

ON THE SATELLITE CAPTURE PROBLEM

Capture and stability regions for planetary satellites

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Abstract.

The stability and capture regions in phase space for retrograde and direct planetary satellites are investigated in the frame of the Circular Restricted Three-Body Problem. We show that a second integral of motion furnishes an accurate description for the stability limit of retrograde satellites.

The distribution of heliocentric orbital elements is studied, and possible candidates to be temporary Jovian satellites are investigated.

Previous results, limited to orbits satisfying the Mirror Theorem, are extended in order to give a complete set of capture conditions in phase-space.

Key words: Restricted Three-Body Problem – Planetary Satellites

1. Introduction

The outer satellites of Jupiter and Phoebe, the outermost satellite of Saturn, share common physical characteristics, resembling asteroids rather than natural planetary satellites. Furthermore, most of them are in non equatorial, elongated and retrograde orbits, being thus unlikely a similar formation process as that of the innermost regular satellites.

Based upon these facts, astronomers have long thought that irregular planetary satellites have a capture origin (Kuiper 1961). Triton, the large moon of Neptune, is perhaps the largest example of satellite capture in the Solar System (Goldreich et al. 1989).

Another interesting application of satellite capture in cosmogony has been recently explored by Brunini (1995), who has shown that capture of planetesimals as planetary satellites has largely increased the efficiency of the accretion process of the outer planets.

Recent numerical studies have shown that short-period comets of the Jupiter family can be temporarily trapped as jovian satellites. Examples revealing the importance of the dynamics of capture are furnished by P / Gehrels 3 (Rickman 1979), P / Oterma which underwent a close encounter with Jupiter in 1934-1939 (Carusi et al. 1985), P / Helin - Roman - Crockett which will perform a long lasting capture of five revolutions about Jupiter (Tancredi et al. 1990) and the recent collision of P / Shoemaker - Levy 9 after being a Jupiter's satellite for several orbital periods.

The first rather complete description of the stability of irregular satellites was done by Hénon (1970) in the frame of Hill's approximation to the Restricted Three-Body Problem (hereafter RTBP). He treated only those orbits satisfying the Mirror Theorem: at the initial instant the minor body is at conjunction, and its velocity vector is perpendicular to the Sun-Planet line. The initial conditions as stated above produce a symmetric motion about both the x axis and the time (Roy and Ovenden 1955). Therefore, in this situation a capture is always followed by an escape.

In a purely gravitational scenario, captures are only of temporary nature. It has been proven by Hopf (1930) that each particle must pass arbitrarily close to its initial position in phase space, making zero the probability of permanent capture. In fact, there exist some initial conditions yielding to permanent capture, but they form a set of Lebesgue measure zero. Nevertheless, the "lifetimes" as temporary satellites may be very large, as the initial conditions approach arbitrarily close to stable periodic orbits (Muriison 1989).

Since at present Jupiter's irregular satellites are stable, some other mechanism is required to transform temporary captures into permanent ones. Several theories involving dissipation of orbital energy have been proposed in order to facilitate permanent captures. Nevertheless, all the non gravitational theories require a temporary capture as a first step. Because dissipative diffusion is in general a slow process, long capture times enlarge the probability of decayment to the condition of permanent satellite. In this context were long-lasting temporary captures are of particular relevance. Longer capture times are in the regions near to stable periodic orbits around the planet, that lie necessarily close to the boundaries of the stability region in phase space, i. e. the region of permanent satellites.

In the frame of the Circular Restricted Three-Body Problem, libration-point capture may be interpreted as the transfer of a body from one mode of motion (heliocentric motion) to another mode (planetocentric motion). Thus, satellite capture can be thought as the competition between the primary masses for the possession of the particle. As expected in such a boundary regime, the structure of phase space is exceedingly complex.

