

The longitudinal and transverse nuclear responses within the RPA framework

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Abstract

The longitudinal and transverse nuclear responses to inclusive electron scattering reactions are analysed within the random phase approximation (RPA) framework. Several residual interactions are considered and it is shown that the exchange terms in the RPA make it very difficult to find an effective residual interaction capable of reproducing simultaneously the quasielastic peak of both the longitudinal and transverse responses. By means of a simplified model it is illustrated that the residual interaction used in a ring approximation must fulfil some restrictions in order to qualitatively reproduce the full RPA results.

1. Introduction

Since the experimental separation of the electromagnetic longitudinal and transverse response functions [1–5], the simultaneous description of both responses in medium and heavy nuclei remains an open problem. The experimental longitudinal response is overestimated and the transverse one is underestimated. A large body of theoretical work has been done in order to solve this puzzle. Let us briefly mention that one point of view ascribes the problem to modifications of the nucleon properties within the nuclear medium (see [6]), while another perspective, followed in the present work, is based on the many-body theory performed in finite nuclei (see [7] and references therein) or in nuclear matter [8–18]. Fortunately, surface effects for heavy and medium nuclei in the energy–momentum region of interest are not very important and the nuclear matter formalism is a good approximation for the nuclear system, once an appropriate Fermi momentum or the local density approximation is used.

It should be mentioned that the many-body problem is very complex: initial and final state interactions, meson exchange currents, etc, should be considered, which implies a huge numerical effort, even in nuclear matter. Taking into consideration all these effects in both channels, Gil *et al* [18], give results for two momentum transfers: $q = 300 \text{ MeV}/c$ for ^{12}C

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and $q = 410 \text{ MeV}/c$ for ^{40}Ca . The last result is shown for energies ranging between 40 and 220 MeV. Even if there is a small tendency to overestimate data, these results certainly represent an improvement over the previous ones. In the recent work of [14] we report results for the ^{40}Ca longitudinal response, at momentum transfers ranging from 300 up to 500 MeV/ c , in good agreement with data. In that work, a full antisymmetric second random phase approximation (SRPA) with the explicit inclusion of the $\Delta(1232)$ was developed. In spite of the complexity of the nuclear models explored, one subject seems to have been overlooked: the choice of the nuclear force. Unfortunately, due to the numerical difficulties of these methods, any attempt to adjust the nuclear interaction using these schemes is a rather involved task.

In this work we study both responses within the random phase approximation (RPA) framework in non-relativistic nuclear matter. In the past, most of the calculations have been performed in the so-called ring approximation (RA). In fact, both the RA and RPA account for the excitations of particle-hole type that can be induced by the electromagnetic probe but, formally, the RA is only the direct contribution of the RPA. It is only recently that two different procedures to evaluate the nuclear responses in the RPA scheme are available for a general finite-range effective interaction [11, 17], and the importance of exchange terms for moderated transferred momenta has been pointed out.

Actually, the exchange terms can be incorporated approximately within the context of the RA, using a contact interaction which accounts implicitly for the antisymmetrization effects. In fact, it has been common to adjust the effective nucleon-nucleon interaction to reproduce both the longitudinal and transverse nuclear responses in the quasielastic peak within the RA. Note that, within the RA, different pieces of the interaction act either in the longitudinal or in the transverse response, facilitating the extraction of the parameters of the interaction for a certain momentum transfer. However, as we will point out in this paper, the parameters of this force should fulfil certain restrictions imposed by the Pauli principle if one wants to recover the original effective force that should be the one to be used in the calculation of additional corrections beyond RPA. These restrictions are not usually obeyed by the effective interactions found in the literature.

In our opinion, any sensible approximation to the nuclear response should at least contain the RPA correlations. In this spirit, a good choice for the effective NN force would be one that, at the level of RPA, gives already an acceptable description of the location of both the longitudinal and transverse quasielastic peaks. This would define an excellent starting point beyond which more sophisticated many-body correlations could be implemented later on. In section 2 we give the details of the nuclear residual interactions used in this work. These forces have been selected to cover the wide variety of interactions found in the literature to specifically study the nuclear response. The results, presented in section 3, show that none of the interactions explored is capable of giving a good simultaneous description of the peaks of the longitudinal and transverse nuclear responses within the RPA. The origin of this failure is studied in section 4 through a simplified model that exploits the relation between the RA and the RPA schemes for contact interactions. We also point out in that section the restrictions that the Pauli principle imposes over a contact interaction to be used in an RA scheme. Finally, our conclusions are given in section 5.

