Annihilation amplitudes and factorization in $B^{\pm} \to \phi K^{*\pm}$

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We study the decay $B^{\pm} \to \phi K^{*\pm}$, followed by the decay of the outgoing vector mesons into two pseudoscalars. The analysis of angular distributions of the decay products is shown to provide useful information about the annihilation contributions and possible tests of factorization.

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I. INTRODUCTION

The analysis of B meson physics offers an attractive opportunity to get a deep insight into the flavor structure of the Standard Model (SM) and the origin of CP violation. In view of the wide variety of decay channels, one can look for many different observables, providing stringent test for the consistency of the model. However, the potential power of the analysis is severely limited by our present theoretical capability of dealing with strong interactions in the intermediate and low energy regimes. In fact, only a limited number of observables are free of theoretical uncertainties within the SM. The main sources of theoretical errors arise from the evaluation of weak transition amplitudes (matrix elements of quark current-current operators between hadron states) and from the estimation of final state interaction (FSI) effects. Thus, the theoretical control of these uncertainties turns out to be a crucial goal.

For nonleptonic B meson decays, the usual procedure to calculate the weak transition amplitudes is based on the effective Hamiltonian approach and the use of Wilson operator product expansion. The Wilson coefficients contain the information from short-distance physics and can be computed perturbatively. This program has been fully carried out already up to next-to-leading order [1], and the main theoretical problem to be addressed in this sense is the analysis of long-distance physics, i.e., the computation of matrix elements of the effective four-quark operators between hadron states. To deal with this, a simple and widely used approach is the so-called factorization approximation (FA) [2]. The extent of the validity of this approximation is however controversial. In the last years, new approaches, such as the so-called QCD factorization (QCDF) [3] and perturbative QCD (PQCD) [4] schemes, have been proposed with the aim of improving the factorization assumption on QCD grounds [5].

In the framework of FA, an extensive analysis of the phenomenology of B decays has been presented by Ali, Kramer and Lü [6], where the authors calculate the branching fractions for charmless non-leptonic two-body B decays and propose a number of tests for the approach. In particular, the authors in [6] take into account the effects of annihilation amplitudes, which are neglected by a priori arguments in most works on the subject. In contrast to the general custom, it is pointed out that the contribution of annihilation diagrams could play a significant and even dominant role, especially in some cases where the non-annihilation amplitudes are suppressed. It is worth to notice that the theoretical control of annihilation amplitudes is very important for the analysis of CP-violating observables, since in many cases the annihilation contribution carries a weak phase different from that provided by the tree or penguin amplitudes. This is e.g. the case of the decays $B^+ \to K^+\pi^0,\ \pi^+K^0$, which have been largely analyzed in connection with the experimental determination of the weak phase angle γ [7]. Moreover, even if in most cases annihilation amplitudes appear to be Cabibbo-suppressed, their presence can be important since they can compete with possible manifestations of new physics, which could be revealed through the analysis of CP-violating observables. On the other hand, the measurement of annihilation contributions is interesting by itself from the point of view of the understanding of low energy dynamics and the viability of the theoretical approaches. For example, annihilation amplitudes are assumed to be suppressed by powers of Λ_{QCD}/m_b in the framework of QCDF, while this is not the case in PQCD.

In this paper, we focus our attention in the annihilation contributions to the process $B \to \phi K^*$, which is the first observed [8] charmless B decay into two vector mesons and has been recently analyzed within both QCDF [9] and PQCD [10]. While annihilation contributions are expected to be highly suppressed in the case of $B \to PP$ decays, an equivalent suppression mechanism is not obvious for $B \to PV$ and $B \to VV$ processes [6]. For example, in the case of the decay $B^+ \to K^{*+}\bar{K}^0$, it has been noticed that once the annihilation part of the amplitude is taken into account, the branching ratio could reach —under reasonable assumptions on form factors— an order of magnitude higher than the value obtanied from the penguin contribution alone [6]. Owing to the large theoretical uncertainties, however, the role of annihilation contributions is in general quite difficult to estimate from the sole measurement of branching ratios. In this sense, B decays into two vector mesons (which subsequently decay into two particles each) present an important feature: the analysis of angular distributions of the final outgoing particles allows to measure both total decay rates and strong and weak phases of the contributing amplitudes. This can be exploited e.g. to get different observables for CP violating parameters [11–13] and solve the so-called discrete ambiguities [14], or to analyze the significance of the contribution of electroweak penguins [15]. We show here that, in the framework of the Standard Model, the analysis of angular distributions in the decay $B^\pm \to \phi K^{*\pm}$ can be used to estimate the annihilation contributions to the process and to test the viability of the factorization assumptions. The process $B^\pm \to \phi K^{*\pm}$ is particularly interesting, since on one hand it is expected to be dominated by penguin-like contributions —thus annihilation amplitudes could be relatively significant— and on the other hand penguin and annihilation contributions carry different weak phases, hence they can be disentangled by looking at CP-odd terms in the angular distribution of final states.