Carusi and others have studied the phenomena of close encounters and temporary captures by means of numerical simulations (Carusi and Pozzi 1978; Carusi, Pozzi and Valsecchi 1979; Carusi and Valsecchi 1979; 1980a,b), whereas the stability and capture regions for retrograde satellites have been numerically explored by Huang and Innanen (1983).

The Jacobi constant C has been frequently used as a criterion for stability (Bailey 1972; Heppenheimer 1975; Brunini 1995) as stated in the theory of forbidden and permitted regions of motion (Szebehely 1967). Hunter (1967) and Hénon (1970) have shown that in fact this is true for direct orbits,

whereas retrograde orbits have been found to be stable for much smaller values of C than the one corresponding to the inner libration point.

In this paper, special attention is paid to the question of the stability limit and the origin of chaos for retrograde satellites. Possible candidates to be captured as jovian satellites are also discussed in section 3 and the distribution of capture times is briefly analyzed in section 4.

The generalization of the previous results characterizing completely the phase-space covered with capture-escape orbits is outlined in section 5. In section 6 we discuss the consequence of considering the problem in the frame of the elliptic RTBP which is more suitable to be applied in Solar System problems.

The last section is devoted to conclusions.

2. Topology of the phase-space

Hénon (1970) studied the stability problem in satellite motion by means of Hill's formulation, which is an approximation to the circular RTBP when the mass ratio is near zero, i. e. well suited to satellites in the Solar System. The equations of the Hill's problem in rotating coordinates may be found in Hénon (1969). They allow a Jacobi integral whose expression is:

$$T = (C - 3)\mu^{-3/2}, \quad (1)$$

where C is the usual Jacobi constant of the RTBP (Moulton 1914) and μ is the mass ratio. In Hill's equations, the planet is in $x = y = 0$.

The circular RTBP, when limited to the plane case, is a system of two degrees of freedom with one integral of motion. The motion is thus confined to a three-dimensional manifold immersed in the four dimensional phase space, suitable to be investigated by the surfaces of section method, which is one of the most powerful tools for revealing the properties of the phase space. The conventional choice for the surface (Hénon 1970) is: $y = 0$, that is the (x, \dot{x}) plane, considering only intersections in the negative direction, i. e. with $\dot{y} < 0$.

Even for the plane circular case, the space of initial conditions to explore is fourth dimensional. Thus, restricting the initial conditions to satisfy the Mirror Theorem reduces it to two.

For $T = 5$ (Figure 2 in Hénon 1970) the transfer between heliocentric orbits and planetocentric orbits is not possible. Points with $x < 0$ represent retrograde satellites, whereas direct orbits are in the $x > 0$ region. At the resolution of the figure, quasi periodic trajectories appear to exist almost in all the accessible region. Retrograde and direct orbits are all stable. For values of $T > 5$ the picture is qualitatively similar.

For Jacobi constants smaller than the one corresponding to the inner libration point (i.e. for $T = 4.326749\dots$), however, the situation changes drastically. For the sake of clarity, the cases of direct and retrograde satellite orbits will be analyzed separately in the next sections.

2.1. DIRECT ORBITS

Surfaces of section for direct orbits were computed by Hénon for a sequence of the Jacobi constant in the range $T = 5$ to $T = 4.35$. For $T = 4.4$ (Fig. 4 in Hénon 1970) we see that the elliptic fixed point has given birth to a period 8 cycle, manifested by 8 stability islands. The main periodic family bifurcates, and consequently, a hyperbolic fixed point appears. This unstable periodic orbit gives rise to the chaotic sea exhibited in Figure 5 of Hénon's paper, which displays the surface of section for the case $T = 4.35$. Nevertheless, direct orbits can not escape from the planet sphere of influence on account that the Hill's curve is closed near the second libration point. For $T < 4.326749\dots$ the central region of the accessible phase space is open in the x direction, and the third body can escape. This is what happens in general, as the chaotic sea occupies almost all the available phase space. Regular orbits are found to fill a small region of phase space. We therefore conclude that escape (and therefore capture) of direct satellites is intimately associated with chaotic motion, and that the Jacobi constant is a useful stability criterion for direct orbits.

