On planar fermions with quartic interaction at finite temperature and density

D.C. Cabra[1,2] and G.L. Rossini[1]
[1] *Departamento de Física, Universidad Nacional de la Plata, C.C. 67, (1900) La Plata, Argentina.*
[2] *Facultad de Ingeniería, Universidad Nacional de Lomas de Zamora, Cno. de Cintura y Juan XXIII, (1832) Lomas de Zamora, Argentina.*
(November 19, 2018)

We study the breaking of parity symmetry in the 2+1 Gross-Neveu model at finite temperature with chemical potential μ, in the presence of an external magnetic field. We find that the requirement of gauge invariance, which is considered mandatory in the presence of gauge fields, breaks parity at any finite temperature and provides for dynamical mass generation, preventing symmetry restoration for any non-vanishing μ. The dynamical mass becomes negligibly small as temperature is raised. We comment on the relevance of our observation for the gap generation of nodal quasi-particles in the pseudo-gap phase of high T_c superconductors.

I. INTRODUCTION

The study of planar fermion systems has become an active area of research in the last years, on its own right and because of the many applications in strongly correlated electron systems as high T_c superconductors, quantum Hall effect systems, etc. In particular, in the last few years several papers [1] have analyzed the role of quasi-particles in transport experiments in cuprates in the under-doped regime in order to provide an explanation for the unusual behaviour observed in [2]. It was pointed there that under the influence of an external magnetic field (transverse to the copper-oxide planes) the thermal conductivity shows a plateau feature for magnetic fields larger than a critical value. One possible scenario to explain the appearance of such a plateau is the opening of a gap in the quasi-particle spectrum [1,3]. More recent experiments however seem to indicate the absence of the plateau in the thermal conductivity for sufficiently clean samples [4]. In the light of this new data, it is the purpose of the present paper to further discuss the issue of the opening of a gap in the theory describing the so-called nodal quasi-particles, in particular in the finite density case.

It is by now established that low temperature properties of planar high T_c superconductors are well described by a d-wave BCS theory [5]. Indeed, in a d-wave superconductor the energy gap (order parameter) vanishes at certain points on the Fermi surface (nodes) and hence low energy quasi-particle excitations are relevant for low temperature transport measurements. Among the quasi-particle excitations above the ground state we are interested in the behaviour of the nodal quasi-particles. These are well defined fermionic relativistic massless quasi-particles, which are assumed to have quartic self interactions [6].

Our main point in the present paper is that in the presence of an external magnetic field, fermionic fields have to be quantized in a gauge-invariant framework and hence a parity anomaly could naturally appear [7]. The parity breaking effect of course depends on the external field configuration and is trivial (unobservable) in the case of constant magnetic fields, unless a non vanishing chemical potential (μ) is considered. Previous analysis [1,3,8,9] have either considered the case with $\mu = 0$ or have duplicated the number of fermion components in such a way that the parity anomaly cancels out. Our analysis, though motivated by high T_c phenomenology, is applicable to any planar fermion system.

We show that, at finite quasi-particle density, the term that breaks parity in the effective action changes dramatically the analysis of the gap equation. In particular, the opening of a gap as a function of the external magnetic field occurs at any finite temperature, thus providing for dynamical mass generation and preventing symmetry restoration for any non-vanishing μ. It should be pointed that the effect of this correction is of order $1/T$ and the gapless behaviour is recovered at high temperatures.

II. THE MODEL

Let us consider the following (Euclidean) 3-dimensional fermionic Lagrangian with four fermion interaction

$$\mathcal{L} = \bar{\psi}^a(\partial\!\!\!/ + ie\, A\!\!\!/ + \gamma^0\mu)\psi^a + \frac{g}{2N}(\bar{\psi}^a\psi^a)^2, \tag{1}$$

where $a = 1, \cdots, N$ is a flavor index and the Fermi fields are in an irreducible two component spinor representation. This model field theory is known as the Gross-Neveu model. For the case of interest in d-wave superconductors, nodal quasi-particles are described by (1) with $N = 4$, corresponding to the four nodes of the energy gap. The gauge field A_ν ($\nu = 0, 1, 2$) represents an external background that for a constant transverse magnetic field B can be chosen as *e.g.* $A_0 = 0$, $A_i = -Bx_2\delta_{i1}$.