Section II includes a general description of angular distributions and observables in $B \to VV$ decays, while in Sect. III we analyze the particular case of $B^\pm \to \phi K^{*\pm}$. The expected results within the factorization approach are discussed in Sect. IV, and in Sect. V we present some concluding remarks.

II. OBSERVABLES AND ANGULAR DISTRIBUTIONS IN $B \to VV$

Let us consider the decay of a B meson into two vector mesons, $B \to V_1 V_2$, followed by the decay of both V_1 and V_2 into two pseudoscalars $P_1 P_1'$ and $P_2 P_2'$ respectively. Following the notation in Ref. [16], the normalized differential angular distribution can be written as

$$\frac{1}{\Gamma_0}\frac{d^3\Gamma}{d\cos\theta_1\, d\cos\theta_2\, d\psi} = \frac{9}{8\pi\mathcal{K}}\left\{K_1 \cos^2\theta_1 \cos^2\theta_2 + \frac{K_2}{2}\sin^2\theta_1 \sin^2\theta_2 \cos^2\psi \right. + \frac{K_3}{2}\sin^2\theta_1 \sin^2\theta_2 \sin^2\psi + \frac{K_4}{2\sqrt{2}}\sin 2\theta_1 \sin 2\theta_2 \cos\psi \left. - \frac{K_5}{2\sqrt{2}}\sin 2\theta_1 \sin 2\theta_2 \sin\psi - \frac{K_6}{2}\sin^2\theta_1 \sin^2\theta_2 \sin 2\psi\right\}, \tag{1}$$

where θ_1 (θ_2) is the angle between the three-momentum of P_1 (P_2) in the V_1 (V_2) rest frame and the three-momentum of V_1 (V_2) in the B rest frame, and ψ is the angle between the planes defined by the $P_1 P_1'$ and $P_2 P_2'$ three-momenta in the B rest frame. The coefficients K_i can be written in terms of three independent amplitudes, A_0, $A_\parallel$ and $A_\perp$, which correspond to the different polarization states of the vector mesons V_1 and V_2 [17]. One has

$$\begin{aligned} K_1 &= |A_0|^2\,, & K_4 &= \mathrm{Re}[A_\parallel A_0^*]\,, \\ K_2 &= |A_\parallel|^2\,, & K_5 &= \mathrm{Im}[A_\perp A_0^*]\,, \\ K_3 &= |A_\perp|^2\,, & K_6 &= \mathrm{Im}[A_\perp A_\parallel^*]\,, \end{aligned} \tag{2}$$

and $\mathcal{K} \equiv K_1 + K_2 + K_3$. Notice that only six from the nine possible observables given by the squared amplitude $A^* A$ can be measured independently. This is due to the fact that both V mesons are assumed to decay into two spin zero particles.

In the literature, $B \to VV$ decays are also frequently described using the helicity basis. According to their Lorentz structure, the amplitudes can be parameterized in general as [11]

$$H_\lambda = \varepsilon^*_{1\mu}(\lambda)\,\varepsilon^*_{2\nu}(\lambda)\left[a g^{\mu\nu} + \frac{b}{m_1 m_2}p^\mu p^\nu + \frac{ic}{m_1 m_2}\epsilon^{\mu\nu\alpha\beta}p_{1\alpha}p_\beta\right], \tag{3}$$

where p is the B meson momentum, λ is the helicity of both vector mesons, and m_i, p_i and ε_i stand for their masses, momenta and polarization vectors respectively. In this way, for $\lambda = 0, \pm 1$ the helicity amplitudes are given by

$$H_{\pm 1} = a \pm c\sqrt{x^2 - 1}\,, \qquad H_0 = -ax - b\,(x^2 - 1)\,, \tag{4}$$

where $x \equiv (m_B^2 - m_1^2 - m_2^2)/(2m_1 m_2)$. The relation between the amplitudes in both schemes is

$$A_\perp = \frac{H_{+1} - H_{-1}}{\sqrt{2}}, \qquad A_\parallel = \frac{H_{+1} + H_{-1}}{\sqrt{2}}, \qquad A_0 = H_0 \tag{5}$$