2. Employed models for the nuclear interaction

As already mentioned, finding a force that gives a reasonable account of the basic features of both the longitudinal and the transverse responses within the RPA, would be particularly interesting to define a proper framework beyond which other excitations, such as ground state correlations, final state interactions and meson exchange currents, are included. In the present

work we have explored several representative interactions commonly used in the literature and, as we will show, no one is able to give a simultaneous satisfactory description of both responses at various momentum transfer values. The reason lies in the fact that, as opposed to the RA, the full interaction is present in the RPA-exchange terms and in addition these terms are very important. Therefore the adjustment of one channel affects the other one and this means that reproducing both channels, longitudinal and transverse, is a far more complex task.

On the other hand, it has been usual to employ a modified residual interaction in the RA scheme trying to account for exchange terms. Another purpose of the present work is to show by making use of a simplified model that, while the RA is able to keep the qualitative trends of the complete RPA response, it is unable to provide a quantitative account of the exchange terms tied to the finite-range part of the force. In addition, the qualitative agreement can only be achieved if some simple constraints over the parameters of the interaction are observed, as will be shown below.

The longitudinal and transverse pieces of the RPA response are calculated following the scheme of [11], but using an HF basis according to the prescriptions given in [16], where the effect of the HF self-energy is adjusted by means of a set of two effective masses (one for particles and the other one for holes) and an energy shift. Each set is chosen to reproduce the exact HF response and depends only on the momentum transferred by the electron. Figure 1 of [11] shows the direct and exchange Goldstone diagrams which contribute to the RPA.

The RPA calculation is performed for three different interactions. The first one, labelled V^I , is described in [9] and contains the exchange of the mesons π , ρ and ω plus a g' term. It is shown there that the overall contribution of the non-renormalized isoscalar ω meson contribution is small.

The structure of the second one has been widely used in the literature and consists of contact terms plus a $(\pi + \rho)$ -meson exchange interaction given by,

$$V^{\text{II}}(k) = \frac{f_\pi^2}{\mu_\pi^2} (f_0 + f'_0 \boldsymbol{\tau} \cdot \boldsymbol{\tau}' + g_0 \boldsymbol{\sigma} \cdot \boldsymbol{\sigma}' + g'_0 \boldsymbol{\tau} \cdot \boldsymbol{\tau}' \boldsymbol{\sigma} \cdot \boldsymbol{\sigma}' + V_\pi(k) + V_\rho(k)) \quad (1)$$

with

$$V_\pi(k) = -\Gamma_\pi^2(k) \frac{k^2}{k^2 + \mu_\pi^2} \boldsymbol{\sigma} \cdot \widehat{\mathbf{k}} \boldsymbol{\sigma}' \cdot \widehat{\mathbf{k}} \boldsymbol{\tau} \cdot \boldsymbol{\tau}', \quad (2)$$

$$V_\rho(k) = -C_\rho \Gamma_\rho^2(k) \frac{k^2}{k^2 + \mu_\rho^2} (\boldsymbol{\sigma} \times \widehat{\mathbf{k}}) \cdot (\boldsymbol{\sigma}' \times \widehat{\mathbf{k}}) \boldsymbol{\tau} \cdot \boldsymbol{\tau}', \quad (3)$$

where μ_π (μ_ρ) is the pion (rho) rest mass and $C_\rho = 2.3$. For the form factor of the πNN (ρNN) vertex we have taken

$$\Gamma_{\pi,\rho}(k) = \frac{\Lambda_{\pi,\rho}^2 - \mu_{\pi,\rho}^2}{\Lambda_{\pi,\rho}^2 + k^2}, \quad (4)$$

with $\Lambda_\pi = 1.3$ GeV and $\Lambda_\rho = 1.75$ GeV. The momentum transferred by the interaction is k and the static limit of the $(\pi + \rho)$ -meson exchange interaction has been taken. The parameters f_0 , f'_0 , g_0 , g'_0 of this force have been left free so as to reproduce both the longitudinal and transverse responses. However, the presence of all terms of the interaction in the exchange contributions makes it very difficult. An acceptable reproduction of both channels, especially the longitudinal one, for momentum transfer $q = 410$ MeV/ c is found with the values $f_0 = -0.1$, $f'_0 = 0.2$, $g_0 = 0$ and $g'_0 = 0.5$.

Finally, we consider the interaction of [16] which utilizes the major components of the Bonn potential, namely the exchange of π , ρ , σ and ω mesons. We note that being a bare

Table 1. Effective masses (m^*/m) and energy shifts ($\Delta\omega$) for the different residual interactions described in the text and for two values of the momentum transfer.