Apart from the usual gauge invariance $A_\nu \to A_\nu^{(\lambda)} = A_\nu + e^{-1}\partial_\nu\lambda$, $\psi \to \psi^{(\lambda)} = \exp(-i\lambda)\psi$, the theory defined by eq. (1) is at the classical level invariant under parity transformations, which are defined as

$$\begin{aligned} (x_0, x_1, x_2) &\to (x_0, -x_1, x_2)\ , \\ (A_0, A_1, A_2) &\to (A_0, -A_1, A_2) \\ \psi &\to \gamma_1\psi\ . \end{aligned} \tag{2}$$

Now, since in odd space-time dimensions the path integration measure cannot be defined in a way that preserves both gauge and parity invariance, a parity violating contribution can arise if one adopts a gauge invariant quantization. Indeed, from the definition of the partition function $\mathcal{Z}[A_\nu]$, if $\mathcal{Z}[A_\nu] = \mathcal{Z}[A_\nu^{(\lambda)}]$ one necessarily has to impose invariance of the fermionic measure under gauge transformations $\psi \to \exp(-i\lambda)\psi$. Due to the presence of an external magnetic field we consider mandatory to quantize the theory in a gauge invariant way, which then leads to the well known parity anomaly [7].

It is also known that the parity anomaly can be overcome by a slight change in the theory, consisting in the use of a suitable reducible four component spinor representation for the Fermi fields (as done in [10]). As we said before, this change not merely duplicates the number of components, but does also change the interaction term [3]. We analyze in the following the case in which an irreducible spinor representation and a gauge invariant regularization are chosen.

In order to study the opening of a gap in this system, which is associated with the breaking of parity symmetry, we use the $1/N$ standard procedure to compute the effective potential for the fermion system. One first introduces an auxiliary field σ trading the quartic interaction term for a linear σ vertex

$$\mathcal{L} = \bar{\psi}^a(\partial\!\!\!/ + ie\, A\!\!\!/ + \gamma^0\mu + \sigma)\psi^a - \frac{N}{2g}\sigma^2; \tag{3}$$

the equation of motion for σ sets the constraint

$$\sigma = \frac{g}{N}\bar{\psi}^a\psi^a. \tag{4}$$

Parity invariance of (3) at the classical level (or alternatively consistency of eq. (4)) requires that the field σ changes as a pseudo-scalar under parity,

$$\sigma \to -\sigma\ . \tag{5}$$

The breaking of parity symmetry at the quantum level would be now signaled by a non vanishing expectation value of the fermion condensate. In order to search for this effect to leading order in $1/N$ it is enough to consider constant values for σ.

The effective potential is defined as

$$V^{eff}_{\beta,\mu}[\sigma] \equiv -\frac{1}{\beta L^2}\log\left(\int \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp - \int_0^\beta d\tau \int d^2x \left(\bar{\psi}^a(\partial\!\!\!/ + ie\, A\!\!\!/ + \gamma^0\mu + \sigma)\psi^a - \frac{N}{2g}\,\sigma^2\right)\right) \tag{6}$$

and the vacuum expectation for the fermion condensate can be found from its minima, that is solving the gap equation $\delta V^{eff}/\delta\sigma = 0$.