and the coefficients K_i can be written in terms of the parameters a, b, c as

$$\begin{aligned} K_1 &= |x\,a + (x^2-1)\,b|^2 & K_4 &= -\sqrt{2}\,\left[x\,|a|^2 + (x^2-1)\,\mathrm{Re}(a^* b)\right] \\ K_2 &= 2\,|a|^2 & K_5 &= \sqrt{2\,(x^2-1)}\,\left[x\,\mathrm{Im}(ac^*) + (x^2-1)\,\mathrm{Im}(bc^*)\right] \\ K_3 &= 2\,(x^2-1)|c|^2 & K_6 &= 2\sqrt{x^2-1}\,\mathrm{Im}(ca^*) \end{aligned} \tag{6}$$

Relative decay rates into V meson states with longitudinal and transverse polarizations are thus given by

$$\begin{aligned} \frac{\Gamma_L}{\Gamma_0} &= \frac{|H_0|^2}{|H_0|^2 + |H_{+1}|^2 + |H_{-1}|^2} = \frac{K_1}{\mathcal{K}}\ , \\ \frac{\Gamma_T}{\Gamma_0} &= \frac{|H_{+1}|^2 + |H_{-1}|^2}{|H_0|^2 + |H_{+1}|^2 + |H_{-1}|^2} = \frac{K_2 + K_3}{\mathcal{K}}\ . \end{aligned} \tag{7}$$

In general, the parameters a, b and c are complex numbers. If it is assumed that the total decay amplitude arises as the sum of several interfering contributions (e.g. different isospin channels), one has

$$a = \sum_i |a_i|\ e^{i\,(\delta_i^a + \varphi_i^a)}\ , \tag{8}$$

where δ and φ stand for “strong” (CP-conserving) and “weak” (CP-violating) phases respectively. Within the Standard Model, the latter arise from the CKM matrix coefficients entering the amplitude, while strong phases receive both contributions from short- and long-distance physics. Similar relations as that in (8) can be written for parameters b and c.

In our analysis we will take into account both the decay $B^+ \to \phi K^{*+}$ and its CP-conjugated process, $B^- \to \phi K^{*-}$. Following standard notation, CP-conjugated amplitudes are denoted as $\bar{A}_\eta$ and $\bar{H}_\lambda$, with $\eta = 0, \parallel, \perp$ and $\lambda = 0, \pm 1$. Accordingly, in the differential decay amplitude (1), one should replace $K_i \to \bar{K}_i$ for $i = 1, \dots 4$ and $K_i \to -\bar{K}_i$ for $i = 5, 6$, which corresponds to replace $a \to \bar{a}$, $b \to \bar{b}$ and $c \to -\bar{c}$ in (3). Since only weak phases change sign after a CP conjugation, one has

$$\bar{a} = \sum_i |a_i|\ e^{i\,(\delta_i^a - \varphi_i^a)}\ , \tag{9}$$

while similar relations hold for $\bar{b}$ and $\bar{c}$.

III. PENGUIN AND ANNIHILATION AMPLITUDES IN $B^\pm \to \phi K^{*\pm}$

Let us now focus on the decay $B^- \to \phi K^{*-}$. In the Standard Model, this process is driven by both penguin and annihilation contributions, with the salient feature that they carry different weak phases. Up to small $\mathcal{O}(\lambda^2)$ corrections ($\lambda = |V_{ud}| \simeq 0.22$), the penguin amplitude is proportional to the V_{CKM} elements $V_{tb}V_{ts}^*$, while the annihilation contribution carries the factor $V_{ub}V_{us}^*$. The relative phase between both terms, up to $\mathcal{O}(\lambda^2)$ corrections, is

$$\arg\left(\frac{V_{ub}V_{us}^*}{V_{tb}V_{ts}^*}\right) \simeq \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right) \equiv \gamma \tag{10}$$

which is one of the angles of the so-called *unitarity triangle*. Though the annihilation contribution is doubly Cabibbo suppressed with respect to the penguin one, this is compensated by the relation between the corresponding Wilson coefficients. We come back to this in the next Section.

As stated in the Introduction, there is a strong theoretical motivation to know the magnitude of annihilation amplitudes. While the penguin contributions can be (at least, roughly) estimated with the aid of the factorization approach, the annihilation contributions in $B \to VV$ decays are much more uncertain, since the corresponding form factors cannot be related to semileptonic decay amplitudes. Since there are no tree amplitudes contributing to $B^\pm \to \phi K^{*\pm}$, this process is a promising one, in the sense that penguin and annihilation contributions can be comparable in size [6] and their relative magnitude can be measured. Moreover, in view of the different weak phase structure, both contributions can be disentangled by looking at CP-odd observables.

