\mu\nu}$ for fermions in the adjoint representation. The dotted vertex coincides with the coupling of A_μ with fermions in the fundamental representation, the cross with that for fermions in the anti-fundamental.

diagram 1c, the resulting contribution is given by

$$\begin{aligned}\Pi_{1c}^{\mu\nu}(p; m) &= e^2 \int \frac{d^3k}{(2\pi)^3} \exp(-ip_\lambda \theta^{\lambda\delta} k_\delta) \\ &\quad \times \text{tr} \left[\gamma^\mu \frac{\not{k} - \not{p} - m}{(k-p)^2 - m^2} \gamma^\nu \frac{\not{k} - m}{k^2 - m^2} \right].\end{aligned}\quad (15)$$

The parity odd part of the above expression is

$$\begin{aligned}\Pi_{1c}^{\mu\nu}(p; m)|_{\text{odd}} &= -2e^2 m \epsilon^{\mu\nu\rho} i p_\rho \\ &\quad \times \int \frac{d^3k}{(2\pi)^3} \frac{\exp(-ip_\lambda \theta^{\lambda\delta} k_\delta)}{(k^2 - m^2)((k-p)^2 - m^2)} \\ &= -2 \frac{m}{|m|} e^2 \epsilon^{\mu\nu\rho} i p_\rho \\ &\quad \times \int \frac{d^3q}{(2\pi)^3} \frac{\exp(-i|m|p_\lambda \theta^{\lambda\delta} q_\delta)}{(q^2 - 1)((q - \frac{p}{|m|})^2 - 1)},\end{aligned}\quad (16)$$

where we have written $k_\mu = |m|q_\mu$. As in the previous section, one should add the regulator contribution.

We are interested in the leading term in a derivative expansion of the effective action. In the ordinary (commutative) case, this amounts to make an expansion in powers of the unique available dimensionless variable, p/m . In the noncommutative case, where one has, apart from the fermion mass, the dimensionfull parameter θ , one can construct a second independent dimensionless variable, $m|p\theta|$. Let us first expand (16) to first order in p/m ,

$$\begin{aligned}\Pi_{1c}^{\mu\nu}(p; m)|_{\text{odd}} &= -\frac{m}{|m|} 2ie^2 \epsilon^{\mu\nu\rho} p_\rho \int \frac{d^3q}{(2\pi)^3} \frac{\exp(-i|m|p_\lambda \theta^{\lambda\delta} q_\delta)}{(q^2 - 1)^2} \\ &\quad - \frac{M}{|M|} 2ie^2 \epsilon^{\mu\nu\rho} p_\rho \\ &\quad \times \int \frac{d^3q}{(2\pi)^3} \frac{\exp(-i|M|p_\lambda \theta^{\lambda\delta} q_\delta)}{(q^2 - 1)^2}\end{aligned}\quad (17)$$

here, the regulator contribution has been explicitly written. Concerning the expansion in powers of the second dimensionless parameter $m|p\theta|$, let us note that, since m is finite, first order in $m|p\theta|$ should be kept in the first term of (17). This gives the same contribution to the effective action as the 1a and 1b graphs. For the second term in (17), the $M \rightarrow \infty$ limit must be taken, then, the oscillating factor makes the integral vanish [11]. Finally, one gets

$$\Pi_{1c}^{\mu\nu}(p; m)|_{\text{odd}} = -i \frac{m}{|m|} \frac{e^2}{4\pi} \epsilon^{\mu\nu\rho} i p_\rho, \quad (18)$$

so that the complete quadratic $\Pi_{\mu\nu}$ for the adjoint representation is then given by

$$\begin{aligned} \Pi_{\text{adj}}^{\mu\nu}(p; m) &= \Pi_{1a}^{\mu\nu}(p; m) + \Pi_{1b}^{\mu\nu}(p; m) \\ &\quad + 2\Pi_{1c}^{\mu\nu}(p; m) \\ &= \frac{e^2}{2\pi} \epsilon_{\mu\nu\rho} p_\rho \frac{M}{|M|}. \end{aligned} \quad (19)$$