We distinguish two different contributions to the effective potential, one even in σ defined as

$$V^{\rm even}[\sigma] \equiv \frac{1}{2}(V^{eff}[\sigma] + V^{eff}[-\sigma]) \tag{7}$$

and the other odd in σ, which signals the breaking of parity, defined as

$$V^{\rm odd}[\sigma] \equiv \frac{1}{2}(V^{eff}[\sigma] - V^{eff}[-\sigma]) \tag{8}$$

The first contribution, V^{even}, can be computed by any method that assumes that V^{eff} depends on σ^2; in particular, a detailed computation was performed in [10] using the Schwinger proper time method. The renormalized result is

$$V^{\rm even}_{\beta,\mu}[\sigma] = \frac{N}{2\pi}\left[\frac{\Lambda}{2\sqrt{\pi}}(\frac{2\sqrt{\pi}}{g}-1)\sigma^2 - \frac{\sqrt{2}}{l^3}\zeta(-\frac{1}{2},\frac{(\sigma l)^2}{2}+1) - \frac{|\sigma|}{2l^2}\right]$$
$$-\frac{N}{4\pi\beta l^2}\left\{\log\left(1+\exp(-2\beta|\sigma|)+2\exp(-\beta|\sigma|)\cosh(\beta\mu)\right)+\right.$$
$$\left. 2\sum_{n=1}^{\infty}\log\left(1+\exp\left(-2\beta\sqrt{\sigma^2+\frac{2n}{l^2}}\right)+2\exp\left(-\beta\sqrt{\sigma^2+\frac{2n}{l^2}}\right)\cosh(\beta\mu)\right)\right\}, \tag{9}$$

where $l = 1/\sqrt{|eB|}$ and Λ is an UV cutoff.

The parity violating contribution to the effective action for a fermion system at finite temperature in a gauge background has been recently computed in exact form for constant field strength configurations in [11]. The result, originally presented without consideration of a chemical potential, can be straightforwardly applied to the case at hand, since the chemical potential μ in eq. (6) plays the same role as an imaginary time component of the gauge field and a constant σ plays the role of a mass term. In fact, replacing the time component eA_0 in [11] by $i\mu$ one gets

$$V^{\rm odd}_{\beta,\mu}[\sigma] = -\frac{N}{2\pi\beta l^2}\ \text{arctanh}\left(\tanh(\frac{\beta\sigma}{2})\tanh(\frac{\beta\mu}{2})\right). \tag{10}$$

The complete expression for the effective potential is of course the sum of both contributions, $V^{eff}_{\beta,\mu}[\sigma] = V^{even}_{\beta,\mu}[\sigma] + V^{odd}_{\beta,\mu}[\sigma]$

It is now easy to see that term in (10) changes the mass generation picture completely at low temperatures. This is because it is smooth at the origin and odd in σ and hence shifts the minimum of the effective potential away from zero, leading to a mass gap. A numerical analysis of the gap equation $\delta V^{eff}/\delta\sigma = 0$ confirms that there is no minimum at $\sigma = 0$, except at very high temperatures, where V^{odd} is subdominant (of order $1/T$) with respect to the even terms.

In order to explore the meaning of V_{eff} we show in the following plots striking qualitative differences between the gauge invariant and the parity conserving effective actions. In Fig. 1 we show the parity conserving effective potential for fixed magnetic field and chemical potential for a range of temperatures where the transition between massless and massive regimes is apparent. In Fig. 2 we plot the gauge invariant effective potential for the same range of parameters, in which case the theory is always massive. In Fig. 3 we include higher temperatures so as to show the tendency to symmetry restoration. In all these figures we plot dimensionless quantities in terms of an arbitrary mass scale. In particular we chose $\Lambda = \sqrt{\pi}$, $g = 0.9\sqrt{\pi}$, $\mu = 0.1$ and $eB = 1$ [10].

III. CONCLUSIONS

In the present paper we have analyzed the consequences of the parity anomaly on the usual picture of dynamical mass generation (associated with parity symmetry breaking) for planar fermions with quartic interactions at finite temperature and density. Our main observation is that the inclusion of a finite chemical potential prevents the appearance of a symmetric phase for arbitrarily small magnetic fields.