According to the general analysis in Section II, the coefficients a, b and c for the case of $B^- \to \phi K^{*-}$ can be written as

$$\begin{aligned} a &= (a_P\, e^{i\delta'_a} + a_A\, e^{i\gamma})\, e^{i\delta_a} \\ b &= (b_P\, e^{i\delta'_b} + b_A\, e^{i\gamma})\, e^{i\delta_b} \\ c &= (c_P\, e^{i\delta'_c} + c_A\, e^{i\gamma})\, e^{i\delta_c} \end{aligned} \tag{11}$$

where the subindices P and A correspond to penguin and annihilation contributions respectively. Without loss of generality, strong phases accompanying both terms have been separated into a global phase δ_i and a relative phase δ'_i, while $a_{P,A}$, $b_{P,A}$ and $c_{P,A}$ are real numbers. For the CP-conjugated decay $B^+ \to \phi K^{*+}$ the corresponding coefficients $\bar{a}$, $\bar{b}$ and $\bar{c}$ are similar to those in (11), just changing $\gamma \to -\gamma$.

Now, in principle, from the angular analysis of $B^\pm \to \phi K^{*\pm}$ decays one can measure 12 observables, K_i and $\bar{K}_i$ with $i = 1$ to 6. Let us first concentrate in the observables given by the transverse modes of the vector mesons ϕ and K^*, that means $i = 2$, 3 and 6. With the above definitions, one has

$$\begin{aligned} K_2 &= 2\left[a_P^2 + a_A^2 + 2\, a_P\, a_A \cos(\delta'_a - \gamma)\right] \\ K_3 &= 2\,(x^2-1)\left[c_P^2 + c_A^2 + 2\, c_P\, c_A \cos(\delta'_c - \gamma)\right] \end{aligned} \tag{12}$$

and similar relations hold for $\bar{K}_2$ and $\bar{K}_3$, changing the sign in front of γ. The relative magnitude of the annihilation contributions can be measured from the combined observables

$$\begin{aligned} K_2 - \bar{K}_2 &= 8\, a_P\, a_A \sin\delta'_a \sin\gamma \\ K_3 - \bar{K}_3 &= 8\,(x^2-1)\, c_P\, c_A \sin\delta'_c \sin\gamma \end{aligned} \tag{13}$$

which are odd under CP. A significant asymmetry provided by any of the quantities in (13) would signal the presence of an important annihilation contribution. This would be e.g. in agreement with the prediction given by PQCD, where annihilation amplitudes are found to enhance the decay width Γ_T by about a factor 2 [10].

Notice that, in order to be different from zero, the quantities defined in (13) require the presence of nonzero relative strong phases $\delta'_{a,c}$. The latter are expected to be nonvanishing even in the absence of final state interaction effects, since in general the penguin amplitudes include absorptive contributions [18]. However, it is possible that these absorptive parts turn out to be suppressed, hence the asymmetries in (13) could be too small to be observed experimentally. This happens e.g. in the framework of factorization, where absorptive contributions entering the effective Wilson coefficients appear to be $\lesssim 20\%$ of the dispersive parts [6,19]. If this is the case, the significance of annihilation contributions can be still estimated by considering the observables K_6 and $\bar{K}_6$, which arise from the interference between the amplitudes $A_\parallel$ and $A_\perp$. In general, the CP-odd observable $K_6 - \bar{K}_6$ is given by

$$K_6 - \bar{K}_6 = 4\sqrt{x^2-1}\;[a_P\, c_A\; \cos(\delta_c - \delta_a - \delta'_a) - a_A\, c_P\; \cos(\delta_c - \delta_a + \delta'_c)] \sin\gamma\;, \tag{14}$$

which is still nonzero in the limit of vanishing strong phases. Moreover, in that case both K_6 and $\bar{K}_6$ provide separate measurements of CP violation, obeying

$$K_6 = -\bar{K}_6 = 2\sqrt{x^2-1}\;[a_P\, c_A - a_A\, c_P] \sin\gamma\;. \tag{15}$$

The validity of this relation would imply the presence of a significant annihilation contribution and support the assumption that strong phases are negligibly small.

The remaining observables K_i and $\bar{K}_i$ with $i = 1$, 4 and 5 can also be analyzed, and once again the measurement of any significant asymmetry $K_i - \bar{K}_i$ would signal the presence of annihilation contributions within the SM. Here we do not enter in the detailed analysis of these observables since the expressions in terms of Lorentz invariant parameters a, b and c, as well as the theoretical analysis of form factors, turn out to be more complicated and do not provide new physical insights.