Note that the whole contribution to $\Pi^{\mu\nu}$ in the adjoint comes from the regulating fields. This accounts for the quadratic part of the CS induced action. Concerning the cubic term, it can be either explicitly computed or adjusted so as to achieve gauge-invariance. In any case, the result for the parity violating effective action for fermions in the adjoint is, to leading order in ∂ ,

$$\Gamma_{\text{odd}}^{\text{ad}}(\hat{A}, m) = \pm \hat{S}_{\text{CS}}(\hat{A}) + O(\partial^2). \quad (20)$$

As before, the result is gauge invariant even under large gauge transformations.

It should be stressed that (20) gives a non-trivial effective action even in the $\theta \rightarrow 0$ limit, in which fermions in the adjoint decouple from the gauge field. As observed in other cases [5,6,13,18], this is due to the fact that this limit does not commute with that of the regulator $M \rightarrow \infty$.

3. The connection between noncommutative CS and chiral WZW theories

As it is well-known, the (ordinary) CS theory can be related with the chiral WZW model following different approaches [24–26]. Here, we shall discuss how such a connection can be established in the noncommutative case.

Consider the action

$$\hat{S}_{\text{CS}}[\hat{A}_0, \hat{A}_i] = \frac{e^2}{4\pi} \text{Tr} \int_{\mathcal{M}} d^3x \epsilon_{ij} (\hat{A}_0 * \hat{F}_{ij} + \hat{A}_i * \hat{A}_j), \quad (21)$$

which differs from the CS action (14) by a surface term. Of course, when \mathcal{M} has no boundary, such surface term is irrelevant. However, in what follows we choose as manifold $\mathcal{M} = \Sigma \times R$ with Σ a two-dimensional manifold. We shall take Eq. (21) as the starting point for quantization of the 2 + 1 theory and follow the steps described in [25,26] in their original derivation of the connection.

Expression (21) can be rewritten as

$$\begin{aligned} \hat{S}_{\text{CS}}[\hat{A}_0, \hat{A}_i] &= \frac{e^2}{4\pi} \text{Tr} \int_{\mathcal{M}} d^3x \epsilon_{ij} (\hat{A}_0 \hat{F}_{ij} + \hat{A}_i * \hat{A}_j) \\ &\quad + \frac{e^2}{4\pi} \text{Tr} \int_{\partial\mathcal{M}} dS_\mu \Lambda^\mu \end{aligned} \quad (22)$$

with

$$\begin{aligned} \Lambda^\mu &= \epsilon_{ij} \sum_{n=1}^{\infty} \frac{1}{n!} \left(\frac{i}{2}\right)^n \theta^{\mu\nu_1} \theta^{\mu\nu_2} \dots \theta^{\mu\nu_n} \partial_{\mu_2} \dots \partial_{\mu_n} \\ &\quad \times \hat{A}_0 \partial_{\nu_1} \partial_{\nu_2} \dots \partial_{\nu_n} \hat{F}_{ij}. \end{aligned} \quad (23)$$

Using action (22), the partition function for the non-commutative CS theory takes the form

$$\begin{aligned} Z &= \int \mathcal{D}\hat{A}_i \mathcal{D}\hat{A}_0 \\ &\quad \times \exp \left(\frac{i\kappa e^2}{4\pi} \text{Tr} \int_{\mathcal{M}} d^3x \epsilon_{ij} (\hat{A}_0 \hat{F}_{ij} + \hat{A}_i * \hat{A}_j) \right. \\ &\quad \left. + \frac{i\kappa e^2}{4\pi} \text{Tr} \int_{\partial\mathcal{M}} dS_\mu \Lambda^\mu \right), \end{aligned} \quad (24)$$

where κ is an integer. For interior points of \mathcal{M} , A_0 acts as a Lagrange multiplier enforcing flatness of the spatial components of the connection

$$\hat{F}_{ij}(x) = 0, \quad \forall x \in \mathcal{M} - \partial\mathcal{M}. \quad (25)$$