Interaction	m^*/m		$\Delta\omega$ (MeV)
	Particle	Hole	
$q = 300$ MeV/ c			
V^I	0.97	0.85	6
V^{II}	0.83	0.67	7
V^{III}	0.75	0.65	9
$q = 410$ MeV/ c			
V^I	1.0	0.85	7
V^{II}	0.85	0.67	9
V^{III}	0.78	0.65	11

interaction, this force, which will be referred to as V^{III} , does not incorporate short-range correlation effects. These short-range correlations could be generated via the summation of ladder diagrams up to infinite order (G -matrix).

For each interaction, the effective masses and energy shifts necessary to calculate the HF responses are shown in table 1.

3. RA and RPA results

In figures 1 and 2, we present the RPA results (solid lines) for a momentum transfer of $q = 300$ MeV/ c and $q = 410$ MeV/ c , respectively, and for the three interaction models. Our results are compared with the data on ^{40}Ca [5]. We also present the HF (long-dashed lines) and the RA (short-dashed lines) responses. We note that the latter is obtained from the ‘contact’ interaction model that already incorporates the exchange terms in an approximated way, as explained in the next section.

In all the calculations the Fermi momentum is taken to be $k_F = 235$ MeV/ c , which is an appropriate value to simulate the results in finite nuclei [19]. The HF response is in fact qualitatively similar to the free Lindhard function but it is hardened by the presence of the single particle spectrum, which increases the energy of particle–hole excitations.

The behaviour of the RPA responses in the longitudinal and transverse channels depends on the interaction considered. For the V^I and V^{II} interactions, when the position of the RPA peak for one channel is moved towards higher energies with respect to the peak in the HF response, the peak in the other channel moves in the opposite direction. This particular feature is a consequence of the exchange terms in the RPA. Only the V^{III} interaction gives a hardening of the response in both channels. We also observe that the interaction V^{II} , whose parameters have been adjusted to account for the response at $q = 410$ MeV/ c , produces a response at $q = 300$ MeV/ c in poor agreement with the experimental results. This means that the interaction should have a richer momentum dependence than just the one provided by π - and ρ -meson exchange.

We further discuss these results in the next section, after analysing the behaviour of the RA and RPA responses with a simple contact interaction model.

4. A simplified model: the contact interaction

In order to clarify the behaviour shown in the last section, let us consider the simpler case of a contact interaction V_C . This has the advantage that the RPA response is calculated as the

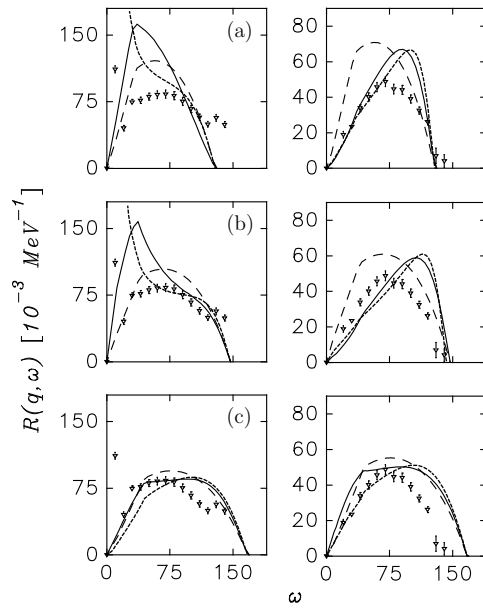


Figure 1. Response function for ^{40}Ca at momentum transfer $q = 300 \text{ MeV}/c$. The panels on the lhs show the longitudinal response and those on the rhs show the transverse one. Results are shown for the HF (long-dashed line), the RA (short-dashed line) and the RPA responses (solid line). Results in rows (a), (b) and (c) are obtained with the V^I , V^{II} and V^{III} interactions, respectively. The experimental data were taken from [5].

