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Order-α_s^2 corrections to one-particle inclusive processes in DIS ☆

A. Daleo [a], C.A. García Canal [b], R. Sassot [c]

[a] *Laboratorio de Física Teórica, Departamento de Física, Facultad de Ciencias Exactas, Universidad Nacional de La Plata, C.C. 67, 1900 La Plata, Argentina*

[b] *Laboratorio de Física Teórica, Departamento de Física, Universidad Nacional de La Plata, C.C. 67, 1900 La Plata, Argentina*

[c] *Departamento de Física, Universidad de Buenos Aires, Ciudad Universitaria, Pab.1 (1428) Buenos Aires, Argentina*

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Abstract

We analyze the order-α_s^2 QCD corrections to semi-inclusive deep inelastic scattering and present results for processes initiated by a gluon. We focus in the most singular pieces of these corrections in order to obtain the hitherto unknown NLO evolution kernels relevant for the non-homogeneous QCD scale dependence of these cross sections, and to check explicitly factorization at this order. In so doing we discuss the prescription of overlapping singularities in more than one variable.

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1. Introduction

In recent years there has been an increasing wealth of interest in semi-inclusive deep inelastic scattering, driven both by crucial breakthroughs in the QCD description of these processes [1–4] and also by an incipient availability of data encompassing polarized, unpolarized, leading baryon, and diffractive deep inelastic phenomena [5].

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E-mail address: daleo@fisica.unlp.edu.ar (A. Daleo).

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From the perturbative QCD standpoint, semi-inclusive deep inelastic scattering (SIDIS) brings before theorists two novel and interesting features. On the one hand, *fracture functions* which, in addition to structure and fragmentation functions, are required for the correct description of hadrons produced in the forward direction and for the factorization of collinear singularities. On the other, *non-homogeneous* Altarelli–Parisi evolution equations, which highlights the interplay of the three intervening parton densities in the scale dependence of these processes [1].

Although the main features related to SIDIS, and specifically to fracture functions, have been studied at the leading order (LO) in QCD (order-α_s in the cross section) [2,4], up to now no computations had been done up to next to leading order (NLO) accuracy, as it is standard in the inclusive case. In particular, there were neither explicit checks of factorization at order-α_s^2 nor indications of how relevant the non-homogeneous evolution might be at NLO.

In LO, non-standard evolution effects although non-negligible, are restricted to a relatively small kinematic region, associated to the fragmentation configurations allowed at that order [6]. This suggests to neglect these effects in many phenomenological analyses of polarized SIDIS [7,8], leading baryon production [9] and diffractive DIS [6], provided some cuts on data are introduced. In NLO the above mentioned kinematical restrictions are no longer present, which in principle may lead to important corrections. In any case, their phenomenological relevance needs to be assessed.

From a theoretical point of view, the computation of the SIDIS NLO corrections, and specifically the explicit check of factorization of collinear singularities involve also some subtleties which need close attention. At variance with the totally inclusive case [10], where after a convenient integration over final states the remaining singularities may be written as distributions in only one variable times a regular function, in the one particle inclusive case at order α_s^2, it is necessary to keep additional variables unintegrated. Consequently one must deal with entangled singularities in more than one variable, corresponding, for example, to three particles becoming collinear simultaneously. As usual for semi-inclusive processes, in order to check factorization on has to keep track of the kinematical origin or configuration which gives rise to the singularity, which represents a non-trivial additional complication and requires a detailed analysis of the singularity structure characteristic of the process.

In this paper we address the above mentioned issues restricting ourselves to processes where the initial state parton is a gluon. This allows to analyze and answer the main issues involved skipping for the moment, and for the sake of clarity, the formidable singularity structure associated with virtual corrections to quark initiated processes, which will be addressed in a forthcoming publication. In doing this, we develop suitable prescription rules for dealing with the SIDIS singularity structure.

As result of our approach we obtain the hitherto unknown NLO non-homogeneous kernels for fracture functions and discuss their distinctive features such as their non-factorizable dependence upon two variables. We also verify explicitly the factorization of collinear singularities up to order α_s^2, and give the expression for the renormalized fracture function in terms of the bare one. In order to asses the relevance of NLO corrections we compare the effects of the new evolution kernels with the already known LO corrections.

The outline of the paper is the following: in the next section we introduce the relevant kinematics and conventions used and we extend the $\mathcal{O}(\alpha_s)$ results for the SIDIS cross sections as required for the later factorization of collinear singularities at $\mathcal{O}(\alpha_s^2)$. In the third section we discuss the computation of amplitudes and phase space integration of the $\mathcal{O}(\alpha_s^2)$ processes. There, we introduce a suitable parameterization for the phase space of the three final state particles, and extend some of the results given in Refs. [11–13] for the angular integration of the corresponding amplitudes. In the fourth section we analyze the SIDIS singularity structure at order α_s^2 and give details about the prescription recipes required for dealing with it. In the fifth we address the issue of factorization and technicalities associated with the convolution of distributions in many variables, present the novel NLO kernels and discuss the evolution of fracture functions. In the last section we present our conclusions.

2. Kinematics and $\mathcal{O}(\alpha_s)$ results

We begin considering the one-particle inclusive process in which a lepton of momentum l scatters off a nucleon of momentum P,

$$l(l) + P(P) \longrightarrow l'(l') + h(P_h) + X, \tag{1}$$

and where in addition to the emerging lepton of momentum l', a hadron h of momentum P_h is tagged in the final state. X stands for all the unobserved particles. For simplicity we consider only the exchange of one photon of momentum $q = l' - l$. In order to characterize the hadronic final state, in addition to the usual DIS variables

$$Q^2 = -q^2 = -(l' - l)^2, \qquad x_B = \frac{Q^2}{2P\cdot q}, \qquad y = \frac{P\cdot q}{P\cdot l}, \qquad S_H = (P + l)^2, \tag{2}$$

we introduce energy and angular variables

$$v_h = \frac{E_h}{E_0(1 - x_B)}, \qquad w_h = \frac{1 - \cos\theta_h}{2}, \tag{3}$$

where E_h and E_0 are the energies of the produced hadron and of the incoming proton in the $\vec{P} + \vec{q} = 0$ frame, respectively. θ_h is the angle between the momenta of the hadron and the virtual photon in the same frame.

The corresponding cross section, differential in the final state lepton and hadron variables, can be written as [2]

$$\begin{aligned}
&\frac{d\sigma}{dx_B\, dy\, dv_h\, dw_h} \\
&= \sum_{i,j=q,\bar{q},g} \int_{x_B}^{1} \frac{du}{u} \int_{v_h}^{1} \frac{dv_j}{v_j} \int_0^1 dw\, f_{i/P}\left(\frac{x_B}{u}\right) D_{h/j}\left(\frac{v_h}{v_j}\right) \frac{d\hat{\sigma}_{ij}}{dx_B\, dy\, dv_j\, dw_j} \delta(w_h - w_j) \\
&\quad + \sum_i \int_{\frac{x_B}{1-(1-x_B)v_h}}^{1} \frac{du}{u} M_{i,h/P}\left(\frac{x_B}{u}, (1 - x_B)v_h\right)(1 - x_B) \frac{d\hat{\sigma}_i}{dx_B\, dy} \delta(1 - w_h),
\end{aligned} \tag{4}$$

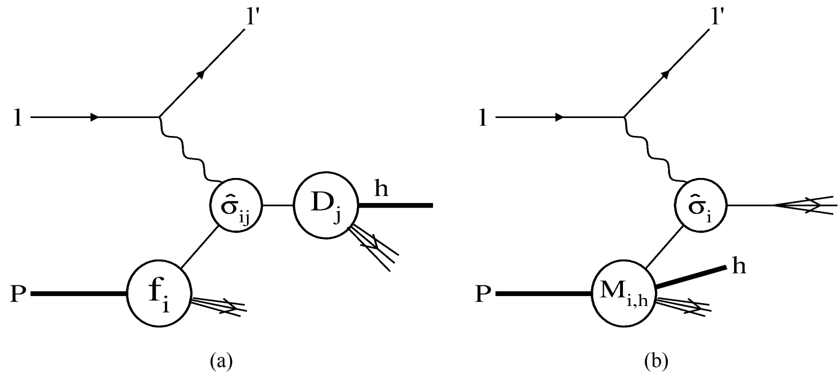

Fig. 1. (a) Current and (b) target fragmentation processes.

where the sum is over all parton species.

In the first term of the r.h.s. of Eq. (4), $d\hat{\sigma}_{ij}$ represents the partonic cross section for the process $l + i \to l' + j + X$, whereas $f_{i/P}$ and $D_{h/j}$ are the usual partonic densities and fragmentation functions. The variable u is related to the fraction of momentum of the incoming parton ξ by $\xi = x_B/u$, while v_j and w_j are the partonic analogs of v_h and w_h. This term, represented in Fig. 1(a), describes a 'current fragmentation' process in which a final state parton j fragments into the final state hadron h, which is produced in the same direction than j.

In the second term of the r.h.s. of Eq. (4), $d\hat{\sigma}_i$ stands for the inclusive partonic cross section initiated by parton i and is convoluted with the fracture functions $M_{i,h/P}$. This term, shown in Fig. 1(b), corresponds to a 'target fragmentation' process, where the initial state nucleon fragments into the final state hadron and a parton, i, which participates in the hard scattering. In the last case, the hadron is produced in the direction of the incoming nucleon.

The above mentioned partonic cross sections can be calculated order by order in perturbation theory and are related to the parton–photon squared matrix elements $\overline{H}_{\mu\nu}^{(n)}(i,j)$ and $\overline{H}_{\mu\nu}^{(n)}(i)$ for the $i + \gamma \to j + X$ and $i + \gamma \to X$ processes, respectively:

$$\frac{d\hat{\sigma}_{ij}}{dx_B\,dy} = \frac{\alpha_{em}^2}{x_B S_H}\left(Y_M\left(-g^{\mu\nu}\right) + Y_L \frac{4x_B^2}{Q^2} P^\mu P^\nu\right)\frac{1}{e^2}\sum_n \overline{H}_{\mu\nu}^{(n)}(i,j) J^{(n)} dv_j\,dw_j,$$

$$\frac{d\hat{\sigma}_{i}}{dx_B\,dy} = \frac{\alpha_{em}^2}{x_B S_H}\left(Y_M\left(-g^{\mu\nu}\right) + Y_L \frac{4x_B^2}{Q^2} P^\mu P^\nu\right)\frac{1}{e^2}\sum_n \overline{H}_{\mu\nu}^{(n)}(i), \tag{5}$$

where n runs over the number of particles in the final state. Matrix elements are averaged over initial state polarizations, summed over final state polarizations and integrated over the phase space of the unobserved particles. $J^{(n)}$ is the Jacobian coming from the phase space integration and depends upon the number of final state particles n. α_{em} stands for the fine structure constant and e is the electron charge. Finally, Y_M and Y_L are the standard

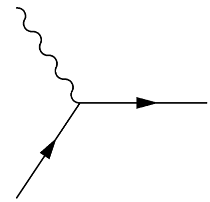

Fig. 2. Born contribution to the cross sections.