IV. FACTORIZATION

In the framework of factorization, the measurement of the observables K_i and $\bar{K}_i$ in the decay $B^\pm \to \phi K^{*\pm}$ can be used not only to estimate the values of form factors related with annihilation amplitudes, but also to test the consistency of the approach itself. As before, we concentrate here in the observables related to the transverse modes of the ϕ and K^*, that means to $i = 2$, 3 and 6.

The penguin amplitudes can be computed within generalized factorization making use of the effective Hamiltonian approach. Once the matrix elements of four-quark operators are factorized, the amplitudes can be written in general in terms of form factors f_V, $V^{B\to V}(q^2)$, $A_i^{B\to V}(q^2)$, $i = 0, 1, 2$, as

$$\begin{aligned}
\langle V(\varepsilon, p')|V_\mu|0\rangle &= f_V\, m_V\, \varepsilon^*_\mu \\
\langle V(\varepsilon, p')|V_\mu|B(p)\rangle &= -\,\frac{2}{m_V + m_B}\,\epsilon_{\mu\nu\alpha\beta}\,\varepsilon^{*\,\nu}\,p^\alpha p'^\beta\,V^{B\to V}(q^2) \\
\langle V(\varepsilon, p')|A_\mu|B(p)\rangle &= i\,\frac{2\,m_V(\varepsilon^*\cdot q)}{q^2}\,q_\mu\,A_0^{B\to V}(q^2) + i\,(m_V + m_B)\left[\varepsilon^*_\mu - \frac{(\varepsilon^*\cdot q)}{q^2}\,q_\mu\right]A_1^{B\to V}(q^2) \\
&\quad - i\left[(p+p')_\mu - \frac{(m_B^2 - m_V^2)}{q^2}\,q_\mu\right]\frac{(\varepsilon^*\cdot q)}{m_V + m_B}\,A_2^{B\to V}(q^2)\;,
\end{aligned} \tag{16}$$

Here $V(\varepsilon, p')$ stands for the outgoing vector meson ϕ or K^*, V_μ and A_μ are the corresponding vector and axial-vector quark currents and $q = p - p'$ is the momentum transfer. The vector and axial-vector form factors can be estimated from the analysis of semileptonic B decays, using the ansatz of pole dominance to account for the momentum dependences in the region of interest.

In this way the penguin amplitudes a_P, b_P and c_P read

$$\begin{aligned}
a_P &= -|C^{(P)}_{eff}|\,m_\phi\,(m_B + m_{K^*})\,f_\phi\,A_1^{B\to K^*}(m_\phi^2) \\
b_P &= |C^{(P)}_{eff}|\,m_\phi\left(\frac{2\,m_{K^*}\,m_\phi}{m_B + m_{K^*}}\right)f_\phi\,A_2^{B\to K^*}(m_\phi^2) \\
c_P &= |C^{(P)}_{eff}|\,m_\phi\left(\frac{2\,m_{K^*}\,m_\phi}{m_B + m_{K^*}}\right)f_\phi\,V^{B\to K^*}(m_\phi^2)
\end{aligned} \tag{17}$$

where

$$C^{(P)}_{eff} = \frac{G_F}{\sqrt{2}}\,V^*_{ts}\,V_{tb}\left[a_3 + a_4 + a_5 - \frac{1}{2}(a_7 + a_9 + a_{10})\right]\,. \tag{18}$$

The coefficients a_i can be calculated by means of renormalization group analysis [1], taking into account the experimental values of the running coupling constants in the SM and the parameters entering the V_{CKM} matrix. They are complex numbers that include absorptive contributions from QCD and electromagnetic penguin diagrams. In general, the theoretical results include some dependence on the renormalization scale (fixed at some value around the b quark mass), which can be reduced through the inclusion of QCD corrections to the quark level matrix elements before the factorization procedure [19]. In the so-called generalized FA, the coefficients are explicitly written as functions of the number of colors N_C, which is treated as a phenomenological parameter (N_C^{eff}) to be adjusted from the analysis of the full pattern of charmless two-body B decays.