By continuity, \hat{F}_{ij} must also vanishes on the boundary. The partition function takes then form

$$Z = \int \mathcal{D}\hat{A}_i \delta(\epsilon_{ij} \hat{F}_{ij})$$

$$\times \exp \left(\frac{i\kappa e^2}{4\pi} \text{Tr} \int_{\mathcal{M}} d^3x \epsilon_{ij} \dot{A}_i * \hat{A}_j \right). \quad (26)$$

Let us discuss the case where Σ is the disk. Then the solution of the flatness condition (25) is $\hat{A}_i = -\frac{i}{e} \hat{g}^{-1} * \partial_i \hat{g}$, and one has reinserting it in (26)

$$Z = \int \mathcal{D}\hat{g} \exp(i\kappa \hat{S}_{\text{CWZW}}[\hat{g}]), \quad (27)$$

where $\hat{S}_{\text{CWZW}}[\hat{g}]$ is the noncommutative, chiral WZW action

$$\begin{aligned} \hat{S}_{\text{CWZW}}[\hat{g}] &= -\frac{1}{4\pi} \text{Tr} \int_{\partial\mathcal{M}} d^2x (\hat{g}^{-1} * \partial_t \hat{g}) * (\hat{g}^{-1} * \partial_\varphi \hat{g}) \\ &\quad - \frac{1}{4\pi} \text{Tr} \int_{\mathcal{M}} d^3x \epsilon_{ij} (\hat{g}^{-1} * \partial_i \hat{g}) * (\hat{g}^{-1} * \partial_j \hat{g}) \\ &\quad * (\hat{g}^{-1} * \partial_j \hat{g}), \end{aligned} \quad (28)$$

here φ is a tangential coordinate which parametrizes the boundary of \mathcal{M}_2 .

With this result in mind and taking into account the connection between commutative and noncommutative WZW models established in [18] through a Seiberg–Witten map, one can advance an analogous connection for the CS theories. The situation can be visualized in the following scheme

$$\begin{array}{ccc} \text{CWZW}[\hat{g}] & \longleftrightarrow & \int d^3x \left(\hat{A} d\hat{A} + \frac{2i}{3} \hat{A}^3 \right) \\ \updownarrow & & \updownarrow? \\ \text{CWZW}[g] & \longleftrightarrow & \int d^3x \left(A dA + \frac{2i}{3} A^3 \right). \end{array} \quad (29)$$

The next section is devoted to the study of this issue.

4. The Seiberg–Witten map

A correspondence between commutative and noncommutative gauge field theories can be defined by the map [3]

$$\begin{aligned} \delta \hat{A}^\mu &= \delta \theta^{\rho\sigma} \frac{\partial}{\partial \theta^{\rho\sigma}} \hat{A}_\mu(\theta) \\ &= -\frac{1}{4} \delta \theta^{\rho\sigma} \{ \hat{A}_\rho, \partial_\sigma \hat{A}_\mu + \hat{F}_{\sigma\mu} \}_+, \end{aligned}$$

$$\begin{aligned} \delta \hat{F}_{\mu\nu}(\theta) &= \delta \theta^{\rho\sigma} \frac{\partial}{\partial \theta^{\rho\sigma}} \hat{F}_{\mu\nu}(\theta) \\ &= \frac{1}{4} \delta \theta^{\rho\sigma} (2 \{ \hat{F}_{\mu\rho}, \hat{F}_{\nu\sigma} \}_+ \\ &\quad - \{ \hat{A}_\rho, \hat{D}_\sigma \hat{F}_{\mu\nu} + \partial_\sigma \hat{F}_{\mu\nu} \}_+). \end{aligned} \quad (30)$$