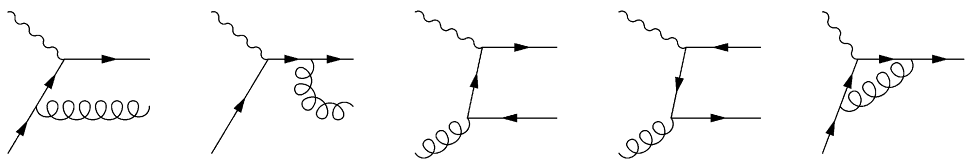

Fig. 3. Real and virtual contributions to the α_s cross sections.

kinematic factors for the contributions of each photon polarization and are given by,

$$Y_M = \frac{1+(1-y)^2}{2y^2}, \qquad Y_L = \frac{1+4(1-y)+(1-y)^2}{2y^2}. \tag{6}$$

The total inclusive cross sections are well known up to order α_s^2 [10], and more recently there have been impressive efforts to go beyond the NLO [14]. The corresponding complete expressions for the singular and finite pieces up to order α_s^2 can be found in Ref. [10].

For the one-particle inclusive cross section, the zeroth-order in α_s comes from the diagram in Fig. 2 giving, in $d = 4+\epsilon$ dimensions, the qq cross section:

$$\frac{d\hat{\sigma}^{(0)}_{qq,M}}{dx_B\, dy\, dv\, dw} = c_q \delta(1-u)\delta(1-v)\delta(w), \tag{7}$$

with

$$c_q = \frac{\alpha^2}{2x_B S_H} 4\pi(2+\epsilon)e_q^2 Y_M. \tag{8}$$

The antiquark cross section $d\hat{\sigma}_{\bar{q}\bar{q}}$ is identical to $d\hat{\sigma}_{qq}$ whereas all the remaining cross sections vanish. The index M refers to the metric terms in Eq. (5), longitudinal contributions are absent at tree level. Notice that at this order the quark is always produced in the backward ($w = 0$) direction implying that forward hadrons ($w = 1$) would come solely from target fragmentation processes, that is, those taken into account by fracture functions.

The first order corrections to the one-particle inclusive cross section are also known. Expressions for the singular and finite terms in dimensional regularization [15] can be found in [2] for the unpolarized case and in [4] for the polarized one. In order to accomplish the factorization of collinear singularities at $\mathcal{O}(\alpha_s^2)$, one also needs the $\mathcal{O}(\alpha_s)$ cross sections up to order ϵ, for this reason we accordingly extend here the results of [2]. The corresponding diagrams are shown in Fig. 3. As it is explained in Ref. [2], the integration region for the cross section, Eq. (4), need to be splitted into two regions, B1 and B2 respectively, in order to account for kinematical constraints in the phase space:

$$\begin{aligned} \mathrm{B1} &= \big\{u \in [x_B, x_u],\ v \in [a, 1],\ w \in [0, w_r]\big\}, \\ \mathrm{B2} &= \big\{u \in [x_u, 1],\ v \in [v_h, 1],\ w \in [0, w_r]\big\}, \end{aligned} \tag{9}$$

with $x_u = x_B/(x_B + (1 - x_B)v_h)$. The metric terms of the unpolarized cross sections can be expressed in the following form:

$$d\hat{\sigma}^{(1)}_{qq(\bar{q}\bar{q}),M}\big|_{\mathrm{B1}} = c_q C_\epsilon \left\{ \frac{2}{\epsilon} P^{(0)}_{q\leftarrow q}(u)\delta(1-v)\delta(w) + C^{(1)}_{1qq,M} + \epsilon D^{(1)}_{1qq,M} \right\}, \tag{10}$$

$$d\hat{\sigma}^{(1)}_{qg(\bar{q}g),M}\big|_{\mathrm{B1}} = c_q C_\epsilon \left\{ \frac{2}{\epsilon} P^{(0)}_{qg\leftarrow q}(u)\delta(a-v)\delta(1-w) + C^{(1)}_{1qg,M} + \epsilon D^{(1)}_{1qg,M} \right\}, \tag{11}$$

$$\begin{aligned} d\hat{\sigma}^{(1)}_{gq(g\bar{q}),M}\big|_{\mathrm{B1}} = c_q C_\epsilon \bigg\{ & \frac{2}{\epsilon} P^{(0)}_{\bar{q}q\leftarrow g}(u)\delta(v-a)\delta(1-w) + \frac{2}{\epsilon} P^{(0)}_{q\leftarrow g}(u)\delta(1-v)\delta(w) \\ & + C^{(1)}_{1gq,M} + \epsilon D^{(1)}_{1gq,M} \bigg\}, \end{aligned} \tag{12}$$

$$\begin{aligned} d\hat{\sigma}^{(1)}_{qq(\bar{q}\bar{q}),M}\big|_{\mathrm{B2}} = c_q C_\epsilon \bigg\{ & \frac{2}{\epsilon} \big(P^{(0)}_{q\leftarrow q}(u)\delta(1-v) + P^{(0)}_{q\leftarrow q}(v)\delta(1-u) \big)\delta(w) \\ & + C^{(1)}_{2qq,M} + \epsilon D^{(1)}_{2qq,M} \bigg\}, \end{aligned} \tag{13}$$

$$d\hat{\sigma}^{(1)}_{qg(\bar{q}g),M}\big|_{\mathrm{B2}} = c_q C_\epsilon \left\{ \frac{2}{\epsilon} P^{(0)}_{g\leftarrow q}(v)\delta(1-u)\delta(w) + C^{(1)}_{2qg,M} + \epsilon D^{(1)}_{2qg,M} \right\}, \tag{14}$$

$$d\hat{\sigma}^{(1)}_{gq(g\bar{q}),M}\big|_{\mathrm{B2}} = c_q C_\epsilon \left\{ \frac{2}{\epsilon} P^{(0)}_{q\leftarrow g}(u)\delta(1-v)\delta(w) + C^{(1)}_{2gq,M} + \epsilon D^{(1)}_{2gq,M} \right\}, \tag{15}$$

where C_ϵ is defined by

$$C_\epsilon = \frac{\alpha_s}{2\pi} f_\Gamma \left(\frac{Q^2}{4\pi\mu^2} \right)^{\epsilon/2}, \qquad f_\Gamma = \frac{\Gamma(1+\epsilon/2)}{\Gamma(1+\epsilon)} \tag{16}$$

and

$$a = \frac{(1-u)x_B}{u(1-x_B)}, \qquad w_r = \frac{(1-v)(1-u)x_B}{v(u-x_B)}. \tag{17}$$

The $P^{(0)}_{i\leftarrow j}$ are the usual LO Altarelli–Parisi kernels [16], whereas the $P^{(0)}_{jk\leftarrow i}$ are the unsubstracted ones. Expressions for them and for the coefficient functions $C^{(1)}$ can be found in Appendix B of Ref. [2]. Notice that in order to match our notation the results of Ref. [2] should be multiplied by a factor $\delta(w_r - w)$. The coefficients $D^{(1)}$ are explicitly given in Appendix A below. Similar expressions for the longitudinal cross sections, which are finite at this order, can be obtained from the results in Ref. [2].

Notice that at this order, the fragmenting parton can be produced in any direction, including the forward one, but all singular contributions come either from the backward or from the forward direction. The former terms are factorized into partonic densities and fragmentation functions, whereas the forward singularities can only be factorized in the redefinition of fracture functions. This factorization gives rise, as we have already mentioned, to non-homogeneous terms in the evolution equations of fracture functions [1]. Notice also that singular terms in the forward direction are always accompanied by a $\delta(v-a)$ factor. This is a characteristic feature of the LO results that in general, will not be present at the α_s^2 order.

3. $\mathcal{O}(\alpha_s^2)$ amplitudes and phase space integration

In this section we outline the computation of the order α_s^2 amplitudes for the one-particle inclusive cross sections. At this order the relevant amplitudes have either two or three final state partons, related to virtual and real contributions, respectively. We have computed the corresponding hadronic tensors $H_{\mu\nu}$ in $d = 4+\epsilon$ dimensions, in the Feynman gauge, and taking all the quarks to be massless.

Algebraic manipulations were performed with the aid of the program MATHEMATICA [17] and the package TRACER [18] to perform the traces over the Dirac indices. In order to obtain the one-particle inclusive cross sections, one has to integrate the resulting matrix elements over the internal loop momenta and over the phase space of the unobserved particles in the final state, which is one of the hardest and most delicate parts of the calculation.

As we mentioned, in the present paper we restrict ourselves to gluon initiated processes, the corresponding $\mathcal{O}(\alpha_s^2)$ real contributions (gg and gq processes) are shown in Fig. 4. For the first of these processes, the phase space integration is over the momenta of the quark–antiquark pair, whereas for the second it is over the gluon–antiquark momenta. To perform this integrals, we choose to work in the center of mass frame of the two unobserved partons and get for the phase space:

$$dPS^3 = Q^2 \frac{1}{\Gamma(1+\epsilon)} \left(\frac{Q^2}{4\pi}\right)^{\epsilon} (4\pi)^{-4} \left(\frac{1-x_B}{x_B}\right)^{1+\epsilon/2} \left(\frac{u-x_B}{x_B}\right)^{\epsilon/2} \times v^{1+3\epsilon/2} \theta(w_r - w)(w_r - w)^{\epsilon/2} w^{\epsilon/2} \times (1-w)^{\epsilon/2} dv\, dw \sin^{1+\epsilon}\beta_1 \sin^{\epsilon}\beta_2\, d\beta_1\, d\beta_2. \tag{18}$$

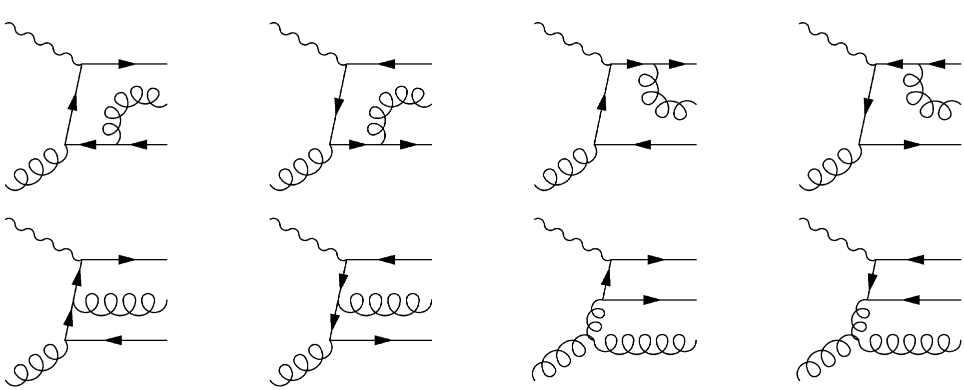

Fig. 4. Real contributions to the α_s^2 cross sections.