One important context where our result could be of relevance is in the interpretation of the plateau feature observed in thermal conductivity curves in [2] and further discussed in [4] where contradicting evidence was reported. Within our point of view for the description of nodal quasi-particles a mass gap is present for any non-vanishing value of the external magnetic field and chemical potential. Then, there could be no sharp changes in the thermal conductivity due to nodal quasi-particle behaviour. This result is consistent with the measures reported in [4] where no clear evidence for a plateau feature was observed. In the case that further experiments confirm the appearance of such a plateau in the thermal conductivity, our result would rule out the mechanism proposed to explain this phenomenon as due to the opening of a quasi-particle mass gap for a magnetic field above a critical value.

We are grateful to C.D. Fosco, E.F. Moreno and F.A. Schaposnik for useful discussions. The authors thank CONICET and Fundación Antorchas (Grants No. A-13622/1-106 and 13887-73) for financial support.

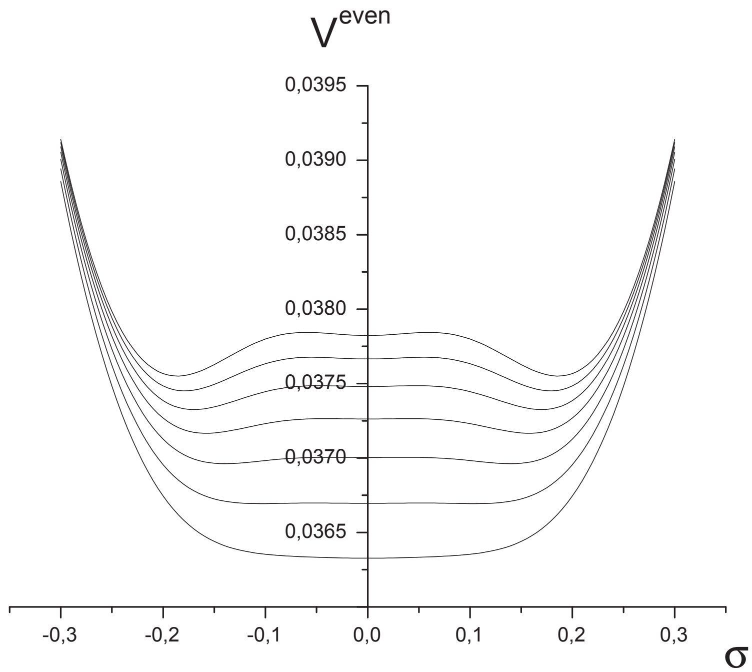

FIG. 1. Parity conserving effective potential, showing the transition from massless to massive regime. Coupling g, magnetic field and chemical potential are kept fixed. Higher temperatures show symmetry restoration (lower curves).