Murison (1989) has paid special attention to the sequence of bifurcations of periodic orbits. He has computed very accurate surfaces of section for the RTBP in the case $\mu = 0.01$. The most interesting surface of section is shown in his Figure 8c, where the oscillations between the stable and unstable manifolds and a subsequent homoclinic tangle are shown. Chaotic regions associated with hyperbolic points are interspersed between island elliptic points. The main feature of the satellite island structure is that they are self-similar.

2.2. RETROGRADE ORBITS

We have computed surfaces of section for retrograde orbits in the RTBP, in the case of μ corresponding to Jupiter's value.

For Jacobi constants greater than the one corresponding to the first collinear point ($C = 3.03844\dots$), regular trajectories fill practically all the accessible phase space. In addition, a new phenomenon is noted: the possibility of collisions and close encounters with the planet.

A frequent end-state of captured satellites is a collision onto the planet surface. Therefore, we have paid special attention to the conditions for collision orbits in the case of Jupiter. The region of phase-space full of collision

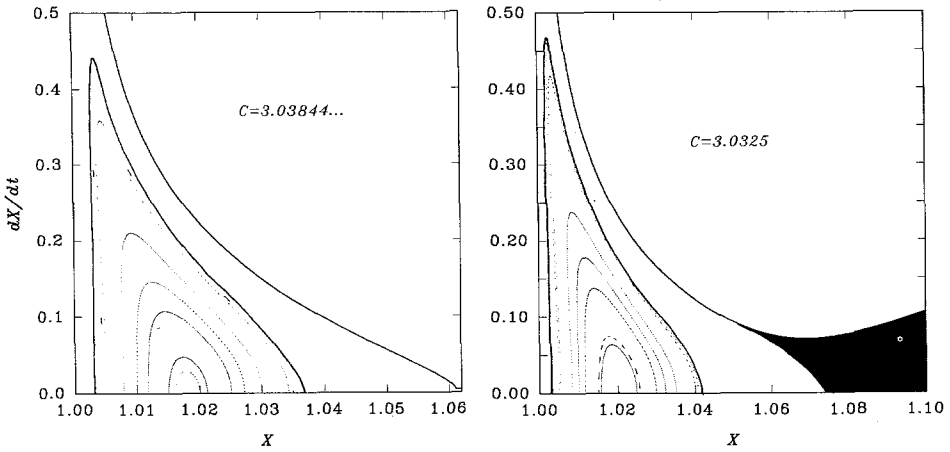


Figure 1. Surfaces of section for retrograde satellite orbits

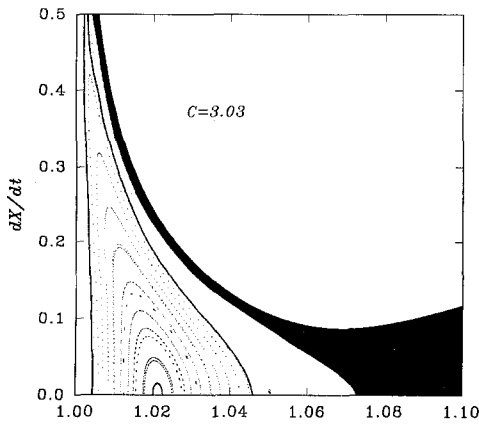


Figure 2. Surface of section for $\mu = 9.54 \times 10^{-4}$. Dark zones represent chaotic orbits. The zone between the chaotic region and the solid line corresponds to collision orbits.

orbits ($r_j \leq R_p$, where r_j is the distance between the particle and Jupiter, and R_p is Jupiter's radius) is represented as the white area, between the solid lines, in the surfaces of section of Figure 1b and Figure 2. They are all retrograde orbits and are placed just in the boundary of the stability region. It thus suggests that collision is the mechanism preventing retrograde orbits from escape, forming a separatrix between the two types of regime: heliocentric and planetocentric motion.