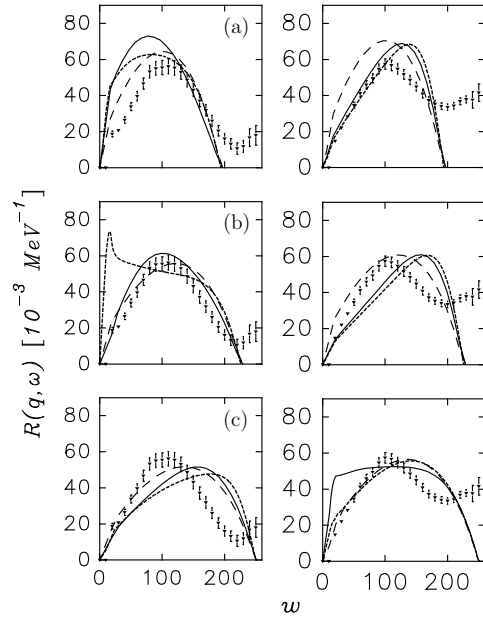


Figure 2. Same as figure 1, but for a momentum transfer $q = 410 \text{ MeV}/c$.

RA one with redefined values for the parameters of the interaction. The non-antisymmetrized interaction can be written as

$$V_C = \frac{f_\pi^2}{\mu_\pi^2} (f + f' \boldsymbol{\tau} \cdot \boldsymbol{\tau}' + g \boldsymbol{\sigma} \cdot \boldsymbol{\sigma}' + g' \boldsymbol{\tau} \cdot \boldsymbol{\tau}' \boldsymbol{\sigma} \cdot \boldsymbol{\sigma}' + h \boldsymbol{\sigma} \cdot \hat{\boldsymbol{q}} \boldsymbol{\sigma}' \cdot \hat{\boldsymbol{q}} + h' \boldsymbol{\sigma} \cdot \hat{\boldsymbol{q}} \boldsymbol{\sigma}' \cdot \hat{\boldsymbol{q}} \boldsymbol{\tau} \cdot \boldsymbol{\tau}') \quad (5)$$

where f, f', g, g', h and h' are all constants. We have preferred to use the spin longitudinal terms h and h' instead of the tensor terms (proportional to S_{12}). This is because, at variance with S_{12} , the spin longitudinal terms do not interfere with the vector terms g and g' in the RA.

This interaction is now used to evaluate direct and exchange terms in the RPA. As mentioned, for such an interaction it is possible to account for exchange terms by a redefinition of these constants as follows:

$$\begin{aligned} f_{\text{ant}} &= f - (f + 3f' + 3g + 9g' + h + 3h')/4 \\ f'_{\text{ant}} &= f' - (f - f' + 3g - 3g' + h - h')/4 \\ g_{\text{ant}} &= g - (f + 3f' - g - 3g' - h - 3h')/4 \\ g'_{\text{ant}} &= g' - (f - f' - g + g' - h + h')/4 \\ h_{\text{ant}} &= h - (h + 3h')/2 \\ h'_{\text{ant}} &= h' - (h - h')/2. \end{aligned} \quad (6)$$

The first term in the rhs of each equation is the direct contribution, while the terms between parentheses come from the action of the exchange operator over the interaction. It is important to keep in mind that solving the RA with this new set of parameters is equivalent to solving the RPA with the original contact interaction of equation (5). At variance with the direct case, only three of the parameters $f_{\text{ant}}, h'_{\text{ant}}, g'_{\text{ant}}$ are independent. They are related through the following relations,

$$\begin{aligned} g'_{\text{ant}} &= -(f_{\text{ant}} + h'_{\text{ant}})/3 \\ g_{\text{ant}} &= -(2f_{\text{ant}} - h'_{\text{ant}})/3 - f'_{\text{ant}} \\ h_{\text{ant}} &= -h'_{\text{ant}}, \end{aligned} \quad (7)$$

which are obtained by simply solving the system (6). Note that the spin longitudinal terms are usually assumed to be proportional to the momentum transfer and they cancel in the Landau limit. Formally, equation (5) with $h = h' = 0$, can be viewed as the zeroth order in the Legendre expansion of the Landau–Migdal interaction. In this sense, equations (7) are a particular case of the more general sum rule results of [20].

Within the RA, f_{ant} and f'_{ant} govern the longitudinal response with the same weight, while for the transverse one only g'_{ant} is relevant (g_{ant} also gives a contribution, but of the order $\mu_s^2/\mu_v^2 \approx 0.035$ with respect to that of g'_{ant}). For this reason, finding a set of parameters that gives a reasonable description of the quasielastic peak for both the longitudinal and transverse responses in an RA scheme is a relatively easy task. However, it is important to emphasize here that the coefficients of the different terms of any force that accounts effectively for exchange terms in RA should obey to a high degree the restrictions imposed by relations (7), which is not usually the case of the interactions found in the literature.