The angles β_1 and β_2 are the polar and azimuthal angles of one of the unobserved partons defined in the mentioned frame. The orientation of the axes in this frame was chosen in order to simplify the functions to be integrated. v and w are the energy and the angle of the fragmenting parton, respectively. In this case, as we anticipated, there is no correlation between them, but the θ function splits the integration region R in the u, v and w volume: $R = \mathrm{B0} \cup \mathrm{B1} \cup \mathrm{B2}$ where B0 is given by

$$\mathrm{B0} = \left\{u \in [x_B, x_u],\ v \in [v_h, a],\ w \in [0, 1]\right\} \tag{19}$$

and B1 and B2 are the LO regions given in Eq. (9).

As it is common practice, in order to perform the angular integration, matrix elements have to be decomposed, via partial fractioning, in such a way that all the angular integrals end in the standard form [12]

$$I(k,l) = \int_0^\pi d\beta_1 \int_0^\pi d\beta_2 \frac{\sin^{1+\epsilon}\beta_1 \sin^\epsilon \beta_2}{(a + b\cos\beta_1)^k (A + B\cos\beta_1 + C\sin\beta_1\cos\beta_2)^l}. \tag{20}$$

This kind of integrals can be classified into four categories according to whether their parameters satisfy either $a^2 = b^2$ or $A^2 = B^2 + C^2$, both relations simultaneously, or neither of them. In the present case, after the partial fractioning we obtained 31 independent integrals. 23 of these integrals were calculated to all orders in ϵ extending the results of Refs. [11–13]. The remaining 8, which we were not able to calculate to all orders, need to be carefully handled before expanding them in a power series in ϵ.

The difficulty with the above mentioned integrals is that ϵ is not only regulating the β integration, but also the singularities in the remaining variables: u, v and w. Although the integrals may be regular functions of these variables, their coefficients may be not. An illustrative example of this situation is given by the integral $I(1,1)|_{A^2=B^2+C^2}$, as in Eq. (20) with $k = 1$, $l = 1$, and satisfying $A^2 = B^2 + C^2$, for which the order by order computation in ϵ gives (Eq. C30 in Ref. [12])

$$I(1,1)|_{A^2=B^2+C^2} = \frac{\pi}{aA - bB}\left\{\frac{2}{\epsilon} + \log\left[\frac{(aA - bB)^2}{(a^2 - b^2)A^2}\right] + \mathcal{O}(\epsilon)\right\}. \tag{21}$$

Let us first consider the case when $a + b \sim (1 - w)$. As long as the coefficient of this integral is regular at $w = 1$, the above expression is integrable as function of w. However, if the coefficient has an extra factor of $(1 - w)^{-1}$ the resulting expression is ill defined due to the logarithm in Eq. (21) which behaves as $\log(1 - w)$. In order to skip this problem, one can recast Eq. (20) using the general methods described in Appendix A of Ref. [11] obtaining:

$$I(1,1)|_{A^2=B^2+C^2} = -\int_0^1 dx\, \frac{\pi(1-x)^{-1+\epsilon/2}}{A(a-b) + b(A-B)x}$$

$$\times\left\{\left(\frac{A}{A(a-b)+b(A-B)x}\right)^{\epsilon/2}\right.$$
$$\left.\times (a+b)^{\epsilon/2}\,{}_2F_1\left[\frac{\epsilon}{2},\epsilon,1+\epsilon;\frac{b(A-B)-2Ab}{A(a-b)+b(A-B)x}\right]-2\right\}. \quad (22)$$

The integral in last equation can be splitted into two pieces, one containing the hypergeometric function, which can be integrated order by order in a power expansion in ϵ after factoring out $(a+b)^{\epsilon/2}$, and the other which can be integrated to all orders in ϵ:

$$I(1,1)|_{A^2=B^2+C^2}=\frac{\pi}{aA-bB}\left\{\frac{4}{\epsilon}+2\log\left[\frac{aA-bB}{(a-b)A}\right]\right.$$
$$\left.+(a+b)^{\epsilon/2}\left[-\frac{2}{\epsilon}+\log[a-b]\right]+\mathcal{O}(\epsilon)\right\}. \quad (23)$$

Notice that factoring out $(a+b)^{\epsilon/2}$ before the expansion in powers of ϵ avoids the appearance of powers of $\log(a+b)$ in the series, which would be singular in the $w\to 1$ limit. In this way we obtain well defined integrals in w, as long as $\epsilon>0$, even if the coefficient has a pole in $w=1$, which is rather frequent. In some cases, the angular integrals have singularities in u, v or w by themselves, but they can be managed in the same way as in the example above.

The procedure just illustrated was performed for all the 8 integrals and for the different combinations of singularities in u, v and w; expressions for them are available upon request. It is also important to stress that, as the singular distributions that show up in the matrix elements after the angular integration give rise to additional poles in ϵ, it is necessary to calculate contributions up to order ϵ^3 in the angular integrals. Fortunately, these poles are always accompanied by one or more δ functions and those higher order terms only need to be calculated in the corresponding limits, what simplifies considerably the integrals.

Virtual contributions for the gq subprocess are obtained from the interference of the one loop graphs in Fig. 5 with the box graphs in Fig. 3. Integration over the loop momentum was done using the standard Passarino–Veltman [19] reduction algorithm and computing the resulting 2, 3 and 4-point scalar integrals. The integration over the phase space of the unobserved antiquark can be trivially performed using the energy–momentum conservation δ function. After this integration, the remaining phase space can be written as

$$dPS^{(2)}=\frac{1}{8\pi\Gamma(1+\epsilon/2)}\left(\frac{Q^2}{4\pi}\right)^{\epsilon/2}\frac{u(1-x_B)}{u-x_B}\left(\frac{1-x_B}{x_B}\right)^{\epsilon/2}$$
$$\times v^{\epsilon}w^{\epsilon/2}(1-w)^{\epsilon/2}\,\delta(w_r-w)\,dv\,dw, \quad (24)$$

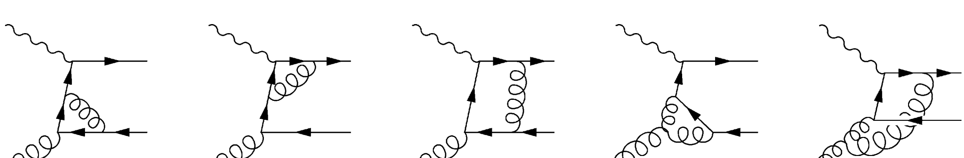

Fig. 5. One loop contributions to the α_s^2 gq cross section. Diagrams obtained from the first four by reversing the quark line must be also taken into account.

where v and w are the energy and angular variables of the hadronizing quark, respectively. As can be seen from the δ function in Eq. (24), these two variables are correlated, which is a distinctive feature of the two particle phase space. It implies an additional constraint over the integration region: as $w = w_r \leqslant 1$ then $v \geqslant a$ should be satisfied and, as it happens for the order α_s corrections, the integration region V has to be splitted into $V = \mathrm{B1} \cup \mathrm{B2}$ where B1 and B2 have been already defined in Eq. (9).

4. Singularity structure

Once the angular integrations are performed, the hadronic tensor shows a rich variety of singularities in the (u, v, w) space, regulated by the parameter ϵ. As it is standard in this kind of calculations, the above mentioned singularities should be *prescribed* in order to get a series expansion in powers of ϵ suitable for making explicit their cancellation. These cancellations are performed by coupling constant renormalization for the UV singularities, by cancellations between virtual and real contributions for the soft ones, and by renormalization of parton densities, fragmentation and fracture functions in the collinear case.

A standard example for the above mentioned *prescriptions* is the appearance of factors like $(1 - u)^{-1+\epsilon}$ in the totally inclusive cross section where, after the phase space integration, u is the only remaining variable. In this case one can use the standard substitution:

$$(1-u)^{-1+\epsilon} \equiv \frac{1}{\epsilon}\delta(1-u) + \left(\frac{1}{1-u}\right)_{+u[0,\underline{1}]} + \mathcal{O}(\epsilon), \tag{25}$$

where $(1/(1-u))_{+u[0,\underline{1}]}$ is the usual 'plus' distribution:

$$\int_0^1 du \left(\frac{1}{1-u}\right)_{+u[0,\underline{1}]} f(u) = \int_0^1 du\, \frac{f(u) - f(1)}{1-u}. \tag{26}$$

However, in the one-particle inclusive case, the structure of the singularities is much more complex, mixing the three variables and consequently this simple prescription is no longer adequate. In Fig. 6 we show the curves along which the singularities in the regions B0 and B1 appear in the v–w plane after the angular integration is performed. We will focus on this two regions because they contain all the singularities in the forward direction which need to be factorized in the redefinition of fracture functions. The case of region B2 is quite similar to that of B1 without the complications of the poles in $w = w_r = 1$ but with additional singularities along the plane $u = 1$, and it will be discussed at the end of this section.

A simple inspection of Fig. 6 allows one to distinguish different possibilities for the singularity structure of the terms in the hadronic tensor. In principle the integration leads to terms that can have none, one, or two poles along the thick curves in the figure, respectively.