On the other hand, the annihilation contributions can be analyzed within FA taking into account form factors f_P, $V_1^{(A)}(q^2)$, $V_2^{(A)}(q^2)$ and $A^{(A)}(q^2)$ defined by

$$\begin{aligned}
\langle 0\,|A_\mu|\,B(p)\rangle &= i\,f_B\,p_\mu \\
p^\mu\,\langle K^*(\varepsilon_1, p_1)\phi(\varepsilon_2, p_2)\,|V_\mu|\,0\rangle &= \left[(\varepsilon_1^*\cdot\varepsilon_2^*)\,p^2\,V_1^{(A)}(p^2) - (\varepsilon_2^*\cdot p_1)\,(\varepsilon_1^*\cdot p_2)\,V_2^{(A)}(p^2)\right] \\
p^\mu\,\langle K^*(\varepsilon_1, p_1)\phi(\varepsilon_2, p_2)\,|A_\mu|\,0\rangle &= i\,\epsilon_{\mu\nu\alpha\beta}\,\varepsilon_1^{*\mu}\,\varepsilon_2^{*\nu}\,p_1^\alpha\,p_2^\beta\,A^{(A)}(p^2)
\end{aligned} \tag{19}$$

where $p = p_1 + p_2$ is the B meson four-momentum, $p^2 = m_B^2$. In this case the magnitude of the form factors cannot be estimated from semileptonic processes, and they are introduced as unknown parameters. From Eqs. (19), annihilation amplitudes a_A, b_A and c_A are given by

$$
\begin{aligned}
a_A &= -|C_{eff}^{(A)}|\, f_B\, m_B^2\, V_1^{(A)}(m_B^2) \\
b_A &= |C_{eff}^{(A)}|\, f_B\, m_\phi\, m_{K^*}\, V_2^{(A)}(m_B^2) \\
c_A &= -|C_{eff}^{(A)}|\, f_B\, m_\phi\, m_{K^*}\, A^{(A)}(m_B^2)
\end{aligned} \tag{20}
$$

where

$$
C_{eff}^{(A)} = \frac{G_F}{\sqrt{2}}\, V_{us}^*\, V_{ub}\, a_1 \ . \tag{21}
$$

The annihilation diagram is dominated by a tree contribution, carrying the coefficient a_1, which is close to one [6,19–21]. In contrast, the a_i coefficients in $C_{eff}^{(P)}$ arise from QCD and electroweak penguin diagrams, and their order of magnitude lies between 10^{-2} and 10^{-4} [6,19,20]. This suppression of penguin amplitudes is however compensated by the ratio between V_{CKM} coefficients in $C_{eff}^{(A)}$ and $C_{eff}^{(P)}$, which is of the order of $\lambda^2 \simeq 0.05$. In addition, annihilation form factors are further suppressed due to the large momentum transfer at $q^2 = m_B^2$, where they have to be evaluated. In view of the theoretical uncertainty on the values of these form factors at $q^2 = 0$, it is not immediate to determine if annihilation contributions are large enough to interfere with penguin ones. This analysis has to be done within a definite model for the underlying QCD dynamics, and can be checked through the measurement of CP-odd observables proposed here.

Let us come back to the observables K_i and $\bar{K}_i$. In the spirit of FA, strong phases originated by final state interactions can be separated from short-distance physics, therefore they should be common to both penguin and annihilation amplitudes. The only relative strong phase between them arises then from the absorptive contributions in the a_i coefficients, which can be estimated perturbatively. Moreover, this phase is the same for the amplitudes a, b and c, since the combination of a_i coefficients in all cases is that in $C_{eff}^{(P)}$. In this way, within FA we have

$$
\delta'_a = \delta'_b = \delta'_c = \arg\left[a_3 + a_4 + a_5 - \frac{1}{2}(a_7 + a_9 + a_{10})\right] \equiv \delta' \ . \tag{22}
$$

and the CP-even and CP-odd combinations of K_i and $\bar{K}_i$ for $i = 2, 3, 6$ read

$$
K_2 + \bar{K}_2 = 4\,(a_P^2 + a_A^2 + 2\, a_P\, a_A \cos\delta' \cos\gamma) \tag{23a}
$$

$$
K_2 - \bar{K}_2 = 8\, a_P\, a_A \sin\delta' \sin\gamma \tag{23b}
$$

$$
K_3 + \bar{K}_3 = 4\,(x^2 - 1)\,(c_P^2 + c_A^2 + 2\, c_p\, c_A \cos\delta' \cos\gamma) \tag{23c}
$$

$$
K_3 - \bar{K}_3 = 8\,(x^2 - 1)\, c_P\, c_A \sin\delta' \sin\gamma \tag{23d}
$$

$$
\begin{aligned}
K_6 + \bar{K}_6 = 4\sqrt{x^2 - 1}\,\Big\{ & [a_P\, c_P + a_A\, c_A + (c_P\, a_A + a_P\, c_A)\, \cos\delta']\, \sin(\delta_c - \delta_a) + \\
& (a_A\, c_P - a_P\, c_A) \sin\delta' \cos(\delta_c - \delta_a)\Big\} \cos\gamma
\end{aligned} \tag{23e}
$$

$$
\begin{aligned}
K_6 - \bar{K}_6 = 4\sqrt{x^2 - 1}\,\Big[& (a_P\, c_A + a_A\, c_P)\, \sin\delta' \sin(\delta_c - \delta_a) + \\
& (a_A\, c_P - a_P\, c_A) \cos\delta' \cos(\delta_c - \delta_a)\Big] \sin\gamma
\end{aligned} \tag{23f}
$$

