Physics of Atomic Nuclei, Vol. 66, No. 3, 2003, pp. 438–443. From Yadernaya Fizika, Vol. 66, No. 3, 2003, pp. 466–471.

Symmetry Violations in Nuclear Hamiltonians and Their Consequences for Electroweak Decays*

D. R. Bes and O. Civitarese[1)]

Departamento de Fisica, CNEA, and Departamento de Fisica, Universidad Favaloro, Argentina

Received February 13, 2002

Abstract—We discuss the results of the treatment of nuclear Hamiltonians in terms of collective and intrinsic variables. The BRST method is adapted to identify spurious and physical sectors of the wave functions and operators. Counterterms are added to the Hamiltonian to enforce the symmetries broken by the single-particle field and/or by the residual two-body interactions. We focus on the study of Fermi and Gamow–Teller transitions, with reference to the nuclear double-beta-decay processes, and on the study of vector operators ($\lambda^\pi = 1^-$), with reference to (μ, e^-) conversion processes. We address the following aspects of the problem: (a) Isospin symmetry and the calculation of 0^+ and 1^+ states; sensitivity of the Fermi and Gamow–Teller response in double-beta-decay processes; (b) Restoration of the translational and Galilean invariance of the nuclear Hamiltonians and the calculation of $I^\pi = 1^-$ states; sensitivity of the nuclear response to the spurious center-of-mass motion and μ-electron lepton-flavor-violation processes.

1. ISOSPIN SYMMETRY AND THE PAIRING HAMILTONIAN

In most cases, the short-range part of the two-body interaction in nuclei is described in terms of monopole pairing forces. From first principles, one may expect that both $T = 0$ and $T = 1$ channels of the interaction should be present [1–3]. The experimental evidence, however, favors the isovector ($T = 1$) pairing, and it is less conclusive about the existence of $T = 0$ (isoscalar) pairing. At the same time, the conventional treatment of the nuclear pairing force implies that the interaction is operative among nucleons with the same charge, and occasionally the interaction between protons and neutrons is also considered. In general, we may say that the pairing channels of the nuclear two-body interaction are not treated in a fully symmetric way. Another source of symmetry breaking is, naturally, the adoption of the empirical single-particle basis. Clearly, the consideration of both sources of symmetry breaking is of some importance, particularly, in dealing with the calculation of observables that are isospin-dependent. Let us start with the analysis of the pairing force problem with reference to isospin-dependent excitations and transitions in nuclei.

The separable pairing Hamiltonian is written as

$$H = H_{\rm sp} + H_{\rm pair}$$

with

$$H_{\rm sp} = \epsilon_{vj}\tau_{vj},$$
$$H_{\rm pair} = -g_p S_p^+ S_p - g_n S_n^+ S_n - \frac{1}{2} g_\perp S_\perp^+ S_\perp,$$
$$S_v^+ = c_{vjm}^+ c_{vj\bar{m}}^+ \quad (\text{with } v = p, n),$$
$$S_\perp^+ = c_{pjm}^+ c_{nj\bar{m}}^+ + c_{njm}^+ c_{pj\bar{m}}^+$$

(with summation over repeated indices). The Hamiltonian allows for differences between proton and neutron single-particle energies and pairing strengths, and for an (arbitrary) strength of the neutron–proton isovector pairing component. As a consequence of the presence of the isovector and isoquadrupole terms in this Hamiltonian, it does not in general conserve isospin. Therefore, we are faced with the problem of discriminating between unphysical violations of the symmetries introduced by the formalism (through the use of the basic set of states determined by the mean field approximation) and violations produced by the lack of invariance of the effective nuclear Hamiltonian. After transforming to collective and intrinsic variables, this Hamiltonian reads

$$H = H_0 + H_1 + H_2,$$

where we define its different terms as

$$H_0 = \epsilon_{aj}\tau_{aj} - g_0 \left(S_p^+ S_p + S_n^+ S_n + \frac{1}{2} S_\perp^+ S_\perp \right),$$
$$H_1 = \epsilon_{0j} D^1_{0\sigma} \tau_{\sigma j} - g_1 D^1_{00} (S_p^+ S_p - S_n^+ S_n)$$

*This article was submitted by the authors in English.