The case of a single pole can easily be handled with minor modifications to the prescription formula in Eq. (25). For terms with more than a single pole, the singular

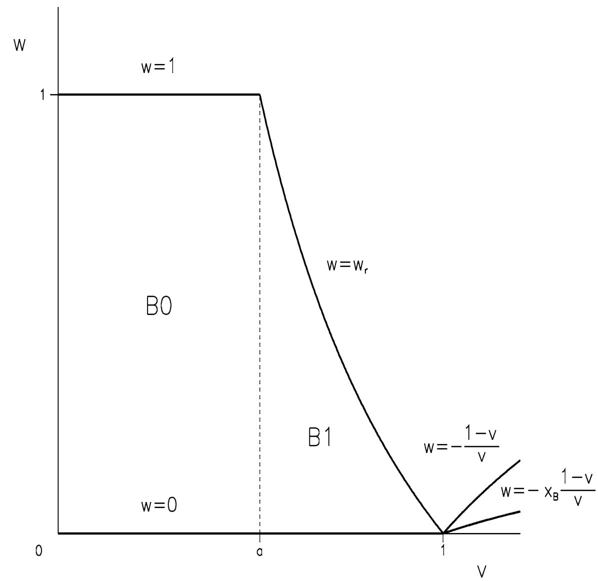

Fig. 6. Position of the singularities in the v–w plane for $x_B \leqslant u \leqslant x_u$. The bold lines represent the curves where the hadronic tensor becomes singular.

curves can either intersect themselves (for example, poles in $w = 1$ and $w = w_r$) or not (for instance, $w = 0$ and $w = 1$).

Terms with poles along two non-intersecting curves can be shown to be always transformed by partial fractioning into two terms with single poles, which reduce to the previous case. On the other hand, the case of two intersecting singular curves cannot be reduced to a simpler one and needs to be treated in a more subtle way.

Overlapping singularities as those mentioned in the previous paragraph can be further classified according to whether: (a) both curves lie in the integration region, like $w = 0$ and $w = w_r$ in region B1; (b) one of them comes from the outside of the integration region but intersects it at some point, as it is the case of $w = 1$ and $w = w_r$ in B0; and (c) both curves converge into a single point of the integration region but coming from the outside, like $w = -(1 - v)/v$ and $w = -x_B(1 - v)/v$.

The first and third occurrences can be cast into the second, by partial fractioning, leaving us with only one case. The technique we employed to treat it is better illustrated by means of an example. Let us consider the two-dimensional integral

$$I(\epsilon) = \int_0^1 dx \int_0^1 dy\, f(x, y)(1 - y)^{-1+\epsilon}(1 - xy)^{-1+\epsilon}, \tag{27}$$

where $f(x, y)$ is a regular function in all the integration region. The integrand has poles along the curves $y = 1$ and $y = 1/x$ which intersect at $x = 1$, $y = 1$. These singularities are regulated if $\epsilon > 0$ (notice that the integral remains finite even if the term ϵ in the exponent of $(1 - xy)$ is absent). If one wrongly uses the recipe in Eq. (25) to prescribe the singularity in $y = 1$ and then again to deal with the pole in $x = 1$ coming from the δ term, one ends with ill-defined terms (more precisely terms with 'plus' distributions which are not integrable) and the leading singularity, in this case a double pole ϵ^{-2}, is accounted twice. The correct way to deal with this integral is to re-write it as

$$I(\epsilon) = \int_0^1 dx \int_0^1 dy\, f(x,1)(1-y)^{-1+\epsilon}(1-xy)^{-1+\epsilon} + \int_0^1 dx \int_0^1 dy\, \big(f(x,y) - f(x,1)\big)(1-y)^{-1+\epsilon}(1-xy)^{-1+\epsilon}. \tag{28}$$

The second term is integrable in the limit $\epsilon \to 0$ whereas in the first one the integration over y can be performed and gives

$$I(\epsilon) = \int_0^1 dx\, (1-x)^{-1+2\epsilon} f(x,1) \frac{{}_2F_1[\epsilon, 2\epsilon, 1+\epsilon; x]}{\epsilon} + \int_0^1 dx \int_0^1 dy\, \frac{f(x,y) - f(x,1)}{(1-y)(1-xy)} + \mathcal{O}(\epsilon). \tag{29}$$

Now, the integral in the first term can be prescribed using (25). Doing that substitution, we end with the following identity:

$$(1-y)^{-1+\epsilon}(1-x\,y)^{-1+\epsilon} = \left\{\frac{1}{2\epsilon^2} + \frac{\pi^2}{6}\right\}\delta(1-x)\delta(1-y) + \left\{\frac{1}{\epsilon}\left(\frac{1}{1-x}\right)_{x[0,\underline{1}]} + 2\left(\frac{\log(1-x)}{1-x}\right)_{x[0,\underline{1}]}\right\}\delta(1-y) + \left(\frac{1}{(1-y)(1-xy)}\right)_{y[0,\underline{1}]} + \mathcal{O}(\epsilon). \tag{30}$$

The 'plus' distribution $1/((1-y)(1-xy))_{y[0,\underline{1}]}$ stands for the second term in the r.h.s. of Eq. (29). The factor $1/2$ in the double pole is a consequence of the fact that the singular curve $y = 1/x$ only intersects the integration region in a single point.

Prescriptions for all the singular (but regular at $u = 1$) terms appearing in the matrix elements can be found, besides some subtleties related to the integration intervals in B1 and B2, with the technique shown in the example. Expressions for the prescriptions relevant in the $w = 1$ region can be found in Appendix B.

The only remaining item is the prescription of singularities in $u = 1$ in B2. These poles always appear as factors $1/(1-u)$ and only give rise to singular integrals (when $\epsilon \to 0$) in terms proportional to $\delta(w)$ or $\delta(w_r - w)$ that come from the prescription of the singularities in the v–w plane. This is so because of the upper limit w_r in the w integration which goes to zero when $u \to 1$. For the δ terms, the prescription of the singularities in $u = 1$ can be done exactly as in Eq. (25).

5. Factorization of singularities

As we mentioned in the previous section, once the angular integration and the prescription of the singularities in the u, v and w variables are accomplished, the partonic cross sections exhibit a complex structure of poles in ϵ. Explicit expressions for this structure in region B0 can be found in Appendix C. Adding virtual and real contributions all IR divergences cancel out, leaving us with the UV and collinear poles. UV poles are canceled by means of coupling constant renormalization:

$$\frac{\alpha_s}{2\pi} = \frac{\alpha_s(M_R^2)}{2\pi}\left(1 + \frac{\alpha_s(M_R^2)}{2\pi} f_\Gamma \frac{\beta_0}{\epsilon}\left(\frac{M_R^2}{4\pi\mu^2}\right)^{\epsilon/2}\right), \tag{31}$$

where M_R is the renormalization scale and β_0 is the lowest order coefficient function in the QCD β function:

$$\beta_0 = \frac{11}{3}C_A - \frac{4}{3}n_f T_F \tag{32}$$

with $C_A = N$ for $SU(N)$ and $T_F = 1/2$ as usual; n_f stands for the number of active quark flavours.

Collinear singularities have to be factorized in the redefinition of parton densities, fragmentation and fracture functions. The redefinition of parton densities is exactly the same as in totally inclusive DIS whereas fragmentation functions are renormalized as they are in one-particle inclusive electron–positron annihilation. Expressions for renormalized parton densities and fragmentation functions, up to order α_s^2 and in the $\overline{MS}$ factorization scheme, can be found in Refs. [10,20], respectively.

Notice that the renormalization of parton densities and fragmentation functions implies convolutions between the evolution kernels and the SIDIS cross sections. At variance with the totally inclusive case, the convolutions between the $\mathcal{O}(\alpha_s)$ cross section and the LO kernels include plus distributions in more than one variable which need to be handled with care. In order to make explicit the cancellations between the $\mathcal{O}(\alpha_s^2)$ cross sections and these counterterms, the results of the above mentioned convolutions need to be expressed in terms of the very same variables used for the cross sections. One way to accomplish this is to retain to all orders in ϵ the $\mathcal{O}(\alpha_s)$ cross sections, that is without replacing the singular factors like $(1-u)^{-1+\epsilon}$ in terms of distributions as described in the previous section, and rewrite the plus distributions in the LO kernels using

$$\left(\frac{1}{1-x}\right)_{+x[0,\underline{1}]} \to \lim_{\epsilon'\to 0}(1-x)^{-1+\epsilon'} - \frac{1}{\epsilon'}\delta(1-x) + \mathcal{O}(\epsilon'). \tag{33}$$

In this way the appearance of plus distributions is avoided and the convolutions can be explicitly performed. The resulting expressions can be prescribed, keeping up to constant terms in ϵ and ϵ', in exactly the same way as the $\mathcal{O}(\alpha_s^2)$ cross sections. Notice that at this point the poles in ϵ' must cancel and the limit $\epsilon' \to 0$ can be safely taken, reflecting the fact that the LO kernels were already regular. The above mentioned procedure allows to extend the results of Ref. [21] to SIDIS.

Once the renormalization of parton densities and fragmentation functions is accomplished, the remaining singularities occur in the region B0 and are proportional to $\delta(1-w)$, that is the forward direction, so they have to be factorized into renormalized fracture functions. Otherwise, factorization would be broken. The bare fracture functions can be written in terms of renormalized quantities as:

$$M_{i,h/P}(\xi,\zeta) = \frac{1}{\xi}\int_{\xi}^{\frac{\xi}{\xi+\zeta}} \frac{du}{u} \int_{\frac{\zeta}{\xi}}^{\frac{1-u}{u}} \frac{dv}{v} \Delta_{ki\leftarrow j}(u,v,M_f) f^r_{j/P}\left(\frac{\xi}{u}, M_f^2\right) D^r_{h/k}\left(\frac{\zeta}{\xi v}, M_f^2\right)$$
$$+ \int_{\frac{\xi}{1-\zeta}}^{1} \frac{du}{u} \Delta_{i\leftarrow j}(u, M_f) M^r_{j,h/P}\left(\frac{\xi}{u}, \zeta, M_f^2\right), \tag{34}$$

where the factorization scale has been chosen to be the same for the three distributions. The functions $\Delta_{i\leftarrow j}$ and $\Delta_{ki\leftarrow j}$ are fixed in order to cancel all the remaining singularities in the cross section.