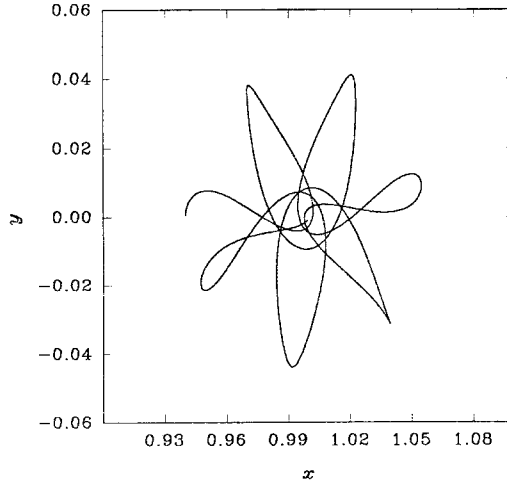


Figure 3. A typical direct-retrograde chaotic orbit. This satellite plows alternatively into the chaotic seas of direct and retrograde motion about the planet.

For C less than 3.025, KAM tori are surrounded by a chaotic region, and escape is then possible, as the chaotic regions extends up to initial conditions corresponding to heliocentric orbits. Collision orbits separate regular and chaotic regions in phase space. It suggests, at first sight, that chaos in retrograde motion would be originated by close encounters. Nevertheless, the chaotic region merges with the chaotic sea corresponding to direct orbits. The orbits of this region are alternatively direct and retrograde. Such a class of orbits is shown in Figure 3.

The stability region for retrograde orbits was explored in the space of initial conditions belonging to the Mirror Theorem.

As in some previous papers (Huang and Innanen 1983), we have defined a satellite as being stable if it is able to survive for more than 1000 orbits around the planet without escaping from the planet sphere of influence. The stable regions are shown in Figure 4.

Dark points represent regular regions, whereas blank spaces correspond to capture-escape regions. We see that regular orbits exist at almost all values of the Jacobi constant. Open circles are initial conditions corresponding to collision orbits. This diagram is somewhat different that the one computed by Huang and Innanen (1983) because they have erroneously used the diameter of Jupiter, rather than its radius, as the collision criterion.

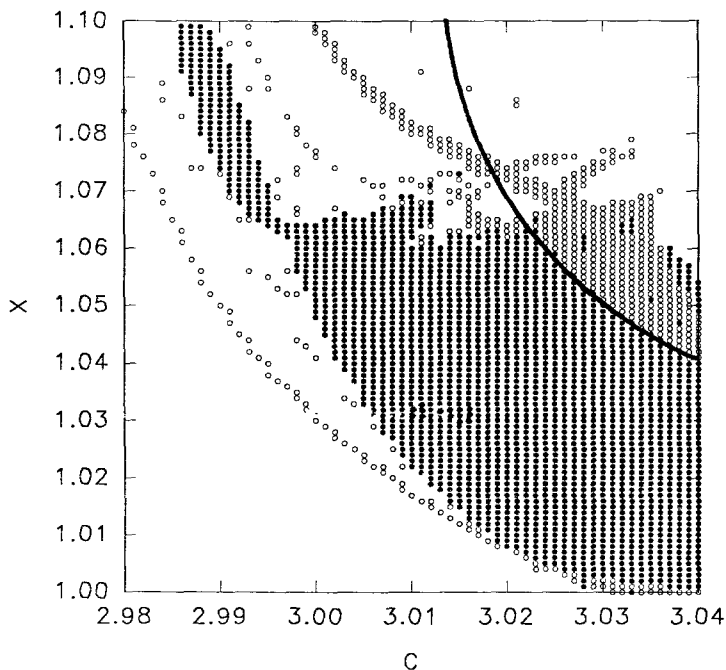


Figure 4. Dark dots: stable region. Open circles: collision orbits.
Full line: theoretical stability limit.