[1)]Departamento de Fisica, UNLP, La Plata, Argentina; e-mail: civitare@venus.fisica.unlp.edu.ar

$$- g_1 \frac{1}{\sqrt{2}} D^1_{01}(S^+_p S_\perp + S^+_\perp S_n)$$
$$+ g_1 \frac{1}{\sqrt{2}} D^1_{0\bar{1}}(S^+_n S_\perp + S^+_\perp S_p),$$
$$H_2 = -g_2 D^2_{00}(S^+_p S_p + S^+_n S_n - S^+_\perp S_\perp)$$
$$+ g_2 \sqrt{\frac{3}{2}} D^2_{01}(S^+_\perp S_n - S^+_p S_\perp)$$
$$+ g_2 \sqrt{\frac{3}{2}} D^2_{0\bar{1}}(S^+_\perp S_p - S^+_n S_\perp)$$
$$+ g_2 \sqrt{6}(D^2_{02} S^+_p S_n + D^2_{0\bar{2}} S^+_n S_p).$$

The coupling constants entering into this expression are defined by

$$g_0 = \frac{1}{3}(g_p + g_n + g_\perp), \quad g_1 = \frac{1}{2}(g_p - g_n),$$
$$g_2 = \frac{1}{6}(g_p + g_n - 2g_\perp),$$
$$\epsilon_{aj} = \epsilon_{pj} + \epsilon_{nj}, \quad \epsilon_{0j} = \epsilon_{pj} - \epsilon_{nj},$$
$$\tau_{aj} = \frac{1}{2}(\tau_{pj} + \tau_{nj}), \quad \tau_{0j} = \frac{1}{2}(\tau_{pj} - \tau_{nj}).$$

1.1. Definition of the Intrinsic Frame

We choose $\mathrm{Im}\langle S_p\rangle = \mathrm{Im}\langle S_n\rangle = \langle S_\perp\rangle = 0$. This choice leads to the usual Bogolyubov–Valatin transformation between identical particles. This selection of a gauge constitutes a violation of isospin and gauge symmetries occuring in the intrinsic frame. The *np* pairing is incorporated through the collective rotations in the isospace and the gauge space [3]. The two remaining expectation values $\langle S_p\rangle$ and $\langle S_n\rangle$ are real and considered to be the order parameters, i.e., the large quantities of the system. If the isospin T is also large, the D functions may be treated within a boson description through generalization of the Holstein–Primakoff algebra (Marshalek's expansion).

1.2. Boson Image of the Hamiltonian

The leading orders of the Hamiltonian are simplified to

$$H_\perp = \omega_{\perp\iota}\Gamma^+_{\perp\iota}\Gamma_{\perp\iota} + \omega_\xi \xi^+\xi$$
$$- g_2 \frac{3}{T}\langle S_p\rangle\langle S_n\rangle(\beta^4\xi^{+2} + \beta^{-4}\xi^2)$$
$$- \left(\beta^2\xi^+\Gamma_{\perp\iota} + \mathrm{h.c.}\right)\phi_{a\iota} - \left(\beta^2\xi^+\Gamma^+_{\perp\iota} + \mathrm{h.c.}\right)\phi_{b\iota}$$
$$- \frac{1}{2\mathcal{I}_\perp}\left[\tau^{(20)}_1, \tau^{(20)}_{\bar{1}}\right]_+.$$

In the present treatment, the proton–neutron interaction is replaced by the isospin-independent nuclear interaction which gives rise to the well-behaved bosons $\Gamma^+_{\perp\iota}$, creating the so-called antianalog states. The operator ξ^+ acts in the collective space by increasing the value of $m = T - M$ (it excites the band of analog states having $M = T$ for the ground state). Therefore, the "badly behaved" operator τ_1 is replaced by the "well-behaved" operator ξ^+. The excitation frequency ω_ξ includes the single-particle Coulomb displacements

$$\omega_\xi = -\frac{1}{T}\epsilon_{0j}\langle\tau_{0j}\rangle + \frac{3g_2 + g_1}{T}\langle S_p\rangle^2 + \frac{3g_2 - g_1}{T}\langle S_n\rangle^2. \quad (1)$$

The spectrum of states associated with the neutron–proton sector is labeled by the quantum numbers $(A, T, M, (n_\xi, n_{\perp\iota}, n_{d\iota}))$. It displays a signature

$$(-1)^{\sum_\iota (n_{\perp\iota} + n_{d\iota}) + T - \frac{1}{2}A} = 1. \quad (2)$$