The homogeneous kernels $\Delta_{i\leftarrow j}$ are the same that appear in the inclusive case for parton densities and can be obtained from the corresponding transition functions in Ref. [10], whereas the non-homogeneous $\Delta_{ki\leftarrow j}$ are presented, for the case $j = g$, in this paper for the first time. Explicitly:

$$\Delta_{gg\leftarrow g}(u,v) = -\frac{\alpha_s}{2\pi} f_\Gamma \left(\frac{M_f^2}{4\pi\mu^2}\right)^{\epsilon/2} \frac{2}{\epsilon} \widetilde{P}^{(0)}_{gg\leftarrow g}(u,v), \tag{35}$$

$$\begin{aligned}\Delta_{gq\leftarrow g}(u,v) &= \Delta_{g\bar{q}\leftarrow g}(u,v)\\ &= \left(\frac{\alpha_s}{2\pi}\right)^2 f_\Gamma^2 \left(\frac{M_f^2}{4\pi\mu^2}\right)^{\epsilon}\\ &\quad\times\left\{\frac{2}{\epsilon^2}\left(\widetilde{P}^{(0)}_{gq\leftarrow q}(u,v)\otimes P^{(0)}_{q\leftarrow g}(u) + \widetilde{P}^{(0)}_{\bar{q}q\leftarrow g}(u,v)\otimes P^{(0)}_{g\leftarrow q}(v)\right.\right.\\ &\quad\left.\left. + \widetilde{P}^{(0)}_{gg\leftarrow g}(u,v)\otimes' P^{(0)}_{q\leftarrow g}(u)\right) - \frac{1}{\epsilon} P^{(1)}_{gq\leftarrow g}(u,v)\right\},\end{aligned} \tag{36}$$

$$\begin{aligned}\Delta_{\bar{q}q\leftarrow g}(u,v) &= \Delta_{q\bar{q}\leftarrow g}(u,v)\\ &= -\frac{\alpha_s}{2\pi} f_\Gamma \left(\frac{M_f^2}{4\pi\mu^2}\right)^{\epsilon/2} \frac{2}{\epsilon} \widetilde{P}^{(0)}_{\bar{q}q\leftarrow g}(u,v) + \left(\frac{\alpha_s}{2\pi}\right)^2 f_\Gamma^2 \left(\frac{M_f^2}{4\pi\mu^2}\right)^{\epsilon}\\ &\quad\times\left\{\frac{2}{\epsilon^2}\left(\widetilde{P}^{(0)}_{\bar{q}q\leftarrow g}(u,v)\otimes P^{(0)}_{g\leftarrow g}(u) + \widetilde{P}^{(0)}_{\bar{q}q\leftarrow g}(u,v)\otimes P^{(0)}_{q\leftarrow q}(v)\right.\right.\\ &\quad\left. + \widetilde{P}^{(0)}_{\bar{q}q\leftarrow g}(u,v)\otimes' P^{(0)}_{q\leftarrow q}(u) + \frac{1}{2}\beta_0 \widetilde{P}^{(0)}_{\bar{q}q\leftarrow g}(u,v)\right)\\ &\quad\left. - \frac{1}{\epsilon} P^{(1)}_{\bar{q}q\leftarrow g}(u,v)\right\},\end{aligned} \tag{37}$$

where α_s is the bare coupling constant, the convolutions are defined as

$$f(u,v)\otimes g(u)=\int_u^{\frac{1}{1+v}}\frac{d\bar{u}}{\bar{u}}f(\bar{u},v)g\left(\frac{u}{\bar{u}}\right),$$

$$f(u,v)\otimes g(v)=\int_v^{\frac{1-u}{u}}\frac{d\bar{v}}{\bar{v}}f(u,\bar{v})g\left(\frac{v}{\bar{v}}\right),$$

$$f(u,v)\otimes' g(u)=\int_u^{1-uv}\frac{d\bar{u}}{\bar{u}}\frac{u}{\bar{u}}f\left(\bar{u},\frac{u}{\bar{u}}v\right)g\left(\frac{u}{\bar{u}}\right), \tag{38}$$

and

$$\widetilde{P}^{(0)}_{ki\leftarrow j}(u,v)=P^{(0)}_{ki\leftarrow j}(u)\delta\left(v-\frac{1-u}{u}\right). \tag{39}$$

Finally the $\mathcal{O}(\alpha_s^2)$ kernels are given by

$$\begin{aligned}
&P^{(1)}_{gq\leftarrow g}(u,v)\\
&\quad=C_A T_F\Bigg\{-\frac{(3-8u)u}{2}-\frac{4(1-u)u}{v}-\frac{8u^3}{(1-uv)^4}+\frac{8u^2(1+u)}{(1-uv)^3}\\
&\qquad-\frac{2u(1+4u-3u^2)}{(1-uv)^2}+\frac{2(1-3u)u}{1-uv}+\log\left(\frac{v}{1-u}\right)\frac{2P^{(0)}_{q\leftarrow g}(u)}{v(1-uv)}\\
&\qquad+\log(1+v)\left[-u(1+2u)-u^2v+\frac{2uP^{(0)}_{q\leftarrow g}(-u)}{1-uv}\right]\\
&\qquad+\log\left(\frac{1-u-uv}{v}\right)\left[2(1-3u)u-\frac{6P^{(0)}_{q\leftarrow g}(u)}{v}-3u^2v\right.\\
&\qquad\left.+\frac{2u(1+4u)}{1-uv}-\frac{2u(1+2u+4u^2)}{(1-uv)^2}+\frac{4u^2(1+u)}{(1-uv)^3}-\frac{4u^3}{(1-uv)^4}\right]\\
&\qquad+\log(u)\left[4(-1+u)u+2u^2v+\frac{4P^{(0)}_{q\leftarrow g}(u)(2-uv)}{v(1-uv)}\right]\\
&\qquad+\log(1-uv)\left[u(3+2u)+\frac{2P^{(0)}_{q\leftarrow g}(u)}{v}+u^2v+\frac{8u^3}{(1-uv)^4}\right.\\
&\qquad\left.-\frac{8u^2(1+u)}{(1-uv)^3}+\frac{4u(1+2u+4u^2)}{(1-uv)^2}-\frac{4u(1+3u)}{1-uv}\right]\Bigg\}
\end{aligned}$$

$$
\begin{aligned}
&+ C_F T_F \bigg\{ 4u + \frac{u^2(1-4v)}{1-u} - \frac{u^3 v}{(1-u)^2} + \frac{3}{(1+v)^2} - \frac{2+5u}{1+v} \\
&\quad + \log(v)\bigg[\frac{u^3 v}{(1-u)^2} + \frac{1}{(1+v)^2} + \frac{1-2u}{1+v} - \frac{u^2(2+v)}{1-u}\bigg] \\
&\quad + \log(1+v)\bigg[\frac{4P^{(0)}_{q\leftarrow g}(u)}{v} - \frac{2}{(1+v)^2} - \frac{2(1-2u)}{1+v} + 2u^2(2+v)\bigg] \\
&\quad + \log(1-u)\bigg[\frac{-4u^3}{1-u} + \frac{2u^4 v}{(1-u)^2} + 4P^{(0)}_{q\leftarrow g}(u)\bigg(\frac{4u}{1-u} - \frac{3}{v} \\
&\quad - \frac{2u^2 v}{(1-u)^2}\bigg)\bigg] + \log(1-u-uv)\bigg[-2u - \frac{2u^3 v}{(1-u)^2} - \frac{1}{(1+v)^2} \\
&\quad - \frac{1-2u}{1+v} + \frac{2u^2(2+v)}{1-u} + 6P^{(0)}_{q\leftarrow g}(u)\bigg(\frac{2}{v} - \frac{2u}{1-u} + \frac{u^2 v}{(1-u)^2}\bigg)\bigg]\bigg\},
\end{aligned}
\tag{40}
$$

and

$$
\begin{aligned}
&P^{(1)}_{\bar{q}q\leftarrow g}(u,v) \\
&= C_A T_F \bigg\{ -u(1-4u) - \frac{8}{u(1+v)^4} + \frac{8(1+u)}{u(1+v)^3} - \frac{2(1+4u-3u^2)}{u(1+v)^2} \\
&\quad + \frac{2(1-3u)}{1+v} + \log(1-uv)\frac{(1-2uv+u^2(1+v^2))}{1+v} \\
&\quad + \log(1-u)\frac{P^{(0)}_{q\leftarrow g}(u)}{1+v} + \log\bigg(\frac{1-u-uv}{1-u}\bigg) \\
&\quad \times \bigg[\frac{2}{1+v} - \frac{3u}{1-u-uv}\bigg]P^{(0)}_{q\leftarrow g}(u) + \log(u)\bigg[2(1-u)u + 2u^2 v \\
&\quad - \frac{2uP^{(0)}_{q\leftarrow g}(u)}{1-u-uv}\bigg] + \log(1+v)\bigg[u(4+u) - u^2 v + \frac{8}{u(1+v)^4} \\
&\quad - \frac{8(1+u)}{u(1+v)^3} + \frac{4(1+2u+4u^2)}{u(1+v)^2} - \frac{4(1+3u)}{1+v} + \frac{2uP^{(0)}_{q\leftarrow g}(u)}{1-u-uv}\bigg] \\
&\quad + \log\bigg(\frac{1-u-uv}{v}\bigg)\bigg[u(1+3u) - 3u^2 v + \frac{4}{u(1+v)^4} - \frac{4(1+u)}{u(1+v)^3} \\
&\quad + \frac{2(1+2u+4u^2)}{u(1+v)^2} - \frac{2(1+4u)}{1+v} + \frac{3uP^{(0)}_{q\leftarrow g}(u)}{1-u-uv}\bigg] \\
&\quad - 4\big[u(1-u) + \log(a_f)P^{(0)}_{q\leftarrow g}(u)\big]\bigg(\frac{1}{a_f - v}\bigg)_{v[0,\underline{a_f}]} \\
&\quad + 4P^{(0)}_{q\leftarrow g}(u)\bigg(\frac{\log(a_f - v)}{a_f - v}\bigg)_{v[0,\underline{a_f}]} + \big[u - 4(1-u)u\log(1-u)
\end{aligned}
$$