2.3. ANALYTIC DERIVATION OF THE STABILITY LIMIT

A first approximation to the limit of stability for retrograde orbits has been given by Huang and Innanen (1983). They have shown that at the distance to the planet

$$r \sim 0.84\mu^{1/3}, \quad (2)$$

the total acceleration in the rotating frame falls to zero.

For the case of Jupiter $r = 0.084$, that should be compared with the stability boundary of Figure 4.

A more accurate description of the stability limit for retrograde orbits may be stated on rigorous dynamical grounds as follows: For regular orbits, a second uniform integral confines the motion to an invariant manifold, equivalent to a two dimensional torus in phase space. Such a second integral of motion, besides the Jacobi constant, was formally found by Contopoulos

(1965) and Bozis (1966). Proceeding as in the two-body problem, we may write the angular momentum integral as

$$\dot{x}y - \dot{y}x - r^2 = C_2 - \mu(1 - \mu) \int_0^t y \left(\frac{1}{r_1^3} - \frac{1}{r_2^3} \right) dt. \quad (3)$$

The last integral in this expression exhibits small oscillations around a mean value, whose main frequency is half a planet year. The amplitude of the oscillations is in general small (Benner and McKinnon 1995). Neglecting this term, we may write, in usual polar coordinates:

$$C_2 = r^2 \dot{\theta} + r^2. \quad (4)$$

The Jacobi integral may also be written in polar coordinates (Moulton, 1914):

$$C = \dot{r}^2 + r^2 \dot{\theta}^2 - (1 - \mu)(r_1^2 + 1/r_1) + \mu(r_2^2 + 1/r_2). \quad (5)$$

Combining these two equations, we can eliminate $\dot{\theta}$ getting thus an expression for the limiting radius for regular orbits (we shall not write this rather complex expression here, which is only an algebraic exercise). In Figure 4, the theoretical limit (solid line) may be compared with the empirical stability limit for retrograde orbits. The agreement is remarkably good up to $C \sim 3.024$. The disagreement for $C \geq 3.024$ is originated by the presence of collision orbits. These orbits have a small angular momentum, and the integral term in the right hand side of eq. (3) can no longer be neglected. A first order approximation of this integral, might furnish a better description of the stability limit for retrograde orbits. Such an approximation may be found in the paper by Bozis (1966).

3. Heliocentric orbital elements

The results of the previous sections, suggest that captured satellites, such as Jupiter's irregular ones, originated in regions of chaotic motion about the Sun. In this section, we briefly discuss the possible candidates to be the progenitors of Jovian irregular satellites.

Sufficiently long time after a particle has escaped from Jupiter, its heliocentric orbital elements might be taken as the final elements (i.e. stables for a long time). We have thus computed heliocentric (a, e) elements for a sample of 5000 test escaping particles in the circular RTBP, Jupiter and the Sun being the primaries. The elements were computed at the instant 100 jovian years after escape. They are shown in Figure 5.

This figure should be compared with those obtained for the elliptic RTBP (Figure 1 of Huang and Innanen; 1983). The small eccentricity of Jupiter's