The operator β^2 increases the value of the isospin by one unit ($\beta^2|T\rangle = |T+1\rangle$). Therefore, the product $\beta^2\xi^+$ conserves the projection M in the laboratory frame. The operator $\beta^4\xi^{+2}$ mixes the ground state of a nucleus having isospin $T-2$ and projection $T-2$ with the double IAS with spin T. It is proportional to the isoquadrupole strength g_2. The operator $\beta^2\xi^+\Gamma^+_{\perp\iota}$ creates the antianalog states $\Gamma^+_{\perp\iota}|0\rangle$ in the neighboring odd–odd nucleus with isospin $T-1$, laboratory projection $M = T-1$, simultaneously with the IAS carrying isospin T, $M = T-1$. There are also transitions in which the antianalog state is destroyed, while the analog is created.

1.3. Transition Operators

The β^- transition operators, being isovectors, should also be transformed to the intrinsic frame. For the Fermi and Gamow–Teller (GT) operators, we obtain

$$\beta^{(\mathrm{F}-)} = -\sqrt{2}\tau_1 \to -\sqrt{2T}\xi^+, \quad (3)$$

$$\beta^{(\mathrm{GT}-)}_q = \sigma_{q1} \equiv \frac{1}{\sqrt{3}}\langle j_1||\sigma||j_2\rangle[c^+_{pj_1}c_{nj_2}]^1_q \quad (4)$$
$$\to \beta^{-2}\left(q_{f\iota}\Gamma^+_{d\iota q} + (-1)^q_{b\iota}\Gamma_{d\iota(-q)}\right).$$

We note that for Fermi transitions the operator τ_1 is again replaced by ξ^+. Therefore, there are only Fermi beta-decay processes within the isobaric analog band. However, the isospin mixing admixtures make possible the transition between the initial state (ATT) and the final state $(A(T-2)(T-2))$. It may proceed through either the intermediate IAS $(AT(T-1)(1,0,0))$ or through the states $(A(T-1)(T-1)(0,1,0))$. The reduced matrix elements of the spin operator are included in the GT operator. In that expression, there appears the isospin-decreasing operator β^{-2}, which allows occurrence of

the double-beta-decay process without recourse to the isospin nonconserving terms of the Hamiltonian. The coefficients $q_{f\iota}, q_{b\iota}$ are obtainable within the RPA calculation of $I^\pi = 1^+$ excitations.

1.4. Mixing between Components

The mixing between different components for each of the states belonging to the above-discussed level scheme may be calculated by direct diagonalization. As an example, we discuss the perturbative approach. The mixing is therefore expressed as

$$\delta|AT(T-1)(n_\xi=1)\rangle = -a_\iota|A(T-1)(T-1)(n_{\perp\iota}=1)\rangle,$$
$$\delta|A(T-2)(T-2)\rangle = b_\iota|A(T-1)(T-2)(n_\xi=1)(n_{\perp\iota}=1)\rangle + c|AT(T-2)(n_\xi=2)\rangle,$$

where

$$a_\iota = \frac{\phi_{a\iota}}{\omega_{\perp\iota} - 2\kappa T - \omega_\xi},$$
$$b_\iota = -\frac{\phi_{b\iota}}{2\kappa(T-1) + \omega_\xi + \omega_{\perp\iota}},$$
$$c = \frac{3g_2\langle S_n\rangle\langle S_p\rangle}{\sqrt{2}T\,(\kappa(2T-1)+\omega_\xi)}.$$

1.5. Transition Matrix Elements

With the wave functions of the previous subsection, we have calculated the matrix elements for allowed Fermi transitions from the initial $(A, T, M = T)$ state. The results are

$$M_{1\xi} = \langle AT(T-1)(n_\xi=1)|\beta^{l(\mathrm{F}-)}|ATT\rangle = -\sqrt{2T},$$
$$M_{1\perp\iota} = \langle AT(T-1)(n_{\perp\iota}=1)|\beta^{l(\mathrm{F}-)}|ATT\rangle = -\sqrt{2T}a_\iota,$$
$$M_{2\xi} = \langle A(T-2)(T-2)|\beta^{l(\mathrm{F}-)}|AT(T-1)(n_\xi=1)\rangle = -\sqrt{2T}(c - a_\iota b_\iota),$$
$$M_{2\perp\iota} = \langle A(T-2)(T-2)|\beta^{l(\mathrm{F}-)}|AT(T-1)(n_{\perp\iota}=1)\rangle = -\sqrt{2T}b_\iota.$$