$$
\begin{aligned}
&\quad - \left(\log(1-u)^2 + 2\log(1-u)\log(u) + 2\,\mathrm{Li}_2(u)\right) P^{(0)}_{q\leftarrow g}(u)\Big]\delta(a_f - v)\Bigg\} \\
&+ C_F T_F \Bigg\{ \frac{4u^2 v}{1-u} + \frac{u^3 v}{(1-u)^2} + \frac{3u^2}{(1-uv)^2} - \frac{2u+5u^2}{1-uv} + \log(1-u-uv) \\
&\quad \times \left[-\frac{u^3 v}{(1-u)^2} + \frac{u^2}{(1-uv)^2} + \frac{(1-2u)u}{1-uv} - \frac{u(1-uv)}{1-u} \right] \\
&\quad + 2\log(1-uv)\Bigg[u(1+u) - u^2 v - \frac{u^2}{(1-uv)^2} - \frac{(1-2u)u}{1-uv} \\
&\quad + \frac{u P^{(0)}_{q\leftarrow g}(u)}{1-u-uv} \Bigg] + 4\log(u)\left[-u(1-u) - u^2 v + \frac{u P^{(0)}_{q\leftarrow g}(u)}{1-u-uv} \right] \\
&\quad + \log(v)\Bigg[\frac{2u^2(1-v)}{1-u} + \frac{2u^3 v}{(1-u)^2} - \frac{u^2}{(1-uv)^2} - \frac{(1-2u)u}{1-uv} \\
&\quad + \frac{6u^3 v^2 P^{(0)}_{q\leftarrow g}(u)}{(1-u)^2}(1-u-uv) \Bigg] \\
&\quad + 2\log(1-u)\Bigg[\left(\frac{4u}{1-u} - \frac{3u}{1-u-uv} + \frac{4u^2 v}{(1-u)^2} \right) P^{(0)}_{q\leftarrow g}(u) \\
&\quad - \frac{u^4 v}{(1-u)^2} - \frac{u^3}{(1-u)} \Bigg] + 4\Bigg[-\frac{u}{4} + (1-u)u\log(a_f) \\
&\quad + \left(\zeta(2) + \frac{\log(1-u)^2}{4} + \mathrm{Li}_2(u) \right) P^{(0)}_{q\leftarrow g}(u) \Bigg] \delta(a_f - v) \Bigg\}, \qquad (41)
\end{aligned}
$$

where $a_f = (1-u)/u$. Although $\Delta_{qg\leftarrow g}$ is formally a NLO kernel, it occurs for the first time at order α_s^3 thus it does not show up in the present calculation. Notice that the NLO kernels depend on both u and v variables and that this dependence cannot be factorized.

Once obtained the explicit expressions for the relations between renormalized and bare fracture functions, we can easily derive the evolution equations for the renormalized fracture functions, which can be written as

$$
\begin{aligned}
&\frac{\partial M^r_{i,h/P}(\xi,\zeta,M^2)}{\partial \log M^2} \\
&= \frac{\alpha_s(M^2)}{2\pi} \int_{\frac{\xi}{1-\zeta}}^{1} \frac{du}{u} \left[P^{(0)}_{i\leftarrow j}(u) + \frac{\alpha_s(M^2)}{2\pi} P^{(1)}_{i\leftarrow j}(u) \right] M^r_{j,h/P}\left(\frac{\xi}{u}, \zeta, M^2 \right) \\
&\quad + \frac{\alpha_s(M^2)}{2\pi} \frac{1}{\xi} \int_{\xi}^{\frac{\xi}{\xi+\zeta}} \frac{du}{u} \int_{\frac{\zeta}{\xi}}^{\frac{1-u}{u}} \frac{dv}{v} \left[\widetilde{P}^{(0)}_{ki\leftarrow j}(u,v) + \frac{\alpha_s(M^2)}{2\pi} P^{(1)}_{ki\leftarrow j}(u,v) \right] \\
&\qquad \times f^r_{j/P}\left(\frac{\xi}{u}, M^2 \right) D^r_{h/k}\left(\frac{\zeta}{\xi v}, M^2 \right), \qquad (42)
\end{aligned}
$$

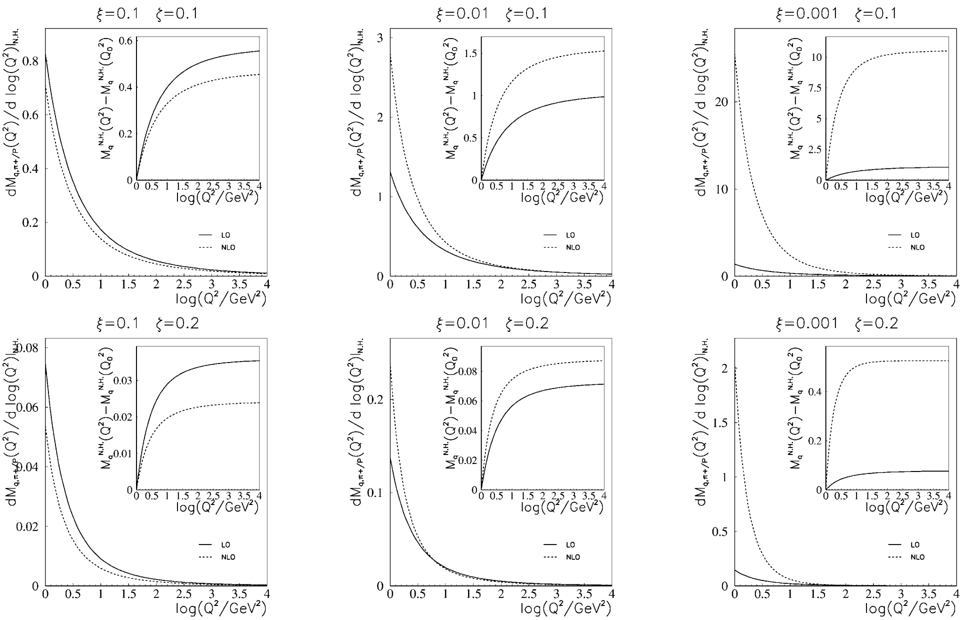

Fig. 7. Non-homogeneous contributions to the derivative of M_q for different values of ξ and ζ. Inset plots show the integral over Q^2 of this contributions taking $M_q^{\mathrm{N.H.}}(Q_0^2) = 0$ with $Q_0 = 1$ GeV as a reference.

where the NLO kernels $P^{(1)}_{i\leftarrow j}(u)$ are 1/8 of those given in Ref. [22] due to the different conventions implemented. At variance with the LO case where the kernels are proportional to $\delta(v - (1-u)/u)$, the NLO kernels have support in all the integration region in the non-homogeneous term of Eq. (42). Due to this fact, at NLO, the non-homogeneous terms in the evolution equations do not take the familiar form given in Eq. (12) of Ref. [1]. In terms of moments, Eq. (42) can be written as:

$$\frac{\partial M^r_{i,h/P}[m,n]}{\partial \log M^2} = M^r_{i,h/P}[m,n] P_{i\leftarrow j}[m] + f^r_{j/P}[m+n-1] D^r_{h/k}[n] \widehat{P}_{ki\leftarrow j}[m-1,n], \tag{43}$$

where the moments are defined as

$$F[m,n] = \int_0^1 \frac{d\xi}{\xi} \int_0^{1-\xi} \frac{d\zeta}{\zeta} \xi^m \zeta^n F(\xi,\zeta), \qquad F[m] = \int_0^1 \frac{d\xi}{\xi} \xi^m F(\xi), \tag{44}$$

and

$$\widehat{P}_{ki\leftarrow j}[m,n] = \int_0^1 \frac{d\xi}{\xi} \int_0^{1-\xi} \frac{d\zeta}{\zeta} \xi^m \zeta^n P_{ki\leftarrow j}\left(\xi, \frac{\zeta}{\xi}\right). \tag{45}$$

Fig. 7 compares (for different values of ξ and ζ) the relative size of the LO and NLO contributions to the non-homogeneous term in the evolution equation (42) computed with standard sets of parton distributions [23] and fragmentation functions [24] for the case $i = q$ and $h = \pi^+$. The inset plots show the integral over Q^2 of this contributions. Notice that only those terms proportional to $f_{g/P}$ were taken into account in the $\mathcal{O}(\alpha_s^2)$ pieces. In reference [9] it was found that the LO non-homogeneous contribution falls rapidly as ζ grows. This behavior is related to the shrinkage of the integration region and with the fall of fragmentation functions, $D_{h/i}(z)$, in the limit $z \to 1$. This is also the case of the NLO contributions. At moderate and large values of ξ ($\xi \geqslant 0.1$) the $\mathcal{O}(\alpha_s^2)$ contributions are typically one order of magnitude smaller than the $\mathcal{O}(\alpha_s)$ ones so NLO and LO results differ only by a few percents. This can be traced back to the extra power of α_s and the small size of the integration region since the interval of the v integral in Eq. (42) shrinks to the point $(1-u)/u$ when $\xi \to 1-\zeta$. However, when ξ diminishes the integration region expands and NLO contributions grow considerably faster than the LO ones which are kinematically restricted to the curve $v = (1-u)/u$. The remarkable growth of the $\mathcal{O}(\alpha_s^2)$ terms makes these contributions even larger than the constrained $\mathcal{O}(\alpha_s)$ pieces at lower values of ξ and thus a priori non-negligible in the evolution equations.

Of course, in order to assess the actual relevance of the NLO non-homogeneous effects in the full evolution of fracture functions, one needs a realistic (based on actual data) estimate for the size and shape for these functions at a given scale, and compute the evolution taking into account all the appropriate kernels, but our present results suggest that non-homogeneous NLO effects could be relevant.

6. Summary and conclusions

In this paper we have computed the $\mathcal{O}(\alpha_s^2)$ gluon initiated QCD corrections to one particle inclusive deep inelastic processes. At variance with the inclusive case, in one particle inclusive processes the kinematical characterization of the final state particle requires to preserve the full dependence of the amplitude in the relevant variables. This impedes the cancellation of some singularities to be later factorized into fracture functions and leads to a non-trivial singularity structure. In order to deal with this we have highlighted the importance of collecting to all orders the potentially singular factors in the 3-particle final state angular integrals and implemented a general approach for the prescription of overlapping singularities.

By the explicit replacement of the bare parton densities, fragmentation and fracture functions with the corresponding renormalized quantities in both the $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(\alpha_s^2)$ cross sections, we have explicitly verified the factorization of collinear singularities obtaining for the first time the relevant kernels at this order. In doing so, we give a recipe for dealing with convolutions of distributions in more than one variable which occur in the computation of the α_s^2 contribution coming from the convolution of $\mathcal{O}(\alpha_s)$ cross sections and renormalized functions. We also derived the evolution equations for fracture functions valid at NLO.

Regarding the phenomenological consequences of these corrections, we have found that the $\mathcal{O}(\alpha_s^2)$ contributions to the evolution equations are mild in most of the kinematical range, however, they are as important or even larger than the α_s ones for small values of x_B, where these last contributions are suppressed by the available phase space. This behaviour, at variance with the LO case, allows the non-homogeneous effects to be sizeable even at larger hadron momentum fractions, thus being relevant for the scale dependence for diffractive and leading baryon processes.