This set of equations deserves some attention. First of all, as stated in the preceding Section, the observables in Eqs. (23b), (23d) and (23f) are CP-odd, thus they vanish in the limit of vanishing annihilation amplitudes. Notice that, in the framework of factorization, the annihilation coefficients a_A, c_A and the strong FSI phases δ_a, δ_c are the only unknown parameters (the former, due to the uncertainty in the estimation of form factors), whereas there is some allowed range for the values of δ' and γ (the latter given by experimental measurements of CP violation in K physics and the golden plate $B \to J/\Psi K_s$). In this way, the six-equation system (23) is overdetermined, and the experimental information on the observables K_i and $\bar{K}_i$ can be used both to get a measurement of the magnitude of annihilation contributions an to test the consistency of the approach. In particular, the expressions in Eqs. (23a) to (23d) do not depend from strong phases $\delta_{a,c}$. With the measurement of this four observables (which corresponds to the measurement of $|A_\parallel|$ and $|A_\perp|$ for $B^- \to \phi K^{*-}$ and $B^+ \to \phi K^{*+}$), and getting the estimation of penguin amplitudes from Eqs. (17), it would be possible to extract the values of annihilation coefficients a_A and c_A as well as the phases δ' and γ, and to check the consistency of the values of these phases with the theoretical and experimental bounds. Then, Eqs. (23e) and (23f) provide a further check of the results with the additional possibility of getting information on the strong phase difference $\delta_a - \delta_c$. From the values of the coefficients a_A and c_A it is immediate to obtain estimations for the unknown annihilation form factors $V_1^{(A)}$ and $A^{(A)}$.

Within factorization one would also expect the strong FSI phases δ_a and δ_c to be relatively small. In this limit (or in the case in which they are approximately equal) Eqs. (23e) and (23f) reduce to

$$\begin{aligned} K_6 + \bar{K}_6 &= 4\sqrt{x^2-1}\;(a_A\, c_P - a_P\, c_A)\sin\delta'\cos\gamma \\ K_6 - \bar{K}_6 &= 4\sqrt{x^2-1}\;(a_A\, c_P - a_P\, c_A)\cos\delta'\sin\gamma \end{aligned} \tag{24}$$

and the ratio between them is given by

$$\frac{K_6 - \bar{K}_6}{K_6 + \bar{K}_6} = \frac{\tan\gamma}{\tan\delta'} \tag{25}$$

which does not depend on the assumptions on form factors. This relation allows a simple test of the significance of strong FSI phases within FA, provided that the interference between penguin and annihilation amplitudes is strong enough to give measurable values for the observables in (24).

The above equations include two approximations that are worth to be mentioned. In fact, penguin contributions should also include the so-called annihilation penguin diagrams, which carry the same weak phase as in $C_{eff}^{(P)}$. It can be seen that the corresponding combination of a_i coefficients is different from that in Eq. (18), even if the order of magnitude is not significantly modified [20]. Within FA, these amplitudes involve annihilation matrix elements, therefore their contributions to a_P, b_P and c_P are proportional to annihilation form factors. Although the inclusion of these terms does not introduce more unknown parameters in Eqs. (23), the disentanglement of annihilation form factors becomes more complicated. Here the contribution of annihilation penguins has been neglected for simplicity. However, they should be incorporated into the set of equations (23) if the effect of annihilation amplitudes is found to be relatively large. A second approximation has been done when assuming that penguin contributions carry a global weak phase arising from the V_{CKM} combination $V_{ts}^* V_{tb}$. Here we have neglected the contribution of a virtual u quark in the penguin loop, which carries a factor $V_{us}^* V_{ub}$ and could lead to an observable signal of CP violation due to the presence of absorptive strong phases [18]. This contribution is doubly-Cabibbo suppressed with respect to the dominant one, and the final effect is expected to be below 1% [22]. Thus a clear evidence of the presence of annihilation amplitudes would require a minimum signal of a few percent level.