The matrix element corresponding to double Fermi transitions is

$$M^{(\mathrm{F})}_{2\nu} = \frac{M_{1\xi}M_{2\xi}}{\Delta + E(\mathrm{IAS})} + \frac{M_{1\perp\iota}M_{2\perp\iota}}{\Delta + \omega_{\perp\iota}}.$$

In the same way, we can write the expression for the double Gamow–Teller matrix element

$$M_{1\diamond jq} = \sigma_{fj}, \qquad M_{2\diamond jq} = \sigma_{bj},$$
$$M^{(\mathrm{GT})}_{2\nu} = \frac{(-1)^q M_{1\diamond\iota q}M_{2\diamond\iota(-q)}}{\Delta + \omega_{\diamond\iota}} = \frac{3\sigma_{f\iota}\sigma_{b\iota}}{\Delta + \omega_{\diamond\iota}}.$$

To summarize, we have developed a collective treatment for motion in the isospace and the gauge space. The system is described within a moving frame of reference by using both collective variables (determining the orientation of the moving frame) and intrinsic variables (describing the motion of the particles with respect to the moving frame) [1]. Two difficulties have to be overcome: (i) a zero-frequency RPA mode and subsequent infrared problems should be expected if the Hamiltonian is an isoscalar, and (ii) the Hamiltonian is not an isoscalar. Thus, we must disentangle the real isospin mixing effects produced by the Hamiltonian from those produced by our isospin-violating treatment. The solution has been checked against exact results for the case of particles moving in a single-j shell and coupled by the isovector pairing interaction [2]. The agreement is very satisfactory. This is not the case for other procedures used previously to treat the same problem (as the renormalized version of the RPA, etc.).

2. CENTER-OF-MASS EFFECTS IN ELECTROWEAK DECAYS

We now turn to the discussion of spurious center-of-mass effects in nuclear Hamiltonians [4]. The problem is perhaps one of the most studied ones and, together with the particle number and angular momentum symmetries, received a lot of attention in the past, mainly in connection with the development of projection techniques. We shall discuss the spurious center-of-mass motion as we have discussed the isospin case, by the way of the separation between collective and intrinsic variables. We shall also discuss the structure of the counterterms that should be introduced to fulfill translational and Galilean invariances [4].

2.1. The Hamiltonian

The degree of violation of the translational and Galilean invariances of a given Hamiltonian H is measured by the commutators

$$\pi_\mu \equiv [H, p_\mu], \qquad \rho_\mu \equiv [H, r_\mu] - \frac{i}{m}p_\mu,$$

which are, in general, nonvanishing operators.

2.2. Counterterms

In order to restore the invariances, we may add counterterms to the Hamiltonian; they are formally defined by

$$H_p = \mathbf{P}\cdot\mathbf{r}, \qquad H_r = \mathbf{R}\cdot\mathbf{p},$$
$$0 = \pi_\mu + iAP_\mu + (-1)^\nu[P_{-\nu}, p_\mu]r_\nu + (-1)^\nu[R_{-\nu}, p_\mu]p_\nu,$$
$$0 = \rho_\mu - iAR_\mu + (-1)^\nu[P_{-\nu}, r_\mu]r_\nu + (-1)^\nu[R_{-\nu}, r_\mu]p_\nu,$$

and these equations have to be solved for each order of the expansion in powers of $1/A$. The systematic application of the previous procedure associates residual interactions to any single-particle Hamiltonian. In what follows we study some applications to the most frequent single-particle contributions, namely, the harmonic oscillator potential, the spin–orbit and l^2 terms, and the empirical single-particle Hamiltonian.