Now we turn back to the analysis of the RPA response with a general finite-range interaction. For direct RPA terms, the external momentum fixes the momentum of the particle–hole interaction. This means that, for direct RPA terms, the interaction behaves as a contact one for each momentum transfer. In table 2 we have extracted the ‘contact’ terms of the interactions V^{I-III} for $q = 300 \text{ MeV}/c$ and $q = 410 \text{ MeV}/c$. Working as if these parameters belonged to a contact interaction we have obtained, using equations (6),

Table 2. For a fixed momentum the interaction for direct RPA terms is a constant. Columns $f-h'$ are the values of these constants for the interactions $V^{\text{I-III}}$ described in the text. Columns $f_{\text{ant}}-h'_{\text{ant}}$ result from applying equations (6) using the values of columns $f-h'$ as direct contributions.

$q = 300 \text{ MeV}/c$						
Interaction	f	f'	g	g'	h	h'
V^{I}	0.000	0.000	-0.094	0.375	0.094	-0.465
V^{II}	-0.100	0.200	0.000	0.351	0.000	-0.578
V^{III}	-0.630	0.085	-0.085	-0.163	0.085	-0.570
	f_{ant}	f'_{ant}	g_{ant}	g'_{ant}	h_{ant}	h'_{ant}
V^{I}	-0.448	0.212	-0.162	0.397	0.744	-0.744
V^{II}	-0.581	0.394	-0.295	0.483	0.868	-0.868
V^{III}	0.300	0.042	-0.541	0.199	0.898	-0.898
$q = 410 \text{ MeV}/c$						
Interaction	f	f'	g	g'	h	h'
V^{I}	0.000	0.000	-0.148	0.282	0.148	-0.386
V^{II}	-0.100	0.200	0.000	0.267	0.000	-0.493
V^{III}	-0.442	0.041	-0.134	-0.251	0.134	-0.482
	f_{ant}	f'_{ant}	g_{ant}	g'_{ant}	h_{ant}	h'_{ant}
V^{I}	-0.270	0.189	-0.226	0.308	0.653	-0.653
V^{II}	-0.455	0.352	-0.295	0.398	0.739	-0.739
V^{III}	0.630	-0.080	-0.603	0.053	0.789	-0.789

the corresponding values $f_{\text{ant}}-h'_{\text{ant}}$, also shown in table 2. We have then performed an RA calculation with these values and the results are given by the short-dashed lines in figures 1 and 2. This simple model illustrates that if we represent a general interaction by a contact one by ignoring the momentum dependence of the finite-range terms, the corresponding RPA, which is in fact an RA with the parameters redefined according to equations (6), gives a qualitative agreement with the results obtained using the complete interaction, displayed by the solid lines in figures 1 and 2. The qualitative similarity refers here to the fact that the positions of the peaks of both longitudinal and transverse channels are moved, within the RA scheme using a modified antisymmetrized interaction, towards the same direction as in the case of the complete RPA calculation. However, we observe some quantitative differences between both methods, which are tied to the momentum dependence of the interaction in the exchange terms. This emphasizes the importance of performing an explicit evaluation of the exchange terms within an RPA scheme before other many-body effects, usually more difficult to handle, are considered.

5. Conclusions

We can summarize our results by stating that the momentum dependence of the interaction in the exchange terms gives rise to quantitative differences between the RPA and the RA responses, the latter being calculated with the antisymmetrized contact version of the original force. This makes it advisable to use the RPA with the complete interaction before attempting more complicated studies of other types of correlations. Correlations beyond RPA are certainly necessary since, to the best of our knowledge, no work is able to reproduce the quasielastic

response at any momentum transfer for both longitudinal and transverse channels, using the same interaction for medium and heavy nuclei. We too have unsuccessfully attempted to find one interaction that at least reproduces the position of the peaks for both channels within the RPA framework.

Another observation of the present work is that if one still wants to account for exchange terms within a simpler RA approach one should make sure that the effective interaction is such that relations (7) are preserved to a high degree, since only in this case the RPA and RA are in qualitative agreement. The implementation of these relations is as simple as the RA itself. The capability of the RA to account approximately for exchange terms is a significant observation, because of the extreme simplicity of the ring propagator in nuclear matter, in contrast to the difficult evaluation of the full RPA.

It is also important to note that, while it is possible to find an interaction that adjusts the longitudinal and transverse channels in an RA scheme, it will not obey relations (7) and, therefore, the corresponding original direct interaction cannot be recovered. Such type of interactions cannot be blindly used in calculations of other types of many-body correlations that explicitly require the knowledge of the original interaction. In particular, one should avoid replacing the effective interaction by dressing it with the ring propagator if relations (7) are ignored.

Acknowledgments

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