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Appendix A

In this appendix we present the results obtained for the coefficients $D^{(1)}$ in the $\mathcal{O}(\alpha_s)$ cross sections. They are

$$\begin{aligned} D^{(1)}_{1qq,M} = \frac{C_F}{2}\Bigg\{ & \Bigg[(1-u)\log\bigg(\frac{1-u}{u}\bigg) + \bigg(\frac{\pi^2}{6} + \frac{\log(\frac{1-u}{u})^2}{2}\bigg) p_{gq\leftarrow q}(u)\Bigg] \\ & \times \delta(1-v)\delta(w) + \delta(w_r - w)\Bigg[p_{gq\leftarrow q}(u)\bigg(\frac{\log(1-v)}{1-v}\bigg)_{+v[a,\underline{1}]} \\ & + \bigg(1-u+\log\bigg(\frac{1-u}{(1-a)u}\bigg)p_{gq\leftarrow q}(u)\bigg)\bigg(\frac{1}{1-v}\bigg)_{+v[a,\underline{1}]} \\ & - \frac{2(v-a)}{(1-a)^2(1-u)} + \frac{(1-v-2(1-a)u)}{(1-a)^2(1-u)}\Bigg[\log\bigg(\frac{(1-u)(1-v)}{(1-a)u}\bigg) \\ & + \log\bigg(\frac{v-a}{1-a}\bigg) - 1\Bigg] + \frac{1}{1-v}\log\bigg(\frac{v-a}{1-a}\bigg)p_{gq\leftarrow q}(u)\Bigg]\Bigg\}, \end{aligned} \tag{A.1}$$

$$\begin{aligned} D^{(1)}_{1qg,M} = \frac{C_F}{2}\Bigg\{ & \Bigg[(1-u)\log\bigg(\frac{1-u}{u}\bigg) + \bigg(\frac{\pi^2}{6} + \frac{\log(\frac{1-u}{u})^2}{2}\bigg) p_{gq\leftarrow q}(u)\Bigg] \\ & \times \delta(v-a)\delta(1-w) + \delta(w_r - w)\Bigg[p_{gq\leftarrow q}(u)\bigg(\frac{\log(v-a)}{v-a}\bigg)_{+v[\underline{a},1]} \\ & + \bigg(1-u+\log\bigg(\frac{1-u}{(1-a)u}\bigg)p_{gq\leftarrow q}(u)\bigg)\bigg(\frac{1}{v-a}\bigg)_{+v[\underline{a},1]} \\ & - \frac{2(1-v)}{(1-a)^2(1-u)} + \frac{(v-a-2(1-a)u)}{(1-a)^2(1-u)}\Bigg[\log\bigg(\frac{(1-u)(1-v)}{(1-a)u}\bigg) \\ & + \log\bigg(\frac{v-a}{1-a}\bigg) - 1\Bigg] + \frac{1}{v-a}\log\bigg(\frac{1-v}{1-a}\bigg)p_{gq\leftarrow q}(u)\Bigg]\Bigg\}, \end{aligned} \tag{A.2}$$

$$D^{(1)}_{1gq,M} = \frac{T_F}{2}\Bigg\{\Bigg[\frac{\pi^2}{6} + 2(1-u)u\bigg(\log\bigg(\frac{1-u}{u}\bigg) - 1 - \frac{\pi^2}{6}\bigg)$$

$$
\begin{aligned}
&+\frac{\log(\frac{1-u}{u})^2 p_{\bar{q}q\leftarrow g}(u)}{2}\Big]\big(\delta(v-a)\delta(1-w)+\delta(1-v)\delta(w)\big)\\
&+\Big[-\frac{2(1+a-2v)\log(1-a)}{(1-a)^2}\\
&-\frac{2}{1-a}\left(\log\left(\frac{1-u}{(1-a)u}\right)+\log\left(\frac{1-v}{1-a}\right)+\log\left(\frac{v-a}{1-a}\right)-1\right)\\
&+\left(\frac{1}{v-a}\log\left(\frac{1-v}{1-a}\right)+\frac{1}{1-v}\log\left(\frac{v-a}{1-a}\right)\right)p_{\bar{q}q\leftarrow g}(u)\\
&+\left(2(1-u)u+\log\left(\frac{1-u}{(1-a)u}\right)p_{\bar{q}q\leftarrow g}(u)\right)\\
&\times\left(\left(\frac{1}{1-v}\right)_{+v[a,\underline{1}]}+\left(\frac{1}{v-a}\right)_{+v[\underline{a},1]}\right)\\
&+p_{\bar{q}q\leftarrow g}(u)\left(\left(\frac{\log(1-v)}{1-v}\right)_{+v[a,\underline{1}]}\right.\\
&\left.+\left(\frac{\log(-a+v)}{v-a}\right)_{+v[\underline{a},1]}\right)\Big]\delta(w_r-w)\Big\},
\end{aligned}
\tag{A.3}
$$

$$
\begin{aligned}
D^{(1)}_{2qq,M}=C_F\Big\{\Big[&-\frac{2u(1-x_B)+x_B}{u-x_B}-\frac{1+u}{2(1-v)}\log\left(\frac{v-a}{1-a}\right)\\
&+\frac{1+v}{2(1-u)}\log\left(\frac{u-x_B}{1-x_B}\right)+\frac{1}{2}\left(\frac{2u^2(1-x_B)}{(u-x_B)^2}-\frac{u(1+v)(1-x_B)x_B}{(u-x_B)^2}\right.\\
&\left.-\frac{(1+v)x_B}{u-x_B}\right)\left(1+\log\left(\frac{(1-u)(1-v)}{(1-a)v}\right)+\log\left(\frac{v-a}{1-a}\right)\right)\\
&+\frac{1+v^2}{2(1-u)(1-v)}\log\left(\frac{v-a}{(1-a)v}\right)-\frac{1+v}{2}\left(\frac{\log(1-u)}{1-u}\right)_{+u[0,\underline{1}]}\\
&-\frac{1+u}{2}\left(\frac{\log(1-v)}{1-v}\right)_{+v[0,\underline{1}]}+\left(\frac{1-u}{2}-\frac{(1+u)\log(1-u)}{2}\right.\\
&\left.-\frac{(1+u^2)}{2(1-u)}\log\left(\frac{u-x_B}{1-x_B}\right)+\left(\frac{\log(1-u)}{1-u}\right)_{+u[0,\underline{1}]}\right)\left(\frac{1}{1-v}\right)_{+v[0,\underline{1}]}\\
&+\left(\frac{1-v}{2}-\frac{(1+v)\log(1-v)}{2}+\frac{(1+v^2)\log(v)}{2(1-v)}\right.\\
&\left.+\left(\frac{\log(1-v)}{1-v}\right)_{+v[0,\underline{1}]}\right)\left(\frac{1}{1-u}\right)_{+u[0,\underline{1}]}\Big]\delta(w_r-w)\\
&+\Big[\left(-\frac{\pi^2}{12}(1+v)+\frac{\log(1-v)\log(v)}{1-v}+\frac{\log(v)^2}{2(1-v)}\right.\\
&+\frac{(1-v)\log((1-v)v)}{2}-\frac{(1+v)\log((1-v)v)^2}{4}+\frac{\pi^2}{6}\left(\frac{1}{1-v}\right)_{+v[0,\underline{1}]}
\end{aligned}
$$

$$+\frac{1}{2}\left(\frac{\log(1-v)^2}{1-v}\right)_{+v[0,\underline{1}]}\Bigg)\delta(1-u)+\Bigg(-\frac{\pi^2}{12}(1+u)$$

$$-\frac{\log(1-u)}{1-u}\log\left(\frac{u-x_B}{1-x_B}\right)+\frac{1}{2(1-u)}\log\left(\frac{u-x_B}{1-x_B}\right)^2$$

$$+\frac{(1-u)}{2}\log\left(\frac{1-u}{(1-a)u}\right)+\frac{(1+u)}{4}\log\left(\frac{1-u}{(1-a)u}\right)^2$$

$$+\frac{\pi^2}{6}\left(\frac{1}{1-u}\right)_{+u[0,\underline{1}]}+\frac{1}{2}\left(\frac{\log(1-u)^2}{1-u}\right)_{+u[0,\underline{1}]}\Bigg)\delta(1-v)$$

$$+\delta(1-u)\delta(1-v)\left(8+\frac{\pi^2}{4}-2\zeta(3)\right)\Bigg]\delta(w)\Bigg\}, \qquad \text{(A.4)}$$

$$D^{(1)}_{2qg,M}=\frac{C_F}{2}\Bigg\{\Bigg[v\log\big((1-v)v\big)+\left(\frac{\pi^2}{6}+\frac{\log((1-v)v)^2}{2}\right)p_{g\leftarrow q}(v)\Bigg]$$

$$\times\delta(1-u)\delta(w)+\delta(w_r-w)\Bigg[p_{g\leftarrow q}(v)\left(\frac{\log(1-u)}{1-u}\right)_{u[0,\underline{1}]}$$

$$+\big(v-\log\big((1-v)v\big)p_{g\leftarrow q}(v)\big)\left(\frac{1}{1-u}\right)_{u[0,\underline{1}]}+\frac{(1-u)u^2}{(v-a)(u-x_B)^2}$$

$$-\frac{2u^2v(1-x_B)}{(v-a)(u-x_B)^2}+\frac{vx_B}{u(v-a)(1-x_B)}+\frac{uv^2(1-x_B)x_B}{(v-a)(u-x_B)^2}$$

$$+\frac{v^2x_B}{(v-a)(u-x_B)}-\Bigg(-4u+\frac{u(2+uv^2)}{1-u+uv}$$

$$-\frac{1+u^2}{(v-a)(1-u+uv)(1-x_B)}+\frac{(1-u)u^2(1-v)}{(u-x_B)^2}-\frac{(1-2u)u}{u-x_B}$$

$$+\frac{2u^2(1-v)}{u-x_B}+\frac{1+uv}{1-u+uv}p_{g\leftarrow q}(v)\Bigg)$$

$$\times\left(\log\left(\frac{(1-u)(1-v)}{(1-a)u}\right)+\log\left(\frac{v-a}{1-a}\right)\right)$$

$$+\frac{1}{1-u}\left(\log\left(\frac{v-a}{(1-a)v}\right)-\log\left(\frac{u-x_B}{1-x_B}\right)\right)p_{g\leftarrow q}(v)\Bigg]\Bigg\}, \qquad \text{(A.5)}$$