2.3. Single-Particle Hamiltonians

(i) The harmonic oscillator case is

$$H_{\rm sp}^{\rm ho} = \frac{1}{2m}p^2 + \frac{m\omega^2}{2}r^2.$$

The resulting interaction is

$$H_p^{\rm ho} = -\chi_{\rm ho}\mathbf{r}\cdot\mathbf{r}, \qquad \chi_{\rm ho} = \frac{m\omega^2}{2A}.$$

The single-particle and residual interactions are

$$H^{\rm ho} = \frac{1}{2m}p^2 + \frac{\chi_{\rm ho}}{2}\sum_{ab}|\mathbf{r}_a - \mathbf{r_b}|^2.$$

(ii) Spin–orbit interaction. The single-particle term is

$$H_{\rm sp}^{so} = -\chi_{so}(\mathbf{l}\cdot\mathbf{s})_{(1v)}.$$

The two-body interactions are

$$H_p^{so} = \frac{\chi_{so}}{A}\sum_w(\mathbf{p}\times\mathbf{s})_{(1(v-w))}\cdot\mathbf{r}_{(1w)},$$

$$H_r^{so} = -\frac{\chi_{so}}{A}\sum_w(\mathbf{r}\times\mathbf{s})_{(1(v-w))}\cdot\mathbf{p}_{(1w)}.$$

The total Hamiltonian is

$$H^{so} = H_{\rm sp}^{so} + H_p^{so} + H_r^{so}$$
$$= -\frac{\chi_{so}}{2A}\sum_{ab}(\mathbf{r}_a - \mathbf{r}_b)\times(\mathbf{p}_a - \mathbf{p}_b)\cdot(\mathbf{s}_a + \mathbf{s}_b) + \frac{\chi_{so}}{2A}(\mathbf{l}\cdot\mathbf{s})_{(2v)},$$

$$P_\mu = \frac{\chi_{so}}{A}\left((\mathbf{p}\times\mathbf{s})_{\mu(1v)} - \frac{1}{2A}\times\sum_w\left(\mathbf{p}_{(1(v-w))}\times\mathbf{s}_{(1w)}\right)_{\mu(2v)}\right),$$

$$R_\mu = -\frac{\chi_{so}}{A}\left((\mathbf{r}\times\mathbf{s})_{\mu(1v)} - \frac{1}{2A}\times\sum_w\left(\mathbf{r}_{(1(v-w))}\times\mathbf{s}_{(1w)}\right)_{\mu(2v)}\right).$$

(iii) The l^2 terms are

$$H_{\rm sp}^{ll} = -\chi_{ll}(\mathbf{l}\cdot\mathbf{l}).$$

The interactions are

$$H_p^{ll} = \frac{\chi_{ll}}{A}(\mathbf{p}\times\mathbf{l})\cdot\mathbf{r} - \frac{\chi_{ll}}{A}(\mathbf{l}\times\mathbf{p})\cdot\mathbf{r} - \frac{2\chi_{ll}}{3A^2}\langle p^2\rangle\mathbf{r}\cdot\mathbf{r},$$
$$H_r^{ll} = \frac{\chi_{ll}}{A}(\mathbf{l}\times\mathbf{r})\cdot\mathbf{p} - \frac{\chi_{ll}}{A}(\mathbf{r}\times\mathbf{l})\cdot\mathbf{p} - \frac{2\chi_{ll}}{3A^2}\langle r^2\rangle\mathbf{p}\cdot\mathbf{p}.$$

The sum of the contributions is

$$H_{\rm sp}^{ll} + H_p^{ll} + H_r^{ll}$$
$$= -\frac{\chi_{ll}}{2A}\sum_{ab}|(\mathbf{r}_a - \mathbf{r}_b)\times(\mathbf{p}_a - \mathbf{p}_b)|^2 - \frac{2\chi_{ll}}{3A}\langle p^2\rangle\left(\frac{\mathbf{r}\cdot\mathbf{r}}{A} - (\mathbf{r}\cdot\mathbf{r})_{(11)+(10)}\right) - \frac{2\chi_{ll}}{3A}\langle r^2\rangle\left(\frac{1}{A}\mathbf{p}\cdot\mathbf{p} - (\mathbf{p}\cdot\mathbf{p})_{(11)+(10)}\right) + \frac{\chi_{ll}}{A}\mathbf{l}\cdot\mathbf{l} + \frac{\chi_{ll}}{A}\sum_{ab}\mathbf{r}_b\times\mathbf{p}_a\cdot\mathbf{r}_a\times\mathbf{p}_b + \frac{\chi_{ll}}{A}(\mathbf{r}\cdot\mathbf{r})_{(11)+(10)}(\mathbf{p}\cdot\mathbf{p})_{(11)+(10)} - \frac{\chi_{ll}}{A}\times\sum_{\mu,a,b}(-1)^\mu r_{a,\mu}p_{b,\mu}(r_{a,\mu+1}p_{b,\mu+1} + r_{a,\mu-1}p_{b,\mu-1}).$$