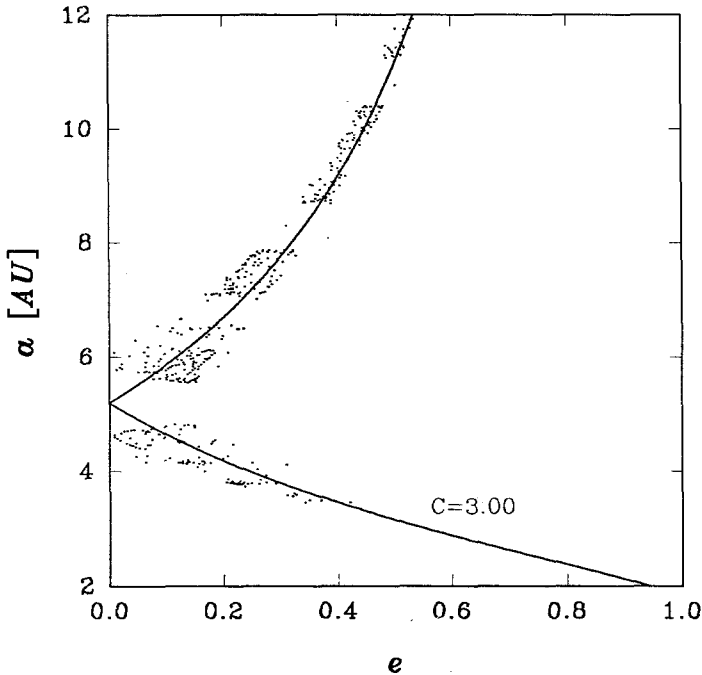


Figure 5. Heliocentric orbital elements of escape-capture bodies for the Jupiter-Sun case.

orbit is the reason for the fairly good concordance of both two distributions. The points in Fig. 5 are approximately distributed along two curves. The motion being chaotic, no other uniform integral of motion exists and therefore, these curves must necessarily be curves of $C = \text{constant}$.

Histograms of a and e are shown in Figure 6a and 6b respectively. We see that the semiaxes distribute in three well definite spikes. The first one corresponds to $a = 3.95$ AU, which is very near the 3 : 2 mean motion resonance with Jupiter, whereas the clusters at 7.3 and 8.6 AU are associated, though not exactly, with the 2 : 3 and 1 : 2 exterior mean motion resonances. The reason of these accumulations remains to be investigated.

The histogram of eccentricities reveals that the preferred eccentricity of capture is ~ 0.3 . The reason on this fact is that this eccentricity corresponds to close approaches with Jupiter of particles whose semiaxes are near the accumulations in 3.95 AU and 7.3 UA.

It is instructive to compare the distribution of the semiaxes and eccentricities shown in Fig. 5 with those of the actual asteroids and short-period and intermediate-period comets. They are plotted in Figure 7 where the curve for $C = 3$ is also plotted. Among asteroids, some Hilda's overlap with this curve. However, they are protected against capture because of the nature

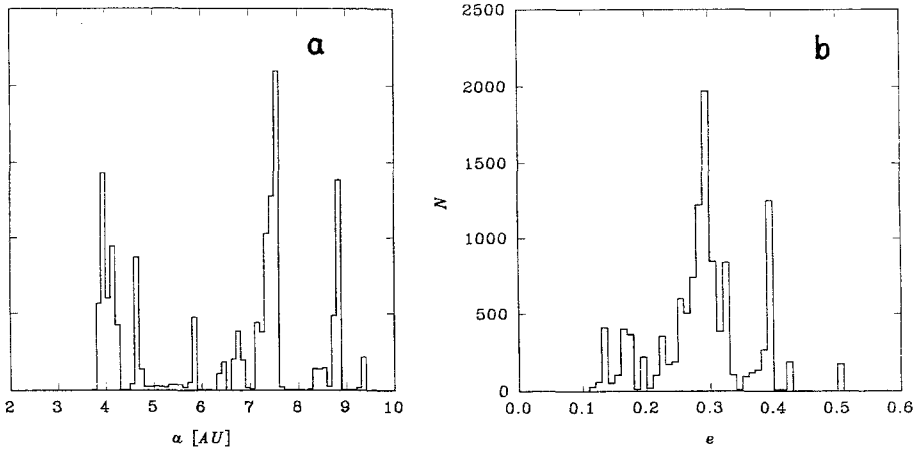


Figure 6. Distribution of semi-axes and eccentricities of escape-capture bodies in the Jupiter-Sun case.

of their resonant motion. We may conjecture that the present asteroids of the Hilda group are survivors of a more dense primordial population. Most of them were probably captured by Jupiter in the past the only steroids remaining are those protected by their libration motion.