$$D^{(1)}_{2gq,M}=T_F\Bigg\{\Bigg[(1-u)u\left(\log\left(\frac{1-u}{(1-a)u}\right)-1\right)$$

$$+\frac{1}{4}\left(\frac{\pi^2}{3}+\log\left(\frac{1-u}{(1-a)u}\right)^2\right)p_{\bar{q}q\leftarrow g}(u)\Bigg]\delta(1-v)\delta(w)$$

$$+\delta(wr-w)\Bigg[\frac{1}{2}\left(\frac{\log(1-v)}{1-v}\right)_{v[0,\underline{1}]}p_{\bar{q}q\leftarrow g}(u)$$

$$+\left((1-u)u+\frac{1}{2}p_{\bar{q}q\leftarrow g}(u)\log\left(\frac{1-u}{(1-a)u}\right)\right)\left(\frac{1}{1-v}\right)_{v[0,\underline{1}]}$$

$$+\frac{1}{2(v-a)}+\left(\frac{1}{2(v-a)}p_{\bar{q}q\leftarrow g}(u)-\frac{1}{1-a}\right)\left(\log\left(\frac{(1-u)(1-v)}{(1-a)u}\right)\right.$$
$$\left.+\log\left(\frac{v-a}{1-a}\right)-1\right)+\frac{1}{2(1-v)}\log\left(\frac{v-a}{1-a}\right)p_{\bar{q}q\leftarrow g}(u)\Bigg]\Bigg\}, \tag{A.6}$$

where

$$p_{gq\leftarrow q}(x)=2\frac{1}{1-x}-1-x, \tag{A.7}$$

$$p_{\bar{q}q\leftarrow g}(x)=1-2x+2x^2, \tag{A.8}$$

$$p_{g\leftarrow q}(x)=\frac{2}{x}-2+x. \tag{A.9}$$

Appendix B

The redefinition of fracture functions can only factorize singularities in the forward region. In the hadronic tensor these singularities show up after the angular integration as $(1-w)^{-1+\epsilon}$ factors which have to be prescribed as explained in Section 3. As we mentioned there, special care has to be taken with overlapping singularities. In this case, inspection of Fig. 6 shows that the only problematic configurations are terms singular along $w=1$ and $w=w_r$ in regions B0 and B1. Using the procedure described in Section 3 we obtained suitable prescriptions in both regions:

$$(1-w)^{-1+\epsilon_1}(w_r-w)^{-1+\epsilon_2}$$
$$\xrightarrow{\text{B0}}\frac{1}{\epsilon_1(\epsilon_1+\epsilon_2)}\frac{\Gamma(1+\epsilon_1)\Gamma(1-\epsilon_1-\epsilon_2)}{\Gamma(1-\epsilon_2)}\delta(1-w)\delta(a-v)(a-z)^{\epsilon_1+\epsilon_2}$$
$$\times\big(a(1-a)\big)^{1-\epsilon_1-\epsilon_2}+\frac{1}{\epsilon_1}\delta(1-w)\big((a-v)^{-1+\epsilon_1+\epsilon_2}\big)_{+v[z,\underline{a}]}$$
$$\times\big(v(1-a)\big)^{1-\epsilon_1-\epsilon_2}w_r^{-\epsilon_1}{}_2F_1\left[\epsilon_1,\epsilon_1+\epsilon_2,1+\epsilon_1;\frac{1}{w_r}\right]$$
$$+\big((1-w)^{-1+\epsilon_1}(w_r-w)^{-1+\epsilon_2}\big)_{+w[0,\underline{1}]}, \tag{B.1}$$

$$(1-w)^{-1+\epsilon_1}(w_r-w)^{-1+\epsilon_2}$$
$$\xrightarrow{\text{B1}}\frac{1}{\epsilon_2(\epsilon_1+\epsilon_2)}\frac{\Gamma(1+\epsilon_2)\Gamma(1-\epsilon_1-\epsilon_2)}{\Gamma(1-\epsilon_1)}\delta(1-w)\delta(v-a)(1-a)^{\epsilon_1+\epsilon_2}$$
$$\times\big(a(1-a)\big)^{1-\epsilon_1-\epsilon_2}+\frac{1}{\epsilon_2}\delta(w_r-w)\big((v-a)^{-1+\epsilon_1+\epsilon_2}\big)_{+v[\underline{a},1]}$$
$$\times\big(v(1-a)\big)^{1-\epsilon_1-\epsilon_2}w_r^{-\epsilon_2}{}_2F_1[\epsilon_2,\epsilon_1+\epsilon_2,1+\epsilon_2;w_r]$$
$$+\big((1-w)^{-1+\epsilon_1}(w_r-w)^{-1+\epsilon_2}\big)_{+w[0,\underline{w_r}]}, \tag{B.2}$$

where ϵ_1 and ϵ_2 are multiples of the regulator ϵ. Notice that these expressions are valid to all orders in ϵ. Terms singular only along $w=1$ can be prescribed using the standard rule in Eq. (25). Terms singular in $w=1$ and $w=0$ can be managed by partial fractioning:

$$\frac{1}{1-w}\frac{1}{w}=\frac{1}{1-w}+\frac{1}{w} \tag{B.3}$$

and be prescribed also as in Eq. (25).

Appendix C

The singular pieces of the order α_s^2 partonic cross sections $\hat{\sigma}_{gg}$ and $\hat{\sigma}_{gq}$ in region B0 can be written as

$$\begin{aligned} d\hat{\sigma}^{(2)}_{gg,M}\big|_{\mathrm{B0}} = \sum_q c_q\, C_\epsilon^2 \bigg\{ & \frac{1}{\epsilon^2}\big[8P^{(0)}_{q\leftarrow g}(u)P^{(0)}_{g\leftarrow q}(v)\delta(w) \\ & + 4\big(P^{(0)}_{q\leftarrow g}(u)\otimes \widetilde{P}^{(0)}_{gq\leftarrow q}(u,v) + P^{(0)}_{g\leftarrow q}(v)\otimes \widetilde{P}^{(0)}_{\bar{q}q\leftarrow g}(u,v) \\ & + P^{(0)}_{q\leftarrow g}(u)\otimes' \widetilde{P}^{(0)}_{gg\leftarrow g}(u,v)\big)\delta(1-w)\big] + \frac{1}{\epsilon}\big[2P^{(1)}_{gq\leftarrow g}(u,v)\delta(1-w) \\ & + 2P^{(0)}_{q\leftarrow g}(u)\otimes C^{(1)}_{1qg,M}(u,v,w) + 2P^{(0)}_{g\leftarrow q}(v)\otimes C^{(1)}_{1gq,M}(u,v,w) \\ & + 2\frac{1-x_B}{x_B}\widetilde{P}^{(0)}_{gg\leftarrow g}(u,v)\otimes' C^{(1)}_{g,M}(u)\delta(1-w)\big] + \mathcal{O}\big(\epsilon^0\big)\bigg\}, \end{aligned} \tag{C.1}$$

$$\begin{aligned} d\hat{\sigma}^{(2)}_{gq,M}\big|_{\mathrm{B0}} = c_q C_\epsilon^2 \bigg\{ & \frac{1}{\epsilon^2}\bigg[2\bigg(\widetilde{P}^{(0)}_{\bar{q}q\leftarrow g}(u,v)\otimes' P^{(0)}_{q\leftarrow q}(u) + \widetilde{P}^{(0)}_{\bar{q}q\leftarrow g}(u,v)\otimes P^{(0)}_{q\leftarrow q}(v) \\ & + \widetilde{P}^{(0)}_{\bar{q}q\leftarrow g}(u,v)\otimes P^{(0)}_{g\leftarrow g}(u) - \frac{1}{2}\beta_0\widetilde{P}^{(0)}_{\bar{q}q\leftarrow g}(u,v)\bigg)\delta(1-w) \\ & + 4P^{(0)}_{q\leftarrow g}(u)P^{(0)}_{q\leftarrow q}(v)\delta(w)\bigg] + \frac{1}{\epsilon}\bigg[P^{(1)}_{\bar{q}q\leftarrow g}(u,v)\delta(1-w) \\ & - \beta_0 C^{(1)}_{1gq,M}(u,v,w) + 2P^{(0)}_{g\leftarrow g}(u)\otimes C^{(1)}_{1gq,M}(u,v,w) \\ & + 2P^{(0)}_{q\leftarrow q}(v)\otimes C^{(1)}_{1gq,M}(u,v,w) \\ & + 2\frac{1-x_B}{x_B}\widetilde{P}^{(0)}_{\bar{q}q\leftarrow g}(u,v)\otimes' C^{(1)}_{q,M}(u)\delta(1-w)\bigg] + \mathcal{O}\big(\epsilon^0\big)\bigg\}, \end{aligned} \tag{C.2}$$

where the $\widetilde{P}^{(0)}_{ki\leftarrow j}(u,v)$ are defined in Eq. (39) and the functions $C^{(1)}_{i,M}(u)$ are the coefficient functions of totally inclusive DIS at $\mathcal{O}(\alpha_s)$, they can be found in Refs. [2,10]. Convolutions between kernels are as in Eq. (38) with the replacement $v\to(1-x_B)v/x_B$ whereas the convolutions between kernels and coefficient functions are given by

$$P_{i\leftarrow j}(u)\otimes C(u,v,w) = \int_{\frac{x_B}{x_B+(1-x_B)v}}^{\frac{x_B}{x_B+(1-x_B)z}} \frac{d\bar{u}}{\bar{u}}P_{i\leftarrow j}\left(\frac{u}{\bar{u}}\right)C\big(\bar{u},v,w\big),$$

$$P_{i\leftarrow j}(v)\otimes C(u,v,w)=\int\limits_{a(u)}^{1}\frac{d\bar{v}}{v}P_{i\leftarrow j}\left(\frac{v}{\bar{v}}\right)C\big(u,\bar{v},w\big),\tag{C.3}$$

$$\widetilde{P}_{ki\leftarrow j}(u,v)\otimes' C(u)=\int\limits_{\frac{x_B-uv(1-x_B)}{x_B}}^{1}\frac{u}{\bar{u}}\frac{d\bar{u}}{\bar{u}}\widetilde{P}_{ki\leftarrow j}\left(\bar{u},\frac{vu(1-x_B)}{x_B\bar{u}}\right)C\left(\frac{u}{\bar{u}},v,w\right).\tag{C.4}$$

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