2.4. The Collective Formalism

The above-described solution guarantees that there is a zero-frequency RPA boson for each direction of space. This consequence of the homogeneity of space gives rise to infrared divergences, which should be taken into account. One way to solve the problem is to introduce collective coordinates, which in the present case represent the coordinates $\mathcal{R}_\mu$ determining the position of the moving frame of reference relative to the laboratory frame. Within this description, there is no way to distinguish between the motion of the body in one direction and the displacement of the frame of reference in the opposite one. This gauge-type invariance is expressed by the constraint $p_\mu - \mathcal{P}_\mu = 0$, where $\mathcal{P}_\mu$ is the generator of displacements of the moving frame, hereon the collective momentum ($[\mathcal{R}_{-\mu}, \mathcal{P}_\nu] = i(-1)^\mu\delta_{\mu\nu}$). Physical

states $|\text{phys}\rangle$ are annihilated by the constraint, and physical operators $\mathcal{O}_{\text{phys}}$ commute with it. As is well known, the constraints may be taken into account by adding to the Hamiltonian the terms proportional to the Lagrange multipliers $\mathbf{\Omega}$

$$H \to H - \mathbf{\Omega} \cdot (\mathbf{p} - \boldsymbol{\mathcal{P}})$$

and requiring the vanishing of $\mathbf{B}$, the momentum conjugate to $\mathbf{\Omega}$ ($[\Omega_{-\mu}, B_\nu] = i(-1)^\mu \delta_{\mu\nu}$).

The BRST Hamiltonian reads

$$H_{\text{BRST}} = H - \mathbf{\Omega} \cdot (\mathbf{p} - \boldsymbol{\mathcal{P}}) + i\boldsymbol{\pi} \cdot \bar{\boldsymbol{\pi}} + \omega^2 \left(\frac{\mathbf{r} \cdot \mathbf{B}}{A} - \frac{\mathbf{B} \cdot \mathbf{B}}{2mA} - i\bar{\boldsymbol{\eta}} \cdot \boldsymbol{\eta} \right)$$

and the different elements entering into the BRST Hamiltonian are the Hermitian and nilpotent operator $\mathcal{Q}$, the fermion ghost operators, and their conjugate momenta. The BRST charge is given by

$$\mathcal{Q} = (\mathbf{p} - \boldsymbol{\mathcal{P}}) \cdot \boldsymbol{\eta} + \mathbf{B} \cdot \bar{\boldsymbol{\pi}}.$$

In the same manner, we can write the operator for the transformation to a moving frame

$$T = \exp\left[\frac{i}{A} \boldsymbol{\mathcal{P}} \cdot \left(\frac{\mathbf{B}}{m} - \mathbf{r} \right) \right],$$

$$T H_{\text{BRST}} T^+ = H'_{\text{BRST}} + \frac{1}{2mA} \mathcal{P}^2.$$

By applying this procedure, we may transform the coordinate to (transformed operator)

$$TF(\mathbf{r}_i + \boldsymbol{\mathcal{R}})T^{-1} = F\left(\mathbf{r}_i + \frac{1}{A}\left(\frac{\mathbf{B}}{m} - \mathbf{r} \right) + \boldsymbol{\mathcal{R}} \right) \to F\left(\mathbf{r}_i - \frac{\mathbf{r}}{A} + \boldsymbol{\mathcal{R}} \right).$$

In the moving frame of reference, the collective variables $\boldsymbol{\mathcal{R}}$ are considered to be real, and thus, as a tradeoff, some original degrees of freedom must join the spurious sector. At the level of elementary modes of excitation, these are given by the RPA zero-frequency modes. In addition to the spurious sector $|n_{0\mu}, n_{1\mu}\rangle$, the intrinsic sector displays elementary modes of excitation, which are represented by the finite-frequency RPA modes $|n_\nu\rangle$ ($\omega_\nu > 0$). The physical operator is $\mathbf{r} + A\boldsymbol{\mathcal{R}}$, and it reduces to $A\boldsymbol{\mathcal{R}}$ after being boosted: the vector $\mathbf{r}$ disappeares from the calculation (and the associated infrared divergencies as well). The problem is reduced to the calculation of the well-behaved operator $\boldsymbol{\mathcal{R}}$ within the collective sector of the Hilbert space.