Let us conclude this Section by presenting a brief numerical analysis of the expected results within the framework of generalized FA. Theoretical estimations of effective coefficients for penguin amplitudes have been performed in previous works [6,19,20], leading to the approximate values quoted in Table I for different choices of the parameter N_C^{eff}. The values for a_P, b_P and c_P in the Table have been estimated following Ref. [6], where the relevant form factors at $q^2 = 0$ are calculated combining lattice QCD results at a high q^2 scale with light-cone QCD sum rule analysis. As it can be seen, a_P turns out to be kinematically enhanced with respect to c_P. However, in the expressions for the observables in Eqs. (23) this enhancement is compensated by the factors (x^2-1) and $\sqrt{x^2-1}$, where $x \simeq 14$ for the process under consideration. In Table I we have also included the estimations for the absorptive phases δ', as well as the results for the total branching ratio for $B^\pm \to \phi K^{*\pm}$ arising from penguin contributions alone. In favor of generalized FA, the latter appear to be in agreement with recent experimental measurements [8], which quote a decay branching fraction of 10^{-5} with an error of about 50%. Nevertheless, the theoretical results in Table I should be taken only as estimative, and even if the experimental error in the measurement of $BR(B^\pm \to \phi K^{*\pm})$ is expected to be reduced in the future, it is unlikely that from the sole measurement of the branching ratio one could evaluate the interference of penguin amplitudes with other possible contributions.

Concerning the theoretical predictions for the absorptive phase δ', it can be seen that within the approach of generalized FA its value lies in a range between 10 and 20 degrees. The remaining parameter to be taken into account in Eqs. (23) is the CP-violating phase γ, which can be constrained by considering the present measurements of V_{CKM} matrix elements and the experimental results for CP-violating observables in K and B physics. We quote here the recent estimation in Ref. [23],

$$\gamma = 63.5^\circ \pm 7.0^\circ\ . \tag{26}$$

These ranges for δ' and γ can be used to constrain the expected result for the ratio in Eq. (25). Notice however that this expression holds only in the limit in which penguin annihilation amplitudes are neglected.

Finally, since we have concentrated here in observables related to B decays into transversely polarized vector mesons ϕ and K^*, it is important to notice that the values in Table I lead to a relative decay fraction $\Gamma_T/\Gamma_0 \simeq 0.14$. Once again this value corresponds to the penguin contribution alone, therefore it does not depend on the global factor $|C^{(P)}_{eff}|$ which carries the dependence on N_C^{eff}. If this ratio is not significantly reduced after the inclusion of annihilation amplitudes, the analysis of $B^\pm \to \phi K^{*\pm}$ decays would include enough statistics so as to allow the measurements of the observables K_i and $\bar{K}_i$ in Eqs. (23) in the near future.

V. CONCLUSIONS

We study the decay $B^\pm \to \phi K^{*\pm}$, showing that the analysis of angular distributions of the final outgoing particles can be used to estimate the significance of annihilation contributions to the decay amplitude. The magnitude of these contributions represents an interesting subject from the theoretical point of view, in view e.g. of the different predictions obtained from QCD-based approaches such as PQCD or QCDF.

In general, due to the existing hadronic uncertainties in the estimation of amplitudes, annihilation contributions are quite difficult to evaluate from the experimental information on total branching ratios. Here we point out that the decay $B^\pm \to \phi K^{*\pm}$ offers an interesting opportunity in this sense, since annihilation amplitudes may be relatively large, and they can be disentangled by looking at certain CP-odd observables. In particular, in the framework of factorization, the experimental information can be used to measure annihilation form factors and strong final state interaction phases. The analysis also serves as a test of the consistency of the factorization approach, taking into account the theoretical estimation of the coefficients in the effective $\Delta B = 1$ Hamiltonian and the experimental information on the angle γ of the unitarity triangle.

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N_C^{eff}	$(G_F/\sqrt{2})^{-1}\vert C_{eff}^{(P)}\vert$	a_P [GeV]	b_P [GeV]	c_P [GeV]	δ'	BR
2	2.2×10^{-3}	-0.9×10^{-8}	4.2×10^{-10}	0.6×10^{-9}	10°	2.0×10^{-5}
3	1.6×10^{-3}	-0.7×10^{-8}	3.1×10^{-10}	4.3×10^{-10}	11°	1.1×10^{-5}
∞	3.7×10^{-4}	-1.6×10^{-9}	0.7×10^{-10}	1.0×10^{-10}	18°	0.6×10^{-6}

TABLE I. Results for penguin effective coefficients and amplitudes within the generalized factorization approach.