For the case of noncommutative Yang–Mills action, this map leads to a complicated non-polynomial commutative action. Remarkably, in the Chern–Simons case, the action remains (up to surface terms) invariant under the map (30). Let us write the noncommutative Chern–Simons action in the form (14)

$$\begin{aligned} \hat{S}_{\text{CS}}(\hat{A}) &= \frac{e^2}{4\pi} \int_{\mathcal{M}} d^3x \epsilon^{\mu\nu\rho} \\ &\quad \times \left(\hat{A}_\mu * \partial_\nu \hat{A}_\rho + \frac{2ie}{3} \hat{A}_\mu * \hat{A}_\nu * \hat{A}_\rho \right), \end{aligned} \quad (31)$$

where we choose for \mathcal{M} either R^3 or $\Sigma \times R$ with Σ a manifold without boundary. Action (31) can be rewritten in the form

$$\hat{S}_{\text{CS}}(\hat{A}) = \frac{e^2}{4\pi} \int_{\mathcal{M}} d^3x \epsilon_{ij} (\hat{A}_0 \hat{F}_{ij} + \dot{\hat{A}}_i \hat{A}_j). \quad (32)$$

In order to investigate the variation of this action under Seiberg–Witten map, let us differentiate it with respect to $\theta_{\mu\nu}$

$$\begin{aligned} \frac{\partial \hat{S}_{\text{CS}}(\hat{A})}{\partial \theta_{\mu\nu}} &= \frac{e^2}{4\pi} \int_{\mathcal{M}} d^3x \epsilon_{ij} \frac{\partial}{\partial \theta_{\mu\nu}} (\hat{A}_0 \hat{F}_{ij} + \dot{\hat{A}}_i \hat{A}_j) \\ &= \frac{e^2}{4\pi} \int_{\mathcal{M}} d^3x \epsilon_{ij} \\ &\quad \times \left(\frac{\partial \hat{A}_0}{\partial \theta_{\mu\nu}} \hat{F}_{ij} + \hat{A}_0 \frac{\partial \hat{F}_{ij}}{\partial \theta_{\mu\nu}} + 2 \frac{\partial \hat{A}_j}{\partial \theta_{\mu\nu}} \dot{\hat{A}}_i \right). \end{aligned} \quad (33)$$

Now, we can use (30) in order to rewrite the θ -derivatives. Keeping only the terms which are antisymmetric with respect to the indices μ, ν and i, j , we get

$$\frac{\partial \hat{S}_{\text{CS}}(\hat{A})}{\partial \theta_{\mu\nu}} = 0 \implies \hat{S}_{\text{CS}}(\hat{A}) = S_{\text{CS}}(A). \quad (34)$$

Here $S_{\text{CS}}(A)$ is the ordinary (commutative) CS action. It is interesting to note that in the $U(1)$ case the SW

map cancels out the cubic term which is present in $\hat{S}_{\text{CS}}(\hat{A})$.

In summary, we see that the SW transformation (30) maps the noncommutative Chern–Simons action into the commutative one.

5. Conclusions

We have computed the effective action for fermions in noncommutative space, for different representations, showing that a gauge invariant answer (even for large gauge transformations) is obtained when regulator contributions are taken in account. In particular, for the adjoint representation, the non-trivial gauge invariant result (20) is completely due to the regulator fields, showing that the commutative $\theta \rightarrow 0$ limit does not commute with the $M \rightarrow \infty$ limit.

We have shown that the noncommutative Chern–Simons action can be related to the chiral noncommutative WZW model in the usual way. It is important to note that for deriving this relation we needed to define the Chern–Simons theory from action (21), which shows A_0 as a Lagrange multiplier enforcing the flatness constraint (24). Finally, we showed that the Chern–Simons action is mapped into the standard (commutative) action under the Seiberg–Witten map (30).

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