2.5. Empirical Single-Particle Energies

We have performed three sets of RPA calculations using the empirical single-particle energies (as in ^{208}Pb) and introducing as interaction (i) an $\mathbf{r} \cdot \mathbf{r}$ term with a self-consistent strength; (ii) the same interaction as in (i) but with such a strength that there is an eigenvalue as close to zero as allowed by the computational facilities; and (iii) the counterterms with the values of $\mathbf{P}_{(11)}, \mathbf{R}_{(11)}$. The results are given in [4], where the matrix elements of the operator $\mathbf{r}$ corresponding to transitions between the ground state and the excited states are calculated as a function of the excitation energy. Although calculation (i) with the self-consistent strength displays a prominent peak at low energies, the peak is finite and located at an energy significantly larger than zero. On the contrary, results (ii) and (iii) show the peak at zero energy (within the numerical accuracy of the calculation). This similarity apparently supports the use of procedure (ii) appearing in the literature. Let us consider now the matrix elements to finite frequency modes, which are the ones that interest us from the physical point of view. In this case, calculations (i) and (ii) yield very similar results, while the scale is smaller by two orders of magnitude for (iii), although the excitation pattern is quite similar. We conclude that the admixture of the spurious and the finite-frequency modes is not changed significantly by varying the strength of the $\mathbf{r} \cdot \mathbf{r}$ interaction and thus the use of this interaction does not insure that we obtain correct matrix elements for excited states [4].

2.6. Transition Operator

Conversion of muons into electrons may proceed according to the lepton-flavor-violating processes. Such a process is mainly of interest because of necessary mixing of muon and electron neutrinos. So far, there are experimental upper limits for this process. Considering the dominance of the contributions due to excitation of 1^- states in the context of the RPA diagonalization, it is obvious that from the nuclear structure point of view one has to estimate the nuclear matrix elements involved in the transitions as accurately as possible. The vector operator exciting $I^\pi = 1^-$ states may be written as

$$j_1(qr) Y_{1\mu}|_{(11)} = \sum_{n_\nu} \langle n_\nu | j_1 Y_1 | \rangle \left(\gamma^+_{n_\nu,\mu} - (-1)^\mu \gamma_{n_\nu,(-\mu)} \right) - \frac{i}{A}(-1)^\mu \langle [j_1 Y_{1\mu}, p_{-\mu}] \rangle r_{\mu(11)}, \quad (5)$$

where n_ν denotes the finite-frequency RPA mode and a similar expression should be used for the dipole axial-vector term of the weak current. Its contribution is not affected by the treatment of the spurious sector. The amplitude in the second line may be regularized. The results corresponding to the transition matrix elements of the shifted operator, obtained in the RPA

diagonalization, are given in [4]. The similarity of the results obtained with the three different Hamiltonians supports the claim about the validity of the procedure. The same effects are expected to materialize in the case of realistic calculations, and work is in progress to include the counterterms obtained in the previous sections, starting with realistic two-body forces.

3. CONCLUSIONS

We have treated the problem of broken symmetries in two steps: (i) reconstruction of the invariance of the Hamiltonian by introducing counterterms, and (ii) inclusion of collective variables in order to eliminate infrared singularities.

The comparison between the available exact results and those obtained by using the present approach shows the accuracy of the method. We thus suggest the use of the present approach in cases where, as in (μ^-, e^-) conversion, the dominance of the $I^\pi = 1^-$ channels is apparent or, as in the case of double Fermi and Gamow–Teller transitions in double-beta-decay processes, the strong dependence of the theoretical results on model parameters may limit the predictive power of the theory.

REFERENCES

1. D. R. Bes and J. Kurchan, *The Treatment of Collective Coordinates in Many-Body Systems* (World Sci., Singapore, 1990).
2. D. R. Bes, O. Civitarese, and N. N. Scoccola, Phys. Lett. B **446**, 93 (1999).
3. D. R. Bes, O. Civitarese, E. Maqueda, and N. N. Scoccola, Phys. Rev. C **61**, 024315 (2000).
4. D. R. Bes and O. Civitarese, Phys. Rev. C **63**, 044323 (2001).