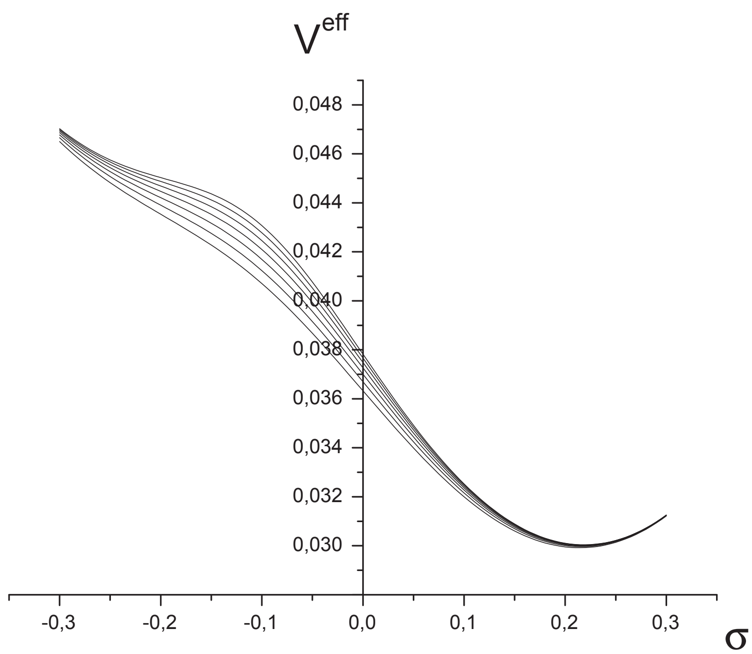

FIG. 2. Complete effective potential, showing persistence of symmetry breaking for the same range parameters of Fig. 1. Temperature grows from top to bottom.

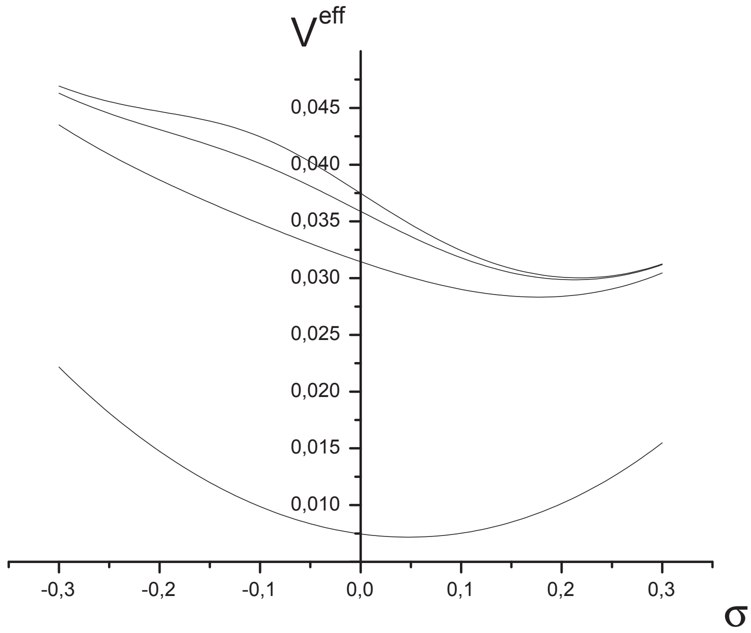

FIG. 3. Complete effective potential for a wider range of temperatures, indicating the irrelevance of the parity breaking contribution at high temperature (lower curve).

[1] See *e.g.* E.J. Ferrer, V.P. Gusynin, V. de la Incera, preprint hep-ph/0101308 and references therein.
[2] K. Krishana *et al*, Science **277**, 83 (1997).
[3] W.V. Liu, Nucl. Phys. **B 556**, 563 (1999).
[4] Y. Ando *et al*, Phys. Rev. B **62**, 626 (2000).
[5] P.A. Lee, preprint condmat/9812226.
[6] P.A. Lee, X.-G. Wen, Phys. Rev. Lett. **78**, 4111 (1997)
[7] N. Redlich, Phys. Rev. Lett. **52**, 18 (1984); Phys. Rev. D **29**, 2366 (1984).
[8] G.W. Semenoff, I.A. Shovkovy, L.C.R. Wijewardhana, Mod. Phys. Lett. **A13**, 1143 (1998).
[9] V. Ch. Zhukovsky, K. G. Klimenko, V. V. Khudyakov, D. Ebert, JETP Lett. 73 (2001) 121-125.
[10] V.P. Gusynin, V.A. Miransky, I.A. Shovkovy, Phys. Rev. Lett. **73**, 3499 (1994); Phys. Rev. D **52**, 4718 (1995).
[11] C. Fosco, G.L. Rossini, F.A. Schaposnik, Phys. Rev. Lett. **79**, 1980 (1997); **79**, 4296(E) (1997); Phys. Rev. D **56**, 6547 (1997).