Short period comets of the Jupiter family are plotted as open circles in Fig. 7. We observe that most of them are distributed very near the curve corresponding to capture conditions. Nevertheless, this diagram should not be taken too rigorously, as we have neglected the orbital inclination, which in fact decreases the probability of capture (Carusi and Valsecchi 1979).

4. Capture times

Capture times are defined as the number of orbits around the planet before the particle escapes into motion around the Sun. If the initial conditions are those satisfying the Mirror Theorem, the motion is symmetric with respect to the time and the x axis. In this situation, if the particle starts its motion in the sphere of influence of the smallest mass μ , the capture time is then twice the time elapsed before escape.

Initial conditions causing long captures are near the stability boundaries of phase space, and are associated with families of periodic orbits. As a consequence of this fact, the small-scale structure of capture times exhibits the same bifurcation sequence and self-similarity pattern as the periodic orbits (Murison, 1989). It is worth noting that there exist short lived orbits that are also periodic about both masses. Interesting examples of these class

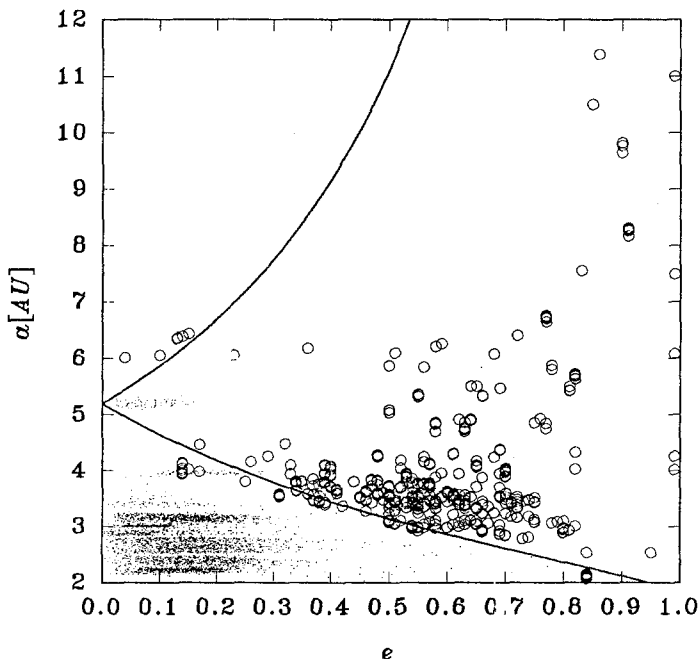


Figure 7. Distribution of actual asteroids (dots) and short period comets (circles)

are shown in Szebehely (1967, Fig. 9.24(b)) for the particular case of $\mu = 1/81.45$, i. e. very near the Earth-Moon system. These class of periodic orbits are strongly unstable.

5. Complete set of capture conditions

As far as we know, all previous papers on satellite capture deal with particular sets of initial conditions, i. e. those satisfying the Mirror Theorem (Roy and Ovenden 1955). However, in order to estimate the probability of satellite capture, the complete set of conditions leading to capture is required.

A first attempt to resolve this problem has been carried out by Brunini et al. (1995).

In this paper, the massless particle has been initially placed on a straight line normal to the Sun-planet direction

$$x = x_0 = 1 - \mu - 1.1\rho \quad (6)$$

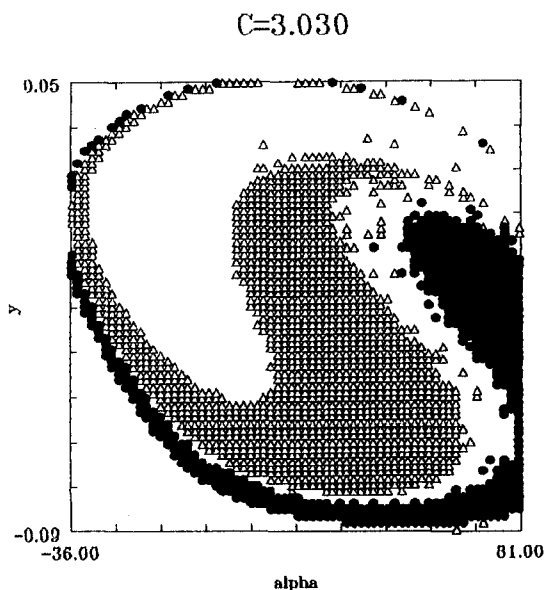


Figure 8. Capture initial conditions for the Jupiter-Sun case. Open triangles: direct orbits. Dark triangles: retrograde orbits. Collision orbits separate direct and retrograde orbits. In this figure $C = 3.024$.

where the value 1.1 - though arbitrary - guaranties the particle being initially outside the planetary sphere of influence (Danby 1988), since ρ is the distance between second libration point L_2 and the planet:

$$\rho = \left(\frac{\mu}{3}\right)^{\frac{1}{3}} - \frac{1}{3}\left(\frac{\mu}{3}\right)^{\frac{2}{3}} + \dots, \quad (7)$$

In fact every heliocentric particle to be captured by the planet must necessarily pass through $x = x_0$. Given C , the only degree of freedom left is that of the direction of the velocity vector, defined by the angle with respect to the Sun-planet line, α , which may take values in the interval $(-90^\circ, 90^\circ)$.

For given C and μ , the initial conditions leading to capture cover the interior of a region in the (y, α) plane as the one shown in Figure 8, referred to as the "capture domain". Open circles correspond to direct orbits, whereas dark circles represent retrograde satellites. The empty area between retrograde and direct satellites are collision orbits. Brunini et al. (1995) have performed a great number of numerical simulations, characterizing capture conditions in terms of μ and C , and also generalizing these results to the 3D case.

6. Capture conditions in the elliptic RTBP

The main difficulty to study the problem in the frame of the elliptic RTBP is that an integral of motion, equivalent to Jacobi's, does no longer exist, and the convenient reduction of the degrees of freedom of the system is not allowed anymore. Since the Jacobi integral is not conserved, a particle crossing the plane $x = x_0$ with conditions belonging to the "capture domain" for a given value of C , could never be captured because of the shrinking of the neck around the second lagrangian point. Conversely, particles that would not be captured in the circular case could be captured when the planetary orbital eccentricity is considered.

As expected, these particles come from a thin bounding ring in the capture domain.

The net effect of the eccentricity of the primaries is the erosion and consequently the reduction of the stability region. Some numerical simulations (Brunini et al. 1995) have also revealed that the size of the stability region is properly scaled by the pericentric distance of the secondary rather than the orbital semimajor axis.

These results, though not conclusive, may be useful from a statistical point of view, guarantying that capture domains in the circular and actual cases will not differ much in the Solar System. A similar argument may be applied to capture times.

7. Conclusions

In this paper, we have made a revision of the main known aspects of the problem of satellite capture. In addition, we have shown that chaos for retrograde satellites is not originated in close encounters, but to an overlapping of capture conditions with those corresponding to direct motion about the planet. The chaotic regime at large values of the Jacobi constant is populated by a mixture between direct and retrograde chaotic orbits. Collisions play the formal role of separating direct and retrograde satellites.

Regular retrograde satellites exist for almost any value of the Jacobi constant, and the persistence of the angular momentum integral is the effective mechanism preventing from escape in this case.

We have also shown possible ways to generalize the results in order to characterize complete sets of capture conditions in phase space.

The effect of the eccentricity, in the range of interest for Solar System, does not affect much the results, at least from the statistical point of view.

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